DOT/SCALAR PRODUCT

Dot Product: Let $\vec{a} = (a_1, a_2, a_3), \vec{b} = (b_1, b_2, b_3)$ be two vectors in $\mathbb{R}^3$. The scalar product of $\vec{a}$ and $\vec{b}$ is $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\vec{a}||\vec{b}|\cos(\theta)$ .. **Angle between two vectors:** $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos(\theta)$, θ is the angle between $\vec{a}, \vec{b}$. Alternatively: $\cos(\theta) = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$. **Orthogonal Vectors:** Two vectors $\vec{a}, \vec{b} \in \mathbb{R}^3$ are orthogonal if $\vec{a} \cdot \vec{b} = 0$ i.e. $\theta = \frac{\pi}{2}$. **Unit Vector:** Direction which is given by the unit vector $\frac{\vec{a}}{|\vec{a}|}$

VECTOR/CROSS PRODUCT

Cross product: $\vec{a} \times \vec{b} = (a_2b_3 - a_3b_2)\hat{\imath} + (a_3b_1 - a_1b_3)\hat{\jmath} + (a_1b_2 - a_2b_1)\hat{k} = (|\vec{a}||\vec{b}|\cos(\theta))\hat{n}\ (n \perp \vec{a}, \vec{b})$

Properties: 1 $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$ for all $\vec{a}, \vec{b}, \in \mathbb{R}^3$, in particular, $\vec{a} \times \vec{a} = 0$ **Theorem**: $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$ and $\vec{b}$. Suppose we have chosen a unit vector $\vec{n}$ perpendicular to the plane by the right hand rule: this means we choose $\vec{n}$ to be the unit (normal) vector that points the way your right thumb points when your fingers curl through the angle θ from $\vec{a}$ to $\vec{b}$. Then: $\vec{a} \times \vec{b} = (|\vec{a}||\vec{b}|\sin(\theta))\vec{n}$ **Parallel**: $\vec{a}, \vec{b} \neq 0$ are parallel iff $\vec{a} \times \vec{b} = 0$

GAUSS LAW ELECTRIC:

Integral form	Differential form
$\Phi = \iint_S \overrightarrow{E_{\text{net}}} \circ \hat{n} dA = \frac{Q_{enc}}{\varepsilon_0}$	$\vec{\nabla} \circ \overrightarrow{E_{\text{net}}} = \frac{\rho}{\varepsilon_0} = 4\pi k\rho \quad \rho = charge\ density$

Method of Gaussian Surfaces: For highly symmetric distributions of charges you can find electric field itself by constructing a Gaussian surface on which: (1) Electric field is parallel or perpendicular to the surface normal, allowing you to convert dot product into algebraic multiplication $|\mathbf{A}|\mathbf{B}|\cos(\theta)$. (2) Electric field is 0 or constant over sections of the surface, for example no θ *or* φ dependence in case of sphere (allows you to remove the electric field from the integral)

GAUSS LAW MAGNETIC:

Integral form: $\oint \vec{B} \circ \hat{n}da = 0$

Force on Q due to B: $\vec{F_b} = Q\vec{v} \times \vec{B}$
$\lvert\vec{B}\rvert = \frac{\lvert\vec{F_b}\rvert}{q\lvert\vec{v}\rvert\sin(\theta)}$,
θ = angle between $\vec{v}$ and $\vec{B}$

Differential form: $\vec{\nabla} \circ \vec{B} = 0$ – The divergence of the magnetic field at any point is 0

Relation of $\vec{B}$ to Magnetic Force: $\vec{B}$ is directly proportional to magnetic force but perpendicular in direction.

Biot-Savart Law: The contribution $d\vec{B}$ to the magnetic field at a specified point P from a small element of electric current is given by the Biot-Savart law: $\vec{B} = \frac{\mu_0}{4\pi} * \frac{Id\vec{l} \times \hat{r}}{r^2} = \frac{Q\vec{V}\times\hat{r}}{r^2}$, where $d\vec{l}$ = vector with length of the infinitesimal current element (length of conductor) in the direction of the current, $\hat{r}$ = unit vector from the current element to P (at which field is being calculated), r = distance between current element and P.

FARADAY LAW:

Induced EMF: $\mathcal{E} = \oint_{\partial S}(\vec{E} + \vec{v} \times \vec{B}) \cdot dl - \frac{d}{dt}\int_S \vec{B} \circ \hat{n}da = -\frac{d}{dt}\Phi_B$, $\mathcal{E}$ is the induced EMF, and Φ_B is the magnetic flux

Integral Form (For Loop Area S): $\oint_{\partial S}(\vec{E} + \vec{v} \times \vec{B}) \cdot dl = -\frac{d}{dt}\iint_S \vec{B} \cdot d\vec{a}$. <u>If loop isn't moving</u>: $\oint E \cdot d\vec{l} = -\frac{d}{dt}\iint_S -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$

Integral Form : $\oint_{\partial\Sigma} \vec{E} \cdot d\vec{l} = -\int_\Sigma \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$, where Σ is a surface bounded by the closed contour $\partial\Sigma$. RHS is an expression of magnetic flux. Correct when $\vec{E}$ represents the electric field in the rest frame of each segment $d\vec{l}$ of the path of integration. F**or LHS:** Induced electric field lines produced by changing magnetic fields must form complete loops. *The net electric field at any point is the vector sum of all electric fields present at that point. Electric field lines can never cross.* $\oint_C \vec{E} \circ d\vec{l} = \oint_C \frac{\vec{F}}{Q} \circ d\vec{l} = \frac{\oint_C \vec{F} \cdot d\vec{l}}{Q} = \frac{W}{Q}$ **For RHS:** Change in magnitude of $\vec{B}$, or the angle between the surface and $\vec{B}$, as well as change in surface's area all cause the RHS of Faraday's law to become non-zero.

Maxwell-Faraday Equation Differential form: $\iint_S(\nabla \times \vec{E}) \cdot d\vec{a} = \iint_S -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$, by Stokes Theorem: $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Addendum: $\mathcal{E} = -\varphi_B$

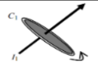
Lenz's Law: The direction of Induced current is such that it opposes the change in Φ_B responsible for its creation.

AMPERE MAXWELL LAW:

Displacement Current Density: $\vec{J}_D = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

Without Maxwell's Correction		With Maxwell's Correction	
Integral Form	Differential form	Integral Form	Differential Form
$\oint_C \vec{B} \cdot d\vec{l} = \iint_S \vec{J} \cdot d\vec{S} = \mu_0 I_{\text{enc}}$	$\nabla \times \vec{B} = \mu_0\vec{J}$	$\oint_C \vec{B} \cdot d\vec{l} = \iint_S \left(\mu_0\vec{J} + \mu_0\epsilon_0 \frac{\partial \vec{E}}{\partial t}\right) \cdot d\vec{S}$	$\nabla \times \vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0 \frac{\partial \vec{E}}{\partial t}$

$\iint_S$	2D Surface integral over S enclosed by C	$\oint_C$	Closed line integral around closed curve C
I_{enc}	net current passing through a membrane/surface created by the path		



Validity Of Uncorrected Ampere's Law: Only valid in a magnetostatic situation, where the system is static except possibly for continuous steady currents within closed loops. For time-dependent $\vec{E}$, use Maxwell's Correction.

Integral form (with Maxwell's Correction): $\int_S \nabla \times \Phi_B \cdot d\vec{a} = \oint_C \vec{B} \cdot d\vec{l} = \mu_0\left(I_{\text{enc}} + \epsilon_0 \frac{d}{dt}\int_S \vec{E} \cdot \hat{n}da\right)$

ELECTROSTATICS:

Coulomb's law: The force between two charged particles is: $\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}\hat{r}_{12}$

Charge density: Electric charge per unit length, surface area, or volume. $\lambda = \frac{dQ}{dl}, \sigma = \frac{dQ}{ds}, \rho = \frac{dQ}{dV}$. To find total charge, given charge density, take the integral of the charge density multiplied by the surface area of the charged object. A charge Q for some area A $dQ = \lambda dl = \sigma dA = \rho dV$, and in general, $Q_{\text{total}} = \int \lambda dl = \int \sigma dA = \int \rho dV$

Dielectric Constant (Relative Permittivity): The dielectric constant is the ratio of permittivity of a substance to permittivity of free space.

Relative Permittivity of free space: $\epsilon_0 = k\epsilon_0$, where $k = 1$ =dielectric constant.

ELECTRIC FIELDS:

Electric Field: The electric force felt by a charge of 1C had it been kept there. $\vec{E} = \frac{\vec{F}}{Q}$ (Induced $\vec{E} = \frac{d\Phi}{dt}$)

Standard Electric Fields	
Point Charge	$\frac{q}{4\pi\epsilon_0 r^2}$
Infinite Line of Charge	$\frac{\lambda}{2\pi\epsilon_0 r}$
Ring of charge	$\frac{\lambda Rz}{2\epsilon_0(z^2+R^2)^{3/2}}$
Disk of Charge	$\frac{\sigma}{2\epsilon_0}\left(1 - \frac{x}{\sqrt{z^2+R^2}}\right)$
Infinite Plane of Charge	$\frac{\sigma}{2\epsilon_0}$

Electrical work: $W = Q\int_a^b \vec{E} \circ d\vec{r} = Q\int_a^b \frac{\vec{F_E}}{Q} \circ d\vec{r} = \int_a^b \vec{F_E} \circ d\vec{r}$

Electric flux: $\Phi_E = \int_S \vec{E} \circ \hat{n}da$

Uniform Electric Field: Constant at every point, approximated by two plates separated by $E = -\frac{\Delta\varphi}{d}$, where $\Delta\varphi$ is potential difference between plates.

