

Contents

1 basic	1	10 other	17
1.1 default	1	10.1cdq	17
1.2 godcode	1	10.2DeBruijnSequence	17
1.3 random	1	10.3SmallestLexicographic	17
1.4 run.bat	1		
1.5 run.sh	1	11 random	18
		11.1XORShift	18
2 binarysearch	1	12 string	18
2.1 二分搜	1	12.1KMP	18
2.2 三分搜	2	12.2minRotation	18
		12.3PalindromeTree	18
3 dataStructure	2	12.4RollingHash	18
3.1 DSU	2	12.5SuffixArray	18
3.2 fenwickTree	2	12.6trie	19
3.3 segmentTree1	2	12.7Z-algorithm	19
3.4 segmentTree2	3	12.8馬拉車	19
3.5 persistantSegTree	3		
3.6 countMinimumSeg	4	13 tree	19
3.7 LiChaoSegTree	4	13.1DSUONTREE	19
3.8 2Dbit	4	13.2EularTour	19
		13.3LCA	20
4 dp	5	13.4treehash	20
4.1 digit	5	13.5HeavyLightDecomposition	20
4.2 p_median	5	13.6VirtualTree	20
4.3 sosdp	5		
4.4 MinimumSteinerTree	5	1 basic	
4.5 lowConvexHull	5	1.1 default	
5 flow	5	<pre>#include <bits/stdc++.h> using namespace std; #define masterspark ios::sync_with_stdio(0), cin.tie(0) ,cout.tie(0),cin.exceptions(cin.failbit); #define int long long #define pp pair<int, int> #define ff first #define ss second #define forr(i,n) for(int i = 1; i <= n; ++i) #define rep(i,j,n) for(int i = j; i < n; ++i) #define PB push_back #define PF push_front #define EB emplace_back #define all(v) (v).begin(), (v).end() #define FZ(x) memset(x, 0, sizeof(x)) //fill zero #define SZ(x) ((int)x.size()) bool chmin(auto &a, auto b) { return (b < a) and (a = b , true); } bool chmax(auto &a, auto b) { return (a < b) and (a = b , true); } using i128 = __int128_t; using i64 = __int64_t; using i32 = __int32_t; void solve(){ } } signed main() { masterspark int t = 1; // freopen("stdin","r",stdin); // freopen("stdout","w",stdout); // cin >> t; while(t--){ solve(); } return 0; }</pre>	
5.1 Dinic	5		
5.2 isap	5		
5.3 KM	6		
5.4 匈牙利	6		
5.5 對偶建圖	6		
5.6 最小花費最大流 dijkstra 不能負值	7		
5.7 最小花費最大流 SPFA	7		
		1.2 godcode	
6 geometry	7	<pre>#pragma GCC optimize("O3,unroll-loops") #pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt") 編譯指令: g++ -std=c++20 -w -Wfatal-errors -Wall - Wshadow -fsanitize=undefined mt19937 gen(chrono::steady_clock::now(). time_since_epoch().count()); int randint(int lb, int ub) { return uniform_int_distribution<int>(lb, ub)(gen); } #define SECs ((double)clock() / CLOCKS_PER_SEC)</pre>	
6.1 Point	7		
6.2 Line	7		
6.3 Circle	8		
6.4 圓多邊形面積	8		
6.5 圓三角形面積	8		
6.6 半平面交	8		
6.7 圓線交	8		
6.8 圓圓交	9		
6.9 線線交	9		
6.10ConvexHull	9		
6.11Hulltrick	9		
6.12點線距	9		
6.13MEC	9		
6.14MEC2	10		
6.15旋轉卡尺	10		
6.16Minkowski	10		
6.17PointInPolygon	10		
6.18UnionOfCircles	10		
6.19UnionOfPolygons	11		
6.20圓公切線	11		
6.21點圓切線	11		
6.22最近點對	11		
7 graph	11		
7.1 BCC	11		
7.2 SCC	12		
7.3 支配樹	12		
7.4 最大團	12		
7.5 最小團	13		
7.6 kShortestPath	13		
7.7 結論	13		
8 math	13		
8.1 DiscreteSqrt	13		
8.2 excrt	14		
8.3 exgcd	14		
8.4 FFT	14		
8.5 josephus	14		
8.6 Theorem	14		
8.7 Primes	15		
8.8 millerrabin	15		
8.9 phi	15		
8.10pollardrho	15		
8.11primes	15		
8.12Euler	15		
8.13quickeuler	16		
8.14sieve	16		
8.15NTT	16		
9 polynomial	16		
9.1 Definition	16		
9.2 Inverse Series	17		
9.3 Poly Shift	17		
9.4 Exponential	17		
9.5 Nature log	17		

```
struct KeyHasher {
    size_t operator()(const Key& k) const {
        return k.first + k.second * 100000;
    }
};
typedef unordered_map<Key,int,KeyHasher> map_t;

__builtin_popcountll    // 二進位有幾個1 (記得這是long
                        long)
__builtin_clzll         // 左起第一個1之前0的個數
__builtin_parityll      // 1的個數的奇偶性
__builtin_mul_overflow(a,b,&h) // a*b是否溢位, h = a * b
;
__builtin_add_overflow(a,b,&h)
```

1.3 random

```
mt19937 mt(chrono::steady_clock::now().time_since_epoch
().count());
//mt19937_64 mt() -> return randnum
int randint(int l, int r){
    uniform_int_distribution<> dis(l, r); return dis(mt
);
}
```

1.4 run.bat

```
@echo off
g++ ac.cpp -o ac.exe
g++ wa.cpp -o wa.exe
set /a num=1
:loop
    echo %num%
    python gen.py > input
    ac.exe < input > ac
    wa.exe < input > wa
    fc ac wa
    set /a num=num+1
if not errorlevel 1 goto loop
```

1.5 run.sh

```
set -e
for ((i=0;;i++))
do
    echo "$i"
    python gen.py > in
    ./ac < in > ac.out
    ./wa < in > wa.out
    diff ac.out wa.out || break
done
```

2 binarysearch

2.1 二分搜

```
int bsearch_1(int l, int r)
{
    while (l < r)
    {
        int mid = l + r >> 1;
        if (check(mid)) r = mid;
        else l = mid + 1;
    }
    return l;
}
// .....0000000000

int bsearch_2(int l, int r)
{
    while (l < r)
    {
        int mid = l + r + 1 >> 1;
        if (check(mid)) l = mid;
        else r = mid - 1;
    }
    return l;
}
// 00000000.....

int m = *ranges::partition_point(views::iota(0LL,(int)1
e9+9),[&](int a){
    return check(a) > k;
```

```
});
//[begin,last)
//111111100000000000
//搜左邊數過來第一個 0
//都是 1 會回傳 last

int partitionpoint(int L,int R,function<bool(int)> chk)
{
    int l = L,r = R-1;
    while(r - l > 10){
        int m = l + (r-l)/2;
        if(chk(m)) l = m;
        else r = m;
    }
    int m = l;
    while(m <= r){
        if(!chk(m)) break;
        ++m;
    }
    if(!chk(m)) return m;
    else return R;
}
```

//手工

2.2 三分搜

```
int l = 1,r = 100;
while(l < r) {
    int lmid = l + (r - l) / 3; // l + 1/3區間大小
    int rmid = r - (r - l) / 3; // r - 1/3區間大小
    lans = cal(lmid),rans = cal(rmid);
    // 求凹函數的極小值
    if(lans <= rans) r = rmid - 1;
    else l = lmid + 1;
}
```

3 dataStructure

3.1 DSU

```
struct DSU {
    vector<i32> f, sz;
    DSU(i32 n) : f(n), sz(n) {
        for (i32 i = 0; i < n; i++) {
            f[i] = i;
            sz[i] = 1;
        }
    }
    i32 find(i32 x) {
        if (x == f[x]) return x;
        f[x] = find(f[x]);
        return f[x];
    }
    void merge(i32 x, i32 y) {
        x = find(x), y = find(y);
        if (x == y) return;
        if (sz[x] < sz[y])
            swap(x, y);
        sz[x] += sz[y];
        f[y] = x;
    }
    bool same(i32 a, i32 b) { return (find(a) == find(b
)); }
};
```

3.2 fenwickTree

```
// [0, n] -> fenwick(n)
struct fenwick {
    #define lowbit(x) (x & -x)
    i32 n; // [0, n)
    vector<i64> v;
    fenwick() {}
    fenwick(i32 _n) : n(_n + 1), v(_n + 2, 0) {}
    void _add(i32 x, i64 u){
        for(; x <= n; x += lowbit(x)) v[x] += u;
    }
    i64 _qry(i32 x){
        i64 ret = 0;
        for(; x ; x -= lowbit(x)) ret += v[x];
        return ret;
    }
};
```

```

}
i32 _lowerbound(i64 k) {
    i64 sum = 0;
    i32 p = 0;
    for (i32 i = (1 << __lg(n)); i; i >>= 1) {
        i32 nxt = p + i;
        if (nxt <= n && sum + v[nxt] < k) {
            sum += v[nxt];
            p = nxt;
        }
    }
    if (p + 1 > n) return -1;
    return p;
}
void add(i32 x, i64 v) { _add(x + 1, v); }
i64 qry(i32 x) { return _qry(x + 1); }
i64 qry(i32 l, i32 r) { return qry(r) - qry(l - 1); }
i32 lower_bound(i64 k) { return _lowerbound(k); }
};

```

3.3 segmentTree1

```

// [l, r)
template<class Info>
struct SegmentTree {
    inline i32 cl(i32 x) { return x << 1; }
    inline i32 cr(i32 x) { return (x << 1) | 1; }
    i32 n;
    vector<Info> info;
    SegmentTree() : n(0) {}
    SegmentTree(i32 n_, Info v_ = Info()) { init(n_, v_); }
    template<class T>
    SegmentTree(vector<T> init_) { init(init_); }

    void init(i32 n_, Info v_ = Info()) { init(vector<
        n_, v_); }
    template<class T>
    void init(vector<T> init_) {
        n = init_.size();
        info.assign(4 << __lg(n), Info());
        function<void(i32, i32, i32)> build = [&](i32 p
            , i32 l, i32 r) {
            if (r - l == 1) {
                info[p] = init_[l];
                return;
            }
            i32 m = (l + r) >> 1;
            build(cl(p), l, m);
            build(cr(p), m, r);
            pull(p);
        };
        build(1, 0, n);
    }

    void pull(i32 p) { info[p] = merge(info[cl(p)],
        info[cr(p)]); }

    void modify(i32 p, i32 l, i32 r, i32 x, const Info
        &v) {
        if (r - l == 1) {
            info[p] = v;
            return;
        }
        i32 m = (l + r) >> 1;
        if (x < m) modify(cl(p), l, m, x, v);
        else modify(cr(p), m, r, x, v);
        pull(p);
    }
    void modify(i32 p, const Info &v) { modify(1, 0, n,
        p, v); }

    Info rangeQuery(i32 p, i32 l, i32 r, i32 x, i32 y)
    {
        if (l >= y || r <= x) return Info();
        if (l >= x && r <= y) return info[p];
        i32 m = (l + r) >> 1;
        return merge(rangeQuery(cl(p), l, m, x, y),
            rangeQuery(cr(p), m, r, x, y));
    }
    Info rangeQuery(i32 l, i32 r) { return rangeQuery
        (1, 0, n, l, r); }
};

```

```

template<class F>
i32 findFirst(i32 p, i32 l, i32 r, i32 x, i32 y, F
    &&pred) {
    if (l >= y || r <= x) return -1;
    if (l >= x && r <= y && !pred(info[p])) return
        -1;
    if (r - l == 1) return l;
    i32 m = (l + r) >> 1;
    i32 res = findFirst(cl(p), l, m, x, y, pred);
    if (res == -1) res = findFirst(cr(p), m, r, x,
        y, pred);
    return res;
}
template<class F>
i32 findFirst(i32 l, i32 r, F &&pred) { return
    findFirst(1, 0, n, l, r, pred); }

template<class F>
i32 findLast(i32 p, i32 l, i32 r, i32 x, i32 y, F
    &&pred) {
    if (l >= y || r <= x) return -1;
    if (l >= x && r <= y && !pred(info[p])) return
        -1;
    if (r - l == 1) return l;
    i32 m = (l + r) >> 1;
    i32 res = findLast(cr(p), m, r, x, y, pred);
    if (res == -1) res = findLast(cl(p), l, m, x, y
        , pred);
    return res;
}
template<class F>
i32 findLast(i32 l, i32 r, F &&pred) { return
    findLast(1, 0, n, l, r, pred); }
};

```

3.4 segmentTree2

```

// [l, r)
template<class Info, class Tag>
struct segTree {
    inline i32 cl(i32 x) { return x << 1; }
    inline i32 cr(i32 x) { return (x << 1) | 1; }
    i32 n;
    vector<Info> info;
    vector<Tag> tag;
    segTree() : n(0) {}
    segTree(i32 n_, Info v_ = Info()) {
        init(n_, v_);
    }
    template<class T>
    segTree(vector<T> init_) {
        init(init_);
    }
    void init(i32 n_, Info v_ = Info()) {
        init(vector<Info>(n_, v_));
    }
    template<class T>
    void init(vector<T> init_) {
        n = init_.size();
        info.assign(4 << __lg(n), Info());
        tag.assign(4 << __lg(n), Tag());
        function<void(i32, i32, i32)> build = [&](i32 p
            , i32 l, i32 r) {
            if (r - l == 1) {
                info[p] = init_[l];
                return;
            }
            i32 m = (l + r) >> 1;
            build(cl(p), l, m);
            build(cr(p), m, r);
            pull(p, l, r);
        };
        build(1, 0, n);
    }
    void pull(i32 p, i32 l, i32 r) {
        i32 m = (l + r) >> 1;
        push(cl(p), l, m);
        push(cr(p), m, r);
        info[p] = merge(info[cl(p)], info[cr(p)]);
    }
};

