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1 basic

1.1 default

```

1 #include <bits/stdc++.h>
1 using namespace std;
1 #define masterspark ios::sync_with_stdio(0), cin.tie(0)
1 ,cout.tie(0),cin.exceptions(cin.failbit);
2
2 #define int long long
2 #define pp pair<int, int>
2 #define ff first
2 #define ss second
2
2 #define forr(i,n) for(int i = 1; i <= n;++i)
2 #define rep(i,j,n) for(int i = j; i < n;++i)
3 #define PB push_back
3 #define PF push_front
3 #define EB emplace_back
3 #define all(v) (v).begin(), (v).end()
3 #define FZ(x) memset(x, 0, sizeof(x)) //fill zero
3 #define SZ(x) ((int)x.size())
3 bool chmin(auto &a, auto b) { return (b < a) and (a = b
3 , true); }
4 bool chmax(auto &a, auto b) { return (a < b) and (a = b
4 , true); }
4 using i128 = __int128_t;
4 using i64 = __int64_t;
5 using i32 = __int32_t;
5
5 void solve(){
6 }
6 signed main()
6 {
6     masterspark
6     int t = 1;
6     // freopen("stdin","r",stdin);
6     // freopen("stdout","w",stdout);
7     // cin >> t;
7     while(t--){
7         solve();
7     }
8     return 0;
8 }

```

1.2 godcode

```

9 #pragma GCC optimize("O3,unroll-loops")
9 #pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")

```

1.3 random

```

10 mt19937 mt(chrono::steady_clock::now().time_since_epoch
10 ().count());
10 //mt19937_64 mt()-> return randnum
11 int randint(int l, int r){
11     uniform_int_distribution<> dis(l, r); return dis(mt
11 );
11 }

```

1.4 run.bat

```

12 @echo off
12 g++ ac.cpp -o ac.exe
12 g++ wa.cpp -o wa.exe
12 set /a num=1
13 :loop
13     echo %num%
13     python gen.py > input
13     ac.exe < input > ac
13     wa.exe < input > wa
13     fc ac wa
14     set /a num=num+1
14 if not errorlevel 1 goto loop

```

1.5 run.sh

```

14 set -e
14 for ((i=0;;i++))
15 do
15     echo "$i"

```

```
python gen.py > in
./ac < in > ac.out
./wa < in > wa.out
diff ac.out wa.out || break
done
```

2 binarysearch

2.1 二分搜

```
int bsearch_1(int l, int r)
{
    while (l < r)
    {
        int mid = l + r >> 1;
        if (check(mid)) r = mid;
        else l = mid + 1;
    }
    return l;
}
// .....0000000000

int bsearch_2(int l, int r)
{
    while (l < r)
    {
        int mid = l + r + 1 >> 1;
        if (check(mid)) l = mid;
        else r = mid - 1;
    }
    return l;
}
// 00000000.....

int m = *ranges::partition_point(views::iota(0LL, (int)1
e9+9), [&](int a){
    return check(a) > k;
});
//[begin, last)
//1111111000000000000
//搜左邊數過來第一個 0
//都是 1 會回傳 last
```

2.2 三分搜

```
int l = 1, r = 100;
while (l < r) {
    int lmid = l + (r - l) / 3; // l + 1/3 區間大小
    int rmid = r - (r - l) / 3; // r - 1/3 區間大小
    lans = cal(lmid), rans = cal(rmid);
    // 求凹函數的極小值
    if (lans <= rans) r = rmid - 1;
    else l = lmid + 1;
}
```

3 dataStructure

3.1 DSU

```
struct STRUCT_DSU {
    vector<int> f, sz;
    void init(int n) {
        f.resize(n), sz.resize(n);
        for (int i = 0; i < n; i++) {
            f[i] = i;
            sz[i] = 1;
        }
    }
    int find(int x) {
        if (x == f[x]) return x;
        f[x] = find(f[x]);
        return find(f[x]);
    }
    void merge(int x, int y) {
        x = find(x), y = find(y);
        if (x == y) return;
        if (sz[x] < sz[y])
            swap(x, y);
        sz[x] += sz[y];
        f[y] = x;
    }
}
```

```
bool same(int a, int b) {
    return (find(a) == find(b));
}
};
```

3.2 fenwickTree

```
struct fenwick{
#define lowbit(x) (x&-x)
    int n;
    vector<int> v;
    fenwick(int _n) : n(_n+1), v(_n+2){}
    void add(int x, int u){
        ++x;
        for(; x < n; x += lowbit(x)) v[x] += u;
    }
    int qry(int x){
        ++x; int ret = 0;
        for(; x; x -= lowbit(x)) ret += v[x];
        return ret;
    }
    int qry(int l, int r) { return qry(r) - qry(l-1); }
    int kth(int k){ // lower_bound(k)
        int x = 0; --k;
        for(int i = (1<<__lg(n)); i; i >>= 1){
            if(x + i <= n and k >= v[x + i]) x += i; k -= v[x
            ];
        }
        return x;
    }
};
```

3.3 segTree

```
#define cl(x) (x << 1)
#define cr(x) (x << 1) + 1

struct segTree {
#define MXN 200500
    int n;
    vector<int> seg;
    vector<int> arr, tag;
    // int seg[MXN], arr[MXN], tag[MXN];
    void init(int a) {
        n = a;
        seg.resize(4 * (n + 5), 0);
        tag.resize(4 * (n + 5), 0);
        arr.resize(n + 5, 0);
        for (int i = 0; i < n + 5; i++)
            arr[i] = 0;
        for (int i = 0; i < 4 * (n + 5); i++)
            seg[i] = tag[i] = 0;
    }
    void push(int id, int l, int r) {
        if (tag[id] != 0) {
            seg[id] += tag[id] * (r - l + 1);
            if (l != r) {
                tag[cl(id)] += tag[id];
                tag[cr(id)] += tag[id];
            }
            tag[id] = 0;
        }
    }
    void pull(int id, int l, int r) {
        int mid = (l + r) >> 1;
        push(cl(id), l, mid);
        push(cr(id), mid + 1, r);
        int a = seg[cl(id)];
        int b = seg[cr(id)];
        seg[id] = a + b;
    }
    void build(int id, int l, int r) {
        if (l == r) {
            seg[id] = arr[l];
            return;
        }
        int mid = (l + r) >> 1;
        build(cl(id), l, mid);
        build(cr(id), mid + 1, r);
        pull(id, l, r);
    }
}
```

```

void update(int id, int l, int r, int ql, int qr,
            int v) {
    push(id, l, r);
    if (ql <= l && r <= qr) {
        tag[id] += v;
        return;
    }
    int mid = (l + r) >> 1;
    if (ql <= mid)
        update(cl(id), l, mid, ql, qr, v);
    if (qr > mid)
        update(cr(id), mid + 1, r, ql, qr, v);
    pull(id, l, r);
}
int query(int id, int l, int r, int ql, int qr) {
    push(id, l, r);
    if (ql <= l && r <= qr) {
        return seg[id];
    }
    int mid = (l + r) >> 1;
    int ans1, ans2;
    bool f1 = 0, f2 = 0;
    if (ql <= mid) {
        ans1 = query(cl(id), l, mid, ql, qr);
        f1 = 1;
    }
    if (qr > mid) {
        ans2 = query(cr(id), mid + 1, r, ql, qr);
        f2 = 1;
    }
    if (f1 && f2)
        return ans1 + ans2;
    if (f1)
        return ans1;
    return ans2;
}
void build() { build(1, 1, n); }
int query(int ql, int qr) { return query(1, 1, n,
    ql, qr); }
void update(int ql, int qr, int val) { update(1, 1,
    n, ql, qr, val); }
};

```

4 dp

4.1 digit

```

ll dp[MXN_BIT][PRE_NUM][LIMIT][F0]; // 字串位置，根據題目
    的值，是否上界，前導0
ll dfs(int i, int pre, bool lim, bool f0, const string&
    str) {
    if (v[i][pre][f0][lim]) return dp[i][pre][f0][lim];
    v[i][pre][f0][lim] = true;

    if (i == str.size())
        return dp[i][pre][f0][lim] = 1;

    ll ret = 0, h = lim ? str[i] : '9';

    for (int j = '0'; j <= h; j++) {
        if (abs(j - pre) >= 2 || f0) {
            ret += dfs(i + 1, j, j == h && lim, f0 && j == '0',
                str);
        }
    }
    return dp[i][pre][f0][lim] = ret;
}

```

4.2 p_median

```

void p_Median() {
    for (int i = 1; i <= N; ++i)
        for (int j = i; j <= N; ++j) {
            m = (i + j) / 2; d[i][j] = 0; // m 是中位
            // 數，d[i][j] 為距離的總和
            for (int k = i; k <= j; ++k) d[i][j] += abs(arr
                [k] - arr[m]);
        }
    for (int p = 1; p <= P; ++p)
        for (int n = 1; n <= N; ++n) {
            dp[p][n] = 1e9;
            for (int k = p; k <= n; ++k)
                if (dp[p - 1][k - 1] + d[k][n] < dp[p][n]) {

```

```

                    dp[p][n] = dp[p - 1][k - 1] + d[k][n];
                    r[p][n] = k; // 從第k個位置往右
                    // 到第j個位置
                }
            }
        }
    }
}

