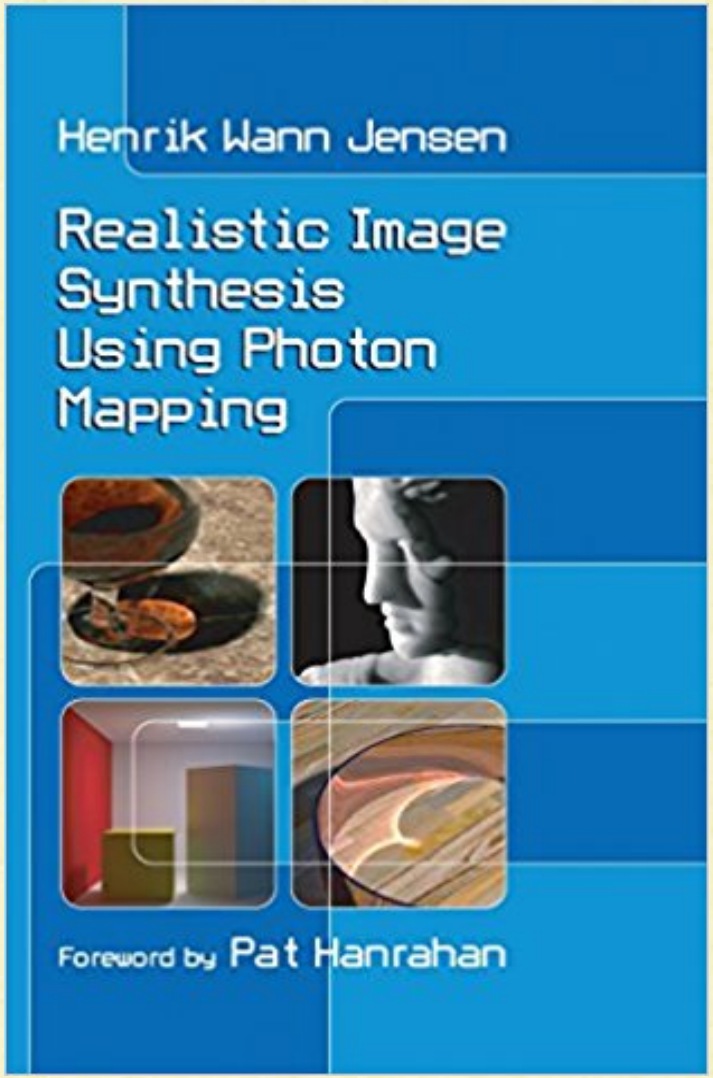
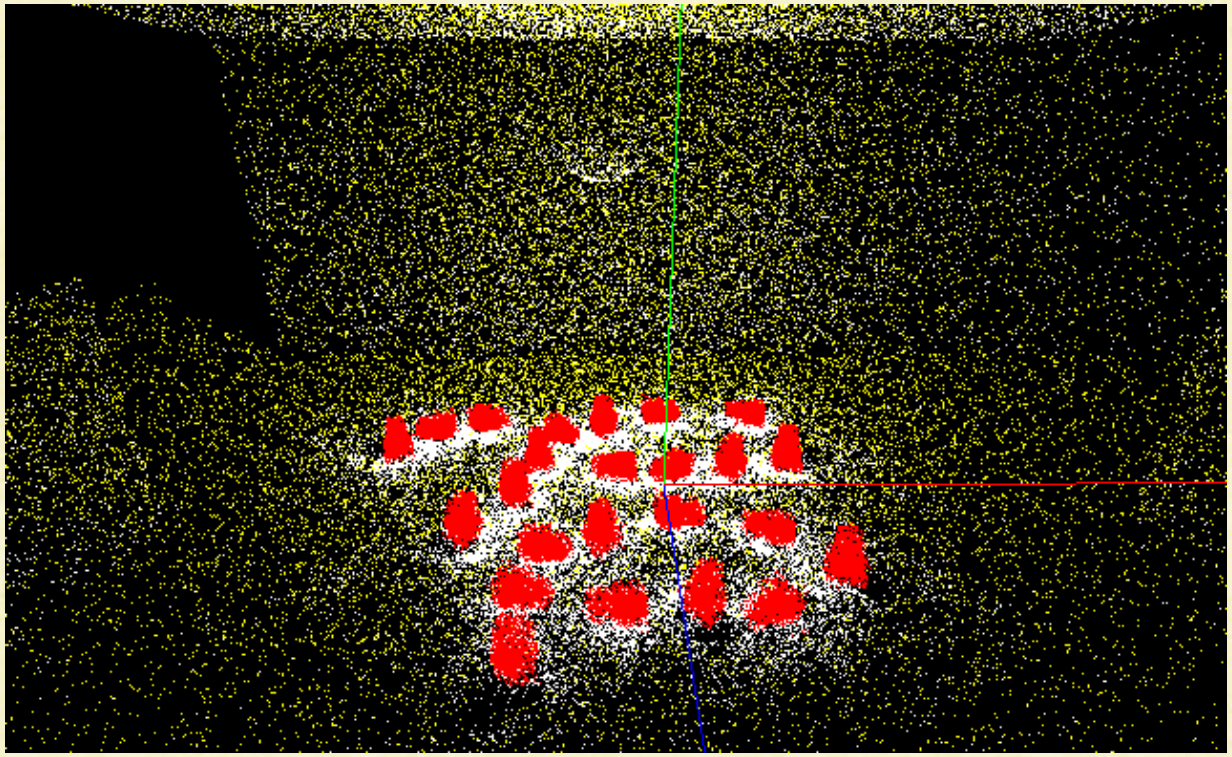
Photon Mapping