Electric Field Continuity: A charge surface causes a "jump" in the electric field

CAPACITORS:

Capacitance: let the charges $+Q$ and $-Q$ be on the opposite terminals of a capacitor. The ratio between Q and the potential difference between the terminals of the capacitor is called capacitance: $C = \frac{Q}{V}$

Parallel plate capacitors: Two plates of area A separated by distance d. Charge distribution on the plates is σ and $-\sigma$ respectively. If $d \ll A$, the plates are considered infinite, so fields between the plates add up. <u>Net field inside capacitor</u> is: $\vec{E} = \begin{cases} \frac{\sigma}{\varepsilon_0}\hat{z}, & 0 < z < d \\ 0, & z < 0 \text{ or } z > d \end{cases}$ Therefore: $C = \frac{Q}{V} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0}d} = \frac{A\varepsilon_0}{d}$

Concentric spherical capacitor: Conducting sphere radius R_1 and charge +Q is surrounded by a concentric conducting shell of radius R_2 and charge –Q. The potential difference between any point at R_1 and R_2 from the center is: $V = \varphi(R_1) - \varphi(R_2) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ Capacitance: $C = 4\pi\varepsilon_0\left(\frac{R_1R_2}{R_2-R_1}\right)$

Cylindrical Capacitor: For a cylindrical geometry like a coaxial cable, the capacitance is usually stated as capacitance per unit length. Charge resides on the outer surface of the inner conductor and the inner wall of the outer conductor. The capacitance expression is: $\frac{C}{L} = \frac{2\pi k\epsilon_0}{\ln\left(\frac{b}{a}\right)}$. The voltage between the cylinders can be found by integrating $\vec{E}$ along a radial line: $\Delta V = \frac{\lambda}{2\pi\epsilon_0}\int_a^b \frac{1}{r}dr = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)$. From the definition of capacitance, and including the case where the volume is filled by a dielectric of dielectric constant k, the capacitance per unit length is defined as $\frac{C}{L} = \frac{\lambda}{\Delta V} = \frac{2\pi k\epsilon_0}{\ln(b/a)}$

Capacitors in Series	Capacitors in Parallel
$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$	$C_{eq} = C_1 + C_2$

Energy Stored in a Capacitor: Capacitor with potential difference V, charge Q. Energy required to move an elemental charge dQ from the negatively to the positively charged plate: $U = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{1}{2}CV^2$

Energy density for capacitor: $u = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2$

Energy density for magnetic field: $u = \frac{1}{2\mu_0}B^2$

$\vec{\mu}_{ring} = I\vec{A}$
Dipole moment W induced I

ELECTRIC DIPOLES:

Dipole moment: If two charges q and $-q$ are separated by a distance d, the dipole moment is defined as $\vec{P} = q \cdot \vec{d}$, where $\vec{d}$ is the vector of length d, pointing from $-q$ to q

Dipole Electric Field: The field at an equatorial point w.r.t a dipole of moment $\vec{P}$ is: $\vec{E} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{P}}{\left(\left(\frac{d}{2}\right)^2 + x^2\right)^{3/2}}$

$\vec{B}_{bar}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{(3(\vec{\mu}_{bar}\cdot\hat{r})\hat{r} - \vec{\mu}_{bar})}{r^3}$

ELECTRIC POTENTIAL:

Electric Potential: Work needed to move a unit positive charge from a reference point to a specific point inside the field without producing any acceleration. It is a scalar quantity denoted by V or φ. **Poisson equation:** $\Delta\varphi = \nabla^2\varphi = -\frac{\rho}{\varepsilon_0}$

Poisson Equation in Spherical and Cylindrical Coordinates: in Spherical: $\nabla \cdot E = \nabla^2\varphi = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\varphi}{dr}\right) = -\frac{\rho(r)}{\epsilon_0}$. For cylindrical, replace r^2 with r.

Potential Due to Point Charge: The electric potential due to a point charge Q is $\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

Work to move charge from A to B: $W_{A\to B} = \int_{\vec{r_A}}^{\vec{r_B}} \vec{E} \cdot d\vec{r} = \left[-\frac{1}{4\pi\epsilon_0}\frac{q}{r}\right]_{\vec{r_A}}^{\vec{r_B}} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_A} - \frac{q}{r_B}\right)$. Also, $W_{A\to B} = \varphi(\vec{r_A}) - \varphi(\vec{r_B})$

Definition through Movement of Charge: If Q is moved from point A to B then $\varphi(\vec{r_A} - \vec{r_B}) = -\int_{-\vec{r_A}}^{\vec{r_B}} \vec{E} \cdot d\vec{r}$

Electric Potential Inside static $\vec{E}$: The potential at a point $\vec{r}$ in a static electric field $\vec{E}$ is: $\varphi_E = -\int_C \vec{E} \cdot d\vec{l}$, where C is an arbitrary path connecting the point with zero potential to $\vec{r}$. When the curl $\nabla \times \vec{E}$ is zero (field is conservative), the integral above is independent of path C, but only on the end points.

Potential of Conservative Electric Field: If the electric field is conservative, then: $\vec{E} = -\nabla\varphi_E$.

Potential of Conservative $\vec{E}$ and Poisson Equation: $\nabla \cdot \vec{E} = \nabla \cdot (-\nabla\varphi_E) = -\nabla^2\varphi_E = \frac{\rho}{\epsilon_0}$

$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$; $\frac{\partial u}{\partial t} = -\vec{\nabla} \cdot \vec{S} - \vec{J} \cdot \vec{E}$
Poynting Th. (Local form)

Electric potential energy: Energy in the system is the amount of energy required to build the system by bringing each of the charges from infinity to its position, one by one. $U = -\int_{-\infty}^{\vec{r_B}} \vec{F} \cdot d\vec{r}$

Electric Potential Energy of test charge: A test charge q has potential energy $U_E = q\varphi$

Potential Due to a System of Point Charges: Simply the sum of point charges' individual potentials.

Conductors: In an electrostatic condition, the field inside the conductor is 0. If it is not, as the conductor allows movement of charged particles, there will be a current and the condition will not be electrostatic. Inside a perfect conductor: potential = const., $\vec{E} = \vec{0}$

Electric Field Inside a Current Carrying conductor: $E = V/L$

Dielectrics: Material permittivity $K_E = \frac{E_0}{E}$ (E_0=E in vacuum, E=E in material). **Permittivity of a dielectric**: $\varepsilon = \varepsilon_0 K_E$

Gauss' Law in Dielectrics: $\iint K_E\epsilon_0\vec{E} \cdot d\vec{A} = Q_{\text{free}}$

Electric Displacement field: $\vec{D} = \epsilon_0 K_E\vec{E} = \varepsilon\vec{E}$, therefore, $\nabla \cdot \vec{D} = \rho_{\text{free}}$

Uniformly Charged Spherical Shell: <u>Outside</u>: $\vec{E}_{\text{out}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$, $\varphi_{\text{out}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$. <u>Inside</u>: $\varphi_{\text{inside}} = \text{const.} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$. <u>Surface</u>: $\varphi_{\text{surface}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R_0}$

ELECTRODYNAMICS:

Currents: rate of charge passing through a cross sectional area: $I = \frac{dQ}{dt} = \int_S \vec{J} \circ \hat{n} da$

Adjustment for Steady Current: When currents have no time variation s.t. $\frac{d\rho}{dt} = 0$ we must have $\vec{\nabla} \circ \vec{J} = 0$

Current density: $\vec{J} = \frac{I}{A}$ (current is the flux of J) . In materials with finite resistance, $\vec{J} = \frac{\vec{E}}{\rho}$ where E is inside the medium.

Current Density In Terms Of Charge Density And Velocity: $\vec{J} = \rho\vec{v}$, where $\vec{v}$ is the velocity of the charges, and ρ is charge density (can be σ as well).

Continuity law: $Q = \iiint \rho(x,y,z,t) d^3\vec{r}$, $I = \iint_{\partial V} \vec{J} d\vec{a}$, $\iint_{\partial V} \vec{J} \cdot d\vec{a} = \iiint_V -\frac{\partial\rho}{\partial t} d^3r$. Therefore, $\iiint_V \nabla \cdot \vec{J} d^3r = \iiint_V -\frac{\partial\rho}{\partial t} d^3r$. This leads to: $\vec{\nabla} \cdot \vec{J} = -\frac{\partial\rho}{\partial t}$

Current Density at an Angle: If current density $\vec{J}$ passes through the area at an angle θ to the area normal $\hat{n}$, then: $\vec{J} \cdot \hat{n} = Jcos(\theta)$. In other words, the component of $\vec{J}$ passing through the surface is $Jcos(\theta)$, while the component passing tangential to the area is $Jsin(\theta)$. However, there is no current actually passing through the area in the tangential direction, so the only component of density that is relevant is the cosine.

