Equity valuation with Discounted Cash Flows

Research internship at Capital Fund Management

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M2 Mathématiques, Vision, Apprentissage

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Motivation:

- Define investment and trading strategies by predicting prices in the short-term.
- Update present value estimations of equities more often than what analysts report information.

Objectives:

- Analyse the Discounted Cash Flows formula to use it as an equity valuation model.
- Try to explore and propose new models using DCF as basis and adding complexity to it.
- Use these models to estimate prices after analysts speak.

Discounted Cash Flows (DCF)

- DCF is a financial valuation method used to estimate the present value of an investment by discounting its expected future cash flows.
- **Time value of money:** Money received today is worth more than the same amount received in the future.
- Since we do not have estimations for all the future cash flows we extrapolate using a constant long-term growth (g)

$$PV = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} + \frac{CF_n}{(1+r)^n} \frac{(1+g)}{(r-g)}$$

- CF_t is the expected Cash Flow at the year t
- r is the discount rate
- g is the long term growth

- We give an example where we have 3 cash flows and we extrapolate the following ones.
- The discount rate gives less weight to those cash flows distant in time

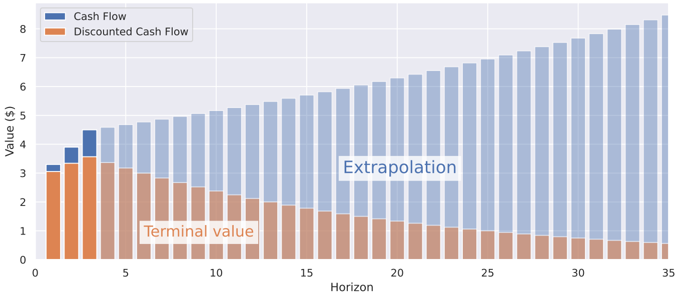

Figure 1: Series of cash flows and its discounted version.

Internal Rate of Return (IRR)

- We define the Net Present Value as:

$$NPV = \sum_t \frac{CF_t}{(1+r)^t} - C_0$$

where C_0 is the initial investment, in our case the price of the equity.

- The **internal rate of return (IRR)** is the discount rate that makes the NPV equal to zero.
- For us, the IRR is the rate that makes the Present Value given our estimated cash flows be equal to the price.

Data for DCF

- We will employ the DCF formula to estimate the present value of a single share of a given equity.
- What data do we need for the DCF formula?
  - CF_t: For the series of cash flows we will use **Earnings Per Share (EPS)** forecasts for the following years (fiscal periods).
  - g: The long term growth is set to the historical economy GDP growth, an average of 2% ($g = 0.02$).
  - r: The Internal Rate of Return (IRR) will be used as the discount rate (r) calculated using the close price of the present day as the present value.

Our universe and data

- Our universe is composed of approximately 2900 equities from America, mostly USA.
- For most of the equities we have $n = 3$ EPS forecasts. Estimations come from Institutional Brokers' Estimate System (IBES) analysts.
- Analysts "speak" with varying frequency depending on company and date, some equities have frequent updates and others longer gaps.

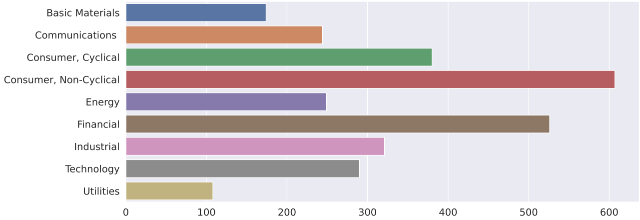

Figure 2: Sectors breakdown of the equities.

Baseline model

- For each equity, every time an analyst speaks we calculate the IRR using the close price of the date.
- To predict the price in the following days, we use the last EPS estimated and the same discount rate (IRR).
- Since we are not changing any parameter in the DCF formula, our estimation of the price is forward filling it.
- Despite its simplicity and being naive, it is useful as **a benchmark**.

Modelling with DCF

- We want to modify the DCF formula in order to model better the development of prices.
- Many equities are **exposed in different countries** with diverse economies.
- Each country could have a different discount rate, we could reformulate DCF to take this into account.
- We will examine the correlation and correspondence between some **macroeconomic variables** and the prices.
- Using this data we will add complexity to the DCF formula.

Country re-summation with DCF

- The classic DCF model can be modified to make a re-summation by country.

$$PV = \sum_{c} \left(\sum_{t=1}^{n} \frac{CF_t w_c}{(1 + r_0 + r_c)^t} + \frac{CF_n w_c}{(1 + r_0 + r_c)^n} \frac{(1 + g)}{(r_0 + r_c - g)} \right)$$

- We have a compound rate $r = r_0 + r_c$ where
  - r_0 is a company-intrinsic component of the rate
  - r_c is a country specific component of the rate.
- For the weight of the country w_c, we use the country exposure reported by the equity.