```

4.3 sosdp

```

// 求子集和 或超集和 -> !(mask & (1 << i))
for (int i = 0; i < (1 << N); ++i) F[i] = A[i]; // 預處理 狀
    態權重
for (int i = 0; i < N; ++i)
    for (int s = 0; s < (1 << N); ++s)
        if (s & (1 << i))
            F[s] += F[s ^ (1 << i)];

```

5 flow

5.1 Dinic

```

struct Dinic {
    struct Edge { int v, f, re; };
    int n, s, t, level[MXN];
    vector<Edge> E[MXN];
    void init(int _n, int _s, int _t) {
        n = _n; s = _s; t = _t;
        for (int i = 0; i < n; ++i) E[i].clear();
    }
    void add_edge(int u, int v, int f) {
        E[u].PB({v, f, SZ(E[v])});
        E[v].PB({u, 0, SZ(E[u]) - 1});
    }
    bool BFS() {
        for (int i = 0; i < n; ++i) level[i] = -1;
        queue<int> que;
        que.push(s);
        level[s] = 0;
        while (!que.empty()) {
            int u = que.front(); que.pop();
            for (auto it : E[u]) {
                if (it.f > 0 && level[it.v] == -1) {
                    level[it.v] = level[u] + 1;
                    que.push(it.v);
                }
            }
        }
        return level[t] != -1;
    }
    int DFS(int u, int nf) {
        if (u == t) return nf;
        int res = 0;
        for (auto &it : E[u]) {
            if (it.f > 0 && level[it.v] == level[u] + 1) {
                int tf = DFS(it.v, min(nf, it.f));
                res += tf; nf -= tf; it.f -= tf;
                E[it.v][it.re].f += tf;
                if (nf == 0) return res;
            }
        }
        if (!res) level[u] = -1;
        return res;
    }
    int flow(int res = 0) {
        while (BFS())
            res += DFS(s, 2147483647);
        return res;
    }
} flow;

```

5.2 isap

```

struct Maxflow {
    static const int MAXV = 20010;
    static const int INF = 1000000;
    struct Edge {
        int v, c, r;
        Edge(int _v, int _c, int _r) :
            v(_v), c(_c), r(_r) {}
    };
    int s, t;
    vector<Edge> G[MAXV * 2];
    int iter[MAXV * 2], d[MAXV * 2], gap[MAXV * 2], tot;
    void init(int x) {
        tot = x + 2;
    }
}

```

```

s = x+1, t = x+2;
for(int i = 0; i <= tot; i++) {
    G[i].clear();
    iter[i] = d[i] = gap[i] = 0;
} }
void addEdge(int u, int v, int c) {
    G[u].push_back(Edge(v, c, SZ(G[v])));
    G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
}
int dfs(int p, int flow) {
    if(p == t) return flow;
    for(int &i = iter[p]; i < SZ(G[p]); i++) {
        Edge &e = G[p][i];
        if(e.c > 0 && d[p] == d[e.v]+1) {
            int f = dfs(e.v, min(flow, e.c));
            if(f) {
                e.c -= f;
                G[e.v][e.r].c += f;
                return f;
            } }
    }
    if (--gap[d[p]] == 0) d[s] = tot;
    else {
        d[p]++;
        iter[p] = 0;
        ++gap[d[p]];
    }
    return 0;
}
int solve() {
    int res = 0;
    gap[0] = tot;
    for(res = 0; d[s] < tot; res += dfs(s, INF));
    return res;
}
void reset() {
    for(int i=0;i<=tot;i++) {
        iter[i]=d[i]=gap[i]=0;
    } } }flow;
} } }

```

5.3 KM

```

struct KM{ // max weight, for min negate the weights
    int n, mx[MXN], my[MXN], pa[MXN];
    ll g[MXN][MXN], lx[MXN], ly[MXN], sy[MXN];
    bool vx[MXN], vy[MXN];
    void init(int _n) { // 1-based, N個節點
        n = _n;
        for(int i=1; i<=n; i++) fill(g[i], g[i]+n+1, 0);
    }
    void addEdge(int x, int y, ll w) {g[x][y] = w;} //左
    //邊的集合節點x連邊右邊集合節點y權重為w
    void augment(int y) {
        for(int x, z; y; y = z) {
            x=pa[y], z=mx[x], my[y]=x, mx[x]=y;
        }
    }
    void bfs(int st) {
        for(int i=1; i<=n; ++i) sy[i]=INF, vx[i]=vy[i]=0;
        queue<int> q; q.push(st);
        for(;;) {
            while(q.size()) {
                int x=q.front(); q.pop(); vx[x]=1;
                for(int y=1; y<=n; ++y) if(!vy[y]){
                    ll t = lx[x]+ly[y]-g[x][y];
                    if(t==0){
                        pa[y]=x;
                        if(!my[y]){augment(y);return;}
                        vy[y]=1, q.push(my[y]);
                    }else if(sy[y]>t) pa[y]=x, sy[y]=t;
                }
            }
            ll cut = INF;
            for(int y=1; y<=n; ++y)
                if(!vy[y]&&cut>sy[y]) cut=sy[y];
            for(int j=1; j<=n; ++j){
                if(vx[j]) lx[j] -= cut;
                if(vy[j]) ly[j] += cut;
                else sy[j] -= cut;
            }
            for(int y=1; y<=n; ++y) if(!vy[y]&&sy[y]==0){
                if(!my[y]){augment(y);return;}
                vy[y]=1, q.push(my[y]);
            }
        }
    }
    ll solve() { // 回傳值為完美匹配下的最大總權重

```

```

        fill(mx, mx+n+1, 0); fill(my, my+n+1, 0);
        fill(ly, ly+n+1, 0); fill(lx, lx+n+1, -INF);
        for(int x=1; x<=n; ++x) for(int y=1; y<=n; ++y) //
            1-base
            lx[x] = max(lx[x], g[x][y]);
        for(int x=1; x<=n; ++x) bfs(x);
        ll ans = 0;
        for(int y=1; y<=n; ++y) ans += g[my[y]][y];
        return ans;
    } }graph;
}

```

5.4 匈牙利

```

bool dfs(int u){
    for(int i : edge[u]){
        if(!vis[i]){ // 有連通且未拜訪
            vis[i] = true; // 紀錄是否走過
            if(match[i]==-1 || dfs(match[i])){
                match[i] = u; match[u] = i; // 紀錄匹配
                return true;
            }
        }
    }
    return false;
}
int hungarian(){
    int ans = 0;
    memset(match, -1, sizeof(match));
    for(int i = 1; i <= lhs; i++){
        // 記得每次使用需清空vis陣列
        memset(vis, 0, sizeof(vis));
        if(dfs(i)) ans++;
    }
    return ans;
}
}

```

5.5 最小花費最大流 dijkstra 不能負值

```

struct MinCostMaxFlow{
    typedef int Tcost;
    static const int MAXV = 20010;
    static const int INFf = 1000000;
    static const Tcost INFc = 1e9;
    struct Edge{
        int v, cap;
        Tcost w;
        int rev;
        Edge(){}
        Edge(int t2, int t3, Tcost t4, int t5)
            : v(t2), cap(t3), w(t4), rev(t5) {}
    };
    int V, s, t;
    vector<Edge> g[MAXV];
    void init(int n, int _s, int _t){
        V = n; s = _s; t = _t;
        for(int i = 0; i <= V; i++) g[i].clear();
    }
    void addEdge(int a, int b, int cap, Tcost w){
        g[a].push_back(Edge(b, cap, w, (int)g[b].size()));
        g[b].push_back(Edge(a, 0, -w, (int)g[a].size()-1));
    }
    Tcost d[MAXV];
    int id[MAXV], mom[MAXV];
    bool inqu[MAXV];
    queue<int> q;
    pair<int,Tcost> solve(){
        int mx = 0; Tcost mnc = 0;
        while(1){
            fill(d, d+V, INFc);
            fill(inqu, inqu+V, 0);
            fill(mom, mom+V, -1);
            mom[s] = s;
            d[s] = 0;
            q.push(s); inqu[s] = 1;
            while(q.size()){
                int u = q.front(); q.pop();
                inqu[u] = 0;
                for(int i = 0; i < (int) g[u].size(); i++){
                    Edge &e = g[u][i];
                    int v = e.v;
                    if(e.cap > 0 && d[v] > d[u]+e.w){
                        d[v] = d[u]+e.w;

```

```

        mom[v] = u;
        id[v] = i;
        if(!inqu[v]) q.push(v), inqu[v] = 1;
    } }
    if(mom[t] == -1) break;
    int df = INFF;
    for(int u = t; u != s; u = mom[u])
        df = min(df, g[mom[u]][id[u]].cap);
    for(int u = t; u != s; u = mom[u]){
        Edge &e = g[mom[u]][id[u]];
        e.cap -= df;
        g[e.v][e.rev].cap += df;
    }
    mxf += df;
    mnc += df*d[t];
}
return {mxf,mnc};
} }flow;