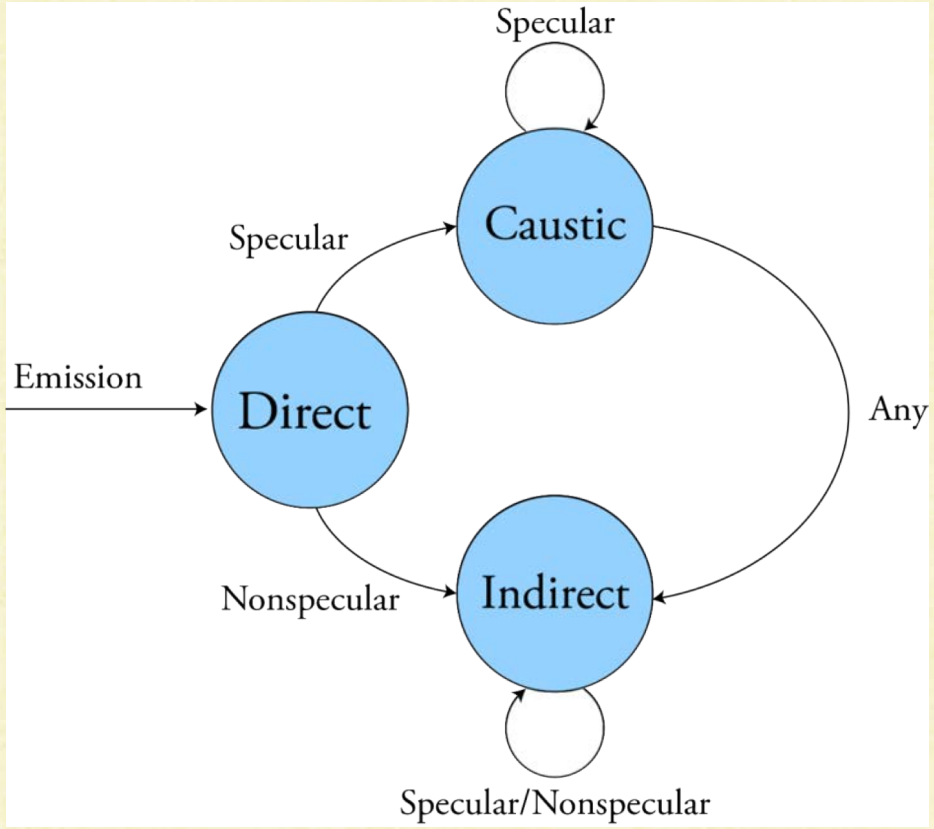
Photon Maps

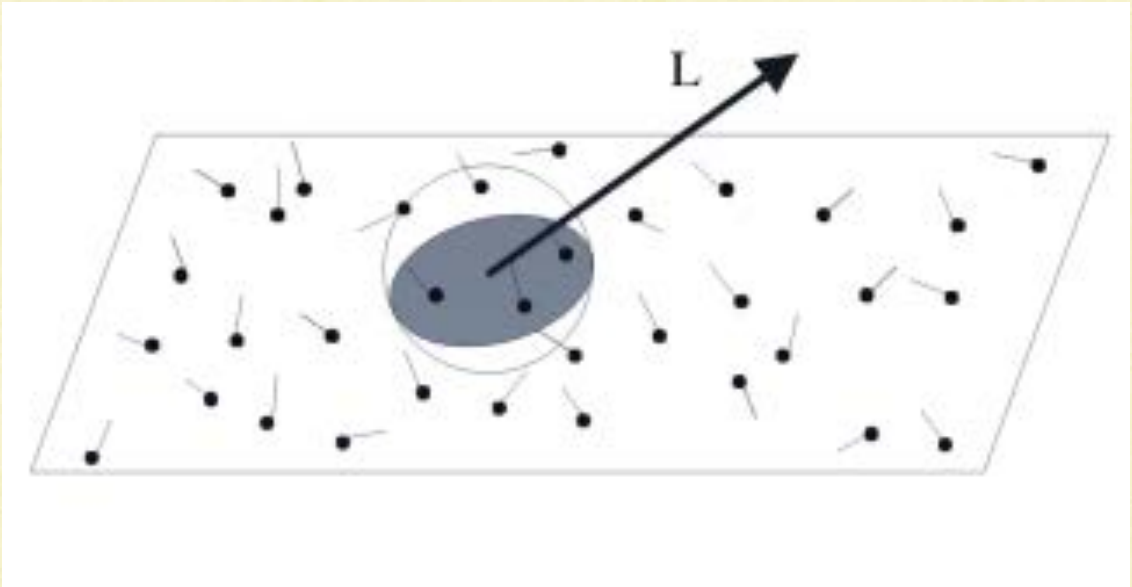
- Photon maps store lighting information on points or “photons” in 3D space
  - Stored either on or near 2D surfaces
- Recall: Radiosity stored information on surfaces patches/triangles (last lecture)



