Introduction to Fuzzy Logic

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What is Fuzzy logic?

- Fuzzy logic is a mathematical language to express something.

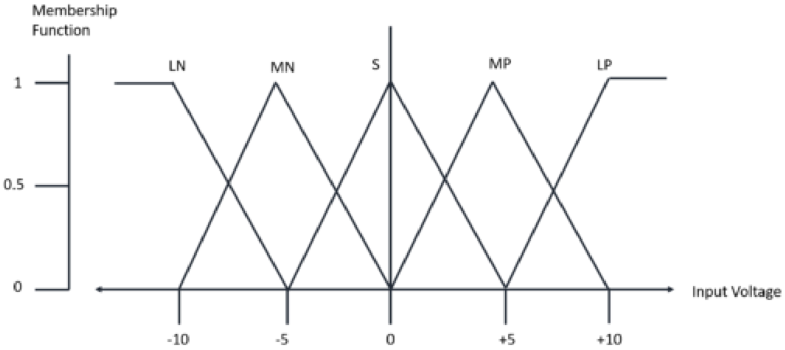
  This means it has grammar, syntax, semantic like a language for communication.

- There are some other mathematical languages also known
  - **Relational algebra** (operations on sets)
  - **Boolean algebra** (operations on Boolean variables)
  - **Predicate logic** (operations on well formed formulae (wff), also called predicate propositions)

- **Fuzzy logic deals with Fuzzy set .**

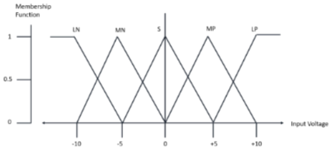
A brief history of Fuzzy Logic

- First time introduced by Lotfi Abdelli Zadeh(1965), University of California, Berkley, USA (1965).



- He is fondly nick-named as **LAZ**

A brief history of Fuzzy logic



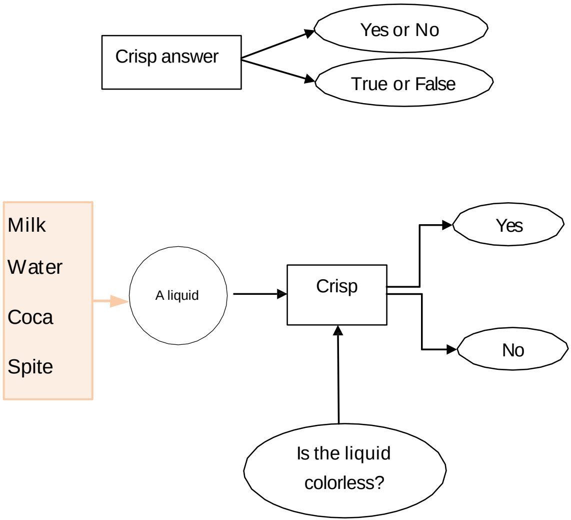
1. Dictionary meaning of **fuzzy** is not clear, noisy etc.

   Example: Is the picture on this slide is fuzzy?

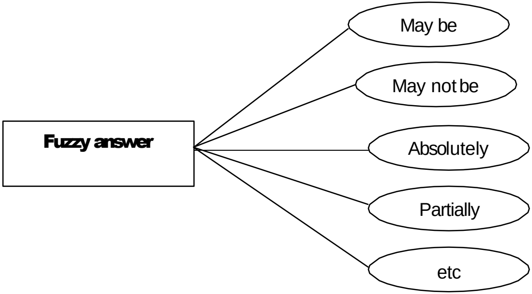
2. Antonym of fuzzy is **crisp**

   Example: Are the chips crisp?

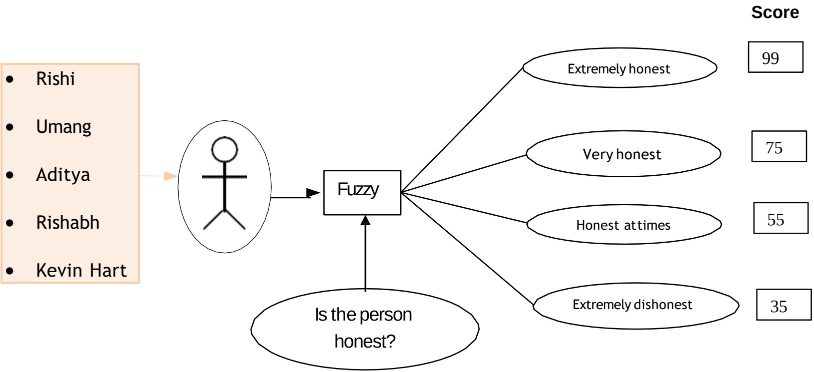
Example : Fuzzy logic vs. Crisp logic



Example : Fuzzy logic vs. Crisp logic



Example : Fuzzy logic vs. Crisp logic



World is fuzzy!