```

5.6 最小花費最大流 SPFA

```

struct zkwflow{
    static const int maxN=10000;
    struct Edge{ int v,f,re; ll w;};
    int n,s,t,ptr[maxN]; bool vis[maxN]; ll dis[maxN];
    vector<Edge> E[maxN];
    void init(int _n,int _s,int _t){
        n=_n,s=_s,t=_t;
        for(int i=0;i<n;i++) E[i].clear();
    }
    void addEdge(int u,int v,int f,ll w){
        E[u].push_back({v,f,(int)E[v].size(),w});
        E[v].push_back({u,0,(int)E[u].size()-1,-w});
    }
    bool SPFA(){
        fill_n(dis,n,LLONG_MAX); fill_n(vis,n,false);
        queue<int> q; q.push(s); dis[s]=0;
        while (!q.empty()){
            int u=q.front(); q.pop(); vis[u]=false;
            for(auto &it:E[u]){
                if(it.f>0&&dis[it.v]>dis[u]+it.w){
                    dis[it.v]=dis[u]+it.w;
                    if(!vis[it.v]){
                        vis[it.v]=true; q.push(it.v);
                    }
                }
            }
            return dis[t]!=LLONG_MAX;
        }
    }
    int DFS(int u,int nf){
        if(u==t) return nf;
        int res=0; vis[u]=true;
        for(int &i=ptr[u];i<(int)E[u].size();i++){
            auto &it=E[u][i];
            if(it.f>0&&dis[it.v]==dis[u]+it.w&&!vis[it.v]){
                int tf=DFS(it.v,min(nf,it.f));
                res+=tf,nf-=tf,it.f-=tf;
                E[it.v][it.re].f+=tf;
                if(nf==0){ vis[u]=false; break; }
            }
        }
        return res;
    }
    pair<int,ll> flow(){
        int flow=0; ll cost=0;
        while (SPFA()){
            fill_n(ptr,n,0);
            int f=DFS(s,INT_MAX);
            flow+=f; cost+=dis[t]*f;
        }
        return{ flow,cost };
    }
    // reset: do nothing
} flow;

```

6 geometry

6.1 Point

```

using ld = long double;
template<class T>
struct pt{
    T x,y;
    pt(T _x,T _y):x(_x),y(_y){}
    pt():x(0),y(0){}

```

```

    pt operator * (T c){ return pt(x*c,y*c);}
    pt operator / (T c){ return pt(x/c,y/c);}
    pt operator + (pt a){ return pt(x+a.x,y+a.y);}
    pt operator - (pt a){ return pt(x-a.x,y-a.y);}
    T operator * (pt a){ return x*a.x + y*a.y;}
    T operator ^ (pt a){ return x*a.y - y*a.x;}

```

```

    auto operator<=>(pt o) const { return (x != o.x) ? x
        <=> o.x : y <=> o.y; } // c++20
    bool operator < (pt a) const { return x < a.x || (x
        == a.x && y < a.y);};
    bool operator==(pt a) const { return x == a.x and y
        == a.y};
    friend T ori(pt a, pt b, pt c) { return (b - a) ^ (c
        - a); }
    friend T abs2(pt a) { return abs(a * a); }
};

```

```

using numbers::pi; // c++20
const ld pi = acos(-1);
const ld eps = 1e-8L;
using Pt = pt<ld>;
int sgn(ld x) { return (x > -eps) - (x < eps); } //
    dcmp == sgn
ld abs(Pt a) { return sqrt(abs2(a)); }
ld arg(Pt x) { return atan2(x.y, x.x); }
bool argcmp(Pt a, Pt b) { // arg(a) < arg(b)
    int f = (Pt{a.y, -a.x} > Pt{} ? 1 : -1) * (a != Pt
        {});
    int g = (Pt{b.y, -b.x} > Pt{} ? 1 : -1) * (b != Pt
        {});
    return f == g ? (a ^ b) > 0 : f < g;
}
Pt unit(Pt x) { return x / abs(x); }
Pt rotate(Pt u) { // pi / 2
    return {-u.y, u.x};
}
Pt rotate(Pt u, ld a) {
    Pt v{sin(a), cos(a)};
    return {u ^ v, u * v};
}

istream &operator>>(istream &s, Pt &a) { return s >> a.
    x >> a.y; }
ostream &operator<<(ostream &s, Pt &a) { return s << "("
    << a.x << ", " << a.y << ")";}

bool collinearity(Pt a, Pt b, Pt c) { // 三點共線
    return ((b - a) ^ (c - a)) == 0;
}

```

6.2 Line

```

struct Line {
    Pt a, b;
    Pt dir() const { return b - a; }
};
int PtSide(Pt p, Line L) {
    return sgn(ori(L.a, L.b, p) / abs(L.a - L.b));
}
bool PtOnSeg(Pt p, Line L) {
    return PtSide(p, L) == 0 and sgn((p - L.a) * (p - L
        .b)) <= 0;
}
Pt proj(Pt p, Line l) {
    Pt dir = unit(l.b - l.a);
    return l.a + dir * (dir * (p - l.a));
}

```

6.3 Circle

```

struct Cir {
    Pt o;
    ld r;
};
bool disjunct(const Cir &a, const Cir &b) {
    return sgn(abs(a.o - b.o) - a.r - b.r) >= 0;
}
bool contain(const Cir &a, const Cir &b) {
    return sgn(a.r - b.r - abs(a.o - b.o)) >= 0;
}

```

6.4 圓多邊形面積

```
double CirclePoly(Cir C, const vector<Pt> &P) {
    auto arg = [&](Pt p, Pt q) { return atan2(p ^ q, p * q); };
    double r2 = C.r * C.r / 2;
    auto tri = [&](Pt p, Pt q) {
        Pt d = q - p;
        auto a = (d * p) / abs2(d), b = (abs2(p) - C.r * C.r) / abs2(d);
        auto det = a * a - b;
        if (det <= 0) return arg(p, q) * r2;
        auto s = max(0., -a - sqrt(det)), t = min(1., -a + sqrt(det));
        if (t < 0 or 1 <= s) return arg(p, q) * r2;
        Pt u = p + d * s, v = p + d * t;
        return arg(p, u) * r2 + (u ^ v) / 2 + arg(v, q) * r2;
    };
    double sum = 0.0;
    for (int i = 0; i < P.size(); i++)
        sum += tri(P[i] - C.o, P[(i + 1) % P.size()] - C.o);
    return sum;
}
```

6.5 圓三角形面積

```
double CircleTriangle(Pt a, Pt b, double r) {
    if (sgn(abs(a) - r) <= 0 and sgn(abs(b) - r) <= 0)
        return abs(a ^ b) / 2;
    if (abs(a) > abs(b)) swap(a, b);
    auto I = CircleLineInter({{a, b}}, r, {a, b});
    erase_if(I, [&](Pt x) { return !PtOnSeg(x, {a, b}); });
    if (I.size() == 1) return abs(a ^ I[0]) / 2 + SectorArea(I[0], b, r);
    if (I.size() == 2) {
        return SectorArea(a, I[0], r) + SectorArea(I[0], b, r) + abs(I[0] ^ I[1]) / 2;
    }
    return SectorArea(a, b, r);
}
```

6.6 半平面交

```
bool cover(Line L, Line P, Line Q) {
    // PtSide(LineInter(P, Q), L) <= 0 or P, Q parallel
    i128 u = (Q.a - P.a) ^ Q.dir();
    i128 v = P.dir() ^ Q.dir();
    i128 x = P.dir().x * u + (P.a - L.a).x * v;
    i128 y = P.dir().y * u + (P.a - L.a).y * v;
    return sgn(x * L.dir().y - y * L.dir().x) * sgn(v) >= 0;
}

vector<Line> HPI(vector<Line> P) {
    // line P.a -> P.b 的逆時針是半平面
    sort(all(P), [&](Line l, Line m) {
        if (argcmp(l.dir(), m.dir()) return true;
        if (argcmp(m.dir(), l.dir()) return false;
        return ori(m.a, m.b, l.a) > 0;
    });
    int n = P.size(), l = 0, r = -1;
    for (int i = 0; i < n; i++) {
        if (i and !argcmp(P[i - 1].dir(), P[i].dir())) continue;
        while (l < r and cover(P[i], P[r - 1], P[r])) r--;
        while (l < r and cover(P[i], P[l], P[l + 1])) l++;
        P[++r] = P[i];
    }
    while (l < r and cover(P[l], P[r - 1], P[r])) r--;
    while (l < r and cover(P[r], P[l], P[l + 1])) l++;
    if (r - l <= 1 or !argcmp(P[l].dir(), P[r].dir()))
        return {}; // empty
    if (cover(P[l + 1], P[l], P[r]))
        return {}; // infinity
    return vector(P.begin() + l, P.begin() + r + 1);
}
```

6.7 圓線交

```
vector<Pt> CircleLineInter(Cir c, Line l) {
    Pt H = proj(c.o, l);
    Pt dir = unit(l.b - l.a);
    double h = abs(H - c.o);
    if (sgn(h - c.r) > 0) return {};
    double d = sqrt(max((double)0., c.r * c.r - h * h));
    if (sgn(d) == 0) return {H};
    return {H - dir * d, H + dir * d};
    // Counterclockwise
}
```

