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Kinematics:

$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$	$v_f^2 = v_i^2 + 2ax$
$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$	$v(t) = v_0 + at$
$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$	$v = \frac{dx}{dt}$

Newton's Laws:

I: $\sum \vec{F} = 0 \Leftrightarrow \frac{d\vec{v}}{dt} = 0$ $[m \neq 0, m = \text{const.}]$
II: $\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt}$ ($F = ma$ [if mass is const.])
II: (v.m): $\vec{F} + \vec{v}_{rel}\frac{dm}{dt} = m\frac{d\vec{v}}{dt}$, ejected/accreted mass is part of the system

Oscillations and SHM:

$A = \text{amplitude}$

$T = \text{period} = \frac{1}{f}$

For small oscillations: $\sin(\theta) \cong \theta$ $\cos(\theta) \cong 1 - \frac{\theta^2}{2}$

SHM is undergone by a particle under $F(x) = -kx$

$f = \frac{\omega}{2\pi}$	$\omega = \sqrt{\frac{k}{m}}$
$v(t) = -A\omega\sin(\omega t + \varphi)$	$a = -\omega^2 A\sin(\omega t) = -\omega^2 y$
$E_{total} = KE + PE = \frac{1}{2}kA^2$ [no damp.]	$\vec{a} + \omega^2 x = 0$
$x(t) = A\cos(\omega t + \varphi)$	$x(t) = \frac{v}{\omega}\sin(\omega t)$.
$F_{damp} = -cv \to \lambda = \frac{c}{2m}$	$a + 2\lambda x + w_0^2 x = 0$
$m\lambda^2 + c\lambda + k = \to$ $\to \lambda = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$	$y = A\sin(\omega t) = A\sin\left(\sqrt{\frac{k}{m}}t\right)$ [simple case]

Over damped:

if $c^2 - 4mk > 0 \to$ overdamped	
$\lambda^2 > \omega_0^2$	$\omega_d = \sqrt{\lambda^2 - \omega^2}$

Under damped:

if $c^2 - 4mk < 0 \to$ underdamped	$\lambda^2 < \omega_0^2$
$X(t) = e^{-\lambda t}(A\cos(vdt) + B\sin(vdt))$	$\omega_d = \sqrt{\omega_0^2 - \lambda^2}$

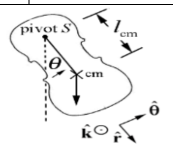
Critical Damped:

$c^2 - 4mk = 0 \to$ crit. damp	$\lambda^2 = \omega_0^2$
$B^2 = 4km$	$B^2 = 4km$

Physical Pendulum:

$\tau_{pivot} = \vec{r}_{s,cm} \times m\vec{g} = l_{cm}\hat{r} \times mg(\cos(\theta)\hat{r} - \sin(\theta)\hat{\theta}) = -l_{cm}mg\sin(\theta)\hat{k}$

$T = 2\pi\sqrt{\frac{I}{mgh}}$ [small amp.]	$\omega = \sqrt{\frac{mgd}{I}}$ [d=distance from pivot to cm]
$-mgl_{cm}\sin(\theta) = I_s\frac{d^2\theta}{dt^2}$	$\frac{d^2\theta}{dt^2} \cong -\frac{mgl_{cm}}{I_s}\theta$ [*for small angles* $\sin(\theta) \cong \theta$]
$\omega_0 = ang.f. = \sqrt{\frac{mgl_{cm}}{I_s}}$	$T = \frac{2\pi}{\omega_0} = \sqrt{\frac{l_{cm}}{g} + \frac{I_{cm}}{mgl_{cm}}}$



Circular Motion:

$v_{cm} = \omega r$	$F_{centripetal} = m\omega^2 R$
$v = \omega r$	$\omega = 2\pi f$
$\vec{L} = I\vec{\omega} = \vec{r} \times \vec{p}$	$L_A = \vec{r} \times m\vec{v}_{cm} + I_{cm}\omega_0$

$\tau_{net\ ext.} = \frac{dL}{dt}$ To find direction of L, curl fingers and look at thumb

E= $\tau\theta$ [$\theta = angle\ moved$]	$\tau = I\alpha$	$\tau = FR\sin(\theta)$

$W = \int_{\theta_1}^{\theta_2} \tau\, d\theta$	$E_{rot.K} = \frac{1}{2}I\omega^2$

<u>Varignon's theorem:</u> the sum of torques due to several forces applied to *a single* point is equal to the torque due to the sum (resultant) of the forces.