Macroeconomic Variables

Some macroeconomic variables we will be using to model the country specific rate component r_c:

- **Government Bond Yield (y):** the return an investor can expect to earn from investing in a government-issued bond.
- **Inflation Swap (IS):** will be used to speculate on long-term inflation.
- **Consumer Price Index (CPI):** another widely used measure of inflation in an economy (short-term).
- **Credit Default Swap (CDS):** used as a measure of risk and to speculate on the creditworthiness of countries.
- **Sovereign credit rating:** another measure of creditworthiness of a country, conducted by credit rating agencies (such as S&P)

Proposed models

- To begin with, we will add inflation swap of the reporting currency (IS_{cur}) to the long term growth: $g = 2\% + IS_{cur}$
- For the country rate component, we want to include the risk of each country.
- We have different models of r_c using different macroeconomic objects:
  1. Government bond yields
  2. Estimate the yield using CDS and IS.
  3. Use CPI in the first horizon.
- Given r_c and g we will solve for r_0 (as done with the IRR) when analysts speak.
- For the following days we can predict the prices using the changes in r_c and inflation.

Model 1

- Our first model uses the government bond yield (in local currency) y_c for each country:

$$PV = \sum_c \left(\sum_{t=1}^{n} \frac{CF_t w_c}{(1 + r_0 + y_c)^t} + \frac{CF_n w_c}{(1 + r_0 + y_c)^n} \frac{(1+g)}{(r_0 + y_c - g)} \right)$$

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- We have bond yield data for 28 countries.
- This model has a problem: EPS in reporting currency and yield in local currency $\rightarrow$ generates distortions for countries with high inflation.

- **Intuition:** for riskier countries, we could model their yields as:

$$y_c' = \text{risk-free yield} + \text{country risk premium}$$

- This would solve the inflation mismatch problem. With our data we have:

$$y_c' \approx y_{US} + CDS_c - CDS_{US}$$

- The non-arbitrage relationship is (π is inflation):

$$y_c \approx y_{US} + CDS_c - CDS_{US} + \pi_c - \pi_{US}$$

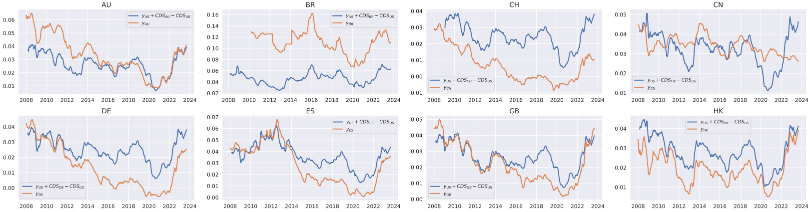

Figure 3: y_c' vs. y_c for some countries.

Model 2

- Our second model takes $r_c = y'_c + IS_{cur} - IS_{US}$ adding only the inflation of the reporting currency.

$$PV = \sum_c \left(\sum_{t=1}^{n} \frac{CF_t w_c}{(1 + r_0 + y'_c + IS_c - IS_{US})^t} + \frac{CF_n w_c}{(1 + r_0 + y'_c + IS_c - IS_{US})^n} \frac{(1+g)}{(r_0 + y'_c + IS_c - IS_{US} - g)} \right)$$

- We have CDS data for 90 countries, but Inflation Swaps for only 5 currencies.

Implied inflation

- Looking at the difference between y_c and y_c' and our approximation for the bond yield we have an estimation for an "implied inflation":

$$\left.\begin{aligned} y_c &\approx y_{US} + CDS_c - CDS_{US} + \pi_c - \pi_{US} \\ y_c' &= y_{US} + CDS_c - CDS_{US} \end{aligned}\right\} \quad \boxed{y_c - y_c' \approx \pi_c - \pi_{US}}$$

- We can add the inflation swap of US to have an estimation of the inflation in absolute terms: $II_c = y_c - y_c' + IS_{US}$