Our world is better described with fuzzily!

Set Theory

A set is an unordered collection of different elements. It can be written explicitly by listing its elements using the set bracket. If the order of the elements is changed or any element of a set is repeated, it does not make any changes in the set.

Example:
A set of all students are Male.
A set of all Courses in BTECH.
A set of all Faculty Teaches Analytics.
A set of all Students interested in Data Science.

If an element x is a member of any set S, it is denoted by $x \in S$ and if an element y is not a member of set S, it is denoted by $y \notin S$.

If CSIT = {Makrand, Neha, Sunil, Dipti, Rishi},
Dipti $\in$ CSIT but
Rishi,Sunil $\notin$ CSIT

Cardinality of Set

Cardinality of a set A, denoted by |A|, this will give the number of elements of the set A

If CSIT = {Makrand, Neha, Sunil, Dipti, Rishi},

|CSIT| = 5

If CSIT = {Makrand, Neha, Sunil, Dipti, Rishi, ...},

|CSIT| = ∞

Types of Set

Sets can be classified into many types; some of which are finite, infinite, subset, universal, proper, singleton set, etc.

Finite Set

A set which contains a definite number of elements is called a finite set.

Example: Set A = {All faculty who teaches Analytics but don't teaches Cyber Security}

Sets can be classified into many types; some of which are finite, infinite, subset, universal, proper, singleton set, etc.

Infinite Set

A set which contains infinite number of elements is called an infinite set.

Example: Set A = {Integer x, x>100, x should be prime, till **N**}

Types of Set

Sets can be classified into many types; some of which are finite, infinite, subset, universal, proper, singleton set, etc.

Proper Set

A Set **M** is a proper subset of set **N** (Written as **M** $\subset$ **N**) if every element of **M** is an element of set **N** and |**M**| < |**N**|

Example:
Set **M** = {Ajay, Amit, Anand, Brijesh, Carol}
Set **N** = {Ajay, Anand}

|**M**| = 5
|**N**| = 2

Types of Set

Sets can be classified into many types; some of which are finite, infinite, subset, universal, proper, singleton set, etc.

Universal Set

A Set **U** is a collection of all elements.

Example:
Set **U** = {All Students in SUAS}
Set **N** = {All Students in CSIT} is subset of U

Types of Set

Sets can be classified into many types; some of which are finite, infinite, subset, universal, proper, singleton set, etc.

Singleton Set (Unit Set)

A Singleton set or Unit set contains only one element.

Example:
Set **S** = {Anand}

Types of Set

Sets can be classified into many types; some of which are finite, infinite, subset, universal, proper, singleton set, etc.

Equal Set

Contain same elements in both set.

Example:
Set **S** = {Anand, Ajay, Rishi}
Set **T** = {Ajay, Rishi, Anand}

Types of Set

Sets can be classified into many types; some of which are finite, infinite, subset, universal, proper, singleton set, etc.

Equivalent Set

If the cardinalities of two sets are same, they are called equivalent sets.

Example:
Set **S** = {Anand, Ajay, Rishi}
Set **T** = {Ajay, Rishi, Umang}

|**S**| = 3
|**T**| = 3

Sets can be classified into many types; some of which are finite, infinite, subset, universal, proper, singleton set, etc.

Overlapping Set

Two sets that have at least one common element are called overlapping sets.

Example:
Set **S** = {Ojas, Ajay, Jaya}
Set **T** = {Ajay, Rishi, Umang}

Common Element = Ajay

Types of Set

Sets can be classified into many types; some of which are finite, infinite, subset, universal, proper, singleton set, etc.

Disjoint Set

Two sets that do not have any common element.

Example:
Set **S** = {Ojas, Ajay, Jaya}
Set **T** = {Aditya, Rishi, Umang}

Common Element = NULL

UNION

the set of elements which are in S, in T, or in both S and T.

Example:
Set **S** = {Ojas, Ajay, Jaya}
Set **T** = {Aditya, Rishi, Umang}

UNION= {Ojas, Ajay, Jaya, Aditya, Rishi, Umang}

INTERSECTION

the set of elements which are in both S and T.

Example:
Set **S** = {Rishi, Ajay, Jaya}
Set **T** = {Ajay, Rishi, Umang}

INTERSECTION= {Ajay, Rishi}

RELATIVE COMPLEMENT

the set of elements which are only in S but not in T.