Charge carriers: $\vec{J} = nQ\vec{v}_{\text{drift}}$; $\vec{v}_{\text{drift}} = \frac{1}{n}\sum \vec{v_i}$; n=number of charge carriers , Q=charge of each charge carrier, $\vec{v_i}$ =velocity of the i[th] carrier

Ohm's Law: $V = IR$. The microscopic version (at any given point): $\vec{J} = \sigma\vec{E}$

EMF due to Magnetic Field: $\text{emf} = \mathcal{E} = \int \vec{E} \cdot d\vec{r} = \int_{r=a}^{r=b} (\bar{u} \times \vec{B}) \cdot d\vec{r})$, where $\bar{u}$ is velocity of charge/conductor/anything w.

RESISTORS:

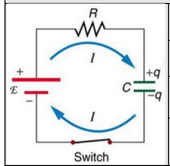
Resistivity: Conductor of cross sectional area A ,length L, potential difference V, current I, the resistivity is $\rho = R\frac{A}{L}, R = \rho\frac{L}{A}$

Resistance from Resistivity: $R = \rho\frac{L}{A}$. Consider splitting into smaller resistors and combining (ex. cylinder into shells)

Method for finding resistance: Given resistivity ρ , find the total resistance of a conductor. To do this, split the conductor into shells, determine if they are in series or in parallel. $R_{\text{series}} = \int dR, 1/R_{\text{parralel}} = \int 1/dR$. Get dR from: $R = \rho\frac{L}{a}. R(r) = \rho(r)\frac{L}{A(r)}. dR = R'(r)$

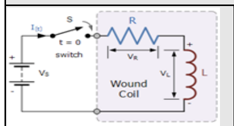
Voltage drop:	Resistors in series:	Resistors in parallel:	Power & Energy:
$\Delta V = -IR$	$R_{eq} = R_1 + R_2$	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$	$P = I^2R = VI$ and $P = \frac{dU}{dt} = V\frac{dQ}{dt}$

RC Series circuits: resistors and capacitors in series (ε =source EMF)



Charge on the capacitor	$Q = \varepsilon C\left(1 - e^{-\frac{t}{RC}}\right)$	Time constant of RC circuit	$\tau = RC$
Current in the circuit	$I = \frac{dQ}{dt} = \frac{\varepsilon}{R}e^{-\frac{t}{RC}}$	Power supplied by the battery	$P_\varepsilon = \varepsilon I = \frac{\varepsilon^2}{R}e^{-\frac{t}{RC}}$
Power dissipated at resistor	$P_R = I^2R = \frac{\varepsilon^2}{R}e^{-\frac{2t}{RC}}$	Energy stored in the capacitor	$U_C = \frac{Q^2}{2C} = \frac{\varepsilon^2 C}{2}\left(1 - e^{-\frac{t}{RC}}\right)^2$
Power in the capacitor	$P_C = \frac{dU_C}{dt} = \varepsilon^2 C\left(1 - e^{\frac{t}{RC}}\right)\frac{e^{-\frac{t}{RC}}}{RC}$	Conservation of Power	$P_\varepsilon = P_R + P_C \rightarrow$ power is conserved

LR Series Circuit: Inductor and resistor in series (ε =source EMF)



Time Constant for LR Circuit	$\tau = L/R$	Current in the Inductor	$I = \frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right)$
Voltage drop across the resistor	$V_R = I \times R$	Voltage drop across the inductor	$V_L = L\frac{di}{dt}$
Total voltage drop	$V_T = I \times R + L\frac{di}{dt}$		

CIRCUIT LAWS:

Kirchhoff's Current Law: Total current /charge entering a junction/node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node. In other words the algebraic sum of ALL the currents entering and leaving a node must be equal to zero, $I_{(\text{exiting})} + I_{(\text{entering})} = 0$. This is the conservation of charge.

Kirchhoff's Voltage Law: In any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop, which is also equal to zero. In other words the algebraic sum of all voltages within the loop must be equal to zero. This is the Conservation of Energy.

Application of Kirchhoff's Circuit Laws: The circuit is said to be "Analyzed", and the basic procedure is as follows: **(1)** Assume all voltages and resistances are given. (If not label them V1, V2,… R1, R2, etc.). **(2)** Label each branch with a branch current. (I1, I2, I3 etc.). **(3)**.Find KCL equations for each node. And KVL equations for each independent loop of the circuit. **(4).** Find the unknown currents using systems of equations.

For multiple sources of EMF: (1) Regard the circuit as collection of independent closed current loops. "Independent" means that the loops may overlap, but each loop has at least one portion that does not overlap with other loops. **(2)** Label the currents in the loops I_1, I_2 … until each loop has some circulating current assigned to it. If direction chosen wrong – answer will just be negative. **(3)** Apply KVL: traverse the loop in the chosen direction of current. Every time you cross a resistor, you get a voltage drop –IR, where I is the total current passing through the resistor. Every time you cross an EMF source, it contributes +V is cross in the "right" direction and –V if crossed in the "wrong direction). Also, Apply KCL at each junction. **(4)** End up with a set of linear equations which can be reduced to find all unknown currents.

VARIABLE CURRENTS:

Discharging a capacitor: $Q = \text{Cap's charge} = CV; \text{Cap's EMF } V = \frac{Q}{C}$.Get $\text{ODE: } \frac{Q}{C} + R\frac{dQ}{dt} = 0$;rearrange into: $\frac{dQ}{Q} = -\frac{dt}{RC}$. Integrate both sides, use the integral to enforce the boundary conditions: initially (t=0) charge separation I sQ_0, later it is $Q(t)$:$\int_{Q=Q_0}^{Q=Q(t)} \frac{dQ}{q} = \int_{t=0}^{t} \frac{dt}{RC} \rightarrow \ln\left(\frac{Q(t)}{Q_0}\right) = -\frac{t}{RC}$.

<u>Solution:</u> $Q(t) = Q_0 e^{-\frac{t}{RC}}$. <u>Current:</u> $I = -\frac{dQ}{dt} = -Q_0\frac{d}{dt}e^{-\frac{t}{RC}} = \frac{Q_0}{RC}e^{-\frac{t}{RC}}$

Charging a capacitor: for capacitor and resistor circuit. Capacitor begins at t=0 with *no charge*. $\text{EMF from battery} = V_b$. Voltage drop across capacitor$= -Q/C$. Voltage drop across resistor$= -IR$. $\text{get ODE: } V_b - \frac{Q}{C} - IR = 0$; I Relates to Q by: $I = +\frac{dQ}{dt}. V_b - \frac{Q}{C} - R\frac{dQ}{dt} = 0$; rearrange into: $\frac{dQ}{dt} = \frac{Q - CV_b}{RC}$ or $\frac{dQ}{Q - CV_b} = -\frac{dt}{RC}$. Integrate (we require Q(t=0)=0), and find: $\ln\left(\frac{CV_b - Q(t)}{CV_b}\right) = -\frac{t}{RC}$. <u>Solution</u>: $Q(t) = CV_b\left(1 - e^{-\frac{t}{RC}}\right)$. <u>Current</u>: $I(t) = CV_b\frac{d}{dt}\left(1 - e^{-\frac{t}{RC}}\right) = \frac{V_b}{R}e^{-\frac{t}{RC}}$. <u>Voltage across capacitor</u>: $V_c(t) = \frac{Q(t)}{C} = V_b\left(1 - e^{-\frac{t}{RC}}\right)$. <u>Voltage across resistor</u>: $V_R(t) = I(t)R = V_b e^{-\frac{t}{RC}}$.<u>Note</u>: $V_c(t) + V_R(t) = V_b$

MAGNETISM:

Magnetic Force: $d\vec{F} = Id\vec{l} \times \vec{B} = dq\bar{v} \times \bar{B}$. **Lorentz force law:** Net force due to $\bar{E}$ and $\bar{B}$ acting on a particle moving with $\vec{V}$: $\vec{F} = Q\vec{E} + Q\vec{V} \times \vec{B}$. At every instant, magnetic force is perpendicular to the charge's velocity – exactly the force needed to cause circular motion.

$\bar{B}$ for an **inf. wire**, at distance r from wire: $\bar{B} = \frac{\mu_0 I}{2\pi r}\hat{\theta}$	**Magnetic Field Inside Solenoid**: $\bar{B} = \mu_0 nI$, and outside→0 as solenoid gets longer		
Toroid: $\vec{B} = \frac{\mu NI}{2\pi r}$	**Current Loop:** $B_{\text{@center}} = \frac{\mu_0 I}{2R}, B_{\text{@central axis}} = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(z^2+R^2)^{3/2}}$	$\bar{\boldsymbol{B}}$ **field density**: $\eta_B = \frac{1}{2}\frac{B^2}{\mu}$	**Force for two steady *I* wires**: $\frac{\lvert\vec{F}\rvert}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$

Inductors: Consider a solenoid (coil) with n turns per unit length. The following formulas then hold:

$B = \mu_0 In$	$\varepsilon = -\frac{d\phi_B}{dt}$, Opposing Emf$= -L\frac{\Delta I}{\Delta t}$	$\varepsilon \propto \frac{dI}{dt}$	**Self-Inductance**: $L = \frac{\varepsilon}{\frac{dI}{dt}}$	**Energy density:** $u_L = \frac{U_L}{V} = \frac{B^2}{2\mu_0} = \frac{1}{2}\frac{1}{\mu_0}B^2$