Photon Maps

- Photons are emitted from light sources and bounce around the scene creating lighting information (left image), which is stored in the photon map
- Later, one can query nearby information from the photon map in order to estimate global illumination (right image)





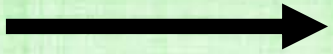
Avoiding Radiosity Simplifications

- The global illumination approach (in the prior lecture) discretized surfaces into triangles, and discretized the hemisphere as well
- This discretization into "elements" is a finite element (FEM) approach to the integral, and similar to Newton-Cotes quadrature
- 3D space + 2D angles = 5D (or 4D ignoring participating media)
- Newton-Cotes quadrature suffers from the curse of dimensionality, and thus we assumed purely diffuse lighting to reduce the dimensionality (for tractability)
- Integrating over angles (radiosity approach) dropped the problem to 2D (or 3D for participating media)
- But then one cannot address the specular term!

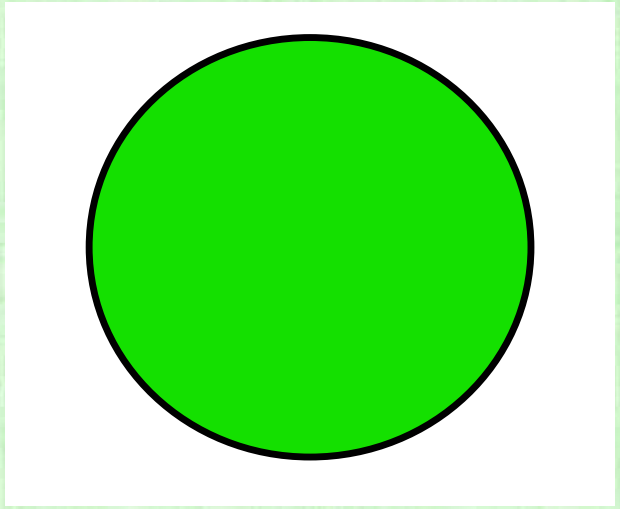
- Alternatively, Monte Carlo integration (which is less accurate than Newton-Cotes quadrature) scales well on higher dimensional problems (no curse of dimensionality)
- Monte Carlo allows one to tackle the full lighting equation in 4D and 5D, without assuming purely diffuse lighting

A Simple Example

- Consider approximating $\pi = 3.1415926535\ldots$
- Use a compass to construct a circle with radius = 1
- Since $A = \pi r^2$, the area of the circle is π
- Setting $f(x, y) = 1$ gives $\iint_A f(x, y)dA = \pi$