6.8 圓圓交

```
vector<Pt> CircleInter(Cir a, Cir b) {
    double d2 = abs2(a.o - b.o), d = sqrt(d2);
    if (d < max(a.r, b.r) - min(a.r, b.r) || d > a.r + b.r) return {};
    Pt u = (a.o + b.o) / 2 + (a.o - b.o) * ((b.r * b.r - a.r * a.r) / (2 * d2));
    double A = sqrt((a.r + b.r + d) * (a.r - b.r + d) * (a.r + b.r - d) * (-a.r + b.r + d));
    Pt v = rotate(b.o - a.o) * A / (2 * d2);
    if (sgn(v.x) == 0 and sgn(v.y) == 0) return {u};
    return {u - v, u + v}; // counter clockwise of a
}
```

6.9 線線交

```
bool isInter(Line l, Line m) {
    if (PtOnSeg(m.a, l) or PtOnSeg(m.b, l) or
        PtOnSeg(l.a, m) or PtOnSeg(l.b, m))
        return true;
    return PtSide(m.a, l) * PtSide(m.b, l) < 0 and
        PtSide(l.a, m) * PtSide(l.b, m) < 0;
}

Pt LineInter(Line l, Line m) {
    double s = ori(m.a, m.b, l.a), t = ori(m.a, m.b, l.b);
    return (l.b * s - l.a * t) / (s - t);
}
```

6.10 ConvexHull

```
vector<Pt> Hull(vector<Pt> P) {
    sort(all(P));
    P.erase(unique(all(P)), P.end());
    P.insert(P.end(), P.rbegin() + 1, P.rend());
    vector<Pt> stk;
    for (auto p : P) {
        auto it = stk.rbegin();
        while (stk.rend() - it >= 2 and \
            ori(*next(it), *it, p) <= 0 and \
            (*next(it) < *it) == (*it < p)) {
            it++;
        }
        stk.resize(stk.rend() - it);
        stk.push_back(p);
    }
    stk.pop_back();
    return stk;
}
```

6.11 Hulltrick

```
struct Convex {
    int n;
    vector<Pt> A, V, L, U;
    Convex(const vector<Pt> &A) : A(A), n(A.size()) {
        // n >= 3
        auto it = max_element(all(A));
        L.assign(A.begin(), it + 1);
        U.assign(it, A.end()), U.push_back(A[0]);
        for (int i = 0; i < n; i++) {
            V.push_back(A[(i + 1) % n] - A[i]);
        }
    }
    int inside(Pt p, const vector<Pt> &h, auto f) {
        auto it = lower_bound(all(h), p, f);
        if (it == h.end()) return 0;
    }
}
```



```

    if (it == h.begin()) return p == *it;
    return 1 - sgn(ori(*prev(it), p, *it));
}
// 0: out, 1: on, 2: in
int inside(Pt p) {
    return min(inside(p, L, less{}), inside(p, U,
        greater{}));
}
static bool cmp(Pt a, Pt b) { return sgn(a ^ b) >
    0; }
// A[i] is a far/closer tangent point
int tangent(Pt v, bool close = true) {
    assert(v != Pt{});
    auto l = V.begin(), r = V.begin() + L.size() -
        1;
    if (v < Pt{}) l = r, r = V.end();
    if (close) return (lower_bound(l, r, v, cmp) -
        V.begin()) % n;
    return (upper_bound(l, r, v, cmp) - V.begin())
        % n;
}
// closer tangent point array[0] -> array[1] 順時針
array<int, 2> tangent2(Pt p) {
    array<int, 2> t{-1, -1};
    if (inside(p) == 2) return t;
    if (auto it = lower_bound(all(L), p); it != L.
        end() and p == *it) {
        int s = it - L.begin();
        return {(s + 1) % n, (s - 1 + n) % n};
    }
    if (auto it = lower_bound(all(U), p, greater{});
        it != U.end() and p == *it) {
        int s = it - U.begin() + L.size() - 1;
        return {(s + 1) % n, (s - 1 + n) % n};
    }
    for (int i = 0; i != t[0]; i = tangent((A[t[0]
        = i] - p), 0));
    for (int i = 0; i != t[1]; i = tangent((p - A[t
        [1] = i]), 1));
    return t;
}
int find(int l, int r, Line L) {
    if (r < l) r += n;
    int s = PtSide(A[l % n], L);
    return *ranges::partition_point(views::iota(l,
        r),
        [&](int m) {
            return PtSide(A[m % n], L) == s;
        }) - 1;
};
// Line A_x A_x+1 intersect with L
vector<int> intersect(Line L) {
    int l = tangent(L.a - L.b), r = tangent(L.b - L
        .a);
    if (PtSide(A[l], L) * PtSide(A[r], L) >= 0)
        return {};
    return {find(l, r, L) % n, find(r, l, L) % n};
}
};

```

6.12 點線距

```

double PtSegDist(Pt p, Line l) {
    double ans = min(abs(p - l.a), abs(p - l.b));
    if (sgn(abs(l.a - l.b)) == 0) return ans;
    if (sgn((l.a - l.b) * (p - l.b)) < 0) return ans;
    if (sgn((l.b - l.a) * (p - l.a)) < 0) return ans;
    return min(ans, abs(ori(p, l.a, l.b)) / abs(l.a - l
        .b));
}
double SegDist(Line l, Line m) {
    return PtSegDist({0, 0}, {l.a - m.a, l.b - m.b});
}

```

6.13 MEC

```

Pt Center(Pt a, Pt b, Pt c) {
    Pt x = (a + b) / 2;
    Pt y = (b + c) / 2;
    return LineInter({x, x + rotate(b - a)}, {y, y +
        rotate(c - b)});
}

```

```

Cir MEC(vector<Pt> P) {
    mt19937 rng(time(0));
    shuffle(all(P), rng);
    Cir C;
    for (int i = 0; i < P.size(); i++) {
        if (C.inside(P[i])) continue;
        C = {P[i], 0};
        for (int j = 0; j < i; j++) {
            if (C.inside(P[j])) continue;
            C = {(P[i] + P[j]) / 2, abs(P[i] - P[j]) /
                2};
            for (int k = 0; k < j; k++) {
                if (C.inside(P[k])) continue;
                C.o = Center(P[i], P[j], P[k]);
                C.r = abs(C.o - P[i]);
            }
        }
    }
    return C;
}

```

6.14 MEC2

```

PT arr[MXN];
int n = 10;
double checky(double x, double y) {
    double cmax = 0;
    for (int i = 0; i < n; i++) { // 過程中回傳距離A2
        // 避免不必要的根號運算
        cmax = max(cmax, (arr[i].x - x) * (arr[i].x - x
            ) + (arr[i].y - y) * (arr[i].y - y));
    }
    return cmax;
}
double checkx(double x) {
    double yl = -1e9, yr = 1e9;
    while (yr - yl > EPS) {
        double ml = (yl + yl + yr) / 3, mr = (yl + yr +
            yr) / 3;
        if (checky(x, ml) < checky(x, mr))
            yr = mr;
        else
            yl = ml;
    }
}
signed main() {
    double xl = -1e9, xr = 1e9;
    while (xr - xl > EPS) {
        double ml = (xl + xl + xr) / 3, mr = (xl + xr +
            xr) / 3;
        if (checkx(ml) < checkx(mr))
            xr = mr;
        else
            xl = ml;
    }
}

```

6.15 旋轉卡尺

```

auto RotatingCalipers(const vector<Pt> &hull) { // 最遠
    // 點對 回傳距離平方
    int n = hull.size();
    auto ret = abs2(hull[0]);
    ret = 0;
    if (hull.size() <= 2) return abs2(hull[0] - hull
        [1]);
    for (int i = 0, j = 2; i < n; i++) {
        Pt a = hull[i], b = hull[(i + 1) % n];
        while(ori(hull[j], a, b) <
            (ori(hull[(j + 1) % n], a, b)))
            j = (j + 1) % n;
        chmax(ret, abs2(a - hull[j]));
        chmax(ret, abs2(b - hull[j]));
    }
    return ret;
}

```

6.16 Minkowski

```

// P, Q, R(return) are counterclockwise order convex
// polygon
vector<Pt> Minkowski(vector<Pt> P, vector<Pt> Q) {

```

```

auto cmp = [&](Pt a, Pt b) {
    return Pt{a.y, a.x} < Pt{b.y, b.x};
};
auto reorder = [&](auto &R) {
    rotate(R.begin(), min_element(all(R), cmp), R.
        end());
    R.push_back(R[0]), R.push_back(R[1]);
};
const int n = P.size(), m = Q.size();
reorder(P), reorder(Q);
vector<Pt> R;
for (int i = 0, j = 0, s; i < n or j < m; ) {
    R.push_back(P[i] + Q[j]);
    s = sgn((P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]));
    if (s >= 0) i++;
    if (s <= 0) j++;
}
return R;
}

```

6.17 PointInPolygon

```

int inPoly(Pt p, const vector<Pt> &P) {
    const int n = P.size();
    int cnt = 0;
    for (int i = 0; i < n; i++) {
        Pt a = P[i], b = P[(i + 1) % n];
        if (PtOnSeg(p, {a, b})) return 1; // on edge
        if ((sgn(a.y - p.y) == 1) ^ (sgn(b.y - p.y) ==
            1))
            cnt += sgn(ori(a, b, p));
    }
    return cnt == 0 ? 0 : 2; // out, in
}