Energy Stored in Inductor: Inductor of length l, cross sectional area A, inductance L: $U_L = \frac{1}{2}LI^2 = \frac{1}{2}\mu_0 n^2 lA\frac{B^2}{\mu_0^2 n^2} = \frac{B^2}{2\mu_0}lA$

Mutual inductance: 2 loops: loop1 and loop2. Current I_1 in loop1 induces magnetic field B_1. Therefore, there will be a magnetic flux ϕ_{B21} due to B_1 in loop2. This will induce potential difference ε_2 in loop 2. Hence there will be current I_2. $\varepsilon_2 = -M_{21} I_1$; $M = M_{12} = M_{21}$

Transformers: 2 coils of equal radii have n_1, n_2 turns. Coil 1 connected to variable voltage ε_1; therefore there is an induced B acting on both coils. Hence there is induced magnetic flux through both coils. Therefore there is induced emf ε_2 across the second coil. $\varepsilon_2 = \varepsilon_1\frac{n_2}{n_1}$. Power is conserved. $\mathcal{E}_1 = -n_1\varphi_B'$, $\mathcal{E}_2 = -n_2\varphi_B'$. $\frac{\mathcal{E}_1}{n_1} = \frac{\mathcal{E}_2}{n_2}$

WAVES:

1D Waves: Let $\psi(x,t) = A\cos(k(x - vt) + \varphi_0) = Acos(kx - \omega t + \varphi_0)$ where A = amplitude, k= wave number, v=propagation speed (only depends on the medium), $\omega = kv$ = angular frequency, $T = \frac{2\pi}{\omega}$= time period, $f = \frac{1}{T} = \frac{\omega}{2\pi}$ =time frequency, φ_0 = initial phase, $\lambda = \frac{2\pi}{k}$= wavelength of the wave.

3D Waves: Let $\hat{n}$ be a unit vector in the direction of propagation of the wave. The equation of the wave is therefore: $\psi(\vec{r}, t) = f(\hat{n} \circ \vec{r} - vt) = Acos(k(\hat{n} \circ \vec{r} - vt) + \varphi_0) = Acos(\vec{k} \circ \vec{r} - \omega t + \varphi_0)$, Where $\vec{k} = k\hat{n}$ is called the wave vector. Therefore: $\nabla^2 \psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$,

Electromagnetic waves: $\nabla^2 \vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$, $\nabla^2 \vec{B} = \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$

2D Wave Equation: $\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + \frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2}$. Since the electric field is in the x-direction only, the propagation is perpendicular to the x-axis and can be in any direction in the yz plane, depending upon the values of the derivatives. This equation is in the general form of the two-dimensional wave equation.

Mutual perpendicularity of EM: $\vec{E}$ and $\vec{B}$ are mutually perpendicular and $E_0 = cB_0$

EM Energy density in a vacuum: $u_{em} = \varepsilon_0 E^2 = c\varepsilon_0 EB$. **|POYNTING VECTOR**=directional energy flux, energy/area/time

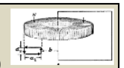
Poyinting vector: $\bar{S} = C\hat{n} = \frac{\vec{E} \times \vec{B}}{\mu_0}$ represents energy flux. Waves propagate in direction of $\bar{S}$. **Momentum due to EM waves:** $\overline{p_{em}} = \frac{\bar{s}}{c^2} = \frac{u_{em}}{c}\hat{n}$

MATH:

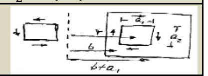
$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}, \quad \nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Circle Area: $A = \pi r^2$	**Sphere Surface A.**: $A = 4\pi r^2$	**Cone Surface Area**: $A = \pi r(r + \sqrt{h^2 + r^2})$
Circle Perimeter: $P = 2\pi r$	**Cylinder Surface A.**: $A = 2\pi rh + 2\pi r^2$	**Prism/Cylinder Volume**: $V = Ah$ (base A. * height)
Sphere Volume: $\frac{4}{3}\pi r^3$	**Pyramid/Cone Vol.**: $V = \frac{1}{3}Ah$	$\iiint \bar{F} \cdot d\bar{V} = \bar{F} \iiint_V d\bar{V} = \bar{F} \cdot V = \bar{F}$ ·volume/perimeter/area of integrated shape

A toroidal coil has a rectangula cross section with inner radius b and outer radius $b + a_1$, and height a_2. It carries a total of N tightly wound turns, and current $I(t) = I_0 + I_1 \cdot t$ in which I_0 and I_1 are positive constants and t is time. A rectangular wire passes through and surrounds the torus. **Sol** Toroid: N_T # turns. $I_t(t) = I_0 + I_1 t$. Rectangular loop: $N_L = 1$ turn. $I_L = I_2\sin(\omega t)$



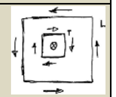
a) *Calculate magnetic field everywhere.* Sol: $\oint \bar{B} \cdot d\bar{l} = \mu_0 I_{enc}$. $B \cdot 2\pi r = \mu_0 N_t I_t(t)$. $B = \frac{\mu_0 N_t I_r}{2\pi r}$



b) *Calculate magnetic energy density inside the toroidal coil..* **Sol**: Magnetic energy density (j/m[2]): $u_b = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 N_T I_T}{2\pi r}\right)^2$

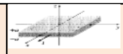
c) *Calculate total magnetic energy stored in the toroidal coil, by integrating the magnetic density over the volume of the torus*. **Sol**: Total magnetic energy: $U_B = \int_b^{b+a_1} u_b 2\pi r a_2 dr = \frac{1}{2\mu_0}\frac{\mu_0^2 N_T^2 I_T^2}{(2\pi)^2} 2\pi a_2 \int_b^{b+a_1}\frac{dr}{r} = \frac{\mu_0 N_T^2 I_T^2 a_2}{4\pi}\ln\left(1 + \frac{a_1}{b}\right)$.

d) *Calculate the current induced in the rectangular loop.* **Sol**: $\mathcal{E}_L = -\frac{d\Phi_{BT}}{dt}$. $\nabla \times \bar{E} = -\frac{\partial B_t}{\partial t}$. $\oint E \cdot dl = -\frac{\partial}{\partial t}\oint \bar{B}_t \cdot d\bar{S}$. $\mathcal{E}_L = -\frac{d}{dt}\left(\frac{\mu_0 N_T a_2}{2\pi}\ln\left(1 + \frac{a_1}{b}\right)(I_0 + I_1 t)\right) = -\frac{\mu_0 N_T a_2}{2\pi}\ln\left(1 + \frac{a_1}{b}\right)I_1$ $(I_1 = \frac{d}{dt}(I_T(t)))$. $\mathcal{E}_L = I_L R_L$, R_L=resistance of loop, I_L =current in loop. Current in loop: L is C.C.W. to oppose increasing flux through toroid.

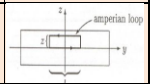


e) *Now assume a current $I(t) = I_2\sin(\omega t)$ is running through the rectangular wire ($I_2\omega$, are positive constants.). Calculate the induced current in the torus.* **Sol**: Mutual Inductance between Loop: L and Torus : T: $\Phi_{LT} = M_{LT}I_T$. $\Phi'_{LT} = M_{LT}I_T'$. $-\mathcal{E}_L = M_{LT}I_T'$. From part (d) $(M_{LT} = M_{TL})$, $M_{LT} = \frac{\mu_0 N_T a_2}{2\pi}\ln\left(1 + \frac{a_1}{b}\right)$. Hence $\mathcal{E}_T = M_{LT}I_L'$. $I'_L = \frac{d}{dt}(I_2\sin(\omega t)) = I_2\omega\cos(\omega t)$. $\mathcal{E}_T = I_T R_T = M_{LT}I_2\omega\cos(\omega t)$. $I)T = \frac{M_{LT}}{R_T}I_2\omega\cos(\omega t)$

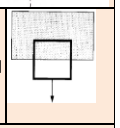
A thick slab extending from $z=-a$ to $z=+a$ carries a uniform volume current $\bar{J}=J\hat{x}$. Find the magnetic field as a function of z, inside and outside the slab.



By the right hand rule, field points in the $-\hat{y}$ direction for $z>0$, and in the $+\hat{y}$ direction for $z<0$. As $z=0$, $B=9$. Use the Amperian loop shown: $\oint B\cdot d\bar{l}=Bl=\mu_0 I_{\text{enc}}=\mu_0 lzJ \Rightarrow \bar{B}=-\mu_0 Jz\hat{y}\ (-a<z<a)$. If $z>a$, $I_{\text{enc}}=\mu_0 laJ$, so $\bar{B}=\mp\mu_0 Ja\hat{y}$ for $z>\pm a$.



