03 December 2023 10:32

Inverse Kinematics:

$$
{}^{0}_{5}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & X \\ r_{21} & r_{22} & r_{23} & Y \\ r_{31} & r_{32} & r_{33} & Z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\begin{aligned} -s\theta_1 &= r_{12} \\ c\theta_1 &= r_{22} \end{aligned} \Rightarrow \boxed{\theta_1 = \mathrm{Atan2}(-r_{12}, r_{22})}
$$

1^{st} Case $\theta_1 \neq 0$

$$
s\theta_1 \left[L_4 c(\theta_2+\theta_4) + e\, s(\theta_2+\theta_4) - d_3 s\theta_2 \right] + (L_2+b)\, c\theta_1 = Y
$$

$$
\Rightarrow \frac{-Y + L_4 s\theta_1 c(\theta_2+\theta_4) + e\, s\theta_1 s(\theta_2+\theta_4) + (L_2+b) c\theta_1}{d_3 s\theta_1} = s\theta_2
$$

$$
\Rightarrow \frac{-Y + L_4 r_{21} + e\, r_{23} + (L_2+b) r_{22}}{-d_3 r_{12}} = \frac{Y - L_4 r_{21} - e\, r_{23} - (L_2+b) r_{22}}{d_3 r_{12}}
$$

$$
(a+L_1) - d_3 c\theta_2 - L_4 s(\theta_2+\theta_4) + e\, c(\theta_2+\theta_4) = Z
$$

$$
\Rightarrow c\theta_2 = \frac{-Z + (a+L_1) - L_4 s(\theta_2+\theta_4) + e\, c(\theta_2+\theta_4)}{d_3}
$$

$$
= \frac{(a+L_1) - Z + L_4 r_{31} + e\, r_{33}}{d_3}
$$

$$
\therefore \boxed{\theta_2 = \mathrm{Atan}\left(\frac{Y - L_4 r_{21} - e\, r_{23} - (L_2+b) r_{22}}{d_3 r_{13}}, \frac{(a+L_1) - Z + L_4 r_{31} + e\, r_{33}}{d_3} \right)}
$$

2^{nd} Case $\theta_1 \neq \frac{\pi}{2}$

$$c\theta_1\left[L_4\, c(\theta_2+\theta_4) + e\, s(\theta_2+\theta_4) - d_3\, s\theta_2\right] - (L_2+b)s\theta_1 = X$$

$$\frac{-X + L_4\, c\theta_1 c(\theta_2+\theta_4) + e\, c\theta_1 s(\theta_2+\theta_4) - (L_2+b)s\theta_1}{d_3\, c\theta_1} = s\theta_2$$

$$\therefore\quad s\theta_2 = \frac{-X + L_4\, r_{11} + e\, r_{13} + (L_2+b)\, r_{12}}{d_3\, r_{22}}$$

$$(a+L_1) - d_3\, c\theta_2 - L_4\, s(\theta_2+\theta_4) + e\, c(\theta_2+\theta_4) = Z$$

$$\frac{(a+L_1) - Z - L_4\, s(\theta_2+\theta_4) + e\, c(\theta_2+\theta_4)}{d_3} = c\theta_2$$

$$\therefore\quad c\theta_2 = \frac{(a+L_1) - Z + L_4\, r_{31} - e\, r_{33}}{d_3}$$

$$\therefore\quad \boxed{\theta_2 = \mathrm{Atan}\left(\frac{-X + L_4 r_{11} + e\, r_{13} + (L_2+b) r_{12}}{d_3 r_{23}}, \frac{(a+L_1) - Z + L_4 r_{31} + e\, r_{33}}{d_3}\right)}$$

$\Rightarrow$ Find d_3

1^{st} Case $\theta_2 \neq 0$ & $\theta_1 \neq 90$

$$c\theta_1\left[L_4\, c(\theta_2+\theta_4) + e\, s(\theta_2+\theta_4) - d_3\, s\theta_2\right] - (L_2+b)s\theta_1 = X$$

$$d_3 = \frac{-X + L_4\, c\theta_1 c(\theta_2+\theta_4) + e\, c\theta_1 s(\theta_2+\theta_4) - (L_2+b)s\theta_1}{c\theta_1\, s\theta_2}$$

$$\boxed{d_3 = \frac{-X + L_4 r_{11} + e\, r_{13} + (L_2+b) r_{12}}{c\theta_1\, s\theta_2}}$$

2^{nd} Case $\theta_2 \neq 90$

$$(a+L_1) - L_4 s(\theta_2+\theta_4) + e\, c(\theta_2+\theta_4) - l_3 c\theta_2 = Z$$

$$l_3 = \frac{(a+L_1) - L_4 s(\theta_2+\theta_4) + e\, c(\theta_2+\theta_4) - Z}{c\theta_2}$$

$$\therefore \boxed{l_3 = \frac{a+L_1 + L_4 r_{31} + e\, r_{33} - Z}{c\theta_2}}$$

$\Rightarrow$ For θ_4 :

$$c(\theta_2+\theta_4) = r_{33}$$
$$s(\theta_2+\theta_4) = -r_{31}$$
$$\Rightarrow \theta_2 + \theta_4 = \mathrm{Atan2}(-r_{31}, r_{33})$$

$$\therefore \boxed{\theta_4 = \mathrm{Atan2}(-r_{31}, r_{32}) - \theta_2}$$