```

6.18 UnionOfCircles

```

// Area[i] : area covered by at least i circle
// TODO:!!!aaa!!!
vector<double> CircleUnion(const vector<Cir> &C) {
    const int n = C.size();
    vector<double> Area(n + 1);
    auto check = [&](int i, int j) {
        if (!contain(C[i], C[j]))
            return false;
        return sgn((C[i].r - C[j].r) > 0 or (sgn((C[i].r
            - C[j].r) == 0 and i < j));
    };
    struct Teve {
        double ang; int add; Pt p;
        bool operator<(const Teve &b) { return ang < b.
            ang; }
    };
    auto ang = [&](Pt p) { return atan2(p.y, p.x); };
    for (int i = 0; i < n; i++) {
        int cov = 1;
        vector<Teve> event;
        for (int j = 0; j < n; j++) if (i != j) {
            if (check(j, i)) cov++;
            else if (!check(i, j) and !disjunct(C[i], C
                [j])) {
                auto I = CircleInter(C[i], C[j]);
                assert(I.size() == 2);
                double a1 = ang(I[0] - C[i].o), a2 =
                    ang(I[1] - C[i].o);
                event.push_back({a1, 1, I[0]});
                event.push_back({a2, -1, I[1]});
                if (a1 > a2) cov++;
            }
        }
        if (event.empty()) {
            Area[cov] += pi * C[i].r * C[i].r;
            continue;
        }
        sort(all(event));
        event.push_back(event[0]);
        for (int j = 0; j + 1 < event.size(); j++) {
            cov += event[j].add;
            Area[cov] += (event[j].p ^ event[j + 1].p)
                / 2.;
            double theta = event[j + 1].ang - event[j].
                ang;

```

```

            if (theta < 0) theta += 2 * pi;
            Area[cov] += (theta - sin(theta)) * C[i].r
                * C[i].r / 2.;
        }
    }
    return Area;
}

```

6.19 UnionOfPolygons

```

// Area[i] : area covered by at least i polygon
vector<double> PolyUnion(const vector<vector<Pt>> &P) {
    const int n = P.size();
    vector<double> Area(n + 1);
    vector<Line> Ls;
    for (int i = 0; i < n; i++)
        for (int j = 0; j < P[i].size(); j++)
            Ls.push_back({P[i][j], P[i][(j + 1) % P[i].
                size()]});
    auto cmp = [&](Line &l, Line &r) {
        Pt u = l.b - l.a, v = r.b - r.a;
        if (argcmp(u, v)) return true;
        if (argcmp(v, u)) return false;
        return PtSide(l.a, r) < 0;
    };
    sort(all(Ls), cmp);
    for (int l = 0, r = 0; l < Ls.size(); l = r) {
        while (r < Ls.size() and !cmp(Ls[l], Ls[r])) r
            ++;
        Line L = Ls[l];
        vector<pair<Pt, int>> event;
        for (auto [c, d] : Ls) {
            if (sgn((L.a - L.b) ^ (c - d)) != 0) {
                int s1 = PtSide(c, L) == 1;
                int s2 = PtSide(d, L) == 1;
                if (s1 ^ s2) event.emplace_back(
                    LineInter(L, {c, d}), s1 ? 1 : -1);
            } else if (PtSide(c, L) == 0 and sgn((L.a -
                L.b) * (c - d)) > 0) {
                event.emplace_back(c, 2);
                event.emplace_back(d, -2);
            }
        }
        sort(all(event), [&](auto i, auto j) {
            return (L.a - i.ff) * (L.a - L.b) < (L.a -
                j.ff) * (L.a - L.b);
        });
        int cov = 0, tag = 0;
        Pt lst{0, 0};
        for (auto [p, s] : event) {
            if (cov >= tag) {
                Area[cov] += lst ^ p;
                Area[cov - tag] -= lst ^ p;
            }
            if (abs(s) == 1) cov += s;
            else tag += s / 2;
            lst = p;
        }
        for (int i = n - 1; i >= 0; i--) Area[i] += Area[i
            + 1];
        for (int i = 1; i <= n; i++) Area[i] /= 2;
        return Area;
    }
}

```

6.20 圓公切線

```

vector<Line> CircleTangent(Cir c1, Cir c2, int sign1) {
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    ld d_sq = abs2(c1.o - c2.o);
    if (sgn(d_sq) == 0) return ret;
    ld d = sqrt(d_sq);
    Pt v = (c2.o - c1.o) / d;
    ld c = (c1.r - sign1 * c2.r) / d;
    if (c * c > 1) return ret;
    ld h = sqrt(max(0.0, 1.0 - c * c));
    for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
        Pt n = Pt(v.x * c - sign2 * h * v.y, v.y * c +
            sign2 * h * v.x);
        Pt p1 = c1.o + n * c1.r;
        Pt p2 = c2.o + n * (c2.r * sign1);
    }
}

```



```

        if (sgn(p1.x - p2.x) == 0 && sgn(p1.y - p2.y) == 0)
            p2 = p1 + rotate(c2.o - c1.o);
        ret.push_back({p1, p2});
    }
    return ret;
}

```

6.21 點圓切線

```

vector<Line> CircleTangent(Cir c, Pt p) {
    vector<Line> z;
    double d = abs(p - c.o);
    if (sgn(d - c.r) == 0) {
        Pt i = rotate(p - c.o);
        z.push_back({p, p + i});
    } else if (d > c.r) {
        double o = acos(c.r / d);
        Pt i = unit(p - c.o);
        Pt j = rotate(i, o) * c.r;
        Pt k = rotate(i, -o) * c.r;
        z.push_back({c.o + j, p});
        z.push_back({c.o + k, p});
    }
    return z;
}

```

7 graph

7.1 BCC

```

#define REP(i, n) for (int i = 0; i < n; i++)
struct BccVertex {
    int n, nScc, step, dfn[MXN], low[MXN];
    vector<int> E[MXN], sccv[MXN];
    int top, stk[MXN];
    void init(int _n) {
        n = _n;
        nScc = step = 0;
        for (int i = 0; i < n; i++) E[i].clear();
    }
    void addEdge(int u, int v) {
        E[u].PB(v);
        E[v].PB(u);
    }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
        stk[top++] = u;
        for (auto v : E[u]) {
            if (v == f) continue;
            if (dfn[v] == -1) {
                DFS(v, u);
                low[u] = min(low[u], low[v]);
                if (low[v] >= dfn[u]) {
                    int z;
                    sccv[nScc].clear();
                    do {
                        z = stk[--top];
                        sccv[nScc].PB(z);
                    } while (z != v);
                    sccv[nScc++].PB(u);
                }
            } else
                low[u] = min(low[u], dfn[v]);
        }
    }
    vector<vector<int>> solve() {
        vector<vector<int>> res;
        for (int i = 0; i < n; i++) dfn[i] = low[i] = -1;
        for (int i = 0; i < n; i++)
            if (dfn[i] == -1) {
                top = 0;
                DFS(i, i);
            }
        REP(i, nScc) res.PB(sccv[i]);
        return res;
    }
} graph;

```

7.2 SCC

```

struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<= n; i++)
            E[i].clear(), rE[i].clear();
    }
    void addEdge(int u, int v){
        E[u].PB(v); rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v : E[u]) if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1; bln[u] = nScc;
        for (auto v : rE[u]) if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        fill(vst, vst+n+1, 0);
        for (int i=0; i<=n; i++)
            if (!vst[i]) DFS(i);
        reverse(vec.begin(), vec.end());
        fill(vst, vst+n+1, 0);
        for (auto v : vec)
            if (!vst[v]){
                rDFS(v); nScc++;
            }
    }
};

```

7.3 支配樹

```

#define REP(i, s, e) for (int i = (s); i <= (e); i++)
#define REPD(i, s, e) for (int i = (s); i >= (e); i--)
struct DominatorTree { // O(N) 1-base
    int n, s;
    vector<int> g[MXN], pred[MXN];
    vector<int> cov[MXN];
    int dfn[MXN], nfd[MXN], ts;
    int par[MXN]; // idom[u] s到u的最後一個必經點
    int sdom[MXN], idom[MXN];
    int mom[MXN], mn[MXN];
    inline bool cmp(int u, int v) { return dfn[u] < dfn[v]; }
    int eval(int u) {
        if (mom[u] == u) return u;
        int res = eval(mom[u]);
        if (cmp(sdom[mn[mom[u]]], sdom[mn[u]])) mn[u] = mn[mom[u]];
        return mom[u] = res;
    }
    void init(int _n, int _s) {
        ts = 0;
        n = _n;
        s = _s;
        REP(i, 1, n) g[i].clear(), pred[i].clear();
    }
    void addEdge(int u, int v) {
        g[u].push_back(v);
        pred[v].push_back(u);
    }
    void dfs(int u) {
        ts++;
        dfn[u] = ts;
        nfd[ts] = u;
        for (int v : g[u])
            if (dfn[v] == 0) {
                par[v] = u;
                dfs(v);
            }
    }
    void build() {
        REP(i, 1, n) {
            idom[i] = par[i] = dfn[i] = nfd[i] = 0;
            cov[i].clear();
            mom[i] = mn[i] = sdom[i] = i;
        }
    }
};