```

```

void rangeModify(i32 p, i32 l, i32 r, i32 x, i32 y,
    const Tag &v) {
    push(p, l, r);
    if (l >= x && r <= y) {
        tag[p] += v;
        return;
    }
    i32 m = (l + r) >> 1;
    if (x < m) rangeModify(cl(p), l, m, x, y, v);
    if (y > m) rangeModify(cr(p), m, r, x, y, v);
    pull(p, l, r);
}
Info rangeQuery(i32 p, i32 l, i32 r, i32 x, i32 y)
{
    push(p, l, r);
    if (l >= y || r <= x) {
        return Info();
    }
    if (l >= x && r <= y) {
        return info[p];
    }
    i32 m = (l + r) >> 1;
    return merge(rangeQuery(cl(p), l, m, x, y),
        rangeQuery(cr(p), m, r, x, y));
}
Info rangeQuery(i32 l, i32 r) { return rangeQuery
    (1, 0, n, l, r); }
void rangeModify(i32 l, i32 r, const Tag &v) {
    rangeModify(1, 0, n, l, r, v); }
void push(i32 p, i32 l, i32 r) { // need complete
    if (tag[p].add != 0) {
        info[p].v += tag[p].add * (r - l);
        if (r - l != 1) {
            tag[cl(p)].add += tag[p].add;
            tag[cr(p)].add += tag[p].add;
        }
        tag[p].add = 0;
    }
}
};

```

3.5 persistantSegTree

```

struct pSeg{
    struct node{
        int v;
        node *l,*r;
    };
    int n;
    vector<node*> ver;
    node* build(int l,int r){
        node* x = new node();
        if(l == r){
            x->v = 0;
            return x;
        }
        int m = (l+r)/2;
        x->l = build(l,m);
        x->r = build(m+1,r);
        x->v = x->l->v + x->r->v;
        return x;
    }
    void init(int _n){
        n = _n;
        ver.PB(build(0,n-1));
    }
    int qry(node* now,int l,int r,int ql,int qr){
        if(ql <= l && r <= qr){
            return now->v;
        }
        int m = (l+r)/2,ret = 0;
        if(ql <= m)ret += qry(now->l,l,m,ql,qr);
        if(qr > m)ret += qry(now->r,m+1,r,ql,qr);
        return ret;
    }
    node* upd(node* prv,int l,int r,int p,int v){
        node* x = new node();
        if(l == r){
            x->v = prv->v + v; //累加
            return x;
        }
        int m = (l+r)/2;

```

```

        if(p <= m) {
            x->l = upd(prv->l,l,m,p,v);
            x->r = prv->r;
        }else{
            x->l = prv->l;
            x->r = upd(prv->r,m+1,r,p,v);
        }
        x->v = x->l->v + x->r->v;
        return x;
    }
    void addver(int p,int v){
        ver.PB(upd(ver.back(),0,n-1,p,v));
    }
    // (a,b] kth //用segTree統計出現次數 //版本當區間 //
    //第 i 個版本為前區間 [0,i] 有統計
    int qurey(node* a,node* b,int l,int r,int k){
        if(l == r) return l;
        int m = (l+r)/2;
        int num = b->l->v - a->l->v;
        if(num >= k) return qurey(a->l,b->l,l,m,k); //
        //左邊大往左搜
        else return qurey(a->r,b->r,m+1,r,k-num);
    }
};

```

3.6 countMinimumSeg

```

//count zeros on segmentTree
struct segTree{
    #define cl (i<<1)
    #define cr ((i<<1)+1)
    pp seg[MXN*4];
    int tag[MXN*4];
    pp comb(pp a,pp b){
        if(a.ff < b.ff) return a;
        if(a.ff > b.ff) return b;
        return pp{a.ff,a.ss+b.ss};
    }
    void push(int i,int l,int r){
        if(tag[i]){
            seg[i].ff += tag[i];
            if(r - l > 1){
                tag[cl] += tag[i];
                tag[cr] += tag[i];
            }
            tag[i] = 0;
        }
    }
    void pull(int i,int l,int r){
        int m = (r-l)/2 + 1;
        push(cl,l,m);
        push(cr,m,r);
        seg[i] = comb(seg[cl],seg[cr]);
    }
    void build(int i,int l,int r){
        if(r - l <= 1){
            seg[i] = pp{0,1};
            return;
        }
        int m = (r-l)/2 + 1;
        build(cl,l,m);
        build(cr,m,r);
        pull(i,l,r);
    }
    void upd(int i,int l,int r,int ql,int qr,int x){
        push(i,l,r);
        if(ql <= l && r <= qr){
            tag[i] += x;
            return;
        }
        int m = (r-l)/2 + 1;
        if(ql < m) upd(cl,l,m,ql,qr,x);
        if(qr > m) upd(cr,m,r,ql,qr,x);
        pull(i,l,r);
    }
    int qry(){
        //count zero
        if(seg[1].ff == 0) return seg[1].ss;
        return 0;
    }
    void upd(int l,int r,int x){
        upd(1,0,MXN,l,r,x);
    }

```

```
}
}st;
```

3.7 LiChaoSegTree

```
const int inf = numeric_limits<i64>::max()/2;
struct Line {
    // y = ax + b
    i64 a{0}, b{-inf};
    i64 operator()(i64 x) {
        return a * x + b;
    }
};

struct Seg{
    int l, r;
    Seg *ls{}, *rs{};
    Line f{};
    Seg(int l, int r) : l(l), r(r) {}
    void add(Line g){
        int m = (l+r)/2;
        if (g(m) > f(m)) swap(g, f);
        if(g.b == -inf || r - l == 1) return;
        if(g.a < f.a){
            if(!ls) ls = new Seg(l,m);
            ls->add(g);
        }else{
            if(!rs) rs = new Seg(m,r);
            rs->add(g);
        }
    }
    i64 qry(i64 x){
        int m = (l+r) / 2;
        i64 y = f(x);
        if(x < m && ls) y = max({y, ls->qry(x)});
        if(x >= m && rs) y = max({y, rs->qry(x)});
        return y;
    }
};

auto add = [&](Line g, int ql, int qr){ //新增線段 [ql,qr)
    auto find = [&](auto &&self, Seg * now, int l, int r)
        -> void {
        if(ql <= l && r <= qr){
            now->add(g);
            return;
        }
        int m = (l+r) / 2;
        if(ql < m) {
            if(!now->ls) now->ls = new Seg(l,m);
            self(self, now->ls, l, m);
        }
        if(qr > m){
            if(!now->rs) now->rs = new Seg(m,r);
            self(self, now->rs, m, r);
        }
    };
    find(find, st, -ninf, ninf);
};
//Seg *st = new Seg(-ninf, ninf); // [l, r)
```

3.8 2Dbit

```
struct fenwick{
    #define lowbit(x) (x&-x)
    int n, m;
    vector<vector<int>>> v;
    fenwick(int _n, int _m) : n(_n+1), m(_m+1), v(_n+2,
        vector<int>(_m+2, 0)){}
    void add(int x, int y, int u){
        ++x, ++y;
        for(; x < n; x += lowbit(x)){
            for(int j = y; j < m; j += lowbit(j)) v[x][j]
                += u;
        }
    }
    int qry(int x, int y){
        ++x, ++y;
        int ret = 0;
        for(; x; x -= lowbit(x)){
            for(int j = y; j; j -= lowbit(j)) ret += v[
                x][j];
        }
    }
};
```

```
}
return ret;
}
// (l, u) <= (r, d)
// d - +
// u + -
// l r
void add(int l, int u, int r, int d, int x){
    ++r, ++d;
    add(l, u, x);
    add(l, d, -x);
    add(r, u, -x);
    add(r, d, x);
}
int qry(int l, int u, int r, int d){
    --l, --u;
    return qry(r, d) - qry(r, u) - qry(l, d) + qry(l, u);
}
};
```

4 dp

4.1 digit

```
ll dp[MXN_BIT][PRE_NUM][LIMIT][F0]; //字串位置，根據題目的值，是否上界，前導0
ll dfs(int i, int pre, bool lim, bool f0, const string& str){
    if(v[i][pre][f0][lim]) return dp[i][pre][f0][lim];
    v[i][pre][f0][lim] = true;

    if(i == str.size())
        return dp[i][pre][f0][lim] = 1;

    ll ret = 0, h = lim ? str[i] : '9';

    for(int j='0'; j<=h; j++){
        if(abs(j-pre)>=2 || f0){
            ret += dfs(i+1, j, j==h && lim, f0 && j=='0', str);
        }
    }
    return dp[i][pre][f0][lim] = ret;
}
```

4.2 p_median

```
void p_Median(){
    for (int i=1; i<=N; ++i)
        for (int j=i; j<=N; ++j){
            m = (i+j)/2, d[i][j] = 0; // m是中位數，d[i][j]為距離的總和
            for (int k=i; k<=j; ++k) d[i][j] += abs(arr[k] - arr[m]);
        }
    for (int p=1; p<=P; ++p)
        for (int n=1; n<=N; ++n){
            dp[p][n] = 1e9;
            for (int k=p; k<=n; ++k)
                if (dp[p-1][k-1] + d[k][n] < dp[p][n]){
                    dp[p][n] = dp[p-1][k-1] + d[k][n];
                    r[p][n] = k; // 從第k個位置往右到第j個位置
                }
        }
}
```

4.3 sosdp

```
// 求子集和 或超集和 -> !(mask & (1 << i))
for(int i = 0; i < (1<<N); ++i) F[i] = A[i]; //預處理 狀態權重

for(int i = 0; i < N; ++i)
    for (int s = 0; s < (1<<N); ++s)
        if (s & (1 << i))
            F[s] += F[s ^ (1 << i)];

//窮舉子集
for(int s = mask; s; s = (s-1)&mask;)
```

4.4 MinimumSteinerTree

```
int dp[MXN][1<<11], vis[MXN];
//dp[i][s] -> 選了前K個點 以第i個點為第K+1個點的 生成
// (1..K+1)的最小生成樹
rep(s, 0, (1<<K)) forr(i, N) dp[i][s] = INF;
rep(j, 0, K) dp[j+1][1<<j] = 0;
rep(s, 0, (1<<K)){
    forr(i, N){
        for(int a = s; a; a=(a-1)&s)
            dp[i][s] = min(dp[i][s], dp[i][s^a] + dp[i][a]);
        // node
    }
    FZ(vis);
    priority_queue<pp, vector<pp>, greater<pp>> Q;
    forr(i, N) Q.emplace(dp[i][s], i);
    while(Q.size()){
        auto [d, u] = Q.top(); Q.pop();
        if(vis[u]) continue;
        vis[u] = 1;
        for(auto [v, w]: E[u]){
            if(dp[u][s]+w < dp[v][s]) {
                dp[v][s] = dp[u][s]+w;
                Q.emplace(dp[v][s], v);
            }
        }
    }
}
rep(i, K+1, N+1) cout << dp[i][(1<<K)-1] << '\n';
```

4.5 lowConvexHull

```
struct Line {
    mutable ll m, b, p;
    bool operator<(const Line& o) const { return m < o.m; }
    bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return false; }
        if (x->m == y->m) x->p = x->b > y->b ? inf : -inf;
        else x->p = div(y->b - x->b, x->m - y->m);
        return x->p >= y->p;
    }
    void insert_line(ll m, ll b) {
        auto z = insert({m, b, 0}); y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll eval(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.m * x + l.b;
    }
};
```

5 flow

5.1 Dinic

```
struct Dinic{
    struct Edge{ int v, f, re; };
    int n, s, t, level[MXN];
    vector<Edge> E[MXN];
    void init(int _n, int _s, int _t){
        n = _n; s = _s; t = _t;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void add_edge(int u, int v, int f){
        E[u].PB({v, f, SZ(E[v])});
        E[v].PB({u, 0, SZ(E[u])-1});
    }
    bool BFS(){
```

```
for (int i=0; i<n; i++) level[i] = -1;
queue<int> que;
que.push(s);
level[s] = 0;
while (!que.empty()){
    int u = que.front(); que.pop();
    for (auto it : E[u]){
        if (it.f > 0 && level[it.v] == -1){
            level[it.v] = level[u]+1;
            que.push(it.v);
        }
    }
}
return level[t] != -1;
}
int DFS(int u, int nf){
    if (u == t) return nf;
    int res = 0;
    for (auto &it : E[u]){
        if (it.f > 0 && level[it.v] == level[u]+1){
            int tf = DFS(it.v, min(nf, it.f));
            res += tf; nf -= tf; it.f -= tf;
            E[it.v][it.re].f += tf;
            if (nf == 0) return res;
        }
    }
    if (!res) level[u] = -1;
    return res;
}
int flow(int res=0){
    while (BFS())
        res += DFS(s, 2147483647);
    return res;
} }
```

5.2 isap

```
struct Maxflow {
    static const int MAXV = 20010;
    static const int INF = 1000000;
    struct Edge {
        int v, c, r;
        Edge(int _v, int _c, int _r):
            v(_v), c(_c), r(_r) {}
    };
    int s, t;
    vector<Edge> G[MAXV*2];
    int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;
    void init(int x) {
        tot = x+2;
        s = x+1, t = x+2;
        for(int i = 0; i <= tot; i++) {
            G[i].clear();
            iter[i] = d[i] = gap[i] = 0;
        }
    }
    void addEdge(int u, int v, int c) {
        G[u].push_back(Edge(v, c, SZ(G[v])));
        G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
    }
    int dfs(int p, int flow) {
        if(p == t) return flow;
        for(int &i = iter[p]; i < SZ(G[p]); i++) {
            Edge &e = G[p][i];
            if(e.c > 0 && d[p] == d[e.v]+1) {
                int f = dfs(e.v, min(flow, e.c));
                if(f) {
                    e.c -= f;
                    G[e.v][e.r].c += f;
                    return f;
                }
            }
        }
        if( (--gap[d[p]]) == 0) d[s] = tot;
        else {
            d[p]++;
            iter[p] = 0;
            ++gap[d[p]];
        }
        return 0;
    }
    int solve() {
        int res = 0;
        gap[0] = tot;
        for(res = 0; d[s] < tot; res += dfs(s, INF));
        return res;
    }
    void reset() {
```



```
for(int i=0;i<=tot;i++){
    iter[i]=d[i]=gap[i]=0;
} } }flow;
```