A square loop is cut out of a thick sheet of aluminum. It is placed so that the top portion is in a uniform magnetic field $\bar{B}$, and allowed to fall under gravity. (in the diagram, shading=field region; field points into page). If the magnetic field is 1 T, find the terminal velocity of the loop. Find the velocity of the loop as function of time. How long does it take to each, say, 90% of terminal velocity? What would happen if you cut a tiny slit in the ring? Note: Dimensions of the loop cancel out; determine actual numbers by using the following values: Aluminum resistivity $\rho=2.8\times10^{-8}$, mass density of aluminum: $\eta=2.7\times10^3$

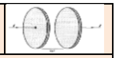


$\mathcal{E}=Blv=IR\Rightarrow I=\frac{Bl}{R}v \Rightarrow$upward magnetic force$=IlB=\frac{B^2l^2}{R}v$, opposing gravitation downward: $mg-\frac{B^2l^2}{R}v=m\frac{dv}{dt};\ \frac{dv}{dt}=g-\alpha v$, where $\alpha\equiv\frac{B^2l^2}{mR}$. $g-\alpha v_t=0\Rightarrow v_t=\frac{g}{\alpha}=\frac{mgR}{B^2l^2}$. $\frac{dv}{g-\alpha v}=dt\Rightarrow-\frac{1}{\alpha}\ln(g-\alpha v)=t+const.\Rightarrow g-\alpha v=Ae^{-\alpha t}$; at $t=0, v=0$, so $A=g$. $\alpha v=g(1-e^{-\alpha t})$; $v=\frac{g}{\alpha}(1-e^{-\alpha t})=v_t(1-e^{-\alpha t})$. At 90% of terminal velocity, $\frac{v}{v_t}=0.9=1-e^{-\alpha t}=1-0.9=0.1$; $\ln(0.1)=-\alpha t$; $\ln(10)=\alpha t$; $t=\frac{1}{\alpha}\ln(10)$, or $t_{90\%}=\frac{v_t}{g}\ln(10)$. Now the numbers: $m=4\eta Al$, where A is crossectional area and l is side length. $R=\frac{4l}{A\sigma}$ where σ is conductivity. If the loop were cut, it would fall freely.

Given the following electric field in a vacuum: $\bar{E}=E_0\sin(\alpha y+\beta z+Dt)\,\hat{x}$, identify the wave amplitude A, the wavenumber $\bar{k}$, wavelength λ, angular frequency ω and the period T. Assuming no constant magnetic field, calculate the magnetic field. Find a relation between α,β and D. Find the energy density and the pointing vector. In which direction this wave propagates?

1. $A=|E_0|$, $\bar{k}=\alpha\hat{y}+b\hat{z}$, $\lambda=\frac{2\pi}{|\bar{k}|}=\frac{2\pi}{(\alpha^2+\beta^2)^{1/2}}$, $\omega=|D|$, $T=\frac{2\pi}{|D|}$. 2. $\nabla\times\bar{E}=E_0(\beta\hat{y}-\alpha\hat{z})\cos(\alpha y+\beta z+Dt)\Rightarrow\bar{B}=\int-\nabla\times\bar{E}dt=-\frac{E_0}{D}[\hat{y}\beta-\hat{z}\alpha]\sin(\alpha y+\beta z+Dz$. 2. $\nabla\times\bar{E}=E_0(\beta\hat{y}-\alpha\hat{z})\cos(\alpha y+\beta z+Dt)\Rightarrow\bar{B}=\int-\nabla\times\bar{E}dt=-\frac{E_0}{d}[\hat{y}\beta-\hat{z}\alpha]\sin(\alpha y+\beta z+Dt)$. 3. $\omega=|\bar{k}|c\Rightarrow|D|=(\alpha^2+\beta^2)^{1/2}c\Rightarrow D^2=(\alpha^2+\beta^2)c, c\equiv\frac{1}{\sqrt{\mu_0\epsilon_0}}$. 4. $u=\frac{E^2}{2}\epsilon_0+\frac{1}{2\mu_0}\bar{B}^2=\frac{E_0^2}{2}\sin^2(\alpha y+\beta z+Dt)\cdot\left[\epsilon_0+\frac{1}{\mu_0}\left(\frac{\alpha^2+\beta^2}{D^2}\right)\right]$. $\bar{S}=\frac{1}{\mu_0}\bar{E}\times\bar{B}=-\frac{1}{\mu_0}\frac{E_0^2}{D}(\beta\hat{z}+\alpha\hat{y})\sin^2(\alpha y+\beta z+Dt)$. 5. EM wave propagates along $\pm\hat{k}$, depending on sign of D.

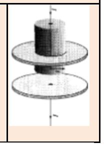
This is an artificial model for the charging capacitor. Current I is constant, the radius of the capacitor is a, separation of plates is $w\ll a$. Assume current flows out over the plates such that σ is uniform at any given time and zero at $t=0$.



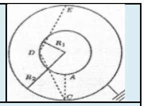
Find the electric field between the plates, as a function of t. $\bar{E}=\frac{\sigma(t)}{\epsilon_0}\hat{z}$; $\sigma(t)=\frac{Q(t)}{\pi a^2}=\frac{It}{\pi a^2}$, $\bar{E}=\frac{It}{\pi\epsilon_0 a^2}\hat{z}$

Find the displacement current through a circle of radius s in the plane midway between the plates. Using this circle as your "Amperian loop" and the flat surface that spans it, find the magnetic field at a distance s from the axis. $I_{D_{\text{enc}}}=J_d\pi s^2=\epsilon_0\frac{dE}{dt}\pi s^2=I\frac{S^2}{a^2}$. $\oint\bar{B}\cdot d\bar{l}=\mu_0 I_{D_{\text{enc}}}\Rightarrow B2\pi s=\mu_0 I\frac{S^2}{a^2}\Rightarrow B=\frac{\mu_0 I}{2\pi a^2}s\hat{\varphi}$

Repeat the previous part, but this time use the cylindrical surface, which extends to the left through the plate and terminates outside the capacitor. Notice the displacement current through this surface is zero, and there are two contributions to I_{enc}: A surface current flows radially outward over the left plate; Let $I(s)$ be the total current crossing a circle of radius s. The charge density (at time t) is: $\sigma(t)=\frac{[I-I(s)]t}{\pi s^2}$. Since we are told this is independent of $s\rightarrow\ \ I-I(s)=\beta s^2$, for some const. β. But $I(a)=0$, so $\beta a^2=I$, or $\beta=\frac{I}{a^2}$. Hence: $I(s)=I\left(1-\frac{s^2}{a^2}\right)$. $B2\pi s=\mu_0 I_{\text{enc}}=\mu_0(I-I(s))=\mu_0\frac{s^2}{a^2}\Rightarrow B=\frac{\mu_0}{2\pi a^2}s\hat{\varphi}$



An insulating full sphere of radius R_1 is charged with the following symmetrical volume charge density: $\rho(r)=\begin{cases}ar^3, & 0\le r\le R_1\\ 0, & r\ge R_1\end{cases}$, where r is the distance from the center of the sphere, and $a>0$. A thin grounded conducting spherical shell of radius R_2, $(R_2>R_1)$ concentrically surrounds the full sphere.



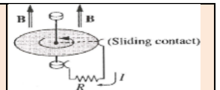
a) *Calculate the total charge on the full sphere.* Answer: $Q_1=\int_0^{R_1}ar^2\cdot4\pi r^2dr=4a\pi\int_0^{R_1}r^5dr=\frac{2}{3}a\pi R_1^6$

b) *Find the total charge on the conducting shell.* Answer: $Q_2=-Q_1=-\frac{2}{3}a\pi R_1^6$

c) *Calculate the Electric field vector everywhere.* Answer: $\bar{E}=\hat{r}\begin{cases}k\cdot\frac{\frac{2}{3}a\pi r^6}{r^2}=\frac{2}{3}ka\pi\cdot r^4, & 0\le r\le R_1\\ k\cdot\frac{\frac{2}{3}a\pi R_1^6}{r^2}, & R_1<r<R_2\end{cases}$, and $0, r>R_2$

d) *What is the work required to translate a positive charge q over the path: $A\rightarrow C\rightarrow D\rightarrow E$ (see figure)?* Answer: The work does NOT depend on the path, so $W_{A\rightarrow E}=q\int_{R_1}^{R_2}\bar{E}\cdot d\bar{r}$. We choose a direct path among the radius: $W_{A\rightarrow E}=q\int_{R_1}^{R_2}\bar{E}\cdot d\bar{r}=q\int_{R_1}^{R_2}\frac{2}{3}a\pi kR_1^6\cdot\frac{dr}{r^2}=\frac{2}{3}qa\pi kR_1^6\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$

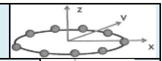
A metal disc of radius a is rotating at angular velocity ω about a vertical axis, through a uniform field $\bar{B}$, pointing up. A circuit is made by connecting one end of a resistor (of resistance R) to the axle, and the other end to a sliding contact, which touches the outer edge of the disk (see figure)



a) *Find the current through the resistor.* Answer: We start with: $\mathcal{E}=\oint_{\text{circuit}}(\bar{E}+\bar{v}\times\bar{B})d\bar{l}$. We know that $\oint_{\text{circuit}}\oint\bar{E}\cdot d\bar{l}=0$ because $\bar{E}$ is conservative here ($\bar{B}$ does not depend on time!). Hence: $\mathcal{E}=\oint_{\text{circuit}}(\bar{E}+\bar{v}\times\bar{B})d\bar{l}=\int_0^a\omega rBdr=\frac{1}{2}\omega Ba^2$. We know that $I=\frac{\mathcal{E}}{R}$, substituting $\mathcal{E}$: $I=\frac{\omega Ba^2}{2R}$

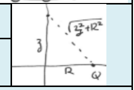
b) *What is the rate of work we need to put into the system to keep the disk rotating at a constant angular velocity ω? Assume there are no losses of energy due to friction, etc.).* Answer: Since there is no loss of energy in the system, the power we invest is directly dissipated in the resistor. Conversely, the power we invest: $P=I^2R=\frac{\omega^2B^2a^4}{4R}$