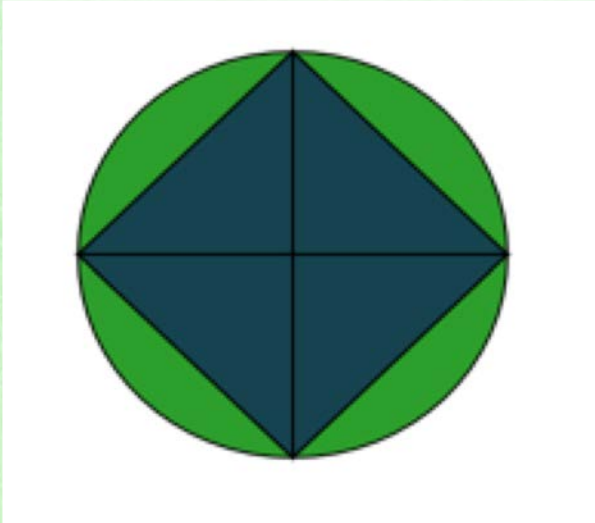


$A = \pi$

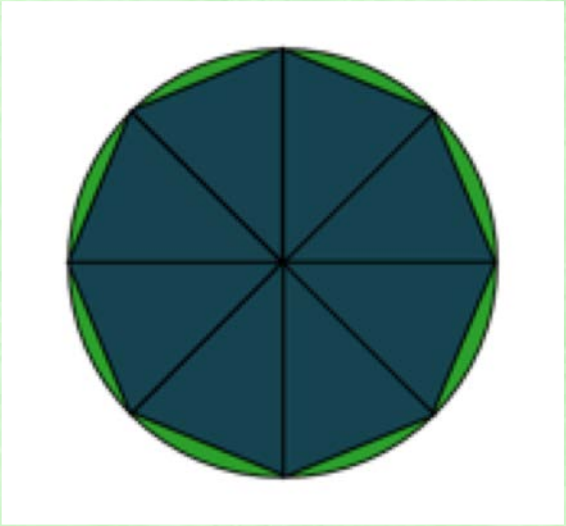


Newton-Cotes Approach

- Inscribe triangles inside the circle
- The function $f(x, y) = 1$ dictates computing the area of each triangle
- The difference between A and its approximation with triangles leads to errors



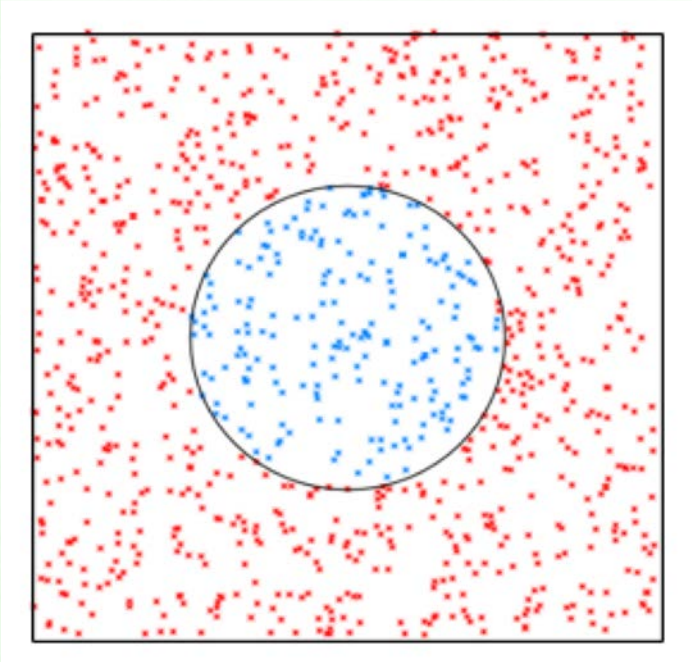
$\pi \approx 2$



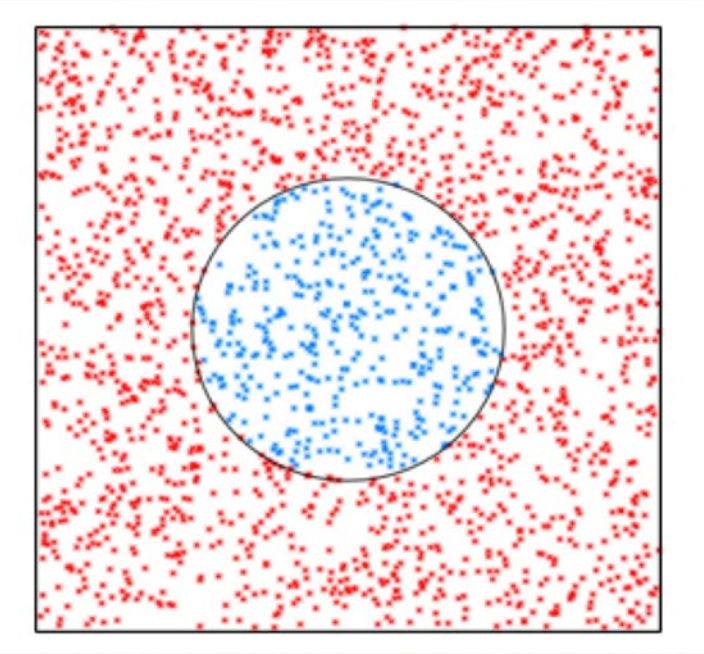
$\pi \approx 2.8284$

Monte Carlo Approach

- Construct a square with side length 4 containing the circle
- Randomly generate N points in the square, and color points inside the circle blue
- Since $\frac{A_{circle}}{A_{box}} = \frac{\pi}{16}$, one can approximate $\pi \approx 16\left(\frac{N_{blue}}{N_{total}}\right)$



$\pi \approx 3.136$



$\pi \approx 3.1440$

Monte Carlo Methods

- Typically used in higher dimensions (5D or more)
- Random (pseudo-random) numbers generate sample "points" that are multiplied by element "size" (e.g. length, area, volume)
- Error decreases like $\frac{1}{\sqrt{N}}$ where N is the number of samples (1/2 order accurate)
  - E.g. 100 times more sample points are needed to gain one more digit of accuracy

- Very slow convergence, but independent of the number of dimensions!
- Not competitive for lower dimensional problems, but the only tractable alternative for higher dimensional problems

