13-Randomized Block Designs

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[1]Thanks to my family, my friend and freedom.

1 The F test for a randomized block design

For randomized block data Y_{ij}, where i represents the i^{th} block and j respresents the j^{th} treatment, the model equation is

$$Y_{ij} = \mu_j + \beta_i + \epsilon_{ij}$$

On the basis of SSTR and SSE, we define the **block sum of squares** (SSB) by

$$SSB = \sum_{i=1}^{b} \sum_{j=1}^{k} (\bar{Y}_{i.} - \bar{Y}_{..})^2$$

Since the variables belong to the same block are dependent, the formula of SSE is defined by

$$SSE = \sum_{i=1}^{b} \sum_{j=1}^{k} (Y_{ij} - \bar{Y}_{.j} - \bar{Y}_{i.} + \bar{Y}_{..})^2$$

The next theorem describes the relation among SSTOT, SSTR, SSE, and SSB.

Theorem 1.1. *Suppose that k treatment levels are measured over a set of b blocks. Then*

1. $SSTOT = SSTR + SSB + SSE$.
2. *$SSTR$, SSB, and SSE are independent random variables.*

Theorem 1.2. *Suppose that k treatment levels, with means $\mu_1, \cdots, \mu_k$, are measured over a set of b blocks, where the block effects are $\beta_1, \beta_2, \cdots, \beta_b$. Then*

1. *When $H_0 : \mu_1 = \mu_2 = \cdots = \mu_k$ is trus, $SSTR/\sigma^2$ has a χ^2 distribution with $k-1$ degrees of freedom.*
2. *When $H_0 : \beta_1 = \beta_2 = \cdots = \beta_k$ is trus, SSB/σ^2 has a χ^2 distribution with $b-1$ degrees of freedom.*
3. *Regardless of whether the μ_j's or the β_i's are equal, SSE/σ^2 has a χ^2 distribution with $(b-1)(k-1)$ degrees of freedom.*

Of course, we can also test the hypothesis $\mu_1 = \cdots = \mu_k$ and $\beta_1 = \cdots = \beta_b$ by F test.

2 Tukey comparisons for randomized block data

Theorem 2.1. *Let $\bar{Y}_{.j}$, $j = 1, 2, \cdots, k$, be the sample means in a $b \times k$ randomized block design. Let μ_j be the true treatment means, $j = 1, 2, \cdots, k$. The probability is $1 - \alpha$ that all $\binom{k}{2}$ pairwise subhypothesis $H_0 : \mu_s = \mu_t$ will simultaneously satisfy the inequalities*

$$\bar{Y}_{.s} - \bar{Y}_{.t} - D\sqrt{MSE} < \mu_s - \mu_t < \bar{Y}_{.s} - \bar{Y}_{.t} + D\sqrt{MSE}$$

where $D = Q_{\alpha,k,(b-1)(k-1)}/\sqrt{b}$. *If, for a given* s *and* t, *zero is not contained in the preceding inequality,* $H_0 : \mu_s = \mu_t$ *can be rejected in favor of* $H_1 : \mu_s \neq \mu_t$ *at the* α *level of significance.*

In the case that $k = 2$, we can use the method similar to two-sample inferences.

Finally, the paired t test and the randomized block test are equal when $k = 2$.