Example:
Set **S** = {Rishi, Ajay, Jaya}
Set **T** = {Ajay, Rishi, Umang}

DIFFERENCE= {Jaya, Umang}

COMPLEMENT SET

is the set of elements which are not in set S.

Example:

If **S** = {Amit, Anand, Sumit} all belongs to SUAS
U = A set of all Students in SUAS

Set **S'** = (**U** – **S**)

Operations on Sets

Cartesian Product

All possible ordered pairs in sets

Example:

If A = {x,y}
B = {3,4}

Then, , A×B = {x3, x4, y3, y4}

Properties of Sets

Commutative Property

$A \cup B = B \cup A$
$A \cap B = B \cap A$

Associative Property

$A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$

Distributive Property

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Idempotency Property

$A \cup A = A$
$A \cap A = A$

Identity Property

$A \cap X = A$
$A \cup X = X$

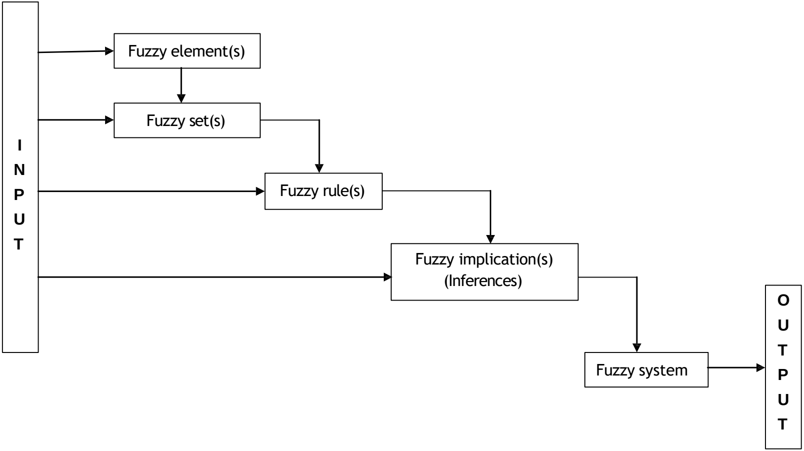
Transitive Property

If $A \subseteq B \subseteq C$, then $A \subseteq C$

De Morgan's Law

$(A \cap B)' = (A)' \cup (B)'$

Concept of fuzzy system



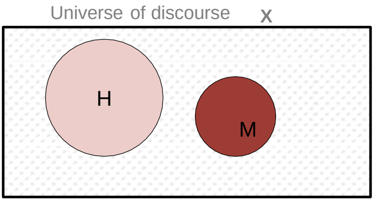
Concept of fuzzy set

To understand the concept of **fuzzy set** it is better, if we first clear our idea of **crisp set**.

X = The entire population of India.

H = All **Hindu** population = $\{h_1, h_2, h_3, \ldots, h_L\}$

M = All **Muslim** population = $\{m_1, m_2, m_3, \ldots, m_N\}$



Here, All are the sets of finite numbers of individuals.

Such a set is called crisp set.

Example of fuzzy set

Let us discuss about fuzzy set.

X = All students in **SEM-VI**.

S = All **Good students**.

S = $\{$(s, g) | s $\in$X $\}$ and g(s) is a measurement of goodness of the student *s*.

Example:

S = $\{$(Rishi, 0.8), (Ketan, 0.7), (Amay, 0.1), (Ankit, 0.9) $\}$ etc.

Fuzzy set vs. Crisp set

Crisp Set	Fuzzy Set
1. $S = \{ s \mid s \in X \}$	1. $F = (s, \mu) \mid s \in X$ and $\mu(s)$ is the degree of s.
2. It is a collection of elements.	2. It is collection of ordered pairs.
3. Inclusion of an element $s \in X$ into S is crisp, that is, has strict boundary **yes** or **no**.	3. Inclusion of an element $s \in X$ into F is fuzzy, that is, if present, then with a degree of **membership**.

Fuzzy set vs. Crisp set

Note: A crisp set is a fuzzy set, but, a fuzzy set is not necessarily a crisp set.

Example:
H = $\{(h_1, 1), (h_2, 1), \ldots, (h_L, 1)\}$

Person = $\{(p_1, 1), (p_2, 0), \ldots, (p_N, 1)\}$

In case of a crisp set, the elements are with extreme values of degree of membership namely either 1 or 0.

How to decide the degree of memberships of elements in a fuzzy set?