N point charges, each one has charge Q, are rigidly fixed and equally spaced on the circumference of a glass rind of radius R, as shown in the figure.



a) *Calculate the electric potential along the z-axis.* Answer: $\varphi(0,0,z)=N\cdot k\frac{Q}{(z^2+R^2)^{1/2}}$

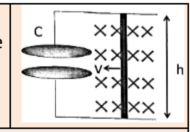
b) *Find the electric field vector at the z-axis.* Answer: The field at z is $\bar{E}(0,0,z)=-\bar{\nabla}\varphi=-\frac{\partial\varphi}{\partial z}\hat{z}=+\frac{NkQz}{(z^2+R^2)^{\frac{3}{2}}}\hat{z}$



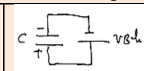
c) *A negative charge q (and mass m) is released from rest at height H above the ring's center, on the z-axis. What is the maximal velocity of the charge, throughout its motion?* Answer: $q<0$, and consequently the particle will be confined to: $-H<z<H$. The energy equation: $q\cdot Nk\frac{Q}{(z^2+R^2)^{1/2}}+\frac{1}{2}mv^2=qNk\frac{Q}{(H^2+R^2)^{1/2}}$. When $z=0$, we will have maximum velocity, so $z=0\Rightarrow v=v_{\max}$. From this, we get the following expression: $v_{\max}^2=\left[\frac{2}{m}\cdot qNkQ\right]\left[\frac{1}{(H^2+R^2)^{\frac{1}{2}}}-\frac{1}{R}\right]>0$. The reason it is >0, is because the terms in the brackets are both negative.

d) *The ring is rotated counterclockwise (↺), with period T. Find the magnetic field vector at the origin.* Frequency of rotation is $\frac{1}{T}$, so the effective current is $I=NQ\frac{1}{T}=\frac{NQ}{T}$. Using the result for a ring carrying current, the magnetic field: $\bar{B}=\hat{z}\frac{\mu_0}{4\pi}\frac{\frac{NQ}{T}\cdot2\pi R}{R^2}=\frac{\mu_0 NQ}{2TR}\hat{z}$.

A perfectly conducting metal bar slides on 2 perfect conductor, zero friction, parallel rails. A p-p capacitor is connected across the rails, and a uniform M.field B is pointing into the page, filling the entire region except the area around the capacitor. The capacitor's plates are circular conducting discs of radius R. Distance between the plates $d, d\ll R$. Bar moves to the left at $v=at$. Bar is far enough from the boundary of the magnetic field's region and stays within this region at all times. Ignore M.field from the wires. Cap is uncharged at $t=0$.



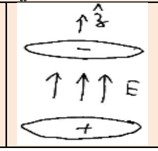
a) *Which of the cap's plates is positive?* Answer: Bottom plate is positive



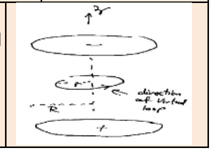
b) *Find the charge density on the cap as a function of time.* Answer: $vBh=\frac{\sigma}{\varepsilon_0}\cdot d,\ atBh=\frac{\sigma}{\varepsilon_0}\cdot d\rightarrow\sigma(t)=\frac{\varepsilon_0 aBh}{d}\cdot t$

c) *Find the surface current density $k(r,t)$ on the cap's plates.* Answer: $Q_1(t)=\sigma(t)\cdot\pi(R^2-r^2)=\frac{\varepsilon_0\pi aBh}{d}\cdot(R^2-r^2)\cdot t$. $2\pi rK(r,t)=\frac{dQ_1}{dt}=\frac{\varepsilon_0\pi aBh}{d}(R^2-r^2)$, $\rightarrow\vec{k}(r,t)=\frac{\varepsilon_0 aBh}{2d}\left(\frac{R^2}{r}-r\right)\hat{r}$

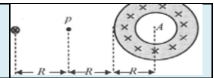
d) *Calculate the Efield inside the cap. Specify its direction explicitly.* Answer: $\vec{E}=\frac{\sigma}{\varepsilon_0}\hat{z}=\frac{aBh}{d}t\hat{z}$



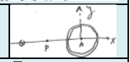
e) *Calculate the Mfield inside the cap. Specify its direction explicitly.* Answer: Using Ampere-Maxwell Law: $\oint_{\partial S}\vec{B}\cdot d\vec{l}=\mu_0 I+\mu_0\mathcal{E}_0\iint_S\frac{\partial\vec{E}}{\partial t}d\vec{a}$. $2\pi rB=0+\mu_0\mathcal{E}_0\cdot\frac{aBh}{d}\cdot\pi r^2.\rightarrow\ \ \vec{B}=\mu_0\mathcal{E}_0\cdot\frac{aBh}{2d}r\hat{\varphi}$



A very long thin pipe of radius R carries current i_0 that is uniformly distributed on its thin cross-section. The direction of i_0 is into the page(the pipe is drawn thick only to illustrate the current density's direction). A wire, carrying current i_1 (in the same direction as i_0), is placed a distance $3R$ from the axis of the pipe.



a) *Calculate the total magnetic field vector at the pipe's axis.* The contribution of the pipe is zero! Hence: $\bar{B}_A=\frac{\mu_0 i_1}{2\pi\cdot3R}(-\hat{y})$

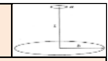


b) *Calculate the total magnetic field vector at point p, at a distance 2R from the center of the pipe (see Figure).* Answer: $\bar{B}_p=\left(-\frac{\mu_0 i_1}{2\pi R}+\frac{\mu_0 i_0}{2\pi\cdot2R}\right)\hat{y}$

c) *What should be the ratio of the currents $\left(\frac{i_1}{i_0}\right)$ if you need the magnetic field at point p to be identical in magnitude to the one at the pipe's axis, but opposite in direction?* Answer: $-\frac{\mu_0 i_1}{2\pi R}+\frac{\mu_0 i_0}{2\pi\cdot2R}=+\frac{\mu_0 i_1}{6\pi R}$. After some algebra to isolate the term, arrive at: $\frac{i_1}{i_0}=\frac{3}{8}$

d) *A negative point charge Q is place at point p, and is given velocity v. Consequently, the charge starts turning towards the wire. In which direction is this velocity given? What is the force (magnitude + direction) acting on this charge at the moment the velocity is given? Assume that the condition you found in subsection (c) holds here!* Answer: $\bar{v}$ has to be in the $-\hat{z}$ direction, i.e. into the page (remember $Q<0$). The force would be: $\bar{F}=-Qv\frac{\mu_0 i_1}{6\pi R}\hat{x}$

A small loop of wire (radius a) lies a distance z above the center of a large loop (radius b), as shown. The planes of the two loops are parallel, and perpendicular to the common axis. Assume $a\ll b$. Resistance of the large loop is R.



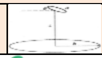
a) *Suppose current I flows in the big loop. Find the flux through the small loop.* Answer: We know the magnetic field at the z-axis: $\bar{B}(0,0,z)=\frac{\mu_0 Ib^2}{2(b^2+z^2)^{3/2}}\hat{z}$. Since $a\ll b$, we can assume that $\bar{B}$ is uniform on the surface bounded by loop a -> $\Phi_B(\text{loop a})=\frac{\mu_0 Ib^2}{2(b^2+z^2)^{3/2}}\cdot\pi a^2$

b) *Find the mutual inductance of the two loops.* Answer: Using the result from (a), we get $M=\frac{\mu_0\pi a^2b^2}{2(b^2+z^2)^{3/2}}$

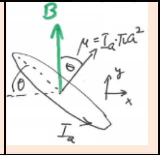
c) *Suppose current $I=kt$ (k is a positive constant and t denoted time) flows in the small loop. Find the induced current in the large loop. Ignore the contributions of self-inductance.* Answer: We start off with the fact that: $\Phi_B(\text{loop b})=MI$. Substituting the result from (b): $\Phi_B(\text{loop b})=MI=\frac{\mu_0\pi a^2b^2}{2(b^2+z^2)^{3/2}}\cdot kt$. Since we know the flux, we can find the induced emf: $\mathcal{E}_b=-\Phi_B'(\text{loop b})=$

$-\frac{\mu_0\pi a^2b^2k}{2(b^2+z^2)^{3/2}}$. Knowing the emf, we can find current: $I_b=\frac{|\mathcal{E}_b|}{R}=\frac{\mu_0\pi a^2b^2k}{2R(b^2+z^2)^{3/2}}$

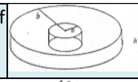
The small loop is tilted wrt the plane of the large loop, by angle θ. Current I_a flows in the small loop, and current I_b in the large one. Both are counter-clockwise, and include all induced contributions due to mutual and self-inductance.



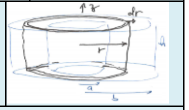
d) Calculate the torque on the small loop, about its center (magnitude and direction!), and explain what will happen if the small loop will be able to move freely about its center. Answer: The magnetic field $B=\frac{\mu_0 I_b b^2}{2(b^2+z^2)^{3/2}}$. Knowing the field, we can find torque as follows: $\bar{\tau}=\bar{\mu}\times\bar{B}=I_a\pi a^2\cdot\frac{\mu_0 I_b b^2}{2(b^2+z^2)^{3/2}}\cdot\sin(\theta)\,\hat{z}=\frac{\mu_0\pi a^2b^2\sin(\theta)I_aI_b}{2(b^2+z^2)^{3/2}}\hat{z}$. The torque "wants" to align $\bar{\mu}$ with $\bar{B}$. If loop a was able to move freely about its center, it will rotate to an equilibrium with the bigger loop, and probably oscillate about this equilibrium.