Review: Random Numbers

- **Random variables** - expressions whose value is the outcome of a random experiment
- **Sample space** - set of all possible outcomes
- **Probability distribution** $p(x)$ - probability of selecting each outcome in the sample space
- **Sample** - value of a random variable chosen from the sample space with probability determined by $p(x)$
- **Pseudo-Random Number Generator** (PRNG) - deterministic algorithm that generate sequences of quasi-"random" numbers based on an initial **seed** (starting point in the pre-determined sequence)
  - PRNGs typically generate a (pseudo) random real number between 0 and 1 with equal (**uniform**) probability
  - Uniformly sampling [0,1] enables the sampling of other sample spaces that have non-uniform probabilities

Monte Carlo Integration (in 1D)

- Consider: $\int_a^b f(x)dx$

- Generate N samples X_i uniformly in the interval $[a, b]$
- A Monte Carlo estimate for the integral is then defined as:

$$F_N = \frac{1}{N}\sum_{i=1}^{N}(b-a)f(X_i)$$

- This is a simple averaging of all the sample results

- The result of Monte Carlo integration is said to be the expected value of the estimate

Importance Sampling

- Suppose $f(x)$ is only nonzero in a subset $[a_1, b_1]$ of $[a, b]$, i.e. $\int_a^b f(x)\mathrm{d}x = \int_{a_1}^{b_1} f(x)\mathrm{d}x$
- Then one only needs samples $X_i \in [a_1, b_1]$, since samples outside $[a_1, b_1]$ do not contribute to the integral
- Change $p(x)$ from a uniform distribution over $[a, b]$ to a uniform distribution over $[a_1, b_1]$ in order to be more prudent/efficient with sampling

- More generally, the probability distribution $p(x)$ should prefer samples in areas with higher contributions (**importance**) to the integral
- For a general $p(x)$ with $\int_a^b p(x)\mathrm{d}x = 1$, the Monte Carlo estimate is given by:

$$F_N = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{p(X_i)}f(X_i)$$

- When uniformly sampling with $p(x) = \frac{1}{b-a}$, this reduces to $F_N = \frac{1}{N}\sum_{i=1}^{N}(b-a)f(X_i)$

Importance Sampling

- Monte Carlo estimates for $\int_0^1 x^2 \mathrm{d}x$ with $N = 100$ samples:

$p(x)$	Estimate using F_N	Relative Error
1	0.33671	1.01%
$2x$	0.33368	0.105%
$3x^2$	$0.3333\overline{3}$	0.000%

- Importance sampling does not necessarily reduce error (and can give worse results)
- The more $p(x)$ "resembles" $f(x)$, the lower the error
  - E.g., setting $p(x) = \frac{f(x)}{\int_a^b f(x)\mathrm{d}x}$ gives the exact solution with a single sample
  - However, this $p(x)$ depends on the integral one is trying to compute
- Choose $p(x)$ based on physical principles or an approximate solution

Photon Emission

- Choose some number of photons, and divide amongst the lights based on their relative power
  - For efficiency/implementation, every photon is set to be the same strength
  - Brighter lights emit more photons instead of higher energy photons
- Using random numbers (Monte Carlo), emit all of a light's photons into the scene
  - Point lights - all photons emitted from a single point
  - Area lights - use random numbers to select a point to emit each photon
    - Semi-random: Divide rectangular light into a uniform 2D grid, and emit a number of photons per grid cell (from a random sub-cell position)
- Emission direction - use random numbers to choose a direction on the sphere, hemisphere, or subset of the sphere (for a spotlight), etc.
- For some scenes (e.g. outdoors - consider the sun), many/most photons will miss the scene entirely
  - As an optimization, ignore those photons (never emit them)
  - Only generate/emit photons for the sub-light region that hits your scene
  - Scale down the energy of the sub-light when dividing up photons

Photon Storage

- Use the ray tracer to find the first piece of geometry a photon intersects
- Every time a photon intersects a surface, its data is added to the photon map to represent **incoming light**
- Create a duplicate photon to store in the photon map
  - don't delete the current photon
- Store the photon's current location in 3D space along with the incoming ray direction that brought it to that location
  - No need to record the energy (since all photons have equal energy)
- Don't delete the original photon, or move it into the photon map
- *May* still need to bounce the photon around a bit more

Photon Absorption

- After storing a photon's data in the photon map, use random numbers to determine what happens to the photon next
- There is some chance that the photon is absorbed by the surface
  - Objects absorb some incoming light, which is why they have a color
- Absorbing a fraction of the photon's energy would result in unequal energy photons
- Instead, use the fraction of light energy that would be absorbed to state a probability that the photon is absorbed
- Generate a uniform random number and compare it to the probability of absorption to see if the photon is absorbed or not (Russian Roulette)
- When absorbed, the process stops
- Otherwise, bounce the current photon, and follow it to the next surface