Center of mass, Inertia:

$X_{cm} = \frac{1}{M_{tot}}\sum_{i=1}^{n} m_i x_i$ [For a system of particles]

$X_{cm} = \frac{\int_0^{end} m*position}{M_{total}} = \frac{\int_0^{end} x\lambda(x)dx}{M_{total}} = \frac{\int_0^{end} xdm}{M_{total}}$

$I = \int_0^M x^2 dm = \int_0^M x^2\lambda(x)dx$; $v_{cm} = \frac{\sum mv}{M}$

<u>Parallel Axis:</u> $I_A = I_{cm} + mr^2$ [$r = dist\ A{\sim}cm$]

<u>Perpendicular Axis:</u> $I_{A_z} = I_{A_y} + I_{A_x}$ [need xyz symmetry]

If linear momentum is conserved, Center of Mass does not move relative to the system. The components of the system will rearrange to preserve CM

Work, Forces, Energy and Momentum:

<u>Isolated system:</u> collection of matter which does not interact with the rest of the universe at all.

<u>Conservative force:</u> Work done is independent of the path taken.

$F_{tension} = mg + ma = \frac{2m_1m_2}{m_1 + m_2}g$ [*for 2 masses and pulley*]

For massless string, tension is *the same* everywhere

$W = \int F(x)dx = Fd\cos(\theta)$

Impulse: $\vec{J} = \int_{\Delta t} \vec{F}\, dt = \Delta\vec{p} = m\Delta\vec{v}$

$\sum F_{which\ point\ to\ rot.center} = F_{centripetal} = \frac{mv^2}{R}$

$p = mv$	$F_{friction} = \mu N$	$W_{total} = \Delta E_k$
$E_p = mgh$	$E_k = \frac{1}{2}mv^2 = \frac{1}{2}kx^2$	

<u>P conserved</u> if $F_{ext} = 0$ <u>L conserved</u> if $\tau_{ext} = 0$ [$\circlearrowleft A$]

<u>E conserved</u> if Work=0 & there's no non-conservative forces

Elastic (bounce)	Inelastic(stick together)
E_k , p are conserved. Solving usually involves simultaneous equations with two variables.	p is conserved , but can't track the kinetic energy through the collision.

<u>Proving F is conservative:</u> Find Curl, if curl=0, conservative

$curl\ F = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ F_x & F_y & F_z \end{vmatrix} = i\left(\frac{d}{dy}F_z - \frac{d}{dz}F_y\right) - j\left(\frac{d}{dx}F_z - \frac{d}{dz}F_x\right) + k\left(\frac{d}{dx}F_x - \frac{d}{dy}F_y\right)$

If curl=0, $W = -\Delta E_p$ where ΔE_p is the change in the potential energy associated with the force. Negative because work done against a force field increases potential energy. <u>Gravity</u>, <u>magnetic</u>, <u>electrostatic</u>, spring forces are conservative. <u>Friction</u> is not.

The maximum angle before one of the items will begin sliding is called the <u>*angle of friction*</u>, defined as $\tan(\theta) = \mu_s$ [θ *from* $h - tl$]

Variable mass:

Identify forces -> if no $F_{external}$ take the changing mass as part of the system. Calculate $p(t)$ and $p(t + \Delta t)$

$$F_{ext} = \frac{p(t + \Delta t) - p(t)}{\Delta t}$$

[*^will be in terms of dm and dt*]

$\overrightarrow{F_{ext}} + \vec{v}_{rel}\frac{dm}{dt} = m\frac{d\vec{v}}{dt}$, ejected/accreted mass is part of the system

Could theoretically not have to use momentum, instead use classical f=ma, making it in the form $F + f_{thrust} = m(t)a$, $f_{thrust} = \vec{v}_{rel}\frac{dm}{dt}$

Rolling without slipping:

$ring \circlearrowleft cm = mr^2$	$disc \circlearrowleft cm = \frac{1}{2}mr^2$
$hollow\ cylinder \circlearrowleft cm = \frac{1}{2}m(r_{inner}^2 + r_{outer}^2)$	$cylinder \circlearrowleft cm = \frac{1}{2}mr^2$
$sphere \circlearrowleft cm = \frac{2}{5}mr^2$	$hollow\ sphere \circlearrowleft cm = \frac{2}{3}mr^2$
$rod \circlearrowleft cm = \frac{1}{12}mr^2$	$rod \circlearrowleft end = \frac{1}{3}mr^2$
$thin\ hoop \circlearrowleft cm = mr^2$	$thin\ disk \circlearrowleft cm = \frac{mr^2}{2}$
$cone \circlearrowleft cm = \frac{3}{10}mr^2$	$cone_x = cone_y = \frac{3}{20}m(r^2 + 4h^2)$
$rectangle \circlearrowleft cm = \frac{a^2+b^2}{12}$	$rectangle = \frac{a^2+b^2}{12}$
$disk \circlearrowleft cm = \frac{R^2}{2}$	$disk \circlearrowleft = \frac{R^2}{4}$

We can view rolling motion as a superposition of pure rotation and pure translation.