Consider a thin hollow cylinder of width h, inner radius a and outer radius b, where $h\ll a$. The cylinder is made of a conducting substance, whose conductivity depends on the distance from the cylinder's axis according to the following formula: $\sigma(r)=\beta r^2$ (β is a positive constant)



a) Calculate the cylinder's resistance between the bases of the cylinder. Answer: Using the connection $R=\rho\frac{L}{A}$ for the thin cylindrical shell of radius r, and the fact that $\rho=\frac{1}{\sigma}$, we get: $dR=\frac{1}{\beta r^2}\cdot\frac{h}{2\pi r dr}$. Then, we add the aforementioned shells together in series: $\frac{1}{R}=\int\frac{1}{dR}=\frac{2\pi\beta}{h}\int_a^b r^3dr=\frac{\pi\beta}{2h}(b^4-a^4)$

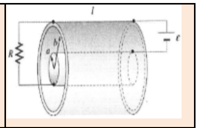


b) If the lower base of the cylinder is held at potential 0 and the upper bases at potential V, what is the volume charge density inside the resistor as a function of r? The electric field is $\bar{E}=\frac{V}{h}\hat{z}$ (constant and uniform!). Hence, $\rho_{\text{inside}}=\epsilon_0\bar{\nabla}\cdot\bar{E}=0$.

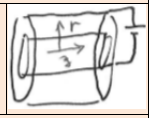
c) Calculate the cylinder's resistance between the inner envelope of the cylinder ($r=a$) and its outer envelope ($r=b$). Answer: We do the same as in (a), but now we have $dR=\frac{1}{\beta r^2}\frac{dr}{2\pi rh}$, and add the shells in parallel: $R=\int dR=\int_a^b\frac{1}{\beta r^2}\frac{dr}{2\pi rh}=\frac{1}{4\beta\pi h}\left[\frac{1}{a^2}-\frac{1}{b^2}\right]$

d) If the inner envelope of the cylinder is held at potential 0 (zero) and the outer envelope is held at potential V, what is the volume charge density inside the resistor as a function of r (the distance from the axis)? Answer: Finding electric field: $I=\frac{V}{R}\Rightarrow\bar{J}(a<r<b)=\frac{V/R}{2\pi rh}\hat{r}$ (this is in cylindrical coordinates). We now use the fact that $\bar{J}=\sigma\bar{E}$, and therefore: $\frac{V}{2\pi rRh}\hat{r}=\beta r^2\bar{E}$. We get the electric field: $\bar{E}=\frac{V}{2\pi R\beta r^2h}\hat{r}$. From electric field, we get: $\rho(a<r<b)=\epsilon_0\bar{\nabla}\cdot\bar{E}$. This is divergence, but we are in cylindrical coordinates, so the appropriate formula: $\rho(a<r<b)=\epsilon_0\bar{\nabla}\cdot\bar{E}=\epsilon_0\frac{1}{r}\frac{\partial}{\partial r}\left(r\cdot\frac{V}{2\pi R\beta r^3h}\right)$. This evaluates to $\rho(a<r<b)=-\epsilon_0\frac{V}{h\cdot\pi R\beta r^4}$

A coaxial cable consists of two concentric long hollow cylinders of zero resistance; the inner has radius a, the outer has radius b, and the length of both is l, with $l\gg b$. The cable transmits DC power from a battery to a load. The battery provides an EMF $\mathcal{E}$ between the two conductors at on end of the cable, and the load is a resistance R between the two conductors at the other end of the cable. Current I flows down the inner conductor and up the outer one. Battery charges the inner conductor to a charge $-Q$, and the outer to $+Q$.



f) Find the magnitude and direction of the electric field $\bar{E}$ everywhere. Answer: Obviously, there is an electric field inside the resistor, and inside the battery, but we ignore them here. We convert to cylindrical coordinate, and use Gauss Law (Integral Form) for the cable. We get (in cylindrical coordinates): $\bar{E}=\hat{r}\begin{cases}-\frac{Q/l}{2\pi\epsilon_0 r}, & a<r<b\\ 0, & 0\le r<a \text{ or } r>b\end{cases}$



g) By integrating over the electric field between the hollow cylinders, find a connection between the voltage $\mathcal{E}$ and the charge Q. Answer: $\mathcal{E}=\int_b^a\bar{E}\cdot d\bar{r}=\int_b^a-\frac{Q}{2\pi\epsilon_0 rl}dr=\frac{Q}{2\pi\epsilon_0 l}\int_a^b\frac{dr}{r}$. Hence: $\mathcal{E}=\frac{Q}{2\pi\epsilon_0 l}\ln(\frac{b}{a})$

h) Find the magnitude and direction of the magnetic field $\bar{B}$ everywhere. Ignore the magnetic field induced by the current in the wires. Consider only the contribution of the cable! Answer: Using Ampere's circuital Law: $\bar{B}=\frac{\mu_0 I}{2\pi r}\hat{\varphi},\ a<r<b$, and $\bar{B}=(0,0,0)$ for $0<r<a$ or $r>b$

i) Calculate the Poynting vector $\bar{S}$ in the cable. Answer: We will be finding the Poynting vector only for $a<r<b$, as everywhere else, $\bar{E}=0$, so $\bar{S}=0$. $\bar{S}=\frac{\bar{E}\times\bar{B}}{\mu_0}=-\hat{z}\cdot\frac{Q\mu_0 I}{4\pi\epsilon_0\mu_0 lr^2}=-\hat{z}\frac{QI}{4\pi^2\epsilon_0 lr^2}$

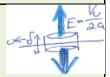
Electric potential of a static configuration of el.charges everywhere in space, given by: $\varphi(\bar{r})=\begin{cases}V_0e^{-z/2a}, & z\ge 0\\ 2V_1e^{z/2a}, & z<0\end{cases}$, a,V_0,V_1 positive constants.

a) Find V_1 in terms of V_0 and a. **Sol**: Due to continuity of the potential: $V_0=2V_1\to V_1=\frac{1}{2}V_0$

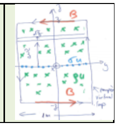
b) Find the electric field everywhere in space. **Sol**: $\bar{E}=-\bar{\nabla}\varphi=\hat{z}\begin{cases}+\frac{V_0}{2a}e^{-z/2a}, & z>0\\ -\frac{V_0}{2a}e^{z/2a}, & z<0\end{cases}$

c) Find the volumetric charge density everywhere. **Sol**: $\rho=\epsilon_0\bar{\nabla}\cdot\bar{E}=\epsilon_0\frac{\partial E_z}{\partial z}=\begin{cases}-\frac{\epsilon_0V_0}{4a^2}e^{-z/2a}, & z>0\\ -\frac{\epsilon_0V_0}{4a^2}e^{z/2a}, & z<0\end{cases}$, note that ρ is negative and symmetric about $z=0$ plane.

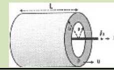
d) Find the surface charge density everywhere. **Sol**: Gauss Law on a thin infinitesimal surface crossing the $z=0$ plane (where there is a gap in $\bar{E}$). $\underbrace{+2\frac{V_0}{2a}\cdot dA}_{\oint_{\partial V}\bar{E}\cdot d\bar{A}}=\underbrace{\frac{1}{\epsilon_0}\sigma\cdot dA}_{(1/\epsilon_0)Q_{\text{enc}}}$. So: $\sigma=+\frac{\epsilon_0V_0}{a}$



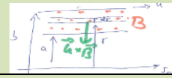
e) All electric charges in the system are now moving at constant uniform velocity $\bar{u}=u\hat{x}$. Find the magnetic field everywhere. **Sol**: Ampere's circuital law: $\oint\bar{B}\cdot d\bar{r}=\mu_0\iint\bar{J}\cdot d\bar{A}$. So: $2B(z)\cdot 1=\mu_0[\sigma\cdot u\cdot 1+\iint\rho u dA]$, where ρ is negative. $2B(z)=\mu_0\left[\frac{\epsilon_0V_0u}{a}-2\int_0^z\frac{\epsilon_0V_0}{4a^2}e^{-\tilde{z}/2a}ud\tilde{z}\right]=\mu_0\left[\frac{\epsilon_0V_0u}{a}+\frac{2\epsilon_0V_0\cdot 2a}{4a^2}\left(e^{-z/2a}-1\right)\right]=\frac{\mu_0\epsilon_0V_0u}{a}e^{-z/2a}$, and $\bar{B}(x,y,z)=\hat{y}\begin{cases}-\frac{\mu_0\epsilon_0V_0u}{a}e^{-z/2a}, & z>0\\ +\frac{\mu_0\epsilon_0V_0u}{a}e^{-z/2a}, & z<0\end{cases}$



A long hollow cylindrical resistor of inner radius a, outer radius b, length L≫b and resistivity ρ, moves in the x-direction with constant velocity u. An infinite wire carrying current I_0 runs along the cylinder's axis (x-axis) as shown in figure. Please note the wire does NOT move in our system of reference.