Photon Bouncing

- Bouncing photons need a new direction
- Compute it by mapping the directions of the BRDF into probabilities
  - E.g. a purely diffuse BRDF has equal probability for all directions on the hemisphere
- A random number is generated and looked-up in the BRDF table to determine the bounce direction
- The photon travels off in this new direction, until it intersects another surface (use the ray tracer)
- The new surface intersection location and the incoming light direction are stored in the photon map
- Again, check the photon for absorption, and if not absorbed, bounce again, etc.
- Set a maximum number of bounces before termination
  - Can be set rather high as photons typically have a diminishing chance of avoiding absorption as the number of bounces increases

Photon Map

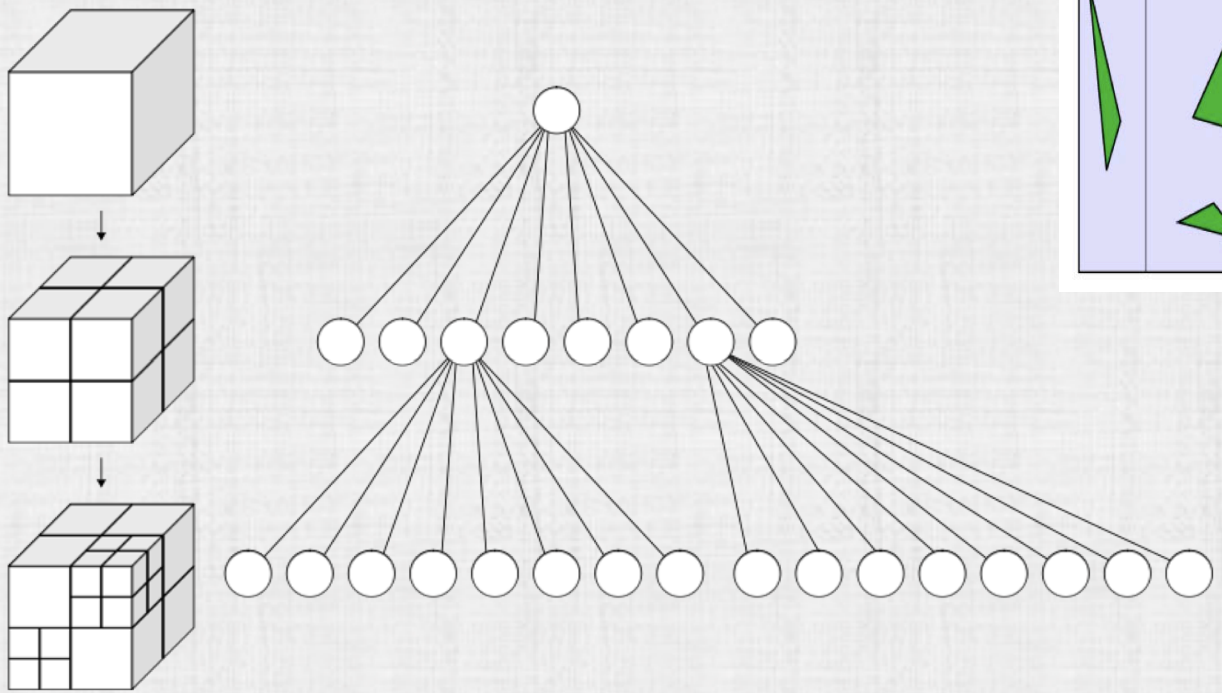
Physically Based Rendering by Pharr and Humphreys