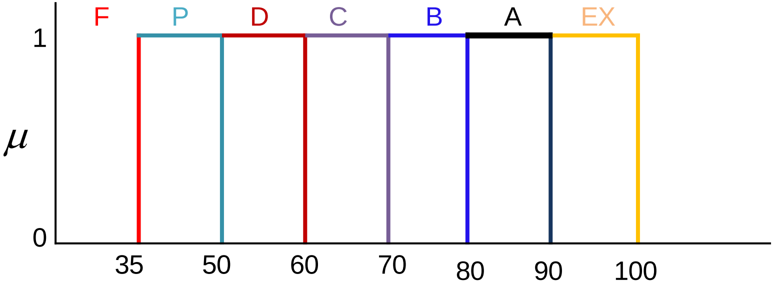
City	Indore	Pune	Bhopal	Patna	Delhi	Goa
DoM	0.95	0.90	0.80	0.01	0.65	0.75

How the cities of comfort can be judged?

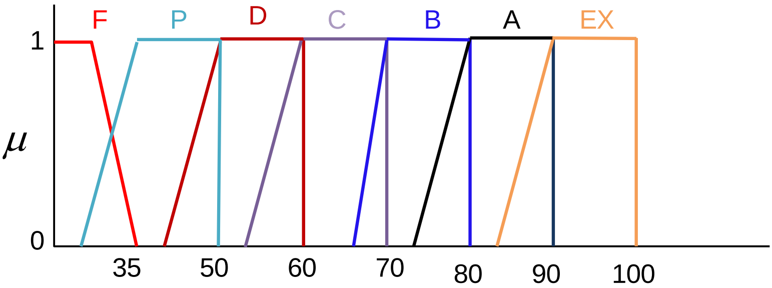
Example: Course evaluation in a crisp way

1. A+ = Marks $\geq$ 90
2. A = 80 $\leq$ Marks $<$ 90
3. B = 70 $\leq$ Marks $<$ 80
4. C = 60 $\leq$ Marks $<$ 70
5. D = 50 $\leq$ Marks $<$ 60
6. P = 35 $\leq$ Marks $<$ 50
7. F = Marks $<$ 35

Example: Course evaluation in a crisp way



Example: Course evaluation in a fuzzy way



Few examples of fuzzy set

- High Temperature
- Bike Mileage
- Color of Apple
- Sweetness of Orange
- Weight of Mango

Note: Degree of membership values always in the range [0...1].

Some basic terminologies and notations

Definition 1: Membership function (and Fuzzy set)

If X is a universe of discourse and $x \in X$, then a fuzzy set A in X is defined as a set of ordered pairs, that is

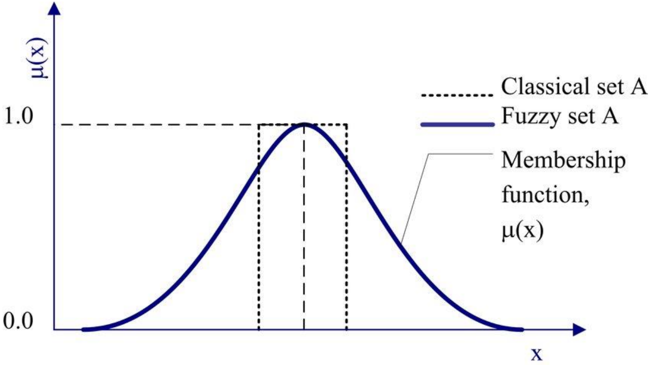
$A = \{(x, \mu_A(x)) | x \in X\}$ where $\mu_A(x)$ is called the membership function for the fuzzy set A.

Note:
$\mu_A(x)$ map each element of X onto a membership grade (or membership value) between 0 and 1 (both inclusive).

Question:
How (and who) decides $\mu_A(x)$ for a Fuzzy set A in X?

Fuzzy set Membership Function

We already know that fuzzy logic is not logic that is fuzzy but logic that is used to describe fuzziness. This fuzziness is best characterized by its membership function. In other words, we can say that membership function represents the degree of truth in fuzzy logic



Fuzzy set Membership Function

Membership functions were first introduced in 1965 by Lofti A. Zadeh in his first research paper “fuzzy sets”.

Membership functions characterize fuzziness (i.e., all the information in fuzzy set), whether the elements in fuzzy sets are discrete or continuous.

Membership functions can be defined as a technique to solve practical problems by experience rather than knowledge.

Membership functions are represented by graphical forms.

Rules for defining fuzziness are fuzzy too.