```

```

dfs(s);
REPD(i, n, 2) {
    int u = nfd[i];
    if (u == 0) continue;
    for (int v : pred[u])
        if (dfn[v]) {
            eval(v);
            if (cmp(sdom[mn[v]], sdom[u])) sdom
                [u] = sdom[mn[v]];
        }
    cov[sdom[u]].push_back(u);
    mom[u] = par[u];
    for (int w : cov[par[u]]) {
        eval(w);
        if (cmp(sdom[mn[w]], par[u]))
            idom[w] = mn[w];
        else
            idom[w] = par[u];
    }
    cov[par[u]].clear();
}
REP(i, 2, n) {
    int u = nfd[i];
    if (u == 0) continue;
    if (idom[u] != sdom[u]) idom[u] = idom[idom
        [u]];
}
}
} domT;

```

7.4 最大團

```

struct MaxClique { // 0-base
    typedef bitset<MXN> Int;
    Int linkto[MXN], v[MXN];
    int n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; i++) {
            linkto[i].reset();
            v[i].reset();
        }
    }
    void addEdge(int a, int b) { v[a][b] = v[b][a] = 1; }
    int popcount(const Int& val) { return val.count(); }
    int lowbit(const Int& val) { return val._Find_first
        (); }
    int ans, stk[MXN];
    int id[MXN], di[MXN], deg[MXN];
    Int cans;
    void maxclique(int elem_num, Int candi) {
        if (elem_num > ans) {
            ans = elem_num;
            cans.reset();
            for (int i = 0; i < elem_num; i++) cans[id[
                stk[i]]] = 1;
        }
        int potential = elem_num + popcount(candi);
        if (potential <= ans) return;
        int pivot = lowbit(candi);
        Int smaller_candi = candi & (~linkto[pivot]);
        while (smaller_candi.count() && potential > ans
            ) {
            int next = lowbit(smaller_candi);
            candi[next] = !candi[next];
            smaller_candi[next] = !smaller_candi[next];
            potential--;
            if (next == pivot || (smaller_candi &
                linkto[next]).count()) {
                stk[elem_num] = next;
                maxclique(elem_num + 1, candi & linkto[
                    next]);
            }
        }
    }
    int solve() {
        for (int i = 0; i < n; i++) {
            id[i] = i;
            deg[i] = v[i].count();
        }
    }
}

```

```

sort(id, id + n, [&](int id1, int id2) { return
    deg[id1] > deg[id2]; });
for (int i = 0; i < n; i++) di[id[i]] = i;
for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
        if (v[i][j]) linkto[di[i]][di[j]] = 1;
Int cand;
cand.reset();
for (int i = 0; i < n; i++) cand[i] = 1;
ans = 1;
cans.reset();
cans[0] = 1;
maxclique(0, cand);
return ans;
}
} solver;

```

7.5 最小圈

```

/* minimum mean cycle O(VE) */
struct MMC{
#define E 101010
#define V 1021
#define inf 1e9
#define eps 1e-6
    struct Edge { int v,u; double c; };
    int n, m, prv[V][V], prve[V][V], vst[V];
    Edge e[E];
    vector<int> edgeID, cycle, rho;
    double d[V][V];
    void init( int _n )
    { n = _n; m = 0; }
    // WARNING: TYPE matters
    void addEdge( int vi , int ui , double ci )
    { e[ m ++ ] = { vi , ui , ci }; }
    void bellman_ford() {
        for(int i=0; i<n; i++) d[0][i]=0;
        for(int i=0; i<n; i++) {
            fill(d[i+1], d[i+1]+n, inf);
            for(int j=0; j<m; j++) {
                int v = e[j].v, u = e[j].u;
                if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                    d[i+1][u] = d[i][v]+e[j].c;
                    prv[i+1][u] = v;
                    prve[i+1][u] = j;
                }
            }
        }
    }
    double solve(){
        // returns inf if no cycle, mmc otherwise
        double mmc=inf;
        int st = -1;
        bellman_ford();
        for(int i=0; i<n; i++) {
            double avg=-inf;
            for(int k=0; k<n; k++) {
                if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i
                    ])/(n-k));
                else avg=max(avg,inf);
            }
            if (avg < mmc) tie(mmc, st) = tie(avg, i);
        }
        fill(vst,0); edgeID.clear(); cycle.clear(); rho.
            clear();
        for (int i=n; !vst[st]; st=prv[i--][st]) {
            vst[st]++;
            edgeID.PB(prve[i][st]);
            rho.PB(st);
        }
        while (vst[st] != 2) {
            if(rho.empty()) return inf;
            int v = rho.back(); rho.pop_back();
            cycle.PB(v);
            vst[v]++;
        }
        reverse(ALL(edgeID));
        edgeID.resize(SZ(cycle));
        return mmc;
    }
}mmc;

```

8 math

8.1 DiscreteSqrt

```
void calcH(i64 &t, i64 &h, const i64 p) {
    i64 tmp=p-1; for(t=0;(tmp&1)==0;tmp/=2) t++; h=tmp;
}
// solve equation x^2 mod p = a
// !!!! (a != 0) !!!!!
bool solve(i64 a, i64 p, i64 &x, i64 &y) {
    if(p == 2) { x = y = 1; return true; }
    int p2 = p / 2, tmp = mypow(a, p2, p);
    if (tmp == p - 1) return false;
    if ((p + 1) % 4 == 0) {
        x=mypow(a,(p+1)/4,p); y=p-x; return true;
    } else {
        i64 t, h, b, pb; calcH(t, h, p);
        if (t >= 2) {
            do {b = rand() % (p - 2) + 2;
                while (mypow(b, p / 2, p) != p - 1);
            } while (1);
            pb = mypow(b, h, p);
            int s = mypow(a, h / 2, p);
            for (int step = 2; step <= t; step++) {
                int ss = (((i64)(s * s) % p) * a) % p;
                for(int i=0;i<t-step;i++) ss=mul(ss,ss,p);
                if (ss + 1 == p) s = (s * pb) % p;
                pb = ((i64)pb * pb) % p;
            } x = ((i64)s * a) % p; y = p - x;
        } return true;
    }
}
```

8.2 excrt

```
typedef __int128 ll;
void exgcd(ll a,ll b,ll &g,ll &x,ll &y) {
    if (b == 0) {
        g = a;
        x = 1;
        y = 0;
        return;
    }
    exgcd(b,a%b,g,y,x);
    y-=(a/b)*x;
}
bool flag = false;
ll a1,a2,n1,n2;
ll abs(ll x) {
    return x>0?x:-x;
}
void china() {
    ll d = a2 - a1;
    ll g,x,y;
    exgcd(n1,n2,g,x,y);
    if (d % g == 0) {
        x = ((x*d/g)%(n2/g)+(n2/g))%(n2/g);
        a1 = x*n1 + a1;
        n1 = (n1*n2)/g;
    }
    else
        flag = true;
}
int n;
long long as[100001]; //算式答案 x
long long ns[100001]; //模数 MOD
ll realchina() {
    a1 = as[0];
    n1 = ns[0];
    for (ll i = 1;i<n;i++) {
        a2 = as[i];
        n2 = ns[i];
        china();
        if (flag)
            return -1;
    }
    return a1;
}
int main() {
    cin>>n;
    flag = false;
    for (ll i = 0;i<n;i++)
        cin>>ns[i]>>as[i];
    cout<<(long long)realchina()<<endl;
}
```

8.3 exgcd

```
int exgcd(int a,int b,int&x,int&y){
    if(b==0)return x=1,y=0,a;
    int d = exgcd(b,a%b,y,x);
    y-=a/b*x;
    return d;
}
```

8.4 FFT

```
const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx; //real() ,imag()
const ld PI = acos(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft(){
    for(int i=0; i<=MAXN; i++)
        omega[i] = exp(i * 2 * PI / MAXN * I);
}
// n must be 2^k
void fft(int n, cplx a[], bool inv=false){
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >>= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN-(i*theta%MAXN) : i*theta%MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
            theta = (theta * 2) % MAXN;
        }
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^= k); k >>= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if(inv) for (i = 0; i < n; i++) a[i] /= n;
}
cplx arr[MAXN+1];
inline void mul(int _n,i64 a[],int _m,i64 b[],i64 ans
    []){
    int n=1,sum=_n+_m-1;
    while(n<sum)
        n<<=1;
    for(int i=0;i<n;i++) {
        double x=(i<_n?a[i]:0),y=(i<_m?b[i]:0);
        arr[i]=complex<double>(x+y,x-y);
    }
    fft(n,arr);
    for(int i=0;i<n;i++)
        arr[i]=arr[i]*arr[i];
    fft(n,arr,true);
    for(int i=0;i<sum;i++)
        ans[i]=(i64)(arr[i].real()/4+0.5);
}
```

8.5 josephus

```
int josephus(int n, int m){ //n人 每m次
    int ans = 0;
    for (int i=1; i<=n; ++i)
        ans = (ans + m) % i;
    return ans;
}
```