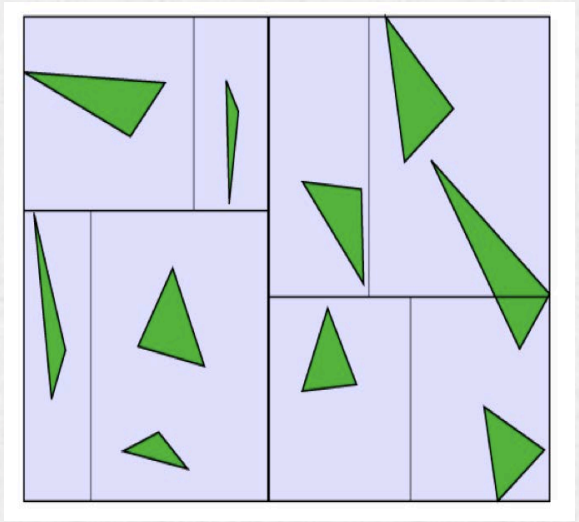
Rendered Image

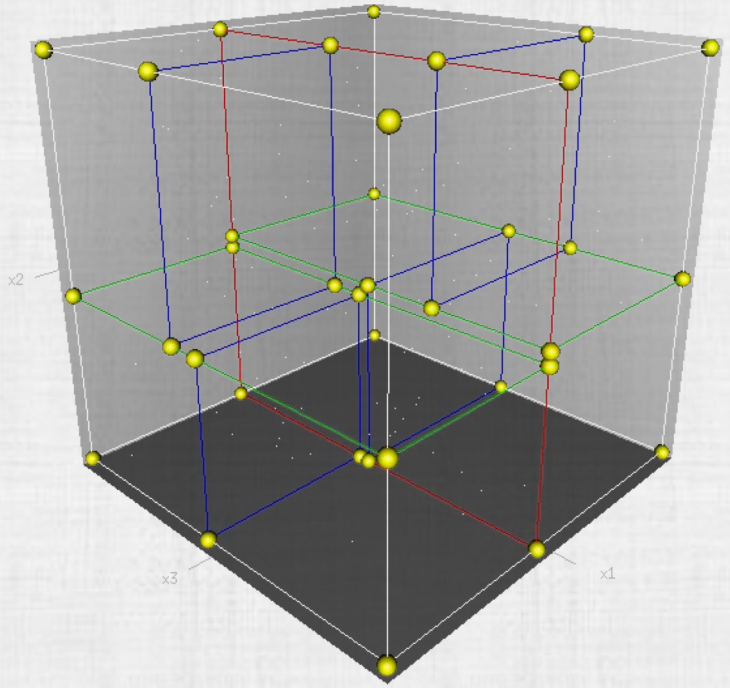
Physically Based Rendering by Pharr and Humphreys

Aside: Code Acceleration

- Photons are typically stored in octree or K-D tree acceleration structures, so that the information they contain is more efficiently retrieved