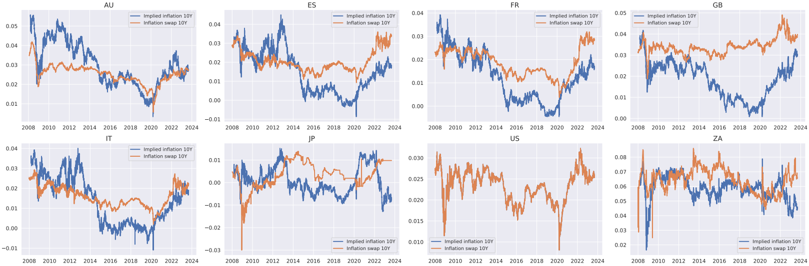

Figure 4: Comparison of inflation swaps and implied inflation

Model with CPI

- We can modify the previous model to include CPI in the first horizon instead of IS.

$$PV = \sum_c \Bigg(\frac{CF_1 w_c}{1 + r_0 + y'_c + CPI_c - CPI_{US}} + \sum_{t=2}^{n} \frac{CF_t w_c}{(1 + r_0 + y'_c + IS_c - IS_{US})^t} + \frac{CF_n w_c}{(1 + r_0 + y'_c + IS_c - IS_{US})^n} \frac{(1+g)}{(r_0 + y'_c + IS_c - IS_{US} - g)} \Bigg)$$

- We have CPI forecasts for 30 countries but data is only available from 2018.
- This model does not consider the same discount rate for each horizon.

Mono-exposed equities

- We consider the particular case were the equity is exposed by 90% or more in only one country.
- In this cases, if the EPS are reported in local currency, we can use directly the yield without having problems with the inflation.
- The formula is almost as simple as the original DCF formula:

$$PV = \sum_{t=1}^{n} \frac{CF_t}{(1 + r_0 + y_C)^t} + \frac{CF_n}{(1 + r_0 + y_C)^n} \frac{(1 + g)}{(r_0 + y_C - g)}$$

- The model becomes much simpler and lets us analyse results and variables more easily

Metrics

- We define metrics to compare PV estimations and prices.
- Absolute Percentage Error (APE):

$$APE(PV, p) = \frac{|PV - p|}{p}$$

- Squared error between normalized ranks (η):

$$\eta(PV, p) = (NR_V(PV) - NR_P(p))^2$$

  where $NR_X(x)$ is the rank of element x in group X normalized in $[0, 1]$.
- For each day we can measure the performance of all equities using these metrics.

Evaluation Methods

- Every time an analyst speaks we update the discount rate (r_0) and then we can evaluate using two methods.
- **Error between analyses:**
  - The model is updated with the macroeconomic variables as days pass by.
  - Evaluations are group according to how many days have passed since the last analysis.
- **Error in next analysis:**
  - When there is a new analysis, we take the average rate r_0 for the five last analyses.
  - The model is evaluated with the new EPS forecasts and updated macroeconomic variables
  - Evaluations are grouped according to the date.

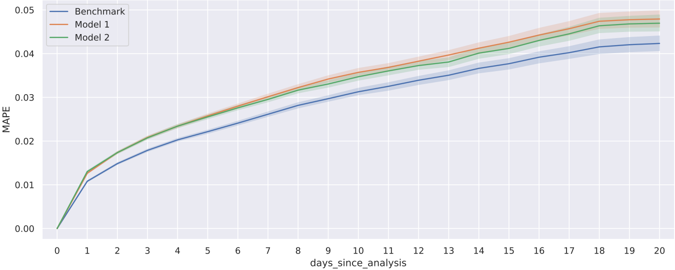

Figure 5: Error between two analyses using MAPE.

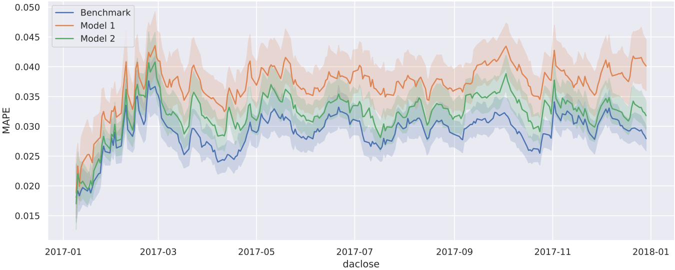

Figure 6: Error in the next analysis using MAPE.

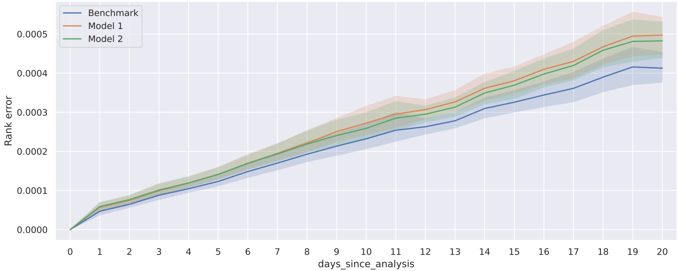

Figure 7: Error between two analyses using rank error.