Some basic terminologies and notations

Example:

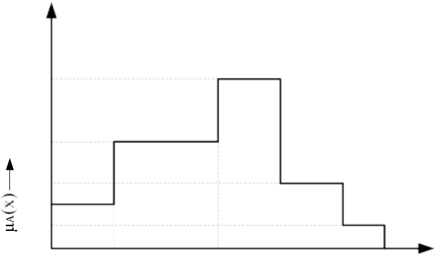
X = All cities in India

A = City of comfort

A=$\{$(Indore, 0.7), (Bhopal, 0.9), (Pune, 0.8), (Ranchi, 0.6), (Goa, 0.3)$\}$

Membership function with discrete membership values

The membership values may be of discrete values.

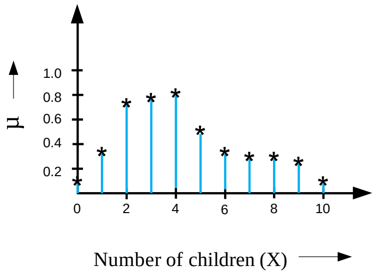


A fuzzy set with discrete values of μ

Example of Discrete : gender, country, language, number of cats
Example of Continuous : Height, Weight, Age

Membership function with discrete membership values

Either elements or their membership values (or both) also may be of discrete values.



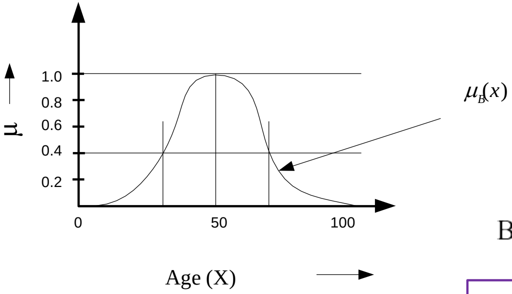
A ={(0,0.1),(1,0.30),(2,0.78)……(10,0.1)}

Note : X = discrete value

How you measure happiness ??

A = "Happy family"

Membership function with continuous membership values



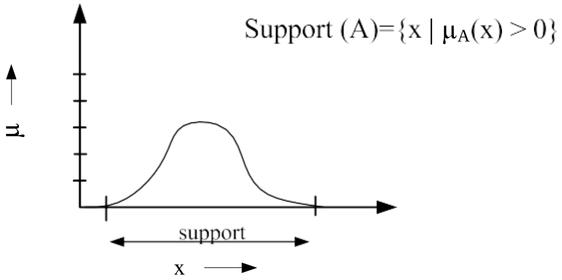
B = "Middle aged"

$$B = \{(x, \mu_B(x))\}$$

Note : x = real value
= R^+

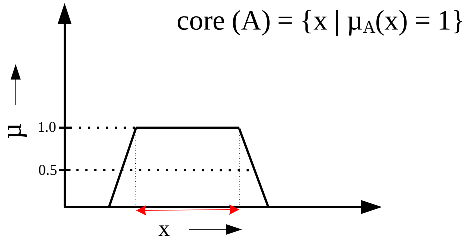
Fuzzy terminologies: Support

Support: The support of a fuzzy set A is the set of all points $x \in X$ such that $\mu_A(x) > 0$



Fuzzy terminologies: Core

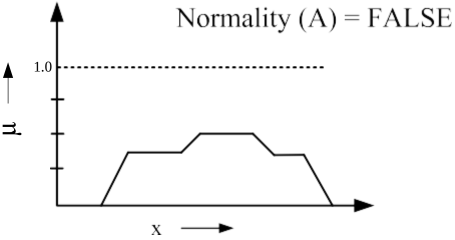
Core: The core of a fuzzy set A is the set of all points x in X such that $\mu_A(x) = 1$



The core of a fuzzy set A is **a crisp subset of X consisting of all elements with membership grades equal to one**

Fuzzy terminologies: Normality

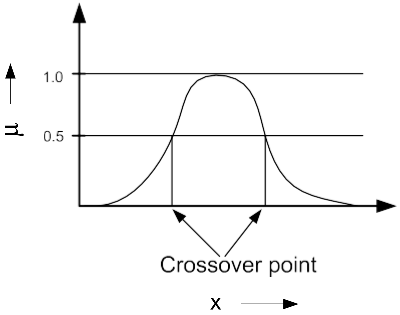
Normality : A fuzzy set *A of X* is a normal if its core is non-empty.



Normality : **A fuzzy set A is a normal if its core is non-empty**

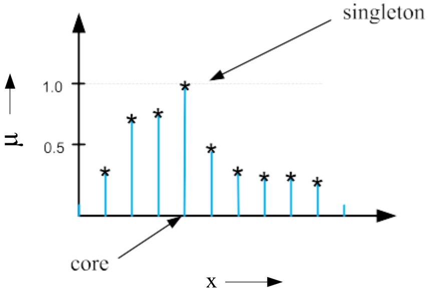
A fuzzy set that is not normal is called Subnormal

Crossover point : A crossover point of a membership function are defined as the elements in the universe for which a particular fuzzy set A has values equal to 0.5



Fuzzy terminologies: Fuzzy Singleton

Fuzzy Singleton : A fuzzy set whose support is a single point in X with $\mu_A(x) = 1$ is called a fuzzy singleton. (Only one element whose membership value is exactly 1.)