When an object is rolling on a plane without slipping, the point of contact of the object with the plane does not move.

$$\frac{dx}{dt} = v_{cm} = \omega R = \frac{d\theta}{dt}R$$
$$x = R\theta$$

Once an object starts rolling on a horizontal surface, there is no need for friction unless there is acceleration. You don't need friction for pure rolling without slipping to occur.

Mathematics:

Cross product: $\vec{u} \times \vec{v} = \begin{pmatrix} u_2v_3 - u_3v_2 \\ u_3v_1 - u_1v_3 \\ u_1v_2 - u_2v_1 \end{pmatrix}$

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \cos\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\csc(\theta)}$$
$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} = \sin\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\sec(\theta)}$$
$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin(\theta)}{\cos(\theta)} = \cot\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\cot(\theta)}$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2} \qquad (uv)' = u'v + v'u$$

$\vec{r} = dist.\ frm\ pt.\ which\ we\ mesur\ \tau\ frm\ to\ impct\ pt.$

Tactic:

Most will be solved by Newton Laws, Conservation of P or L, or conservation of E. Newton's Laws hold everywhere unless the frame of reference moves.

ODEs:

Integrating factor:

$$\text{if } \frac{dy}{dx} + p(x)y = q(x), \text{then :}$$
$$e^{\int p(x)dx} = \int e^{\int p(x)dx} q(x)dx$$
$$y'_x = f(y) \to \int \frac{dy}{f(y)} + c$$
$$y'_x = f(x)g(y) \to \int \frac{dy}{g(y)} = \int f(x)dx + c$$

In terms of	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
$\sin(\theta)$	$\sin(\theta)$	$\pm\sqrt{1-\cos^2(\theta)}$	$\pm\frac{\tan(\theta)}{\sqrt{1+\tan^2(\theta)}}$
$\cos(\theta)$	$= \pm\sqrt{1-\sin^2(\theta)}$	$\cos(\theta)$	$\pm\frac{1}{\sqrt{1+\tan^2(\theta)}}$
$\tan(\theta)$	$\pm\frac{\sin(\theta)}{\sqrt{1-\sin^2(\theta)}}$	$\pm\frac{\sqrt{1-\cos^2(\theta)}}{\cos(\theta)}$	$\tan(\theta)$

$\sin(-\theta) = -\sin(\theta)$	$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$	$\sin(\pi - \theta) = \sin(\theta)$
$\cos(-\theta) = \cos(\theta)$	$\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$	$\cos(\pi - \theta) = -\cos(\theta)$
$\tan(-\theta) = -\tan(\theta)$	$\tan\left(\frac{\pi}{2} - \theta\right) = \cot(\theta)$	$\tan(\pi - \theta) = -\tan(\theta)$
$\sin\left(\theta + \frac{\pi}{2}\right) = \cos(\theta)$	$\sin(\theta + \pi) = -\sin(\theta)$	$\sin(\theta + 2\pi) = \sin(\theta)$
$\cos\left(\theta + \frac{\pi}{2}\right) = -\sin(\theta)$	$\cos(\theta + \pi) = -\cos(\theta)$	$\cos(\theta + 2\pi) = \cos(\theta)$

$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$	$\sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$
$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$	$\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}$
$\sin(2\theta) = 2\sin(\theta)\cos(\theta) = \frac{2\tan(\theta)}{1+\tan^2(\theta)}$	$\sin^2(\theta) + \cos^2(\theta) = 1$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$	$\sin(\theta) = \pm\sqrt{1-\cos^2(\theta)}$ Sign depends on quadrant of θ
$\sin(3\theta) = -4\sin^3(\theta) + 3\sin(\theta)$	$\cos(\theta) = \pm\sqrt{1-\sin^2(\theta)}$
$\cos(3\theta) = 4\cos^3(\theta) - 3\cos(\theta)$	
$\sin^2\left(\frac{\theta}{2}\right) = \frac{(1-\cos(\theta))}{2}$	
$\cos^2\left(\frac{\theta}{2}\right) = \frac{(1+\cos(\theta))}{2}$	
$2\cos(\theta)\cos(\phi) = \cos(\theta - \phi) + \cos(\theta + \phi)$	
$2\sin(\theta)\sin(\phi) = \cos(\theta - \phi) - \cos(\theta + \phi)$	
$2\sin(\theta)\cos(\phi) = \sin(\theta + \phi) + \sin(\theta - \phi)$	
$2\cos(\theta)\sin(\phi) = \sin(\theta + \phi) - \sin(\theta - \phi)$	