8.6 Theorem

- Lucas's Theorem :
For $n, m \in \mathbb{Z}^+$ and prime P , $C(m, n) \bmod P = \prod C(m_i, n_i)$ where m_i is the i -th digit of m in base P .
- Stirling approximation :
$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n e^{\frac{1}{12n}}$$
- Stirling Numbers(permutation $|P| = n$ with k cycles):
 $S(n, k) = \text{coefficient of } x^k \text{ in } \Pi_{i=0}^{n-1} (x+i)$

- Stirling Numbers(Partition n elements into k non-empty set):

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

- Pick's Theorem : $A = i + b/2 - 1$
A: Area, i : grid number in the inner, b : grid number on the side

- Catalan number : $C_n = \binom{2n}{n} / (n+1)$
 $C_n^{n+m} - C_{n+1}^{n+m} = (m+n)! \frac{n-m+1}{n+1}$ for $n \geq m$
 $C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$
 $C_0 = 1$ and $C_{n+1} = 2 \binom{2n+1}{n+2} C_n$
 $C_0 = 1$ and $C_{n+1} = \sum_{i=0}^n C_i C_{n-i}$ for $n \geq 0$

- Euler Characteristic:
planar graph: $V - E + F - C = 1$
convex polyhedron: $V - E + F = 2$
 V, E, F, C : number of vertices, edges, faces(regions), and components

- Kirchhoff's theorem :
 $A_{ii} = \deg(i), A_{ij} = (i, j) \in E ? -1 : 0$, Deleting any one row, one column, and cal the det(A)

- Polya's theorem (c is number of color, m is the number of cycle size):
 $(\sum_{i=1}^m c^{\gcd(i, m)}) / m$

- Burnside lemma:
 $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$

- 錯排公式: (n 個人中, 每個人皆不再原來位置的組合數):
 $dp[0] = 1; dp[1] = 0;$
 $dp[i] = (i-1) * (dp[i-1] + dp[i-2]);$

- Bell 數 (有 n 個人, 把他們拆組的方法總數) :
 $B_0 = 1$
 $B_n = \sum_{k=0}^n s(n, k)$ (second - stirling)
 $B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k$

- Wilson's theorem :
 $(p-1)! \equiv -1 \pmod{p}$

- Fermat's little theorem :
 $a^p \equiv a \pmod{p}$

- Euler's totient function:
 $A^{B^C} \pmod{p} = \text{pow}(A, \text{pow}(B, C, p-1)) \pmod{p}$

- 歐拉函數降冪公式:
 $A^B \pmod{C} = A^{B \pmod{\phi(C)} + \phi(C)} \pmod{C}$

- 6 的倍數:
 $(a-1)^3 + (a+1)^3 + (-a)^3 + (-a)^3 = 6a$

8.7 Primes

Prime	Root	Prime	Root
7681	17	167772161	3
12289	11	104857601	3
40961	3	985661441	3
65537	3	998244353	3
786433	10	1107296257	10
5767169	3	2013265921	31
7340033	3	2810183681	11
23068673	3	2885681153	3
469762049	3	605028353	3

8.8 millerrabin

```
// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pirmses <= 13
// n < 2^64               7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
bool witness(i64 a, i64 n, i64 u, int t){
    if(!a) return 0;
    i64 x=mypow(a,u,n);
    for(int i=0;i<t;i++){
        i64 nx=mul(x,x,n);
        if(nx==1&&x!=1&&x!=n-1) return 1;
        x=nx;
    }
    return x!=1;
}
bool mii64er_rabin(i64 n) {
    int s = 7;
    // iterate s times of witness on n
    if(n<2) return 0;
```

```
if(!(n&1)) return n == 2;
i64 u=n-1; int t=0;
// n-1 = u*2^t
while(!(u&1)) u>=1, t++;
while(s--){
    i64 a=magic[s]%n;
    if(witness(a,n,u,t)) return 0;
}
return 1;
}
```

8.9 phi

```
ll phi(ll n){ // 計算小於n的數中與n互質的有幾個
    ll res = n, a=n; // 0(sqrtN)
    for(ll i=2;i*i<=a;i++){
        if(a%i==0){
            res = res/i*(i-1);
            while(a%i==0) a/=i;
        }
    }
    if(a>1) res = res/a*(a-1);
    return res;
}
```

8.10 pollardrho

```
// does not work when n is prime 0(n^(1/4))
i64 f(i64 x, i64 c, i64 mod){ return add(mul(x,x,mod),c,mod); }
i64 poi64ard_rho(i64 n) {
    i64 c = 1, x = 0, y = 0, p = 2, q, t = 0;
    while (t++ % 128 or gcd(p, n) == 1) {
        if (x == y) c++, y = f(x = 2, c, n);
        if (q = mul(p, abs(x-y), n)) p = q;
        x = f(x, c, n); y = f(f(y, c, n), c, n);
    }
    return gcd(p, n);
}
```

8.11 primes

```
/* 12721, 13331, 14341, 75577, 123457, 222557, 556679
* 999983, 1097774749, 1076767633, 100102021, 999997771
* 1001010013, 1000512343, 987654361, 999991231
* 999888733, 98789101, 987777733, 999991921, 1010101333
* 1010102101, 1000000000039, 100000000000037
* 2305843009213693951, 4611686018427387847
* 9223372036854775783, 18446744073709551557 */
int mu[ N ], p_tbl[ N ];
vector<int> primes;
void sieve() {
    mu[ 1 ] = p_tbl[ 1 ] = 1;
    for( int i = 2 ; i < N ; i ++ ){
        if( !p_tbl[ i ] ){
            p_tbl[ i ] = i;
            primes.push_back( i );
            mu[ i ] = -1;
        }
        for( int p : primes ){
            int x = i * p;
            if( x >= M ) break;
            p_tbl[ x ] = p;
            mu[ x ] = -mu[ i ];
            if( i % p == 0 ){
                mu[ x ] = 0;
                break;
            }
        }
    }
}
vector<int> factor( int x ){
    vector<int> fac{ 1 };
    while( x > 1 ){
        int fn = SZ(fac), p = p_tbl[ x ], pos = 0;
        while( x % p == 0 ){
            x /= p;
            for( int i = 0 ; i < fn ; i ++ )
                fac.PB( fac[ pos ++ ] * p );
        }
    }
    return fac;
}
```

8.12 Euler

```
int Euler(int n){
```

```
int now = n;
for (int i = 2; i * i <= n; i++)
    if (n % i == 0){
        now = now - now / i;
        while (n % i == 0) n = n / i;
    }
if (n > 1) now = now - now / n;
return now;
}
```

8.13 quickeuler

```
vector<int> pri;
bool not_prime[MXN + 10];
int phi[MXN + 10];
void quick_euler(int n) {
    phi[1] = 1;
    for (int i = 2; i <= n; i++) {
        if (!not_prime[i]) {
            pri.push_back(i);
            phi[i] = i - 1;
        }
        for (int pri_j : pri) {
            if (i * pri_j > n)
                break;
            not_prime[i * pri_j] = true;
            if (i % pri_j == 0) {
                phi[i * pri_j] = phi[i] * pri_j;
                break;
            }
            phi[i * pri_j] = phi[i] * phi[pri_j];
        }
    }
}
```

8.14 sieve

```
const int MXN = 1e8 + 50;
const int SQRTMXN = 1e4 + 50;
bitset<MXN> isprime;
void sieve() {
    isprime[1] = 1;
    for (int i = 2; i <= SQRTMXN; i++) {
        if (!isprime[i])
            for (i64 j = i * i; j < MXN; j += i)
                isprime[j] = 1;
    }
}
```

9 string

9.1 KMP

```
vector<int> prefunc(const string& s){
    int n = s.size();
    vector<int> pi(n);
    for(int i=1,j=0;i<n;++i){
        j = pi[i-1];
        while(j && s[j] != s[i]) j = pi[j-1]; //取次小LCP
        if(s[j] == s[i]) ++j;
        pi[i] = j;
    }
    return pi;
}
vector<int> kmp(string str, string s, vector<int>& nxt)
{
    vector<int> ans;
    for (int i = 0, j = 0; i < SZ(str); i++) {
        while (j && str[i] != s[j]) j = nxt[j - 1];
        if (str[i] == s[j]) j++;
        if (j == SZ(s)) {
            ans.push_back(i - SZ(s) + 1);
            j = nxt[j - 1];
        }
    }
    return ans;
}
```

9.2 minRotation

```
// rotate(begin(s),begin(s)+minRotation(s),end(s))
#define rep(i, s, e) for (int i = (s); i < (e); i++)
```

```
int minRotation(string s) {
    int a = 0, N = s.size();
    s += s;
    rep(b, 0, N) rep(k, 0, N) {
        if (a + k == b || s[a + k] < s[b + k]) {
            b += max(0LL, k - 1);
            break;
        }
        if (s[a + k] > s[b + k]) {
            a = b;
            break;
        }
    }
    return a;
}
```