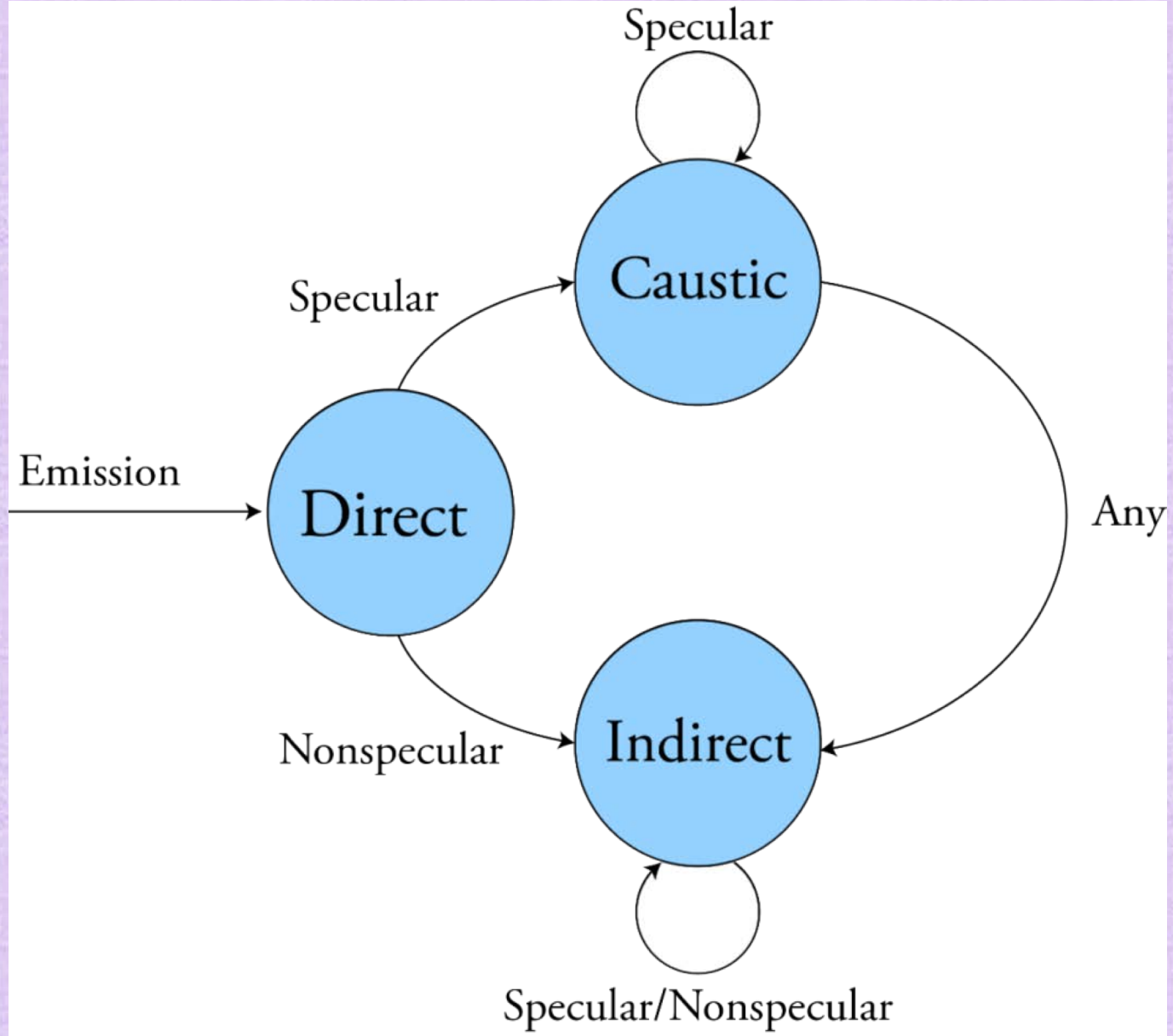
Direct Lighting

- It’s more accurate to evaluate direct lighting using shadow rays, rather than interpolating lighting from the photon map
- Thus, the first time a photon emitted from a light source hits an object, it is not stored in the photon map
- This makes the photon map a lot more efficient, since one doesn’t need to store any photons for direct illumination

Separate Diffuse/Specular Photon Maps

- It's more convenient to create separate BRDFs for diffuse and specular lighting
- When bouncing a photon, first use a random number to determine if the photon undergoes:
  - absorption (deleted), or a diffuse bounce, or a specular bounce
- Then, a second random number is used to pick the direction for the diffuse or specular bounce
- Create two photon maps:
  - Caustic photon map stores photons that have undergone only specular bounces up to the point at which they are stored in the map
  - Indirect lighting map stores any photon that has ever undergone at least one diffuse bounce

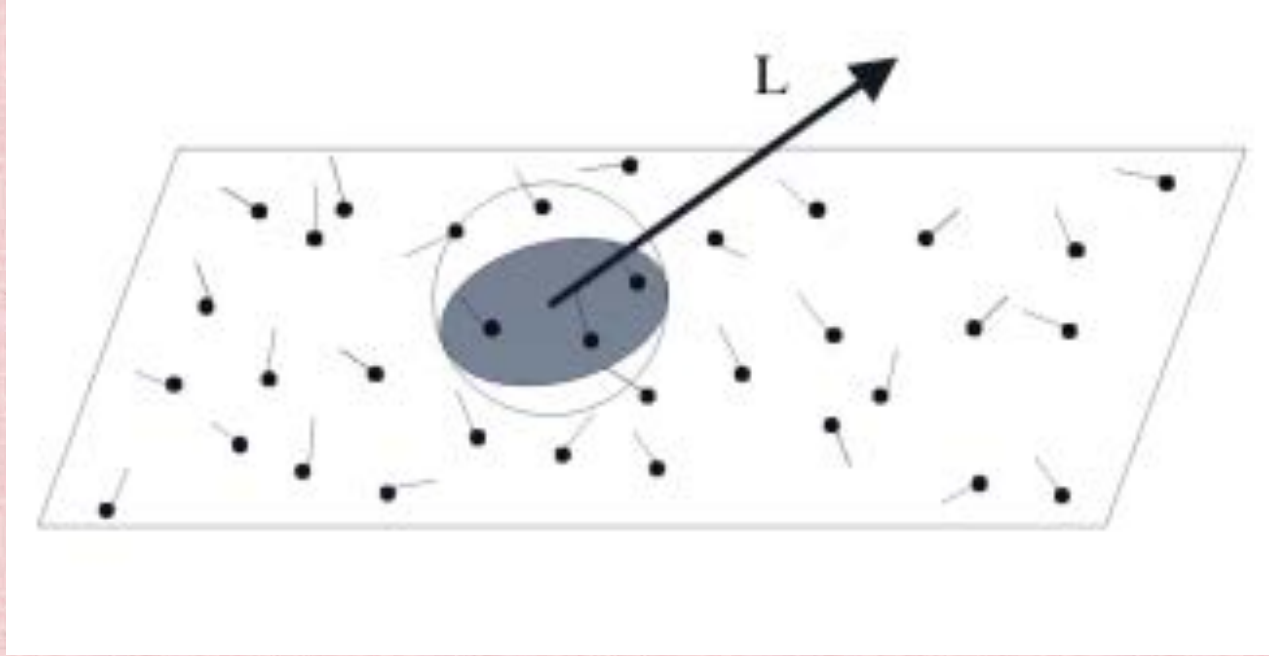
Separate Diffuse/Specular Photon Maps



Specular Photon Map for Caustics

Gathering Radiance

- Trace rays from the camera and intersect with objects (as usual)
- Use shadow rays for direct lighting (as usual)
- Estimate the radiance contribution to the ray from caustics and indirect lighting using the respective photon maps:
  - Find the N closest photons to the point of intersection (with the aid of an acceleration structure to store the photons: an octree or KD tree)



Color

- Create 3 photon maps, one for each color channel: Red, Green, Blue
- Objects of a certain color better absorb photons of differing colors, creating differences in the photon maps
- This gives color bleeding and related effects