5.3 KM

```
struct KM{ // max weight, for min negate the weights
    int n, mx[MXN], my[MXN], pa[MXN];
    ll g[MXN][MXN], lx[MXN], ly[MXN], sy[MXN];
    bool vx[MXN], vy[MXN];
    void init(int _n) { // 1-based, N個節點
        n = _n;
        for(int i=1; i<=n; i++) fill(g[i], g[i]+n+1, 0);
    }
    void addEdge(int x, int y, ll w) {g[x][y] = w;} //左
    //邊的集合節點x連邊右邊集合節點y權重w
    void augment(int y) {
        for(int x, z; y; y = z)
            x=pa[y], z=mx[x], my[y]=x, mx[x]=y;
    }
    void bfs(int st) {
        for(int i=1; i<=n; ++i) sy[i]=INF, vx[i]=vy[i]=0;
        queue<int> q; q.push(st);
        for(;;) {
            while(q.size()) {
                int x=q.front(); q.pop(); vx[x]=1;
                for(int y=1; y<=n; ++y) if(!vy[y]){
                    ll t = lx[x]+ly[y]-g[x][y];
                    if(t==0){
                        pa[y]=x;
                        if(!my[y]){augment(y);return;}
                        vy[y]=1, q.push(my[y]);
                    }else if(sy[y]>t) pa[y]=x, sy[y]=t;
                }
            }
            ll cut = INF;
            for(int y=1; y<=n; ++y)
                if(!vy[y]&&cut>sy[y]) cut=sy[y];
            for(int j=1; j<=n; ++j){
                if(vx[j]) lx[j] -= cut;
                if(vy[j]) ly[j] += cut;
                else sy[j] -= cut;
            }
            for(int y=1; y<=n; ++y) if(!vy[y]&&sy[y]==0){
                if(!my[y]){augment(y);return;}
                vy[y]=1, q.push(my[y]);
            }
        }
    }
    ll solve() { // 回傳值為完美匹配下的最大總權重
        fill(mx, mx+n+1, 0); fill(my, my+n+1, 0);
        fill(ly, ly+n+1, 0); fill(lx, lx+n+1, -INF);
        for(int x=1; x<=n; ++x) for(int y=1; y<=n; ++y) //
            1-base
            lx[x] = max(lx[x], g[x][y]);
        for(int x=1; x<=n; ++x) bfs(x);
        ll ans = 0;
        for(int y=1; y<=n; ++y) ans += g[my[y]][y];
        return ans;
    }
} }graph;
```

5.4 匈牙利

```
struct hungarian {
    int L, R;
    vector<bool> vis;
    vector<int> match;
    vector<vector<int>> E;
    hungarian(int l, int r): //左邊有幾個, 右邊有幾個
        L(l), R(r),
        vis(l+r+1),
        match(l+r+1,-1),
        E(l+r+1){}
    void add_edge(int l, int r){ //左側第幾個(1-base),
        //右側第幾個(1-base)
        r = L+r;
        E[l].push_back(r);
        E[r].push_back(l);
    }
    bool dfs(int u){
        for(int i : E[u]){
            if(vis[i]) continue; // 有連通且未拜訪
            vis[i] = true; // 紀錄是否走過
            if(match[i] == -1 || dfs(match[i])){
                match[i] = u; match[u] = i; // 紀錄匹配
            }
        }
    }
};
```

```
return true;
}
return false;
}
int solve(){
    int ans = 0;
    for(int i = 1; i <= L; i++){
        fill(all(vis),0);
        if(dfs(i)) ans++;
    }
    return ans;
}
};
```

5.5 對偶建圖

```
auto add = [&](int u,int v,int w){
    E[u].EB(v,w);
    E[v].EB(u,w);
};
//A : 橫槓(n*(m-1)); B : 直槓((n-1)*m); C : 斜槓((n-1)
//*(m-1));
//n 列 m 行平面圖 (1-base) S起點 (左上) T 終點 (右下)
forr(s,(n-1)){
    int M = (m-1)*2;
    forr(i,M){
        int id = i + (s-1)*M;
        if(i&1){
            int u = (s < n-1) ? ((i+1) + s*M) : T;
            int e = (i > 1) ? id - 1 : T;
            add(id,e,B[s-1][(i-1)/2]);
            add(id,u,A[s][(i-1)/2]);
        }else{
            if(i == M) add(id,S,B[s-1][m-1]);
            if(s == 1) add(id,S,A[s-1][i/2-1]);
            int w = C[s-1][i/2-1];
            add(id,id-1,w);
        }
    }
}
};
```

5.6 最小花費最大流 dijkstra 不能負值

```
struct MinCostMaxFlow{
    typedef int Tcost;
    static const int MAXV = 20010;
    static const int INFf = 1000000;
    static const Tcost INFc = 1e9;
    struct Edge{
        int v, cap;
        Tcost w;
        int rev;
        Edge(){}
        Edge(int t2, int t3, Tcost t4, int t5)
            : v(t2), cap(t3), w(t4), rev(t5) {}
    };
    int V, s, t;
    vector<Edge> g[MAXV];
    void init(int n, int _s, int _t){
        V = n; s = _s; t = _t;
        for(int i = 0; i <= V; i++) g[i].clear();
    }
    void addEdge(int a, int b, int cap, Tcost w){
        g[a].push_back(Edge(b, cap, w, (int)g[b].size()));
        g[b].push_back(Edge(a, 0, -w, (int)g[a].size()-1));
    }
    Tcost d[MAXV];
    int id[MAXV], mom[MAXV];
    bool inqu[MAXV];
    queue<int> q;
    pair<int,Tcost> solve(){
        int mxf = 0; Tcost mnc = 0;
        while(1){
            fill(d, d+1+V, INFc);
            fill(inqu, inqu+1+V, 0);
            fill(mom, mom+1+V, -1);
            mom[s] = s;
            d[s] = 0;
            q.push(s); inqu[s] = 1;
            while(q.size()){
                int u = q.front(); q.pop();
                for(int i : g[u]){
                    int v = i.v;
                    if(d[v] > d[u] + i.w){
                        d[v] = d[u] + i.w;
                        mom[v] = u;
                    }
                }
            }
            if(d[t] == INFc) break;
            int f = INFf;
            for(int u = t; u != s; u = mom[u])
                f = min(f, g[u].cap);
            mxf += f; mnc += f*d[t];
            for(int u = t; u != s; u = mom[u])
                g[u].cap -= f; g[u].rev.cap += f;
        }
        return mnc;
    }
};
```

```

    inqu[u] = 0;
    for(int i = 0; i < (int) g[u].size(); i++){
        Edge &e = g[u][i];
        int v = e.v;
        if(e.cap > 0 && d[v] > d[u]+e.w){
            d[v] = d[u]+e.w;
            mom[v] = u;
            id[v] = i;
            if(!inqu[v]) q.push(v), inqu[v] = 1;
        }
    }
    if(mom[t] == -1) break ;
    int df = INF;
    for(int u = t; u != s; u = mom[u])
        df = min(df, g[mom[u]][id[u]].cap);
    for(int u = t; u != s; u = mom[u]){
        Edge &e = g[mom[u]][id[u]];
        e.cap -= df;
        g[e.v][e.rev].cap += df;
    }
    mxf += df;
    mnc += df*d[t];
}
return {mxf,mnc};
} }flow;

```

5.7 最小花費最大流 SPFA

```

struct zkwflow{
    static const int maxN=10000;
    struct Edge{ int v,f,re; ll w;};
    int n,s,t,ptr[maxN]; bool vis[maxN]; ll dis[maxN];
    vector<Edge> E[maxN];
    void init(int _n,int _s,int _t){
        n=_n,s=_s,t=_t;
        for(int i=0;i<n;i++) E[i].clear();
    }
    void addEdge(int u,int v,int f,ll w){
        E[u].push_back({v,f,(int)E[v].size(),w});
        E[v].push_back({u,0,(int)E[u].size()-1,-w});
    }
    bool SPFA(){
        fill_n(dis,n,LLONG_MAX); fill_n(vis,n,false);
        queue<int> q; q.push(s); dis[s]=0;
        while (!q.empty()){
            int u=q.front(); q.pop(); vis[u]=false;
            for(auto &it:E[u]){
                if(it.f>0&&dis[it.v]>dis[u]+it.w){
                    dis[it.v]=dis[u]+it.w;
                    if(!vis[it.v]){
                        vis[it.v]=true; q.push(it.v);
                    }
                }
            }
            return dis[t]!=LLONG_MAX;
        }
    }
    int DFS(int u,int nf){
        if(u==t) return nf;
        int res=0; vis[u]=true;
        for(int &i=ptr[u];i<(int)E[u].size();i++){
            auto &it=E[u][i];
            if(it.f>0&&dis[it.v]==dis[u]+it.w&&!vis[it.v]){
                int tf=DFS(it.v,min(nf,it.f));
                res+=tf,nf-=tf,it.f-=tf;
                E[it.v][it.re].f+=tf;
                if(nf==0){ vis[u]=false; break; }
            }
        }
        return res;
    }
    pair<int,ll> flow(){
        int flow=0; ll cost=0;
        while (SPFA()){
            fill_n(ptr,n,0);
            int f=DFS(s,INT_MAX);
            flow+=f; cost+=dis[t]*f;
        }
        return { flow,cost };
    }
    // reset: do nothing
} flow;

```

6 geometry

6.1 Point

```

using ld = long double;
template<class T>
struct pt{
    T x,y;
    pt(T _x,T _y):x(_x),y(_y){}
    pt():x(0),y(0){}

    pt operator * (T c){ return pt(x*c,y*c);}
    pt operator / (T c){ return pt(x/c,y/c);}
    pt operator + (pt a){ return pt(x+a.x,y+a.y);}
    pt operator - (pt a){ return pt(x-a.x,y-a.y);}
    T operator * (pt a){ return x*a.x + y*a.y;}
    T operator ^ (pt a){ return x*a.y - y*a.x;}

    auto operator<=>(pt o) const { return (x != o.x) ? x
        <=> o.x : y <=> o.y; } // c++20
    bool operator < (pt a) const { return x < a.x || (x
        == a.x && y < a.y);}
    bool operator==(pt a) const { return x == a.x and y
        == a.y;}
    friend T ori(pt a, pt b, pt c) { return (b - a) ^ (c
        - a); }
    friend T abs2(pt a) { return a * a; }
};
using numbers::pi; // c++20
const ld pi = acos(-1);
const ld eps = 1e-8L;
using Pt = pt<ld>;
int sgn(ld x) { return (x > -eps) - (x < eps); } //
    dcmp == sgn
ld abs(Pt a){ return sqrt(abs2(a)); }
ld arg(Pt x) { return atan2(x.y, x.x); }
bool argcmp(Pt a, Pt b) { // arg(a) < arg(b)
    int f = (Pt{a.y, -a.x} > Pt{} ? 1 : -1) * (a != Pt
        {});
    int g = (Pt{b.y, -b.x} > Pt{} ? 1 : -1) * (b != Pt
        {});
    return f == g ? (a ^ b) > 0 : f < g;
}
Pt unit(Pt x) { return x / abs(x); }
Pt rotate(Pt u) { // pi / 2
    return {-u.y, u.x};
}
Pt rotate(Pt u, ld a) {
    Pt v{sin(a), cos(a)};
    return {u ^ v, u * v};
}

istream &operator>>(istream &s, Pt &a) { return s >> a.
    x >> a.y; }
ostream &operator<<(ostream &s, Pt &a) { return s << "("
    << a.x << ", " << a.y << ")";}

bool collinearity(Pt a, Pt b, Pt c) { // 三點共線
    return ((b - a) ^ (c - a)) == 0;
}

```

6.2 Line

```

struct Line {
    Pt a, b;
    Pt dir() const { return b - a; }
};
int PtSide(Pt p, Line L) {
    // return sgn(ori(L.a, L.b, p) / abs(L.a - L.b));
    return sgn(ori(L.a, L.b, p));
}
bool PtOnSeg(Pt p, Line L) {
    return PtSide(p, L) == 0 and sgn((p - L.a) * (p - L
        .b)) <= 0;
}
Pt proj(Pt p, Line l) {
    Pt dir = unit(l.b - l.a);
    return l.a + dir * (dir * (p - l.a));
}

```

6.3 Circle

```

struct Cir {
    Pt o;
    ld r;
};

```



```
};
bool disjunct(const Cir &a, const Cir &b) { // 分開 ||
    外切 -> 1
    return sgn(abs(a.o - b.o) - a.r - b.r) >= 0;
}
bool contain(const Cir &a, const Cir &b) {
    return sgn(a.r - b.r - abs(a.o - b.o)) >= 0;
}
```