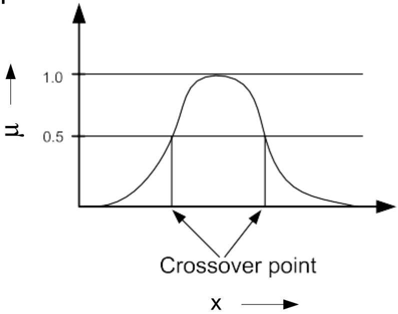


Bandwidth :

For a normal and convex fuzzy set, the bandwidth (or width) is defined as the distance the two unique crossover points:

Bandwidth(A) = | x_1 - x_2 |

where $\mu_A(x_1) = \mu_A(x_2) = 0.5$

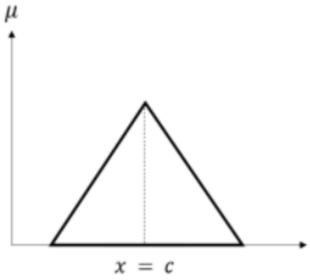


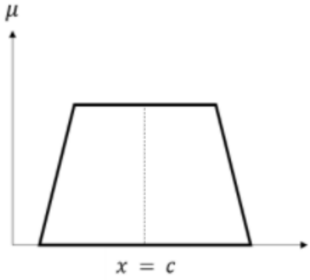
Fuzzy terminologies: Symmetry

Symmetry : Fuzzy set A is symmetric if its membership function around a center point x = c is symmetric

$\mu A(x + c) = \mu A(x - c), \forall x \in X$

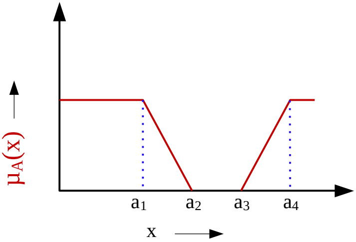
Triangular, Trapezoidal, Gaussian etc. are mostly symmetric. This is more natural to represent the membership then non-symmetric shape.

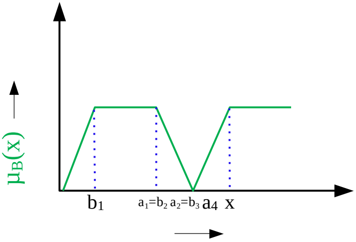




Example 1: Fuzzy Set Operations

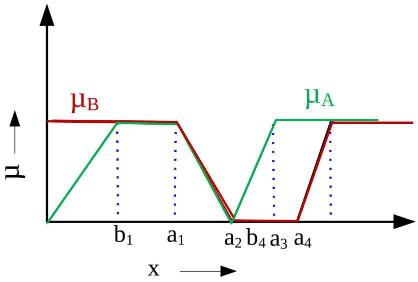
Let A and B are two fuzzy sets defined over a universe of discourse X with membership functions $\mu_A(x)$ and $\mu_B(x)$, respectively. Two MFs $\mu_A(x)$ and $\mu_B(x)$ are shown graphically.





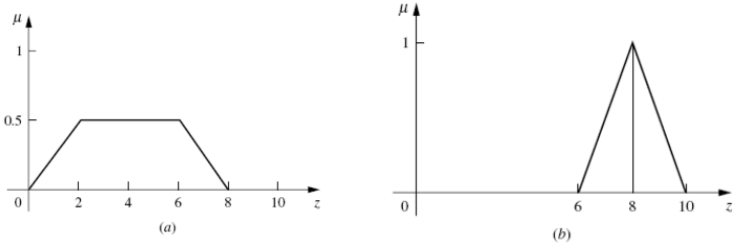
Example 1: Plotting two sets on the same graph

Let's plot the two membership functions on the same graph



Defuzzification

It may be defined as the process of reducing a fuzzy set into a crisp set or to convert a fuzzy member into a crisp member. the process of Defuzzification is also called “rounding it off”.