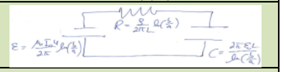
f) Find the emf inside the resistor, between $r=a$ and $r=b$, r is the distance from the x-axis. **Sol** (cylindrical): $\bar{u}\times\bar{B}=uB(-\hat{r})=\frac{\mu_0I_0u}{2\pi r}(-\hat{r})$. emf $=\int_{r=b}^{r=a}(\bar{u}\times\bar{B})\cdot d\bar{r}=\int_a^b-\frac{\mu_0I_0u}{2\pi r}(-\hat{r})\cdot dr\cdot\hat{r}=\int_a^b\frac{\mu_0I_0u}{2\pi r}dr=\frac{\mu_0I_0u}{2\pi}\ln\left(\frac{b}{a}\right)$



g) Find the total resistance of the resistor, between $r=a$ and $r=b$. **Sol**: Split into shells $dR=\rho\frac{dr}{2\pi rL}$. $R=\int dR=\int_a^b\rho\frac{dr}{2\pi rL}=\frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$

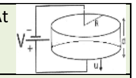
h) Due to the resistor's motion, charge will accumulate on its inner and outer surfaces. (I) Which surface is positive? $r=a$ or $r=b$? Explain! (II) Calculate the capacitance of the cylindrical system. **Sol**: **(I)** $r=a$ will be positive, due to the direction of $\bar{u}\times\bar{B}$ in subsection (a) **(II)**: $\bar{E}(a<r<b)=\frac{\lambda}{2\pi\epsilon_0 r}\hat{r}=\frac{Q}{2\pi\epsilon_0 Lr}\hat{r}$. $V=\int_{r=a}^{r=b}\bar{E}\cdot d\bar{r}=\int_a^b\frac{Q}{2\pi\epsilon_0 L}\frac{dr}{r}=\frac{Q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right)$, so $C=\frac{Q}{V}=\frac{2\pi\epsilon_0 L}{\ln(b/a)}$

i) Find the electric current flowing inside the cylinder, as a function of time. **Sol**: The situation is best described by the RC-circuit to the right, from which we get: $I(t)=\frac{\mathcal{E}}{R}e^{-t/RC}=\frac{\mu_0I_0uL}{\rho}e^{-t/\epsilon_0\rho}$



j) Calculate the current density vector in the resistor (using your sol to (d)). **Sol**: $\bar{J}=\frac{I}{2\pi rL}(-\hat{r})=-\frac{\mu_0I_0u}{2\pi\rho r}e^{-\frac{t}{\epsilon_0\rho}}\hat{r}$

A parallel plate capacitor is connected to a voltage source V. The capacitor's plates are circular conducting disks of radius R. At $t=0$, the capacitor's positive plate starts moving away from the negative plate, at constant speed, s.t the distance between the plates, as a function of time is $d=d_0+ut$ (d_0,u are +ve consts). Throughout the experiment the plates satisfy $d\ll R$.



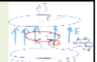
a) Calculate the Charge on the positive plate as a function of time. **Sol**: $Q(t)=CV=\epsilon_0\frac{\pi R^2}{d_0+ut}V$

b) What is the direction of the current in the circuit? Specify it in the figure explicitly, and justify your answer. **Sol**: The direction of I is clockwise due to the fact that charge on plates decreases with time.

c) Calculate the current in the circuit. **Sol**: $I=\left|\frac{dQ}{dt}\right|=\epsilon_0\frac{\pi R^2u}{(d_0+ut)^2}V$

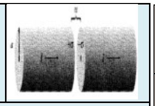
d) Find the electric field vector inside the capacitor as a function of time. **Sol**: $\bar{E}=\frac{Q}{\pi R^2}\frac{1}{\epsilon_0}\hat{z}=\frac{V}{d_0+ut}\hat{z}$

e) Find the magnetic field vector inside the capacitor (everywhere). **Sol**: Maxwell-Ampere Circuital Law in cylindrical coordinates: $\oint\bar{B}\cdot d\bar{r}=\mu_0\iint\bar{J}\cdot d\bar{A}+\mu_0\epsilon_0\iint\frac{\partial\bar{E}}{\partial t}\cdot d\bar{A}$. $B\cdot 2\pi r=0+\mu_0\epsilon_0\left(-\frac{Vu}{(d_0+ut)^2}\right)\pi r^2$. $\bar{B}(r,t)=\frac{\mu_0\epsilon_0Vur}{2(d_0+ut)^2}(-\hat{\varphi})$



f) Calculate the total displacement current inside the capacitor, and show that it is identical to the actual current in the circuit. **Sol**: $I_D=\epsilon_0\iint_{r=R}\frac{\partial\bar{E}}{\partial t}\cdot d\bar{A}=\epsilon_0\left(-\frac{Vu}{(d_0+ut)^2}\right)\pi r^2=-\frac{\epsilon_0Vu\pi R^2}{(d_0+ut)^2}=\frac{dQ}{dt}$. I_D agrees with I in both magnitude and "direction" (+ve flux direction)

A fat wire, radius a carries a constant current I, uniformly distributed over its cross section. A narrow gap in the wire, of width $w\ll a$ forms a parallel-plate capacitor. Find the magnetic field in the gap, at distance $s<a$ from the axis.

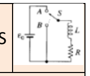


The displacement current density is $\bar{J}_D=\epsilon_0\frac{\partial\bar{E}}{\partial t}=\frac{I}{A}=\frac{I}{\pi a^2}\hat{z}$. Drawing an "Amperian loop" at radius s: $\oint\bar{B}\cdot d\bar{l}=B\cdot 2\pi s=\mu_0I_{D_{\text{enc}}}=\mu_0\frac{I}{\pi a^2}\pi s^2\Rightarrow B=\frac{\mu_0Is^2}{2\pi sa^2}=\frac{\mu_0Is}{2\pi a^2}\hat{\varphi}$

A long solenoid, radius a is driven by an alternating current, so that the field inside is sinusoidal: $\bar{B}(t)=B_0\cos(\omega t)\,\hat{z}$. A circular loop of wire, of radius $a/2$ and resistance R, is placed inside the solenoid, and coaxial with it. Find the current induced in the loop as a function of time.

$\Phi=\pi\left(\frac{a}{2}\right)^2$, $B=\frac{\pi a^2}{4}B_0\cos(\omega t)$; $\mathcal{E}=-\frac{d\Phi}{dt}=\frac{\pi a^2}{4}B_0\omega\sin(\omega t)$. $I(t)=\frac{\mathcal{E}}{R}=\frac{\pi a^2\omega}{4R}B_0\sin(\omega(t)$

Circuit in the figure was connected for a long time, when suddenly, at t=0, switch S is thrown, bypassing the battery.



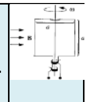
What is the current at any subsequent time ? Initial current: $I_0=\mathcal{E}_0/R$. So $-L\frac{dI}{dt}=IR\Rightarrow\frac{dI}{dt}=-\frac{R}{L}I\Rightarrow$ $I=I_0e^{-Rt/L}$ or $I(t)=\frac{\mathcal{E}_0}{R}e^{-Rt/L}$

What is the total energy delivered to the resistor? $P=I^2R=\left(\frac{\mathcal{E}_0}{R}\right)^2e^{-2Rt/L}R=\frac{\mathcal{E}_0^2}{R}e^{-2Rt/L}=\frac{dW}{dt}$. $W=\frac{\mathcal{E}_0^2}{R}\int_0^\infty e^{-\frac{2Rt}{L}}dt=\frac{\mathcal{E}_0^2}{R}\left[-\frac{L}{2R}e^{-\frac{2Rt}{L}}\right]_0^\infty=\frac{1}{2}L\left(\frac{\mathcal{E}_0}{R}\right)^2$

Show that this equals the energy originally stored in the inductor. $W_0=\frac{1}{2}LI_0^2=\frac{1}{2}\left(\frac{\mathcal{E}_0}{R}\right)^2$

A long solenoid with radius a and n turns per unit length, carries a time dependent current $I(t)$ in the $\hat{\varphi}$ direction. Find the electric field at a distance s from the axis| In the quasistatic approximation, $B=\begin{cases}\mu_0nI\hat{z}, & s<a\\ 0, & s>a\end{cases}$. Inside: For an "Amperian loop" of radius $s<a$, $\Phi=B\pi s^2=\mu_0nI\pi s^2$; $\oint\bar{E}\cdot d\bar{l}=E2\pi s=-\frac{d\Phi}{dt}=-\mu_0n\pi s^2\frac{dI}{dt}$; $\bar{E}=-\frac{\mu_0ns}{2}\frac{dI}{dt}\hat{\varphi}$. Outside: for an "Amperian loop" of radius $s>a$: $\Phi=B\pi a^2=\mu_0nI\pi a^2$; $E2\pi s=-\mu_0n\pi a^2\frac{dI}{dt}$; $\bar{E}=-\frac{\mu_0na^2}{2s}\frac{dI}{dt}\hat{\varphi}$

A square loop (side a) is mounted on a vertical shaft and rotated at angular velocity ω. A uniform magnetic field $\bar{B}$ is pointing to the right. Find $\mathcal{E}(t)$ for this alternating current generator.



$\Phi=\bar{B}\cdot\bar{a}=Ba^2\cos(\theta)$, here $\theta=\omega t$, so $\mathcal{E}=-\frac{d\Phi}{dt}=-Ba^2(-\sin(\omega t))\omega$. $\mathcal{E}=B\omega a^2\sin(\omega t)$