6.4 圓多邊形面積

```
double CirclePoly(Cir C, const vector<Pt> &P) {
    auto arg = [&](Pt p, Pt q) { return atan2(p ^ q, p * q); };
    double r2 = C.r * C.r / 2;
    auto tri = [&](Pt p, Pt q) {
        Pt d = q - p;
        auto a = (d * p) / abs2(d), b = (abs2(p) - C.r * C.r) / abs2(d);
        auto det = a * a - b;
        if (det <= 0) return arg(p, q) * r2;
        auto s = max(0., -a - sqrt(det)), t = min(1., -a + sqrt(det));
        if (t < 0 || 1 <= s) return arg(p, q) * r2;
        Pt u = p + d * s, v = p + d * t;
        return arg(p, u) * r2 + (u ^ v) / 2 + arg(v, q) * r2;
    };
    double sum = 0.0;
    for (int i = 0; i < P.size(); i++)
        sum += tri(P[i] - C.o, P[(i + 1) % P.size()]) - C.o;
    return sum;
}
```

6.5 圓三角形面積

```
double CircleTriangle(Pt a, Pt b, double r) {
    if (sgn(abs(a) - r) <= 0 and sgn(abs(b) - r) <= 0)
        return abs(a ^ b) / 2;
    if (abs(a) > abs(b)) swap(a, b);
    auto I = CircleLineInter({}, r, {a, b});
    erase_if(I, [&](Pt x) { return !PtOnSeg(x, {a, b}); });
    if (I.size() == 1) return abs(a ^ I[0]) / 2 + SectorArea(I[0], b, r);
    if (I.size() == 2) {
        return SectorArea(a, I[0], r) + SectorArea(I[1], b, r) + abs(I[0] ^ I[1]) / 2;
    }
    return SectorArea(a, b, r);
}
```

6.6 半平面交

```
bool cover(Line L, Line P, Line Q) {
    // PtSide(LineInter(P, Q), L) <= 0 or P, Q parallel
    i128 u = (Q.a - P.a) ^ Q.dir();
    i128 v = P.dir() ^ Q.dir();
    i128 x = P.dir().x * u + (P.a - L.a).x * v;
    i128 y = P.dir().y * u + (P.a - L.a).y * v;
    return sgn(x * L.dir().y - y * L.dir().x) * sgn(v) >= 0;
}
vector<Line> HPI(vector<Line> P) {
    // line P.a -> P.b 的逆時針是半平面
    sort(all(P), [&](Line l, Line m) {
        if (argcmp(l.dir(), m.dir()) return true;
        if (argcmp(m.dir(), l.dir()) return false;
        return ori(m.a, m.b, l.a) > 0;
    });
    int n = P.size(), l = 0, r = -1;
    for (int i = 0; i < n; i++) {
        if (i and !argcmp(P[i - 1].dir(), P[i].dir())) continue;
        while (l < r and cover(P[i], P[r - 1], P[r])) r--;
        while (l < r and cover(P[i], P[l], P[l + 1])) l++;
        P[++r] = P[i];
    }
}
```

```

    }
    while (l < r and cover(P[l], P[r - 1], P[r])) r--;
    while (l < r and cover(P[r], P[l], P[l + 1])) l++;
    if (r - l <= 1 or !argcmp(P[l].dir(), P[r].dir()))
        return {}; // empty
    if (cover(P[l + 1], P[l], P[r]))
        return {}; // infinity
    return vector(P.begin() + l, P.begin() + r + 1);
}
```

6.7 圓線交

```
vector<Pt> CircleLineInter(Cir c, Line l) {
    Pt H = proj(c.o, l);
    Pt dir = unit(l.b - l.a);
    double h = abs(H - c.o);
    if (sgn(h - c.r) > 0) return {};
    double d = sqrt(max((double)0., c.r * c.r - h * h));
    if (sgn(d) == 0) return {H};
    return {H - dir * d, H + dir * d};
    // CounterClockwise
}
```

6.8 圓圓交

```
vector<Pt> CircleInter(Cir a, Cir b) {
    double d2 = abs2(a.o - b.o), d = sqrt(d2);
    if (d < max(a.r, b.r) - min(a.r, b.r) || d > a.r + b.r) return {};
    Pt u = (a.o + b.o) / 2 + (a.o - b.o) * ((b.r * b.r - a.r * a.r) / (2 * d2));
    double A = sqrt((a.r + b.r + d) * (a.r - b.r + d) * (a.r + b.r - d) * (-a.r + b.r + d));
    Pt v = rotate(b.o - a.o) * A / (2 * d2);
    if (sgn(v.x) == 0 and sgn(v.y) == 0) return {u};
    return {u - v, u + v}; // counter clockwise of a
}
```

6.9 線線交

```
bool isInter(Line l, Line m) {
    if (PtOnSeg(m.a, l) or PtOnSeg(m.b, l) or
        PtOnSeg(l.a, m) or PtOnSeg(l.b, m))
        return true;
    return PtSide(m.a, l) * PtSide(m.b, l) < 0 and
        PtSide(l.a, m) * PtSide(l.b, m) < 0;
}
Pt LineInter(Line l, Line m) {
    double s = ori(m.a, m.b, l.a), t = ori(m.a, m.b, l.b);
    return (l.b * s - l.a * t) / (s - t);
}
```

6.10 ConvexHull

```
vector<Pt> Hull(vector<Pt> P) {
    sort(all(P));
    P.erase(unique(all(P)), P.end());
    P.insert(P.end(), P.rbegin() + 1, P.rend());
    vector<Pt> stk;
    for (auto p : P) {
        auto it = stk.rbegin();
        while (stk.rend() - it >= 2 and \
            ori(*next(it), *it, p) <= 0 and \
            (*next(it) < *it) == (*it < p)) {
            it++;
        }
        stk.resize(stk.rend() - it);
        stk.push_back(p);
    }
    stk.pop_back();
    return stk;
}
```

6.11 Hulltrick

```
struct Convex {
    int n;
    vector<Pt> A, V, L, U;
    Convex(const vector<Pt> &A) : A(A), n(A.size()) {
        // n >= 3
    }
}
```

```

    auto it = max_element(all(A));
    L.assign(A.begin(), it + 1);
    U.assign(it, A.end()), U.push_back(A[0]);
    for (int i = 0; i < n; i++) {
        V.push_back(A[(i + 1) % n] - A[i]);
    }
}

int inside(Pt p, const vector<Pt> &h, auto f) {
    auto it = lower_bound(all(h), p, f);
    if (it == h.end()) return 0;
    if (it == h.begin()) return p == *it;
    return 1 - sgn(ori(*prev(it), p, *it));
}

// 0: out, 1: on, 2: in
int inside(Pt p) {
    return min(inside(p, L, less{}), inside(p, U, greater{}));
}

static bool cmp(Pt a, Pt b) { return sgn(a ^ b) > 0; }

// A[i] is a far/closer tangent point
int tangent(Pt v, bool close = true) {
    assert(v != Pt{});
    auto l = V.begin(), r = V.begin() + L.size() - 1;
    if (v < Pt{}) l = r, r = V.end();
    if (close) return (lower_bound(l, r, v, cmp) - V.begin()) % n;
    return (upper_bound(l, r, v, cmp) - V.begin()) % n;
}

// closer tangent point array[0] -> array[1] 順時針
array<int, 2> tangent2(Pt p) {
    array<int, 2> t{-1, -1};
    if (inside(p) == 2) return t;
    if (auto it = lower_bound(all(L), p); it != L.end() and p == *it) {
        int s = it - L.begin();
        return {(s + 1) % n, (s - 1 + n) % n};
    }
    if (auto it = lower_bound(all(U), p, greater{}); it != U.end() and p == *it) {
        int s = it - U.begin() + L.size() - 1;
        return {(s + 1) % n, (s - 1 + n) % n};
    }
    for (int i = 0; i != t[0]; i = tangent((A[t[0] = i] - p), 0));
    for (int i = 0; i != t[1]; i = tangent((p - A[t[1] = i]), 1));
    return t;
}

int find(int l, int r, Line L) {
    if (r < l) r += n;
    int s = PtSide(A[l % n], L);
    return *ranges::partition_point(views::iota(l, r), [&](int m) { return PtSide(A[m % n], L) == s; }) - 1;
};

// Line A_x A_x+1 intersect with L
vector<int> intersect(Line L) {
    int l = tangent(L.a - L.b), r = tangent(L.b - L.a);
    if (PtSide(A[l], L) * PtSide(A[r], L) >= 0) return {};
    return {find(l, r, L) % n, find(r, l, L) % n};
}
};

```

6.12 點線距

```

double PtSegDist(Pt p, Line l) {
    double ans = min(abs(p - l.a), abs(p - l.b));
    if (sgn(abs(l.a - l.b)) == 0) return ans;
    if (sgn((l.a - l.b) * (p - l.b)) < 0) return ans;
    if (sgn((l.b - l.a) * (p - l.a)) < 0) return ans;
    return min(ans, abs(ori(p, l.a, l.b)) / abs(l.a - l.b));
}

double SegDist(Line l, Line m) {
    return PtSegDist({0, 0}, {l.a - m.a, l.b - m.b});
}

```

```

};

```

6.13 MEC

```

Pt Center(Pt a, Pt b, Pt c) {
    Pt x = (a + b) / 2;
    Pt y = (b + c) / 2;
    return LineInter({x, x + rotate(b - a)}, {y, y + rotate(c - b)});
}

Cir MEC(vector<Pt> P) {
    mt19937 rng(time(0));
    shuffle(all(P), rng);
    Cir C = {P[0], 0, 0};
    for (int i = 0; i < P.size(); i++) {
        if (C.inside(P[i])) continue;
        C = {P[i], 0};
        for (int j = 0; j < i; j++) {
            if (C.inside(P[j])) continue;
            C = {(P[i] + P[j]) / 2, abs(P[i] - P[j]) / 2};
            for (int k = 0; k < j; k++) {
                if (C.inside(P[k])) continue;
                C.o = Center(P[i], P[j], P[k]);
                C.r = abs(C.o - P[i]);
            }
        }
    }
    return C;
}

```

6.14 MEC2

```

PT arr[MXN];
int n = 10;
double checky(double x, double y) {
    double cmax = 0;
    for (int i = 0; i < n; i++) { // 過程中回傳距離^2
        // 避免不必要的根號運算
        cmax = max(cmax, (arr[i].x - x) * (arr[i].x - x) + (arr[i].y - y) * (arr[i].y - y));
    }
    return cmax;
}

double checkx(double x) {
    double yl = -1e9, yr = 1e9;
    while (yr - yl > EPS) {
        double ml = (yl + yl + yr) / 3, mr = (yl + yr + yr) / 3;
        if (checky(x, ml) < checky(x, mr))
            yr = mr;
        else
            yl = ml;
    }
}

signed main() {
    double xl = -1e9, xr = 1e9;
    while (xr - xl > EPS) {
        double ml = (xl + xl + xr) / 3, mr = (xl + xr + xr) / 3;
        if (checkx(ml) < checkx(mr))
            xr = mr;
        else
            xl = ml;
    }
}

```

6.15 旋轉卡尺

```

auto RotatingCalipers(const vector<Pt> &hull) { // 最遠點對 回傳距離平方
    int n = hull.size();
    auto ret = abs2(hull[0]);
    ret = 0;
    if (hull.size() <= 2) return abs2(hull[0] - hull[1]);
    for (int i = 0, j = 2; i < n; i++) {
        Pt a = hull[i], b = hull[(i + 1) % n];
        while(ori(hull[j], a, b) < ori(hull[(j + 1) % n], a, b))
            j = (j + 1) % n;
        chmax(ret, abs2(a - hull[j]));
    }
}

```

```

        chmax(ret, abs2(b - hull[j]));
    }
    return ret;
}

```

6.16 Minkowski

```

// P, Q, R(return) are counterclockwise order convex
// polygon
vector<Pt> Minkowski(vector<Pt> P, vector<Pt> Q) {
    auto cmp = [&](Pt a, Pt b) {
        return Pt{a.y, a.x} < Pt{b.y, b.x};
    };
    auto reorder = [&](auto &R) {
        rotate(R.begin(), min_element(all(R), cmp), R.
            end());
        R.push_back(R[0]), R.push_back(R[1]);
    };
    const int n = P.size(), m = Q.size();
    reorder(P), reorder(Q);
    vector<Pt> R;
    for (int i = 0, j = 0, s; i < n or j < m; ) {
        R.push_back(P[i] + Q[j]);
        s = sgn((P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]));
        if (s >= 0) i++;
        if (s <= 0) j++;
    }
    return R;
}

```

6.17 PointInPolygon

```

int inPoly(Pt p, const vector<Pt> &P) {
    const int n = P.size();
    int cnt = 0;
    for (int i = 0; i < n; i++) {
        Pt a = P[i], b = P[(i + 1) % n];
        if (PtOnSeg(p, {a, b})) return 1; // on edge
        if ((sgn(a.y - p.y) == 1) ^ (sgn(b.y - p.y) ==
            1))
            cnt += sgn(ori(a, b, p));
    }
    return cnt == 0 ? 0 : 2; // out, in
}

```

6.18 UnionOfCircles

```

// Area[i] : area covered by at least i circle
// TODO:!!!aaa!!!
vector<double> CircleUnion(const vector<Cir> &C) {
    const int n = C.size();
    vector<double> Area(n + 1);
    auto check = [&](int i, int j) {
        if (!contain(C[i], C[j]))
            return false;
        return sgn((C[i].r - C[j].r) > 0 or (sgn(C[i].r
            - C[j].r) == 0 and i < j));
    };
    struct Teve {
        double ang; int add; Pt p;
        bool operator<(const Teve &b) { return ang < b.
            ang; }
    };
    auto ang = [&](Pt p) { return atan2(p.y, p.x); };
    for (int i = 0; i < n; i++) {
        int cov = 1;
        vector<Teve> event;
        for (int j = 0; j < n; j++) if (i != j) {
            if (check(j, i)) cov++;
            else if (!check(i, j) and !disjunct(C[i], C
                [j])) {
                auto I = CircleInter(C[i], C[j]);
                assert(I.size() == 2);
                double a1 = ang(I[0] - C[i].o), a2 =
                    ang(I[1] - C[i].o);
                event.push_back({a1, 1, I[0]});
                event.push_back({a2, -1, I[1]});
                if (a1 > a2) cov++;
            }
        }
        if (event.empty()) {
            Area[cov] += pi * C[i].r * C[i].r;