9.3 PalindromeTree

```
// len[s] 是對應的回文長度
// num[s] 是有幾個回文後綴
// cnt[s] 是這個回文字串在整個字串中的出現次數
// fail[s] 是他長度次長的回文後綴, aba的fail是a
// fail[s] -> s 建邊是顆樹
const int MXN = 1000010;
struct PalT{
    int nxt[MXN][26], fail[MXN], len[MXN];
    int tot, lst, n, state[MXN], cnt[MXN], num[MXN];
    int diff[MXN], sfail[MXN], fac[MXN], dp[MXN];
    char s[MXN]={'-1'};
    int newNode(int l, int f){
        len[tot]=l, fail[tot]=f, cnt[tot]=num[tot]=0;
        memset(nxt[tot], 0, sizeof(nxt[tot]));
        diff[tot]=(l>0?l-len[f]:0);
        sfail[tot]=(l>0&&diff[tot]==diff[f]?sfail[f]:f);
        return tot++;
    }
    int getfail(int x){
        while(s[n-len[x]-1]!=s[n]) x=fail[x];
        return x;
    }
    int getmin(int v){
        dp[v]=fac[n-len[sfail[v]]-diff[v]];
        if(diff[v]==diff[fail[v]])
            dp[v]=min(dp[v], dp[fail[v]]);
        return dp[v]+1;
    }
    int push(){
        int c=s[n]-'a', np=getfail(lst);
        if(!lst || !nxt[np][c]){
            lst=newNode(len[np]+2, nxt[getfail(fail[np])][c]);
            nxt[np][c]=lst; num[lst]=num[fail[lst]]+1;
        }
        fac[n]=n;
        for(int v=lst; len[v]>0; v=sfail[v])
            fac[n]=min(fac[n], getmin(v));
        return ++cnt[lst], lst;
    }
    void init(const char *_s){
        tot=lst=n=0;
        newNode(0, 1), newNode(-1, 1);
        for(; _s[n];) s[n+1]=_s[n], ++n, state[n-1]=push();
        for(int i=tot-1; i>1; i--) cnt[fail[i]]+=cnt[i];
    }
}palt;
```

9.4 RollingHash

```
struct RollingHash{
#define psz 2
    vector<ll> primes={17, 75577};
    vector<ll> MOD={998244353, 1000000007};
    vector<array<ll, psz>> hash, base;
    void init(const string &s){
        hash.clear(); hash.resize(s.size());
        base.clear(); base.resize(s.size());
        for(int i=0; i<psz; i++){
            hash[0][i] = s[0];
            base[0][i] = 1;
        }
        for(int i=1; i<s.size(); i++){
            for(int j=0; j<psz; j++){
```

```

        hash[i][j] = (hash[i-1][j] * primes[j]
            % MOD[j] + s[i]) % MOD[j];
        base[i][j] = base[i-1][j] * primes[j] %
            MOD[j];
    }
}
array<ll, psz> getHash(int l, int r){
    if(l == 0) return hash[r];
    array<ll, psz> ret = hash[r];
    for(int i=0; i<psz; i++){
        ret[i] -= hash[l-1][i] * base[r-l+1][i] %
            MOD[i];
        if(ret[i]<0) ret[i]+=MOD[i];
    }
    return ret;
}
}Hash;

```

9.5 SuffixArray

```

const int N = 300010;
struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
    bool _t[N*2];
    int _s[N*2], _sa[N*2], _c[N*2], x[N], _p[N], _q[N*2],
        hei[N], r[N];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
            hei[r[i]] = ans;
        }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
        int *c, int n, int z){
        bool uniq = t[n-1] = true, neq;
        int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
            lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
        memcpy(x, c, sizeof(int) * z); \
        XD; \
        memcpy(x + 1, c, sizeof(int) * (z - 1)); \
        REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i]
            -1]]++] = sa[i]-1; \
        memcpy(x, c, sizeof(int) * z); \
        for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]
            -1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
        MS0(c, z);
        REP(i,n) uniq &= ++c[s[i]] < 2;
        REP(i,z-1) c[i+1] += c[i];
        if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
        for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i
            +1] ? t[i+1] : s[i]<s[i+1]);
        MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[s[i
            ]]] = p[q[i]=nn++] = i);
        REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
            neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa
                [i])*sizeof(int));
            ns[q[lst=sa[i]]]=nmzx+=neq;
        }
        sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx
            + 1);
        MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[p[
            nsa[i]]]] = p[nsa[i]]);
    }
}sa;
// H[i] 第 i 跟前面的最大共同前綴
// SA[i] 第 i 小是從第幾個字元開始
int H[ N ], SA[ N ];
void suffix_array(int* ip, int len) {
    // should padding a zero in the back

```

```

// ip is int array, len is array length
// ip[0..n-1] != 0, and ip[len] = 0
ip[len++] = 0;
sa.build(ip, len, 128); // 注意字元個數
for (int i=0; i<len; i++) {
    H[i] = sa.hei[i + 1];
    SA[i] = sa._sa[i + 1];
}
// resulting height, sa array \in [0,len)
}

```

9.6 trie

```

//01 bitwise trie
struct trie{
    trie *nxt[2]; // 差別
    int cnt; //紀錄有多少個數字以此節點結尾
    int sz; //有多少數字的前綴包括此節點
    trie():cnt(0),sz(0){
        memset(nxt,0,sizeof(nxt));
    }
};
//創建新的字典樹
trie *root;
void insert(int x){
    trie *now = root; // 每次從根節點開始
    for(int i=22;i>=0;i--){ // 從最高位元開始往低位元走
        now->sz++;
        //cout<<(x>>i&1)<<endl;
        if(now->nxt[x>>i&1] == NULL){ //判斷當前第 i 個
            位元是 0 還是 1
            now->nxt[x>>i&1] = new trie();
        }
        now = now->nxt[x>>i&1]; //走到下一個位元
    }
    now->cnt++;
    now->sz++;
}

```

9.7 Z-algorithm

```

vector<int> zfunc(string &s){ //求 s 跟 s[i..n-1] 的最
    長真共同前綴長度 z[0] = 0
    int n = s.size();
    vector<int> z(n);
    for(int i = 1, l = 0, r = 0; i < n; ++i){
        if(i <= r && z[i - l] < r - i + 1) z[i] = z[i - l];
        else {
            z[i] = max(0LL, r - i + 1);
            while(i + z[i] < n && s[z[i]] == s[i + z[i]]) ++z
                [i];
        }
        if(i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
    return z;
}

```

9.8 馬拉車

```

void z_value_pal(char* s, int len, int* z) {
    len = (len < 1) + 1;
    for (int i = len - 1; i >= 0; i--)
        s[i] = i & 1 ? s[i >> 1] : '@';
    z[0] = 1;
    for (int i = 1, l = 0, r = 0; i < len; i++) {
        z[i] = i < r ? min(z[l + l - i], r - i) : 1;
        while (i - z[i] >= 0 && i + z[i] < len && s[i -
            z[i]] == s[i + z[i]])
            ++z[i];
        if (i + z[i] > r)
            l = i, r = i + z[i];
    }
}

```

10 tree

10.1 DSUONTREE

```

int ans[MXN], color[MXN], son[MXN];
map<int, int> mp[MXN];
void dfs(int x, int f){
    if(son[x]){

```



```

        dfs(son[x], x);
        swap(mp[x], mp[son[x]]);
        ans[x] = ans[son[x]];
    }
    mp[x][color[x]]++;
    ans[x] = max(ans[x], mp[x][color[x]]);
    for(int i : edge[x]){
        if(i == f || i == son[x]) continue;
        dfs(i, x);
        for(auto j : mp[i]){
            mp[x][j.first] += j.second;
            ans[x] = max(ans[x], mp[x][j.first]);
        }
    }
}

```

10.2 EulerTour

```

int timing=0;
int in[N],out[N];
void dfs(int u){
    in[u] = ++timing;//這時進入u
    for(int nxt : g[u]){//跑過所有孩子
        dfs(nxt);
    }
    out[u] = timing;//這時離開u 不會++
}

```

10.3 LCA

```

int n, q;
int anc[MAXN][25], in[MAXN], out[MAXN];
vector<int> edge[MAXN];
int timing = 1;
void dfs(int cur, int fa) {
    anc[cur][0] = fa;
    in[cur] = timing++;
    for (int nex : edge[cur]) {
        if (nex == fa) continue;
        dfs(nex, cur);
    }
    out[cur] = timing++;
}
void init() {
    dfs(1, 0);
    for (int i = 1; i < 25; i++) {
        for (int cur = 1; cur <= n; cur++) {
            anc[cur][i] = anc[anc[cur][i - 1]][i - 1];
        }
    }
}
bool isanc(int u, int v) { return (in[u] <= in[v] &&
    out[v] <= out[u]); }
int lca(int a, int b) {
    if (isanc(a, b)) return a;
    if (isanc(b, a)) return b;
    for (int i = 24; i >= 0; i--) {
        if (anc[a][i] == 0) continue;
        if (!isanc(anc[a][i], b)) a = anc[a][i];
    }
    return anc[a][0];
}

```

10.4 treehash

```

i64 dfs(int u){
    vector<i64> h;
    subtree_sz[u] = 1;
    for(i64 child : edge[u]){
        h.push_back(dfs(child));
        subtree_sz[u] += subtree_sz[child];
    }
    sort(h.begin(), h.end());
    i64 ret = subtree_sz[u];
    for(i64 v : h){
        ret = (ret * base + v) % MOD;
    }
    return ret;
}

```