Problem of Conversion from fuzzy to crisp

Defuzzification

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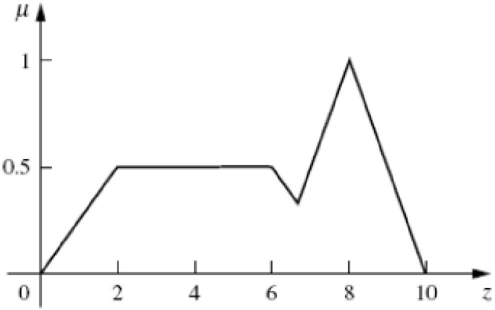

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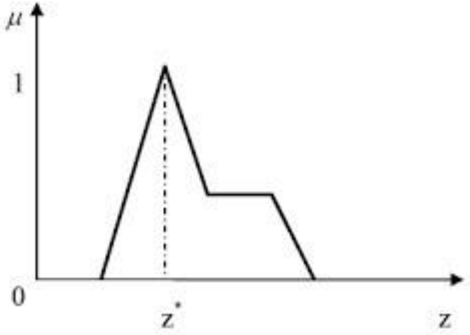
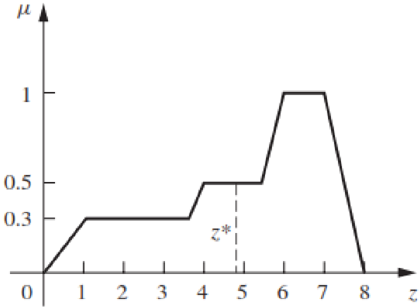
1. Max-Membership Principle

This method is also known as height method and is limited to peak output functions.



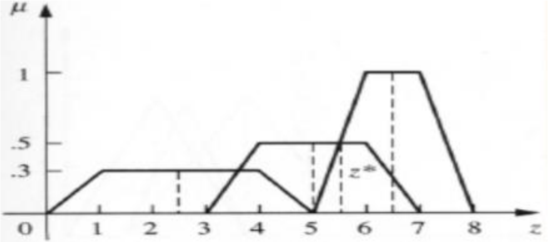
2. Centroid Method

This method is also known as the centre of mass, centre of area or centre of gravity. It is the most commonly used defuzzification method



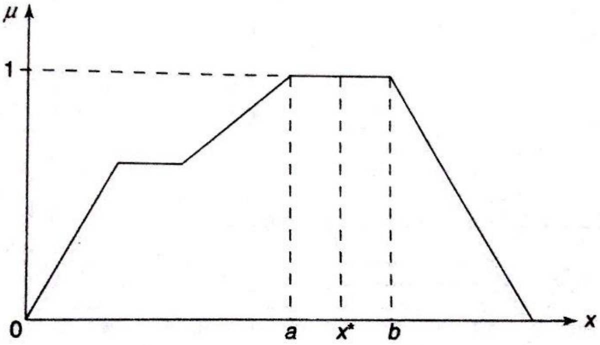
3. Weighted Average Method

This method is valid for symmetrical output membership functions only. Each membership function is weighted by its maximum membership value.



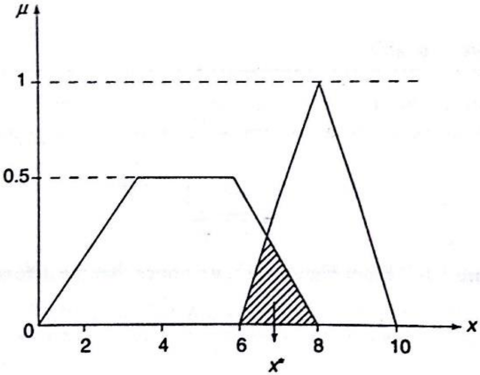
4. Mean-Max Membership

This method is also known as the middle of the maxima. This is closely related to the max-membership method, except that the locations of the maximum membership can be nonunique.



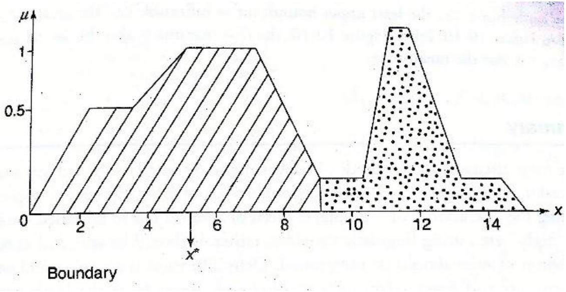
5. Centre of Sums

This method employs the algebraic sum of the individual fuzzy subsets instead of their union. The calculations here are very fast, but the main drawback is that the intersecting areas are added twice.