```

```

        continue;
    }
    sort(all(event));
    event.push_back(event[0]);
    for (int j = 0; j + 1 < event.size(); j++) {
        cov += event[j].add;
        Area[cov] += (event[j].p ^ event[j + 1].p)
            / 2.;
        double theta = event[j + 1].ang - event[j].
            ang;
        if (theta < 0) theta += 2 * pi;
        Area[cov] += (theta - sin(theta)) * C[i].r
            * C[i].r / 2.;
    }
    return Area;
}

```

6.19 UnionOfPolygons

```

// Area[i] : area covered by at least i polygon
vector<double> PolyUnion(const vector<vector<Pt>> &P) {
    const int n = P.size();
    vector<double> Area(n + 1);
    vector<Line> Ls;
    for (int i = 0; i < n; i++)
        for (int j = 0; j < P[i].size(); j++)
            Ls.push_back({P[i][j], P[i][(j + 1) % P[i].
                size()]});
    auto cmp = [&](Line &l, Line &r) {
        Pt u = l.b - l.a, v = r.b - r.a;
        if (argcmp(u, v)) return true;
        if (argcmp(v, u)) return false;
        return PtSide(l.a, r) < 0;
    };
    sort(all(Ls), cmp);
    for (int l = 0, r = 0; l < Ls.size(); l = r) {
        while (r < Ls.size() and !cmp(Ls[l], Ls[r])) r
            ++;
        Line L = Ls[l];
        vector<pair<Pt, int>> event;
        for (auto [c, d] : Ls) {
            if (sgn((L.a - L.b) ^ (c - d)) != 0) {
                int s1 = PtSide(c, L) == 1;
                int s2 = PtSide(d, L) == 1;
                if (s1 ^ s2) event.emplace_back(
                    LineInter(L, {c, d}), s1 ? 1 : -1);
            } else if (PtSide(c, L) == 0 and sgn((L.a -
                L.b) ^ (c - d)) > 0) {
                event.emplace_back(c, 2);
                event.emplace_back(d, -2);
            }
        }
        sort(all(event), [&](auto i, auto j) {
            return (L.a - i.ff) * (L.a - L.b) < (L.a -
                j.ff) * (L.a - L.b);
        });
        int cov = 0, tag = 0;
        Pt lst{0, 0};
        for (auto [p, s] : event) {
            if (cov >= tag) {
                Area[cov] += lst ^ p;
                Area[cov - tag] -= lst ^ p;
            }
            if (abs(s) == 1) cov += s;
            else tag += s / 2;
            lst = p;
        }
    }
    for (int i = n - 1; i >= 0; i--) Area[i] += Area[i
        + 1];
    for (int i = 1; i <= n; i++) Area[i] /= 2;
    return Area;
}

```

6.20 圓公切線

```

vector<Line> CircleTangent(Cir c1, Cir c2, int sign1) {
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    ld d_sq = abs2(c1.o - c2.o);
    if (sgn(d_sq) == 0) return ret;

```

```

ld d = sqrt(d_sq);
Pt v = (c2.o - c1.o) / d;
ld c = (c1.r - sign1 * c2.r) / d;
if (c * c > 1) return ret;
ld h = sqrt(max(0.0, 1.0 - c * c));
for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
    Pt n = Pt(v.x * c - sign2 * h * v.y, v.y * c +
        sign2 * h * v.x);
    Pt p1 = c1.o + n * c1.r;
    Pt p2 = c2.o + n * (c2.r * sign1);
    if (sgn(p1.x - p2.x) == 0 && sgn(p1.y - p2.y)
        == 0)
        p2 = p1 + rotate(c2.o - c1.o);
    ret.push_back({p1, p2});
}
return ret;
}

```

6.21 點圓切線

```

vector<Line> CircleTangent(Cir c, Pt p) {
    vector<Line> z;
    double d = abs(p - c.o);
    if (sgn(d - c.r) == 0) {
        Pt i = rotate(p - c.o);
        z.push_back({p, p + i});
    } else if (d > c.r) {
        double o = acos(c.r / d);
        Pt i = unit(p - c.o);
        Pt j = rotate(i, o) * c.r;
        Pt k = rotate(i, -o) * c.r;
        z.push_back({c.o + j, p});
        z.push_back({c.o + k, p});
    }
    return z;
}

```

6.22 最近點對

```

pair<ld, pair<i32, i32>> ClosestPair(vector<Pt> &P) {
    // ans = dis * dis !!注意ans overflow問題
    if (P.size() == 1) { return {1e200L, {0, 0}}; }
    pair<i32, i32> ansi;
    auto ans = abs2(P[0] - P[1]);
    ansi = {0, 1};
    auto upd = [&](const Pt &a, const Pt &b) {
        auto dis = abs2(a - b);
        if (dis < ans) ans = dis, ansi.FF = a.id, ansi.
            SS = b.id;
    };
    auto cmpy = [&](const Pt &a, const Pt &b) { return a
        .y < b.y; };
    vector<Pt> t(P.size() + 1);
    auto rec = [&](auto &&self, i32 l, i32 r) {
        if (r - l <= 3) {
            for (i32 i = l; i <= r; i++)
                for (i32 j = i + 1; j <= r; j++) upd(P[
                    i], P[j]);
            sort(P.begin() + l, P.begin() + r + 1, cmpy
            );
            return;
        }
        i32 m = (l + r) >> 1;
        auto midx = P[m].x;
        self(self, l, m), self(self, m + 1, r);
        i32 tsz = 0;
        inplace_merge(P.begin() + l, P.begin() + m + 1,
            P.begin() + r + 1, cmpy);
        for (i32 i = l; i <= r; i++) {
            if (abs(P[i].x - midx) * abs(P[i].x - midx)
                >= ans) continue;
            for (i32 j = tsz - 1; j >= 0 && (P[i].y - t
                [j].y) * (P[i].y - t[j].y) < ans; j--)
                upd(P[i], t[j]);
            t[tsz++] = P[i];
        }
        sort(all(P));
        rec(rec, 0, P.size() - 1);
        return make_pair(sqrt(ans), ansi);
    };
}

```

```

}

```

7 graph

7.1 BCC

```

#define REP(i, n) for (int i = 0; i < n; i++)
struct BccVertex {
    int n, nScc, step, dfn[MXN], low[MXN];
    vector<int> E[MXN], sccv[MXN];
    int top, stk[MXN];
    void init(int _n) {
        n = _n;
        nScc = step = 0;
        for (int i = 0; i < n; i++) E[i].clear();
    }
    void addEdge(int u, int v) {
        E[u].PB(v);
        E[v].PB(u);
    }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
        stk[top++] = u;
        for (auto v : E[u]) {
            if (v == f) continue;
            if (dfn[v] == -1) {
                DFS(v, u);
                low[u] = min(low[u], low[v]);
                if (low[v] >= dfn[u]) {
                    int z;
                    sccv[nScc].clear();
                    do {
                        z = stk[--top];
                        sccv[nScc].PB(z);
                    } while (z != v);
                    sccv[nScc++].PB(u);
                }
            } else
                low[u] = min(low[u], dfn[v]);
        }
    }
    vector<vector<int>> solve() {
        vector<vector<int>> res;
        for (int i = 0; i < n; i++) dfn[i] = low[i] =
            -1;
        for (int i = 0; i < n; i++)
            if (dfn[i] == -1) {
                top = 0;
                DFS(i, i);
            }
        REP(i, nScc) res.PB(sccv[i]);
        return res;
    }
} graph;

```

7.2 SCC

```

struct Scc {
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n) {
        n = _n;
        for (int i = 0; i <= n; i++)
            E[i].clear(), rE[i].clear();
    }
    void addEdge(int u, int v) {
        E[u].PB(v); rE[v].PB(u);
    }
    void DFS(int u) {
        vst[u] = 1;
        for (auto v : E[u]) if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u) {
        vst[u] = 1; bln[u] = nScc;
        for (auto v : rE[u]) if (!vst[v]) rDFS(v);
    }
    void solve() {
        nScc = 0;
        vec.clear();
        fill(vst, vst + n + 1, 0);
        for (int i = 0; i <= n; i++)

```

```

    if (!vst[i]) DFS(i);
reverse(vec.begin(), vec.end());
fill(vst, vst+n+1, 0);
for (auto v : vec)
    if (!vst[v]){
        rDFS(v); nScc++;
    }
}
};

```

7.3 支配樹

```

#define REP(i, s, e) for (int i = (s); i <= (e); i++)
#define REPD(i, s, e) for (int i = (s); i >= (e); i--)
struct DominatorTree { // O(N) 1-base
    int n, s;
    vector<int> g[MAXN], pred[MAXN];
    vector<int> cov[MAXN];
    int dfn[MAXN], nfd[MAXN], ts;
    int par[MAXN]; // idom[u] s到u的最後一個必經點
    int sdom[MAXN], idom[MAXN];
    int mom[MAXN], mn[MAXN];
    inline bool cmp(int u, int v) { return dfn[u] < dfn[v]; }
    int eval(int u) {
        if (mom[u] == u) return u;
        int res = eval(mom[u]);
        if (cmp(sdom[mn[mom[u]]], sdom[mn[u]])) mn[u] = mn[mom[u]];
        return mom[u] = res;
    }
    void init(int _n, int _s) {
        ts = 0;
        n = _n;
        s = _s;
        REP(i, 1, n) g[i].clear(), pred[i].clear();
    }
    void addEdge(int u, int v) {
        g[u].push_back(v);
        pred[v].push_back(u);
    }
    void dfs(int u) {
        ts++;
        dfn[u] = ts;
        nfd[ts] = u;
        for (int v : g[u])
            if (dfn[v] == 0) {
                par[v] = u;
                dfs(v);
            }
    }
    void build() {
        REP(i, 1, n) {
            idom[i] = par[i] = dfn[i] = nfd[i] = 0;
            cov[i].clear();
            mom[i] = mn[i] = sdom[i] = i;
        }
        dfs(s);
        REPD(i, n, 2) {
            int u = nfd[i];
            if (u == 0) continue;
            for (int v : pred[u])
                if (dfn[v]) {
                    eval(v);
                    if (cmp(sdom[mn[v]], sdom[u])) sdom[u] = sdom[mn[v]];
                }
            cov[sdom[u]].push_back(u);
            mom[u] = par[u];
            for (int w : cov[par[u]]) {
                eval(w);
                if (cmp(sdom[mn[w]], par[u]))
                    idom[w] = mn[w];
                else
                    idom[w] = par[u];
            }
            cov[par[u]].clear();
        }
        REP(i, 2, n) {
            int u = nfd[i];
            if (u == 0) continue;

```

```

            if (idom[u] != sdom[u]) idom[u] = idom[idom[u]];
        }
    }
} domT;

```

7.4 最大團

```

struct MaxClique { // 0-base
    typedef bitset<MXN> Int;
    Int linkto[MAXN], v[MAXN];
    int n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; i++) {
            linkto[i].reset();
            v[i].reset();
        }
    }
    void addEdge(int a, int b) { v[a][b] = v[b][a] = 1; }
    int popcount(const Int& val) { return val.count(); }
    int lowbit(const Int& val) { return val._Find_first(); }
    int ans, stk[MAXN];
    int id[MAXN], di[MAXN], deg[MAXN];
    Int cans;
    void maxclique(int elem_num, Int candi) {
        if (elem_num > ans) {
            ans = elem_num;
            cans.reset();
            for (int i = 0; i < elem_num; i++) cans[id[stk[i]]] = 1;
        }
        int potential = elem_num + popcount(candi);
        if (potential <= ans) return;
        int pivot = lowbit(candi);
        Int smaller_candi = candi & (~linkto[pivot]);
        while (smaller_candi.count() && potential > ans) {
            int next = lowbit(smaller_candi);
            candi[next] = !candi[next];
            smaller_candi[next] = !smaller_candi[next];
            potential--;
            if (next == pivot || (smaller_candi & linkto[next]).count()) {
                stk[elem_num] = next;
                maxclique(elem_num + 1, candi & linkto[next]);
            }
        }
    }
    int solve() {
        for (int i = 0; i < n; i++) {
            id[i] = i;
            deg[i] = v[i].count();
        }
        sort(id, id + n, [&](int id1, int id2) { return deg[id1] > deg[id2]; });
        for (int i = 0; i < n; i++) di[id[i]] = i;
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                if (v[i][j]) linkto[di[i]][di[j]] = 1;
        Int cand;
        cand.reset();
        for (int i = 0; i < n; i++) cand[i] = 1;
        ans = 1;
        cans.reset();
        cans[0] = 1;
        maxclique(0, cand);
        return ans;
    }
} solver;

```

7.5 最小圈

```

/* minimum mean cycle O(VE) */
struct MMC{
#define E 101010
#define V 1021
#define inf 1e9

