1. (5 pts each, 30 pts total) Mark the following reasoning patterns as S (= sound, carries true premises to true conclusions) or U (= unsound, may carry true premises to false conclusions). Premises are shown above the line, conclusions below the line. Here, " $\Rightarrow$ " means "implies" and " $\neg$ " means "not." The first one is done for you as an example.

a. S

$$\begin{array}{l} P \Rightarrow Q \\ P \\ \hline Q \end{array}$$

b. U

$$\begin{array}{l} P \Rightarrow Q \\ Q \\ \hline P \end{array}$$

c. U

$$\begin{array}{l} P \Rightarrow Q \\ \\ \hline P \text{ or } \neg Q \end{array}$$

d. S

$$\begin{array}{l} P \Rightarrow \neg Q \\ Q \\ \hline \neg P \end{array}$$

e. S

$$\begin{array}{l} P \Rightarrow Q \\ \neg Q \\ \hline \neg P \end{array}$$

f. S

$$\begin{array}{l} P \Rightarrow Q \\ \\ \hline \neg P \text{ or } Q \end{array}$$

g. U

$$\begin{array}{l} \neg P \Rightarrow Q \\ P \\ \hline \neg Q \end{array}$$

2. (5 pts each, 40 pts total) In each of the following, *KB* is a set of sentences, *{}* is the empty set of sentences, and *S* is a single sentence. Recall |= means "entails" and |- means "derives," where |-i means "inference procedure i derives." Use these keys:

Snd = Sound.
Unsnd = Unsound.
C = Complete.
I = Incomplete.
V = Valid.
Sat = Satisfiable.
Unsat = Unsatisfiable.
N = None of the above.

For each blank below, write in the key above that best corresponds to the correct term.

(a) Suppose some inference procedure i has the property, that for some *KB* and some *S*, *KB* |= *S* but not *KB* |-i *S*. Then the inference procedure i is__I_____.

(b) Let *S* be given in advance. Suppose that for some KB_1, KB_1 |= *S*; but that for some other KB_2, KB_2 |= ¬*S*. Then *S* is __Sat___.

(c) Suppose some inference procedure i has the property, that for any *KB* and any *S*, whenever *KB* |= *S* then *KB* |-i *S*. Then the inference procedure i is ___C___.

(d) Suppose inference procedure i has the property, that for some *KB* and some *S*, *KB* |-i *S* but not *KB* |= *S*. Then the inference procedure i is ____Unsnd_.

(e) Let *S* be given in advance. Suppose that *{}* |= *S*. Then *S* is ___V____.

(f) Suppose some inference procedure i has the property, that for any *KB* and any *S*, whenever *KB* |-i *S* then *KB* |= *S*. Then the inference procedure i is __Snd__.

(g) Suppose that *KB* |= *S*, then the sentence (*KB* ⇒ *S*) is ___V_____.

(h) Suppose that *KB* |= *S*, then the sentence (*KB* and ¬*S*) is __Unsat_______.

3. Consider the KB shown below.

a. (5 pts each, 15 pts total) Translate the following *KB* into Conjunctive Normal Form. The first one is done for you as an example (it was already in Conjunctive Normal Form ;-)).

A. $P \vee R$. ______ $P \vee R$ ______

B. $Q \Rightarrow S$. ______ $\neg Q \vee S$ ______

C. $P \Rightarrow Q$. ______ $\neg P \vee Q$ ______

D. $R \Rightarrow S$. ______ $\neg R \vee S$ ______

b. (15 pts total, -5 for each wrong step, but not negative. The order may vary, if proof is correct.) Write a complete resolution proof that *KB* |= *S*. Show the two clauses that you resolve in front of the symbol |-, and the resulting clause after |-. You may not require all of the lines provided. The sentence labeled "E." adds the negated goal. The first one is done for you as an example.

E. $\neg S$

(a) ___ $\neg S$ ___, ___ $\neg Q \vee S$ ___, |- ___ $\neg Q$ ___.

(b) ___ $\neg Q$ ___, ___ $\neg P \vee Q$ ___, |- ___ $\neg P$ ___.

(c) ___ $\neg P$ ___, ___ $P \vee R$ ___, |- ___ R ___.

(d) ___ R ___, ___ $\neg R \vee S$ ___, |- ___ S ___.

(e) ___ S ___, ___ $\neg S$ ___, |- ___ $\square$ ___.

Other proofs are fine if correct. For example, at step (d) above you could have resolved with $\neg S$:

(d) ___ $\neg S$ ___, ___ $\neg R \vee S$ ___, |- ___ $\neg R$ ___.

(e) ___ $\neg R$ ___, ___ R ___, |- ___ $\square$ ___.