6. Centre of Largest Area

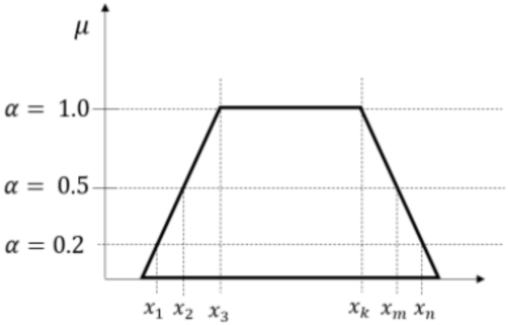
This method can be adopted when the output of at least two convex fuzzy subsets which are not overlapping. The output, in this case, is biased towards a side of one membership function. When output fuzzy st has at least two convex regions, then the center of gravity of the convex fuzzy subregion having the largest are is used to obtain the defuzzified value.



Alpha cut

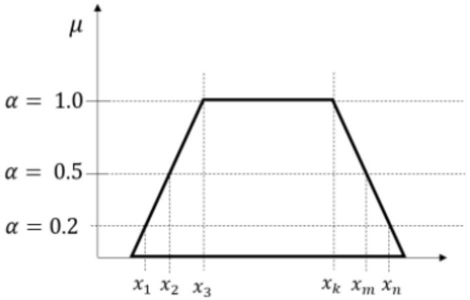
The **α-cut** of a fuzzy set A is a crisp set defined by $A_\alpha = \{ x \mid \mu A(x) \geq \alpha \}$

Strong α-cut of a fuzzy set A is a crisp set defined by $A_\alpha{+} = \{ x \mid \mu A(x) > \alpha \}$



Alpha cut in fuzzy set

Alpha cut



Alpha cut in fuzzy set

For the above diagram,

•The set $A_{\alpha=0.2}$ contains all the elements from x_1 to x_n, including both end values

•The set $A_{\alpha=0.5}$ contains all the elements from x_2 to x_m, including both end values

•The set $A_{\alpha=1.0}$ contains all the elements from x_3 to x_k, including both end values

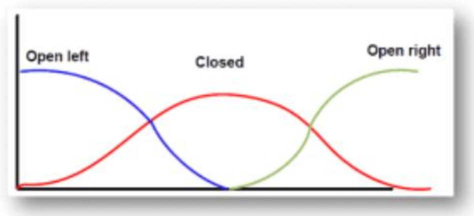
For different values of α , we get different crisp sets.

Open and Closed fuzzy sets

Open left: Open left fuzzy sets have all the elements on left after certain point have membership value 1, and all the elements on right side after certain point have membership value 0.

Open Right: Open right fuzzy sets have all the elements on left after certain point have membership value 0, and all the elements on right side after certain point have membership value 1.

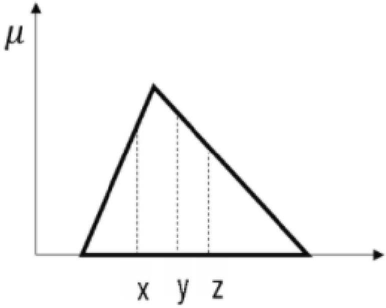
Closed: Closed fuzzy sets have all the elements on left or right side after certain point have membership value 0.



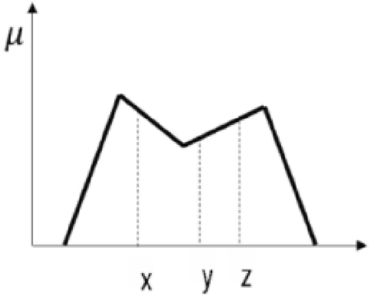
Convexity

Convex Set: for any elements x, y and z in a fuzzy set A, the relation $x < y < z$ implies that: $\mu_A(y) \geq \min(\mu_A(x), \mu_A(z))$. If this condition holds for all points, the fuzzy set is called convex fuzzy set.

Convex fuzzy sets are strictly increasing and then strictly decreasing



Convex



~~Convex~~

Fuzzy vs. Probability

Fuzzy : When we say about certainty of a thing

Example: A patient come to the doctor and he has to diagnose so that medicine can be prescribed.
Doctor prescribed a medicine with certainty 60% that the patient is suffering from flue. So, the disease will be cured with certainty of 60% and uncertainty 40%. Here, in stead of flue, other diseases with some other certainties may be.

Probability: When we say about the chance of an event to occur

Example: India will win the T20 tournament with a chance 60% means that out of 100 matches, India own 60 matches.

Prediction vs. Forecasting

The Fuzzy vs. Probability is analogical to Prediction vs. Forecasting

Prediction : When you start guessing about things.

Forecasting : When you take the information from the past job and apply it to new job.

The main difference:

Prediction is based on the best guess from experiences.
Forecasting is based on data you have actually recorded and packed from previous job.

Thank You