

Proof of the CFD second order has convergence order 1:

$$f(x+h) = f(x) + f'(x)(h) + \frac{f''(x)(h^2)}{2} + \frac{f'''(x)(h^3)}{6} + O(h^4)$$

$$f(x-h) = f(x) - f'(x)(h) + \frac{f''(x)(-h)^2}{2} - \frac{f'''(x)(h^3)}{6} + O(h^4)$$

$$\text{So, } f(x+h) + f(x-h) = 2f(x) + f''(x)(h^2) + \dots$$

$$\text{And, } f(x+h) + f(x-h) - 2f(x) = f''(x)(h^2) + \frac{f^{(4)}(x)(h^4)}{12} + O(h^4)$$

$$\text{So, } \frac{f''(x)(h^2) + O(h^4)}{h^2} = f''(x) + O(h^2)$$