```



```
#define eps 1e-6
struct Edge { int v,u; double c; };
int n, m, prv[V][V], prve[V][V], vst[V];
Edge e[E];
vector<int> edgeID, cycle, rho;
double d[V][V];
void init( int _n )
{ n = _n; m = 0; }
// WARNING: TYPE matters
void addEdge( int vi , int ui , double ci )
{ e[ m ++ ] = { vi , ui , ci }; }
void bellman_ford() {
    for(int i=0; i<n; i++) d[0][i]=0;
    for(int i=0; i<n; i++) {
        fill(d[i+1], d[i+1]+n, inf);
        for(int j=0; j<m; j++) {
            int v = e[j].v, u = e[j].u;
            if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                d[i+1][u] = d[i][v]+e[j].c;
                prv[i+1][u] = v;
                prve[i+1][u] = j;
            }
        }
    }
}
double solve(){
    // returns inf if no cycle, mmc otherwise
    double mmc=inf;
    int st = -1;
    bellman_ford();
    for(int i=0; i<n; i++) {
        double avg=-inf;
        for(int k=0; k<n; k++) {
            if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
            else avg=max(avg,inf);
        }
        if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    fill(vst,0); edgeID.clear(); cycle.clear(); rho.clear();
    for (int i=n; !vst[st]; st=prv[i--][st]) {
        vst[st]++;
        edgeID.PB(prve[i][st]);
        rho.PB(st);
    }
    while (vst[st] != 2) {
        if(rho.empty()) return inf;
        int v = rho.back(); rho.pop_back();
        cycle.PB(v);
        vst[v]++;
    }
    reverse(ALL(edgeID));
    edgeID.resize(SZ(cycle));
    return mmc;
} }mmc;
```

7.6 kShortestPath

```
while(Q.size()){
    auto [dx,x] = Q.top();Q.pop();
    if(dis[x].size() >= k) continue;
    dis[x].PB(dx);
    for(auto [v,w]:E[x]) Q.emplace(w+dx,v);
}
```

7.7 結論

- 2-SAT :
 $(a_i \vee a_j) = \text{true} \quad \forall (i, j)$
 對於任意限制 $(x \vee y)$
 建兩條有向邊 (要多編號 $\neg x$)
 $x \rightarrow \neg y$ and $y \rightarrow \neg x$
 跑 scc
 $\text{scc.blm}[x] < \text{scc.blm}[\neg x] \Leftrightarrow x \text{ is true}$
 $\text{scc.blm}[\neg x] < \text{scc.blm}[x] \Leftrightarrow x \text{ is false}$
 $\exists x \text{ which } \text{scc.blm}[x] == \text{scc.blm}[\neg x] \Leftrightarrow \text{無解}$

- 差分約束:
 n 個變數及 m 個約束條件
 求滿足所有 $x_j - x_i \leq b_k \quad (i, j \in [1, n], k \in [1, m])$
 的一組 $x_1 \dots x_n$
 可轉成 $x_j - x_i \leq b_k \rightarrow x_j \leq x_i + b_k$
 結論就是使得所有 x_j 變小以滿足上式
 建邊跑 SPFA/Bellman
 要多建起點 s 連到所有 i 且邊權 0, $\text{dis}[s] = 0$
 有負環則無解, 否則起點到所有 i 的距離為一組解
 $x_j - x_i \leq k \Rightarrow \text{addEdge } i \xrightarrow{k} j$

$x_j - x_i \geq k \Rightarrow \text{addEdge } j \xrightarrow{-k} i$
 $x_j = x_i \Rightarrow \text{addEdge } i \xrightarrow{0} j \text{ and } j \xrightarrow{0} i$

8 math

8.1 DiscreteSqrt

```
void calcH(i64 &t, i64 &h, const i64 p) {
    i64 tmp=p-1; for(t=0;(tmp&1)==0;tmp/=2) t++; h=tmp;
}
// solve equation x^2 mod p = a
// !!!! (a != 0) !!!!
bool solve(i64 a, i64 p, i64 &x, i64 &y) {
    if(p == 2) { x = y = 1; return true; }
    int p2 = p / 2, tmp = mypow(a, p2, p);
    if (tmp == p - 1) return false;
    if ((p + 1) % 4 == 0) {
        x = mypow(a, (p+1)/4, p); y = p - x; return true;
    } else {
        i64 t, h, b, pb; calcH(t, h, p);
        if (t >= 2) {
            do {b = rand() % (p - 2) + 2;
                while (mypow(b, p / 2, p) != p - 1);
                pb = mypow(b, h, p);
            } int s = mypow(a, h / 2, p);
            for (int step = 2; step <= t; step++) {
                int ss = (((i64)(s * s) % p) * a) % p;
                for(int i=0;i<t-step;i++) ss=mul(ss,ss,p);
                if (ss + 1 == p) s = (s * pb) % p;
                pb = ((i64)pb * pb) % p;
            } x = ((i64)s * a) % p; y = p - x;
        } return true;
    }
}
```

8.2 excrt

```
i128 exgcd(i128 a, i128 b, i128 &x, i128 &y){
    if (b == 0) return x=1, y=0, a;
    int d = exgcd(b, a % b, y, x);
    y -= a / b * x;
    return d;
}
// as -> 算式答案
// ns -> 模數 MOD
i128 CRT(vector<i64> as, vector<i64> ns) {
    i32 n = as.size();
    i128 a1, a2, n1, n2;
    bool flag = false;
    auto china = [&]() {
        i128 d = a2 - a1;
        i128 x, y;
        i128 g = exgcd(n1, n2, x, y);
        if (d % g == 0) {
            x = ((x * d / g) % (n2 / g) + (n2 / g)) % (
                n2 / g);
            a1 = x * n1 + a1;
            n1 = (n1 * n2) / g;
        } else {
            flag = true;
        }
    };
    a1 = as[0], n1 = ns[0];
    for (i32 i = 1; i < n; i++) {
        a2 = as[i], n2 = ns[i];
        china();
        if (flag) return -1;
    }
    return a1;
}
```

8.3 exgcd

```
int exgcd(int a,int b,int&x,int&y){
    if(b==0)return x=1,y=0,a;
    int d = exgcd(b,a%b,y,x);
    y-=a/b*x;
    return d;
}
```


8.4 FFT

```
const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx; //real() ,imag()
const ld PI = acos(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft(){
    for(int i=0; i<=MAXN; i++){
        omega[i] = exp(i * 2 * PI / MAXN * I);
    }
    // n must be 2^k
void fft(int n, cplx a[], bool inv=false){
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN-(i*theta%MAXN) : i*theta%MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
        }
        theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^ k); k >= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if(inv) for (i = 0; i < n; i++) a[i] /= n;
}
cplx arr[MAXN+1];
inline void mul(int _n,i64 a[],int _m,i64 b[],i64 ans
[]){
    int n=1,sum=_n+_m-1;
    while(n<sum)
        n<=<=1;
    for(int i=0;i<n;i++){
        double x=(i<_n?a[i]:0),y=(i<_m?b[i]:0);
        arr[i]=complex<double>(x+y,x-y);
    }
    fft(n,arr);
    for(int i=0;i<n;i++){
        arr[i]=arr[i]*arr[i];
        fft(n,arr,true);
    }
    for(int i=0;i<sum;i++){
        ans[i]=(i64)(arr[i].real()/4+0.5);
    }
}
```

8.5 josephus

```
int josephus(int n, int m){ //n人每m次
    int ans = 0;
    for (int i=1; i<=n; ++i)
        ans = (ans + m) % i;
    return ans;
}
```

8.6 Theorem

- Lucas's Theorem :
For $n, m \in \mathbb{Z}^*$ and prime P , $C(m, n) \bmod P = \prod(C(m_i, n_i))$ where m_i is the i -th digit of m in base P .
- Stirling approximation :
$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n e^{\frac{1}{12n}}$$
- Stirling Numbers(permutation $|P| = n$ with k cycles):
$$S(n, k) = \text{coefficient of } x^k \text{ in } \prod_{i=0}^{n-1} (x+i)$$
- Stirling Numbers(Partition n elements into k non-empty set):
$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$
- Pick's Theorem : $A = i + b/2 - 1$
 A : Area, i : grid number in the inner, b : grid number on the side

- Catalan number : $C_n = \binom{2n}{n} / (n+1)$
$$C_n^{n+m} - C_{n+1}^{n+m} = (m+n)! \frac{n-m+1}{n+1} \quad \text{for } n \geq m$$

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)n!}$$

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = 2 \binom{2n+1}{n+2} C_n$$

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{i=0}^n C_i C_{n-i} \quad \text{for } n \geq 0$$
- Euler Characteristic:
planar graph: $V - E + F - C = 1$
convex polyhedron: $V - E + F = 2$
 V, E, F, C : number of vertices, edges, faces(regions), and components
- Kirchhoff's theorem :
 $A_{ii} = \deg(i), A_{ij} = (i, j) \in E ? -1 : 0$, Deleting any one row, one column, and cal the $\det(A)$
- Polya' theorem (c is number of color, m is the number of cycle size):
$$\left(\sum_{i=1}^m c^{gcd(i, m)} \right) / m$$
- Burnside lemma:
 $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- 錯排公式: (n 個人中, 每個人皆不再原來位置的組合數):
$$dp[0] = 1; dp[1] = 0;$$

$$dp[i] = (i-1) * (dp[i-1] + dp[i-2]);$$
- Bell 數 (有 n 個人, 把他們拆組的方法總數) :
$$B_0 = 1$$

$$B_n = \sum_{k=0}^n s(n, k) \quad (\text{second - stirling})$$

$$B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k$$
- Wilson's theorem :
 $(p-1)! \equiv -1 \pmod{p}$
- Fermat's little theorem :
 $a^p \equiv a \pmod{p}$
- Euler's totient function:
 $A^{B^C} \bmod p = \text{pow}(A, \text{pow}(B, C, p-1)) \bmod p$
- 歐拉函數降幂公式:
 $A^B \bmod C = A^{B \bmod \phi(C) + \phi(C)} \bmod C$
- 6 的倍數:
$$(a-1)^3 + (a+1)^3 + (-a)^3 + (-a)^3 = 6a$$
- Standard young tableau (標準楊表):
 $\lambda = (\lambda_1 \geq \dots \geq \lambda_k), \sum \lambda_i = n$ denoted by $\lambda \vdash n$
 $\lambda \vdash n$ 意思為 λ 整數拆分 n eg. $n = 10, \lambda = (6, 4)$ 此拆分可表示一種楊表形狀。
楊表: 第 1 列 λ_1 行 \dots 第 k 列 λ_k 行的方格圖。
標準楊表: 每列從左到右遞增, 每行從上到下遞增。
Let T 為某一 Permutation 跑 RSK 後的標準楊表, 則此 Permutation 的 LDS、LIS 長度分別為 T 的列、行數。
- RSK Correspondence:
A permutation is bijective to (P, Q) 一對標準楊表
 P : Permutation 跑 RSK 算法的結果, 可為半標準楊表。
 Q : 可用來還原 Permutation (像排列矩陣)。
- Hook length formula (形狀為 λ 的標準楊表個數):
$$f^\lambda = \frac{n!}{\prod h_\lambda(i, j)}$$

 $h_\lambda(i, j) = \text{number of pair } (x, y) \text{ where } (x = i \vee y = j) \wedge (x, y) \geq (i, j)$
且 (x, y) 落在形狀為 λ 的表上。
Recursion:
(i) $f^{(0, \dots, 0)} = 1$
(ii) $f^{(\lambda_1, \dots, \lambda_m)} = \sum_{k=1}^m f^{(\lambda_1, \dots, \lambda_{k-1}, \lambda_k-1, \lambda_{k+1}, \dots, \lambda_m)}$

8.7 Primes

Prime	Root	Prime	Root
7681	17	167772161	3
12289	11	104857601	3
40961	3	985661441	3
65537	3	998244353	3
786433	10	1107296257	10
5767169	3	2013265921	31
7340033	3	2810183681	11
23068673	3	2885681153	3
469762049	3	605028353	3
924844033	5	0	0

8.8 millerrabin

```
// n < 4,759,123,141          3 : 2, 7, 61
// n < 1,122,004,669,633      4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383      6 : pirmses <= 13
// n < 2^64                    7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
bool witness(i64 a,i64 n,i64 u,int t){
```

```

if(!a) return 0;
i64 x=mpow(a,u,n);
for(int i=0;i<t;i++) {
    i64 nx=mul(x,x,n);
    if(nx==1&&x!=1&&x!=n-1) return 1;
    x=nx;
}
return x!=1;
}
bool mii64er_rabin(i64 n) {
    int s = 7;
    // iterate s times of witness on n
    if(n<2) return 0;
    if(!(n&1)) return n == 2;
    i64 u=n-1; int t=0;
    // n-1 = u*2^t
    while(!(u&1)) u>>=1, t++;
    while(s--){
        i64 a=magic[s]%n;
        if(witness(a,n,u,t)) return 0;
    }
    return 1;
}

```

8.9 phi

```

ll phi(ll n){ // 計算小於n的數中與n互質的有幾個
    ll res = n, a=n; // O(sqrtN)
    for(ll i=2;i*i<=a;i++){
        if(a%i==0){
            res = res/i*(i-1);
            while(a%i==0) a/=i;
        }
    }
    if(a>1) res = res/a*(a-1);
    return res;
}

```

8.10 pollardrho

```

// does not work when n is prime O(n^(1/4))
i64 f(i64 x, i64 c, i64 mod){ return add(mul(x,x,mod),c,mod); }
i64 poi64ard_rho(i64 n) {
    i64 c = 1, x = 0, y = 0, p = 2, q, t = 0;
    while (t++ % 128 or gcd(p, n) == 1) {
        if (x == y) c++, y = f(x = 2, c, n);
        if (q = mul(p, abs(x-y), n)) p = q;
        x = f(x, c, n); y = f(f(y, c, n), c, n);
    }
    return gcd(p, n);
}

