Proof of the proposition that BFD has convergence order 1:

$$|\delta_{-h} f(x) - f'(x)| = \left|\frac{f(x)-f(x-h)}{h} - f'(x)\right|.$$

For the expansion of $f(x-h)$, when h is small, we have

$$f(x-h) = f(x) - f'(x)\cdot h + O(h^2)$$

Then, $f(x) - f(x-h) = f'(x)\cdot h - O(h^2)$

$$|\delta_{-h} f(x) - f'(x)| = \left|\frac{f'(x)h - O(h^2)}{h} - f'(x)\right|$$

$$= \left|\frac{\cancel{f'(x)h}}{\cancel{h}} - \frac{O(h^2)}{h} \cancel{- f'(x)}\right|$$

$$= |-O(h)| = O(h)$$

Proof of the proposition that CFD has convergence order 2:

$$|\delta_{\pm h} f(x) - f'(x)| = \left|\frac{1}{2}(\delta_h f(x) + \delta_{-h} f(x)) - f'(x)\right|$$

$$= \left|\frac{1}{2}\left(\frac{f(x+h)-f(x)}{h} + \frac{f(x)-f(x-h)}{h}\right) - f'(x)\right|$$

$$= \left|\frac{1}{2}\left(\frac{f(x+h)-\cancel{f(x)}+\cancel{f(x)}-f(x+h)}{h}\right) - f'(x)\right|$$

$$= \left|\frac{f(x+h)-f(x-h)}{2h} - f'(x)\right|$$

When we expand $\frac{f(x+h)-f(x-h)}{2h}$, we have

$$\frac{f(x+h)-f(x-h)}{2h} = f'(x) + \frac{f'''(x)(h^2)}{6} + \dots$$

There is no h term due to the cancelation. So,

$$|\delta_{\pm h} f(x) - f'(x)| = O(h^2)$$