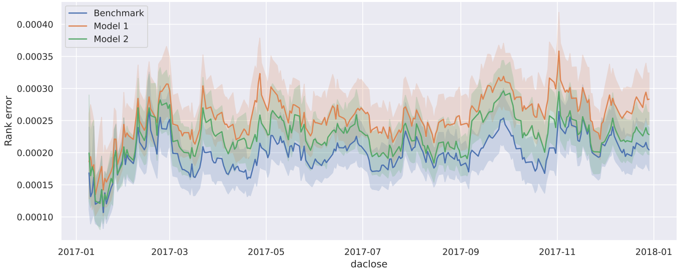

Figure 8: Error in the next analysis using rank error.

- None of the previous models outperforms the benchmark.
- The CPI model gets closer to it but it is still worse.

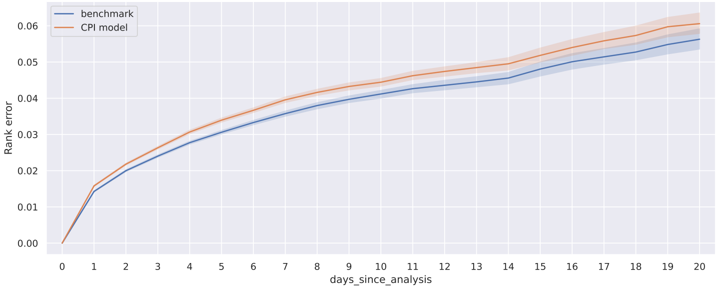

Figure 9: Error between analyses using MAPE.

Mono-exposed equities

- We tried smoothing the yield to reduce the noise of market volatility and daily fluctuations.
- The bigger the smoothing window, the closer the model is to the benchmark (as a model)

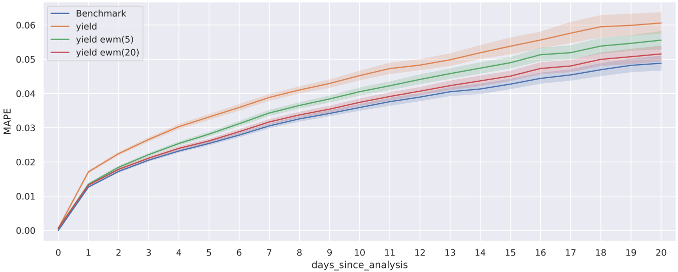

Figure 10: Error for USA using different smoothings of the yield.

- In the case of Brazil we have a more high and variable yield: there is a slight improvement.
- There are only 41 equities mono-exposed in Brazil (in our data).

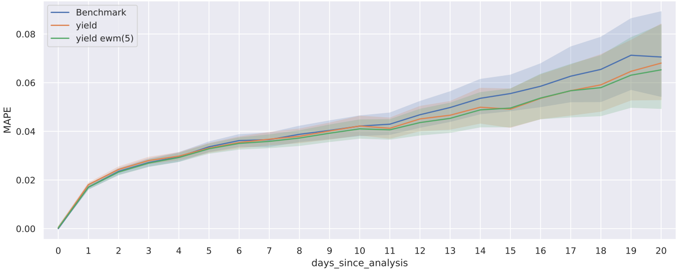

Figure 11: Error for Brazil using different smoothings of the yield.

Sectors Breakdown

- Looking at the different sectors, we see that Utilities outperforms the benchmark after a few days since analysis.
- Companies in the utilities sector could have a stronger correlation with the yield because they are regulated by public commissions, and they usually pay high dividends.

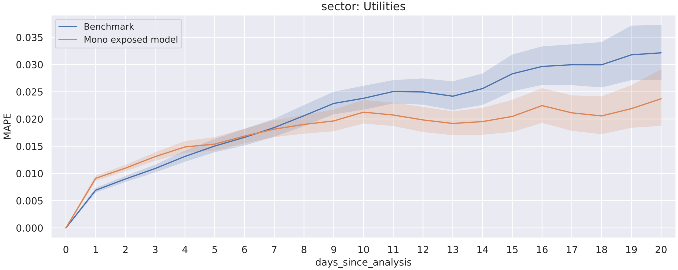

Figure 12: Performance of equities mono-exposed in US belonging to Utilities sector.

Conclusions

- Although most of the models do not outperform our baseline, for some particular cases there is an improvement.
- The use of macroeconomic variables cannot consider all the price changes in the wide variety of equities.
- The models could work better under certain conditions, like Utilities, or others yet to be explored.
- Predicting prices in the very short term can be challenging because of market volatility, but trying to predict trends after a few days seems to be a more achievable task.