```

8.11 primes

```

/* 12721, 13331, 14341, 75577, 123457, 222557, 556679
* 999983, 1097774749, 1076767633, 100102021, 999997771
* 1001010013, 1000512343, 987654361, 999991231
* 999888733, 98789101, 987777733, 999991921, 1010101333
* 1010102101, 10000000000039, 1000000000000037
* 2305843009213693951, 4611686018427387847
* 9223372036854775783, 18446744073709551557 */
int mu[ N ], p_tbl[ N ];
vector<int> primes;
void sieve() {
    mu[ 1 ] = p_tbl[ 1 ] = 1;
    for( int i = 2; i < N; i ++ ){
        if( !p_tbl[ i ] ){
            p_tbl[ i ] = i;
            primes.push_back( i );
            mu[ i ] = -1;
        }
        for( int p : primes ){
            int x = i * p;
            if( x >= M ) break;
            p_tbl[ x ] = p;
            mu[ x ] = -mu[ i ];
            if( i % p == 0 ){
                mu[ x ] = 0;
                break;
            }
        }
    }
}
vector<int> factor( int x ){
    vector<int> fac{ 1 };

```

```

while( x > 1 ){
    int fn = SZ(fac), p = p_tbl[ x ], pos = 0;
    while( x % p == 0 ){
        x /= p;
        for( int i = 0; i < fn; i ++ )
            fac.PB( fac[ pos ++ ] * p );
    }
    return fac;
}

```

8.12 Euler

```

int Euler(int n){
    int now = n;
    for (int i = 2; i * i <= n; i++)
        if (n % i == 0){
            now = now - now / i;
            while (n % i == 0) n = n / i;
        }
    if (n > 1) now = now - now / n;
    return now;
}

```

8.13 quickeuler

```

vector<int> pri;
bool not_prime[MXN + 10];
int phi[MXN + 10];
void quick_euler(int n) {
    phi[1] = 1;
    for (int i = 2; i <= n; i++) {
        if (!not_prime[i]) {
            pri.push_back(i);
            phi[i] = i - 1;
        }
        for (int pri_j : pri) {
            if (i * pri_j > n) break;
            not_prime[i * pri_j] = true;
            if (i % pri_j == 0) {
                phi[i * pri_j] = phi[i] * pri_j;
                break;
            }
            phi[i * pri_j] = phi[i] * phi[pri_j];
        }
    }
}

```

8.14 sieve

```

const int MXN = 1e8 + 50;
const int SQRTMXN = 1e4 + 50;
bitset<MXN> isprime;
void sieve() {
    isprime[1] = 1;
    for (int i = 2; i <= SQRTMXN; i++) {
        if (!isprime[i])
            for (i64 j = i * i; j < MXN; j += i)
                isprime[j] = 1;
    }
}

```

8.15 NTT

```

constexpr i64 power(i64 a, i64 b, i64 m) {
    i64 ret = 1;
    for (; b >>= 1, a = a * a % m; b & 1) ret = ret * a % m;
    return ret;
};
template<i64 M, i64 root>
struct NTT {
    static const int Log = 21;
    array<i64, Log + 1> e{}, ie{};
    NTT() {
        static_assert(__builtin_ctz(M - 1) >= Log);
        e[Log] = power(root, (M - 1) >> Log, M);
        ie[Log] = power(e[Log], M - 2, M);
        for (int i = Log - 1; i >= 0; i--) {
            e[i] = e[i + 1] * e[i + 1] % M;
            ie[i] = ie[i + 1] * ie[i + 1] % M;
        }
    }

```

```

    }
    void operator()(vector<i64> &v, bool inv) {
        int n = v.size();
        for (int i = 0, j = 0; i < n; i++) {
            if (i < j) swap(v[i], v[j]);
            for (int k = n / 2; (j ^= k) < k; k /= 2);
        }
        for (int m = 1; m < n; m *= 2) {
            i64 w = (inv ? ie : e)[_lg(m) + 1];
            for (int i = 0; i < n; i += m * 2) {
                i64 cur = 1;
                for (int j = i; j < i + m; j++) {
                    i64 g = v[j], t = cur * v[j + m] % M;
                    v[j] = (g + t) % M;
                    v[j + m] = (g - t + M) % M;
                    cur = cur * w % M;
                }
            }
        }
        if (inv) {
            i64 in = power(n, M - 2, M);
            for (int i = 0; i < n; i++) v[i] = v[i] * in % M;
        }
    }
};
template<int M, int G> //nlogn f*g
vector<i64> convolution(vector<i64> f, vector<i64> g) {
    static NTT<M, G> ntt;
    int n = ssize(f) + ssize(g) - 1;
    int len = bit_ceil(1ull * n);
    f.resize(len);
    g.resize(len);
    ntt(f, 0), ntt(g, 0);
    for (int i = 0; i < len; i++) {
        (f[i] *= g[i]) %= M;
    }
    ntt(f, 1);
    f.resize(n);
    return f;
}

```

9 polynomial

9.1 Definition

```

using poly = vector<i64>;
poly operator * (poly A, poly B){
    return convolution<mod,G>(A,B);
}
poly operator - (i64 r, poly A){
    r %= mod;
    r = (r + mod) % mod;
    for(auto &c:A){
        c %= mod;
        c = (mod - c) % mod;
    }
    A[0] = (A[0] + r) % mod;
    return A;
}
poly operator + (i64 r, poly A){
    r %= mod;
    r = (r + mod) % mod;
    A[0] = (A[0] + r) % mod;
    return A;
}
poly operator * (i64 r, poly A){
    for(auto &c:A){
        c = r * c % mod;
        c = (c + mod) % mod;
    }
    return A;
}
poly operator - (poly A, poly B){
    int n = max(A.size(), B.size());
    A.resize(n), B.resize(n);
    for(int i = 0; i < n; ++i){
        A[i] = (A[i] - B[i]) % mod;
        A[i] = (A[i] + mod) % mod;
    }
    return A;
}

```

```

}
poly operator + (poly A, poly B){
    int n = max(A.size(), B.size());
    A.resize(n), B.resize(n);
    for(int i = 0; i < n; ++i){
        A[i] = (A[i] + B[i]) % mod;
        A[i] = (A[i] + mod) % mod;
    }
    return A;
}
poly operator % (poly A, int n){
    A.resize(n);
    return A;
}
poly derive(poly P){
    if(!P.size()) return P;
    for(int i = 0; i < P.size(); ++i){
        P[i] = i * P[i] % mod;
        P[i] = (P[i] + mod) % mod;
    }
    P.erase(P.begin());
    return P;
}
poly integr(poly P){
    if(!P.size()) return P;
    for(int i = 0; i < P.size(); ++i){
        P[i] = P[i] * inv(i + 1) % mod;
        P[i] = (P[i] + mod) % mod;
    }
    P.insert(P.begin(), 0);
    return P;
}

```

9.2 Inverse Series

```

poly inverse(poly A){
    int n = A.size();
    int tn = 1;
    poly Q(tn);
    Q[0] = inv(A[0]);
    while(tn < n){
        tn *= 2;
        Q = Q * (2 - A % tn * Q % tn);
        Q = Q % tn;
    }
    return Q;
}

```

9.3 Poly Shift

```

// nlogn f(x) -> f(x + k)
auto shift = [&](vector<i64> f, i64 k) {
    k %= mod;
    k += mod;
    k %= mod;
    int n = f.size() - 1;
    vector<i64> g(n+1);
    for(int i = 0; i <= n; ++i){
        f[i] = f[i] * fac[i] % mod;
        g[n - i] = fpow(k, i) * inv(fac[i]) % mod;
        //x^(-n) -> x^(0)
    }
    auto h = convolution<mod,3>(f, g);
    h.erase(h.begin(), h.begin() + n);
    for(int i = 0; i <= n; ++i) h[i] = h[i] * inv(fac[i] % mod);
    return h;
};

```

9.4 Exponential

```

poly exp(poly P){
    int n = P.size();
    int tn = 1;
    poly Q(tn);
    Q[0] = 1;
    while(tn < n){
        tn *= 2;
        Q = Q * (1 + P % tn - log(Q) % tn);
        Q = Q % tn;
    }
}

```

```
    return Q;
}
```

9.5 Nature log

```
poly log(poly P){
    return integr(derive(P) * inverse(P));
}
```

10 other

10.1 cdq

```
// 三維偏序 (求 arr[j] < arr[i] (每一維嚴格小於), i!=j
// j 的個數)
// 先照 x 排序 merge sort排y 最後BIT動態求z的順序個數
// 左區間的 x < 右區間的
void cdq(int ll,int rr){
    if(ll == rr) return;
    int m = (ll+rr)/2;
    cdq(ll,m),cdq(m+1,rr);
    int i = ll,j = m+1,t = 0;
    auto work = [&]() {
        ans += BIT.qry(arr[j].z); //計數
        temp[t++] = arr[j++];
    };
    while(i <= m && j <= rr){
        if(arr[i].y <= arr[j].y){
            BIT.add(arr[i].z,1); //二維偏序求法
            temp[t++] = arr[i++];
        }
        else work();
    }
    while(i <= m) temp[t++] = arr[i++];
    while(j <= rr) work();
    BIT.reset(); //操作復原
    rep(k,0,t) arr[k+ll] = temp[k];
}
//[l,r)
auto cdq = [&](auto&& self,auto l,auto r){
    if((r - l) <= 1) return;
    auto m = (r - l) / 2 + 1;
    self(self,l,m);
    self(self,m,r);
    auto i = l,j = m;
    auto work = [&]() {
        ++j;
    };
    while(i != m && j != r){
        if(arr[*i][1] <= arr[*j][1]) {
            ++i;
        }
        else work();
    }
    while(j != r) work();
    clear();
    inplace_merge(l,m,r,[&](auto a,auto b){
        return arr[a][1] < arr[b][1];
    });
};
cdq(cdq,all(ord)); //排ord
```

10.2 DeBruijnSequence

```
//求由所有 N 長度bitstring作為substring 最短的字串 B(2,
// N) //B(k,N) : 以k個字元作為N長度字串節點
//00110 -> 00 01 11 10
//建圖 : 點為substrings 邊用 0 1 連接
//走訪 : 000 -1-> 001
// 解為 Hamiltonian 路徑 (剛好所有節點走過一遍)
// 可同構到 N-1 圖上的Eulerian Circuit (每條邊 N-1 圖上
// 的邊 代表 N 圖上的一個點)
vector<int> edges[1<<(N-1)];
vector<int> ans;
void dfs(int x){ // Eulerian Circuit
    while(edges[x].size()){
        int u = edges[x].back();
        edges[x].pop_back();
        ans.push_back(u+1);
        dfs(u);
    }
}
void solve(int n){
```

```
if(n == 1) {
    ans = {1,0};
    return;
}
for(int i = 0; i < (1<<(n-1)); ++i){
    edges[i].push_back(((i<<1)&&((1<<(n-1))-1))); // 0
    // 的邊
    edges[i].push_back(((i<<1)+1)&&((1<<(n-1))-1)));
    // 1 的邊
}
for(int i = 0; i < n-1; ++i) ans.push_back(0); //初
// 始狀態
dfs(0);
}
```

10.3 SmallestLexicographic

```
//對於可化作DAG的回溯問題求最小字典序的選擇
//建反圖 (反著做回來) (把以 i 結尾變成 以 i 開頭)
//結論 : i <- j (i < j) 取最小的 a[j]
for(int j = N; j; --j) {
    for(auto i:E[j])
        dp[i] = min(dp[i],dp[j]);
}
}
```

11 random

11.1 XORShift

```
const i64 mask = std::chrono::steady_clock::now().
    time_since_epoch().count();
//13 17 5
//13 17 7
i64 shift(i64 x) { // XOR shift (1-1 func)
    x ^= x << 13;
    x ^= x >> 7;
    x ^= x << 17;
    x ^= mask;
    return x;
}
```

12 string

12.1 KMP

```
//pi[i] = 最大的 k 使得 s[0...(k-1)] = s[i-(k-1)...i]
vector<int> prefunc(const string& s){
    int n = s.size();
    vector<int> pi(n);
    for(int i=1,j=0;i<n;++i){
        j = pi[i-1];
        while(j && s[j] != s[i]) j = pi[j-1]; //取次小LCP
        if(s[j] == s[i]) ++j;
        pi[i] = j;
    }
    return pi;
}
//找 s 在 str 中出現的所有位子
vector<int> kmp(string str, string s) {
    vector<int> nxt = prefunc(s);
    vector<int> ans;
    for (int i = 0, j = 0; i < SZ(str); i++) {
        while (j && str[i] != s[j]) j = nxt[j - 1];
        if (str[i] == s[j]) ++j;
        if (j == SZ(s)) {
            ans.push_back(i - SZ(s) + 1);
            j = nxt[j - 1];
        }
    }
    return ans;
}
```

12.2 minRotation

```
// rotate(begin(s),begin(s)+minRotation(s),end(s))
#define rep(i, s, e) for (int i = (s); i < (e); i++)
int minRotation(string s) {
    int a = 0, N = s.size();
    s += s;
    rep(b, 0, N) rep(k, 0, N) {
        if (a + k == b || s[a + k] < s[b + k]) {
            b += max(0LL, k - 1);
        }
    }
}
```

```

        break;
    }
    if (s[a + k] > s[b + k]) {
        a = b;
        break;
    }
}
return a;
}