$\frac{d}{dx}e^x = e^x$	$\frac{d}{dx}a^x = a^x\ln(a)$
$\frac{d}{dx}x^x = x^x(1 + \ln(x))$	$\frac{d}{dx}\ln(x) = \frac{1}{x}$
$\frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$	$\frac{d}{dx}\sin(x) = \cos(x)$
$\frac{d}{dx}\cos(x) = -\sin(x)$	$\frac{d}{dx}\tan(x) = \sec^2(x) = \frac{1}{\cos^2(x)}$
$\frac{d}{dx}\cot(x) = -\csc^2(x)$	$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$
$\int \frac{1}{x}dx = \ln\lvert x\rvert$	$\int \ln(x) = \ln(x) - x$
$\frac{d}{dx}\sec(x) = sec(x)\tan(x)$	

Basic Forms

$\int x^n dx = \frac{1}{n+1}x^{n+1}$, $n \neq -1$	$\int u\,dv = uv - \int v\,du$
$\int \frac{1}{x}dx = \ln\lvert x\rvert$	$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln\lvert ax+b\rvert$

Integrals of Rational Functions

$\int \frac{1}{(x+a)^2}dx = -\frac{1}{x+a}$	$\int \frac{1}{1+x^2}dx = \tan^{-1}x$
$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1$	$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$
$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)}$	$\int \frac{x}{a^2+x^2}dx = \frac{1}{2}\ln\lvert a^2+x^2\rvert$
$\int \frac{x^3}{a^2+x^2}dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln\lvert a^2+x^2\rvert$	$\int \frac{x^2}{a^2+x^2}dx = x - a\tan^{-1}\frac{x}{a}$
$\int \frac{1}{ax^2+bx+c}dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}}$	$\int \frac{1}{(x+a)(x+b)}dx = \frac{1}{b-a}\ln\frac{a+x}{b+x}, a \neq b$
$\int \frac{x}{ax^2+bx+c}dx = \frac{1}{2a}\ln\lvert ax^2+bx+c\rvert - \frac{b}{a\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}}$	$\int \frac{x}{(x+a)^2}dx = \frac{a}{a+x} + \ln\lvert a+x\rvert$

Integrals with Roots

$\int \sqrt{x-a}\ dx = \frac{2}{3}(x-a)^{3/2}$	$\int \sqrt{\frac{x}{a+x}}\ dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x} + \sqrt{x+a}\right]$
$\int \frac{1}{\sqrt{x\pm a}}\ dx = 2\sqrt{x\pm a}$	$\int x\sqrt{ax+b}\ dx = \frac{2}{15a^2}(-2b^2 + abx + 3a^2x^2)\sqrt{ax+b}$
$\int \frac{1}{\sqrt{a-x}}\ dx = -2\sqrt{a-x}$	$\int \sqrt{x(ax+b)}\ dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)} - b^2\ln\left\lvert a\sqrt{x} + \sqrt{a(ax+b)}\right\rvert\right]$
$\int \sqrt{ax+b}\ dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b}$	$\int \sqrt{x^3(ax+b)}\ dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right]\sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}}\ln\left\lvert a\sqrt{x} + \sqrt{a(ax+b)}\right\rvert$
$\int (ax+b)^{3/2}\ dx = \frac{2}{5a}(ax+b)^{5/2}$	$\int \sqrt{x^2 \pm a^2}\ dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2\ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert$
$\int \frac{x}{\sqrt{x\pm a}}\ dx = \frac{2}{3}(x \mp 2a)\sqrt{x\pm a}$	$\int \sqrt{a^2 - x^2}\ dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$
$\int \sqrt{\frac{x}{a-x}}\ dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a}$	$\int x\sqrt{x^2 \pm a^2}\ dx = \frac{1}{3}(x^2 \pm a^2)^{3/2}$
$\int \frac{1}{\sqrt{x^2\pm a^2}}\ dx = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert$	$\int \frac{1}{\sqrt{a^2-x^2}}\ dx = \sin^{-1}\frac{x}{a}$
$\int \frac{x}{\sqrt{x^2\pm a^2}}\ dx = \sqrt{x^2 \pm a^2}$	$\int \frac{x}{\sqrt{a^2-x^2}}\ dx = -\sqrt{a^2 - x^2}$
$\int \frac{x^2}{\sqrt{x^2\pm a^2}}\ dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}a^2\ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert$	$\int \sqrt{ax^2+bx+c}\ dx = \frac{b+2ax}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}}\ln\left\lvert 2ax + b + 2\sqrt{a(ax^2+bx^+c)}\right\rvert$
$\int \frac{1}{\sqrt{ax^2+bx+c}}\ dx = \frac{1}{\sqrt{a}}\ln\left\lvert 2ax + b + 2\sqrt{a(ax^2+bx+c)}\right\rvert$	$\int \frac{x}{\sqrt{ax^2+bx+c}}\ dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}}\ln\left\lvert 2ax + b + 2\sqrt{a(ax^2+bx+c)}\right\rvert$
$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$	

Integrals with Exponentials

$\int e^{ax}\ dx = \frac{1}{a}e^{ax}$	$\int xe^x\ dx = (x-1)e^x$