```

12.3 PalindromeTree

```

// len[s] 是對應的回文長度
// num[s] 是有幾個回文後綴
// cnt[s] 是這個回文字串在整個字串中的出現次數
// fail[s] 是他長度次長的回文後綴, aba 的 fail 是 a
// fail[s] -> s 建邊是顆樹
const int MXN = 1000010;
struct PalT {
    int nxt[MXN][26], fail[MXN], len[MXN];
    int tot, lst, n, state[MXN], cnt[MXN], num[MXN];
    int diff[MXN], sfail[MXN], fac[MXN], dp[MXN];
    char s[MXN] = {-1};
    int newNode(int l, int f) {
        len[tot] = l, fail[tot] = f, cnt[tot] = num[tot] = 0;
        memset(nxt[tot], 0, sizeof(nxt[tot]));
        diff[tot] = (l > 0 ? l - len[f] : 0);
        sfail[tot] = (l > 0 && diff[tot] == diff[f] ? sfail[f] : f);
        return tot++;
    }
    int getfail(int x) {
        while (s[n - len[x] - 1] != s[n]) x = fail[x];
        return x;
    }
    int getmin(int v) {
        dp[v] = fac[n - len[sa[v]] - diff[v]];
        if (diff[v] == diff[fail[v]])
            dp[v] = min(dp[v], dp[fail[v]]);
        return dp[v] + 1;
    }
    int push() {
        int c = s[n] - 'a', np = getfail(lst);
        if (!(lst = nxt[np][c])) {
            lst = newNode(len[np] + 2, nxt[getfail(fail[np])][c]);
            nxt[np][c] = lst; num[lst] = num[fail[lst]] + 1;
        }
        fac[n] = n;
        for (int v = lst; len[v] > 0; v = sfail[v])
            fac[n] = min(fac[n], getmin(v));
        return ++cnt[lst], lst;
    }
    void init(const char *_s, int LEN) {
        tot = lst = n = 0;
        newNode(0, 1), newNode(-1, 1);
        for (; n < LEN && _s[n];) s[n + 1] = _s[n], ++n, state[n - 1] = push();
        for (int i = tot - 1; i > 1; i--) cnt[fail[i]] += cnt[i];
    }
} palt;

```

12.4 RollingHash

```

struct RollingHash {
#define psz 2
    vector<ll> primes = {17, 75577};
    vector<ll> MOD = {998244353, 1000000007};
    vector<array<ll, psz>> hash, base;
    void init(const string &s) {
        hash.clear(); hash.resize(s.size());
        base.clear(); base.resize(s.size());
        for (int i = 0; i < psz; i++) {
            hash[0][i] = s[0];
            base[0][i] = 1;
        }
        for (int i = 1; i < s.size(); i++) {
            for (int j = 0; j < psz; j++) {
                hash[i][j] = (hash[i - 1][j] * primes[j]
                    % MOD[j] + s[i]) % MOD[j];
                base[i][j] = base[i - 1][j] * primes[j] %
                    MOD[j];
            }
        }
    }
}

```

```

    }
    array<ll, psz> getHash(int l, int r) {
        if (l == 0) return hash[r];
        array<ll, psz> ret = hash[r];
        for (int i = 0; i < psz; i++) {
            ret[i] -= hash[l - 1][i] * base[r - l + 1][i] %
                MOD[i];
            if (ret[i] < 0) ret[i] += MOD[i];
        }
        return ret;
    }
} Hash;

```

12.5 SuffixArray

```

const int N = 300010;
struct SA {
#define REP(i, n) for (int i = 0; i < (n); i++)
#define REP1(i, a, b) for (int i = (a); i <= (b); i++)
    bool _t[N * 2];
    int _s[N * 2], _sa[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2],
        hei[N], r[N];
    int operator [] (int i) { return _sa[i]; }
    void build(int *s, int n, int m) {
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n) {
        REP(i, n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i, n) if (r[i]) {
            int ans = i > 0 ? max(hei[r[i - 1]] - 1, 0) : 0;
            while (_s[i + ans] == _s[_sa[r[i] - 1] + ans]) ans++;
            hei[r[i]] = ans;
        }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
        int *c, int n, int z) {
        bool uniq = t[n - 1] = true, neq;
        int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
            lst = -1;
#define MS0(x, n) memset((x), 0, n * sizeof(*x))
#define MAGIC(XD) MS0(sa, n); \
        memcpy(x, c, sizeof(int) * z); \
        XD; \
        memcpy(x + 1, c, sizeof(int) * (z - 1)); \
        REP(i, n) if (sa[i] && !t[sa[i] - 1]) sa[x[sa[i]
            - 1]++] = sa[i] - 1; \
        memcpy(x, c, sizeof(int) * z); \
        for (int i = n - 1; i >= 0; i--) if (sa[i] && t[sa[i]
            - 1]) sa[--x[sa[i] - 1]] = sa[i] - 1;
        MS0(c, z);
        REP(i, n) uniq &= ++c[s[i]] < 2;
        REP(i, z - 1) c[i + 1] += c[i];
        if (uniq) { REP(i, n) sa[--c[s[i]]] = i; return; }
        for (int i = n - 2; i >= 0; i--) t[i] = (s[i] == s[i
            + 1] ? t[i + 1] : s[i] < s[i + 1]);
        MAGIC(REP1(i, 1, n - 1) if (t[i] && !t[i - 1]) sa[--x[sa[i]
            - 1]] = p[q[i] = nn++] = i);
        REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
            neq = lst < 0 || memcmp(s + sa[i], s + lst, (p[q[sa[i]] + 1] - sa
                [i]) * sizeof(int));
            ns[q[lst = sa[i]]] = nmzx + neq;
        }
        sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx
            + 1);
        MAGIC(for (int i = nn - 1; i >= 0; i--) sa[--x[p[nsa[i]]]] = p[nsa[i]]);
    }
} sa;

```

```

// H[i] 第 i 跟前面的最大共同前綴
// SA[i] 第 i 小是從第幾個字元開始
int H[N], SA[N];
void suffix_array(int *ip, int len) {
    // should padding a zero in the back
    // ip is int array, len is array length
    // ip[0..n-1] != 0, and ip[len] = 0
    ip[len++] = 0;
    sa.build(ip, len, 128); // 注意字元個數
    for (int i = 0; i < len; i++) {
        H[i] = sa.hei[i + 1];
    }
}

```

```

    SA[i] = sa._sa[i + 1];
}
// resulting height, sa array \in [0,len)
}

```

12.6 trie

```

//01 bitwise trie
struct trie{
    trie *nxt[2]; // 差別
    int cnt; //紀錄有多少個數字以此節點結尾
    int sz; //有多少數字的前綴包括此節點
    trie():cnt(0),sz(0){
        memset(nxt,0,sizeof(nxt));
    }
};
//創建新的字典樹
trie *root;
void insert(int x){
    trie *now = root; // 每次從根節點開始
    for(int i=22;i>=0;i--){ // 從最高位元開始往低位元走
        now->sz++;
        //cout<<(x>>i&1)<<endl;
        if(now->nxt[x>>i&1] == NULL){ //判斷當前第 i 個
            位元是 0 還是 1
            now->nxt[x>>i&1] = new trie();
        }
        now = now->nxt[x>>i&1]; //走到下一個位元
    }
    now->cnt++;
    now->sz++;
}

```

12.7 Z-algorithm

```

//z[i] = s 跟 s[i..n-1] 的最長真共同前綴長度 // z[0] = 0
vector<int> zfunc(string &s){
    int n = s.size();
    vector<int> z(n);
    for(int i = 1, l = 0, r = 0; i < n; ++i){
        if(i <= r && z[i - l] < r - i + 1) z[i] = z[i - l];
        else {
            z[i] = max(0LL, r - i + 1);
            while(i + z[i] < n && s[z[i]] == s[i + z[i]]) ++z[i];
        }
        if(i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
    return z;
}

```

12.8 馬拉車

```

//以每個字元為中心的最長迴文長度
//abc -> @a@b@c
void z_value_pal(char* s, int len, int* z) {
    len = (len << 1) + 1;
    for (int i = len - 1; i >= 0; i--)
        s[i] = i & 1 ? s[i >> 1] : '@';
    z[0] = 1;
    for (int i = 1, l = 0, r = 0; i < len; i++) {
        z[i] = i < r ? min(z[l + l - i], r - i) : 1;
        while (i - z[i] >= 0 && i + z[i] < len && s[i - z[i]] == s[i + z[i]])
            ++z[i];
        if (i + z[i] > r)
            l = i, r = i + z[i];
    }
}

```

13 tree

13.1 DSUONTREE

```

int ans[MXN], color[MXN], son[MXN];
map<int, int> mp[MXN];
void dfs(int x, int f){
    if(son[x]){
        dfs(son[x], x);
        swap(mp[x], mp[son[x]]);
        ans[x] = ans[son[x]];
    }
}

```

```

}
mp[x][color[x]]++;
ans[x] = max(ans[x], mp[x][color[x]]);
for(int i : edge[x]){
    if(i == f || i == son[x]) continue;
    dfs(i, x);
    for(auto j : mp[i]){
        mp[x][j.first] += j.second;
        ans[x] = max(ans[x], mp[x][j.first]);
    }
}
}

```

13.2 EulerTour

```

int timing=0;
int in[N],out[N];
void dfs(int u){
    in[u] = ++timing;//這時進入u
    for(int nxt : g[u]){//跑過所有孩子
        dfs(nxt);
    }
    out[u] = timing;//這時離開u 不會++
}

```

13.3 LCA

```

int n, q;
int anc[MAXN][25], in[MAXN], out[MAXN];
vector<int> edge[MAXN];
int timing = 1;
void dfs(int cur, int fa) {
    anc[cur][0] = fa;
    in[cur] = timing++;
    for (int nex : edge[cur]) {
        if (nex == fa) continue;
        dfs(nex, cur);
    }
    out[cur] = timing++;
}
void init() {
    dfs(1, 0);
    for (int i = 1; i < 25; i++) {
        for (int cur = 1; cur <= n; cur++) {
            anc[cur][i] = anc[anc[cur][i - 1]][i - 1];
        }
    }
}
bool isanc(int u, int v) { return (in[u] <= in[v] && out[v] <= out[u]); }
int lca(int a, int b) {
    if (isanc(a, b)) return a;
    if (isanc(b, a)) return b;
    for (int i = 24; i >= 0; i--) {
        if (anc[a][i] == 0) continue;
        if (!isanc(anc[a][i], b)) a = anc[a][i];
    }
    return anc[a][0];
}

```

```

int t = 0, tt = 0;
vector<int> dfn(n), in(n), out(n), dep(n);
vector anc(n, vector<int>(20));
auto pdfs = [&](auto &&self, int x, int f, int d = 0) ->
    void {
        in[x] = ++t;
        anc[x][0] = f;
        dep[x] = d;
        dfn[x] = ++tt;
        for(auto u:E[x]){
            if(u == f) continue;
            self(self, u, x, d+1);
        }
        out[x] = ++t;
    };
pdfs(pdfs, 0, 0);
for(int k = 1; k < 20; ++k){
    for(int i = 0; i < n; ++i){

```



```

        anc[i][k] = anc[anc[i][k-1]][k-1];
    }
}
auto isanc = [&](int u,int v){
    return in[u] <= in[v] && out[v] <= out[u];
};
auto lca = [&](int x,int y){
    if(isanc(x,y)) return x;
    if(isanc(y,x)) return y;
    for(int i = 19; i >= 0; --i){
        if(!isanc(anc[x][i],y)) x = anc[x][i];
    }
    return anc[x][0];
};
};

```

13.4 treehash

```

map<vector<int>,int> id; //rooted
int dfs(int x,int f){
    vector<int> s;
    for(int u:E[x]){
        if(u == f) continue;
        s.pb(dfs(u,x));
    }
    sort(all(s));
    if(!id.count(s)) id[s] = id.size();
    return id[s];
}

const i64 mask = std::chrono::steady_clock::now().
    time_since_epoch().count();
//13 17 5
//13 17 7
i64 shift(i64 x) { // XOR shift (1-1 func)
    x ^= mask;
    x ^= x << 13;
    x ^= x >> 7;
    x ^= x << 17;
    x ^= mask;
    return x;
}

int dfs(int x,int f){
    int ret = 1; // 需要常數
    for(int u:E[x]){
        if(u == f) continue;
        ret += shift(dfs(u,x));
    }
    // ret ^= rand_mask //如果xor hash被卡
    return ret;
}

```

13.5 HeavyLightDecomposition

```

int t = 0;
vector<int> dep(n+1),p(n+1),sz(n+1),dfn(n+1),son(n+1);
auto dfs = [&](auto &&self,int x,int f,int d = 0) ->
    void {
        ++sz[x],dep[x] = d,p[x] = f;
        for(auto u:E[x]){
            if(u == f) continue;
            self(self,u,x,d+1);
            sz[x] += sz[u];
            if(!son[x] || sz[u] > sz[son[x]]) son[x] = u;
        }
    };
vector<int> top(n+1);
auto dfsa = [&](auto &&self,int x,int f,int now) ->
    void {
        dfn[x] = ++t;
        top[x] = now;
        if(son[x]) self(self,son[x],x,now);
        for(auto u:E[x]){
            if(u == f || u == son[x]) continue;
            self(self,u,x,u);
        }
    };
dfs(dfs,1,1);
dfsa(dfsa,1,1,1);
auto lca = [&](int x,int y){
    while(top[x] != top[y]){
        if(dep[top[x]] < dep[top[y]]) swap(x,y);
    }
}

```

```

        x = p[top[x]];
    }
    return dep[x] < dep[y] ? x : y ;
};
// 如果要開線段樹 要每個鏈都開一顆 (比較快)

```

13.6 VirtualTree

```

//求關鍵點的虛樹
//thm1 : 照dfn (dfs序) 排序後的 "相鄰點" 求lca可求出全
點對的lca
auto virTree = [&](vector<int> key){
    auto cmp = [&](int a,int b){return dfn[a] < dfn[b]
    };};
    sort(all(key),cmp);
    auto res = vector<int>(all(key));
    for(int i = 1; i < key.size();++i){
        res.pb(lca(key[i-1],key[i]));
    }
    sort(all(res),cmp);
    res.erase(unique(all(res)),res.end());
    return res; // res : 全點對lca集 + 關鍵點集
};
//詢問
for(int i = 1; i < ret.size(); ++i){
    int LCA = lca(ret[i-1],ret[i]);
    query(LCA,ret[i]); // 2. LCA -> ret[i] 是一條
        virTree的邊
    //query : 路徑詢問
    //且會全部算到
}

```









