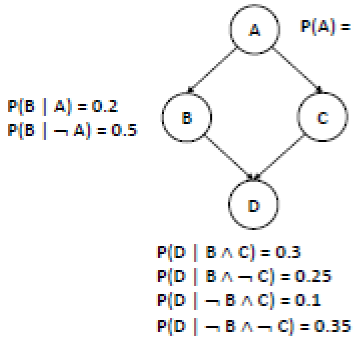
CSC384 Bayesian Networks

Problem 1

Consider the following Bayesian network. A, B, C, and D are Boolean random variables. If we know that A is true, what is the probability of D being true?

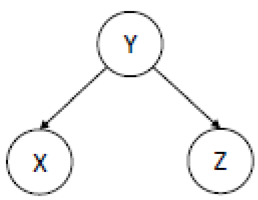


Solution

$P(D|A) = P(A, D)/P(A)$
$= (P(A, B, C, D) + P(A, B, \neg C, D) + P(A, \neg B, C, D) + P(A, \neg B, \neg C, D))/P(A)$
$= P(B|A)P(C|A)P(D|B, C) + P(B|A)P(\neg C|A)P(D|B, \neg C) + P(\neg B|A)P(C|A)P(D|\neg B, C) + P(\neg B|A)P(\neg C|A)P(D|\neg B, \neg C)$
$= 0.197.$

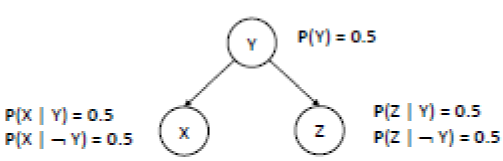
Problem 2.

For the following Bayesian network



we know that X and Z are not guaranteed to be independent if the value of Y is unknown. This means that, depending on the probabilities, X and Z can be independent or dependent if the value of Y is unknown. Construct probabilities where X and Z are independent if the value of Y is unknown, and show that they are indeed independent.

Solution.

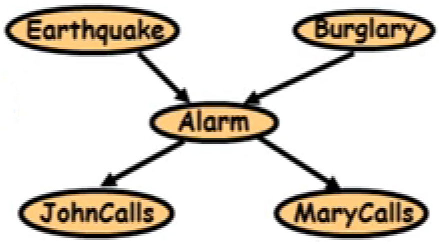


$P(X) = P(Y)P(X|Y) + P(\neg Y)P(X|\neg Y) = 0.5$
$P(Z) = P(Y)P(Z|Y) + P(\neg Y)P(Z|\neg Y) = 0.5$
$P(X, Z) = P(X, Y, Z) + P(X, \neg Y, Z)$
$= P(Y)P(X|Y)P(Z|Y) + P(\neg Y)P(X|\neg Y)P(Z|\neg Z)$
$= 0.25$
Therefore, $P(X)P(Z) = P(X, Z)$.

We can similarly show that $P(X)P(\neg Z) = P(X, \neg Z)$, $P(\neg X)P(Z) = P(\neg X, Z)$ and $P(\neg X)P(\neg Z) = P(\neg X, \neg Z)$ to prove that X and Z are independent if the value of Y is unknown.

Problem 3.

Recall the burglary problem from the lecture.



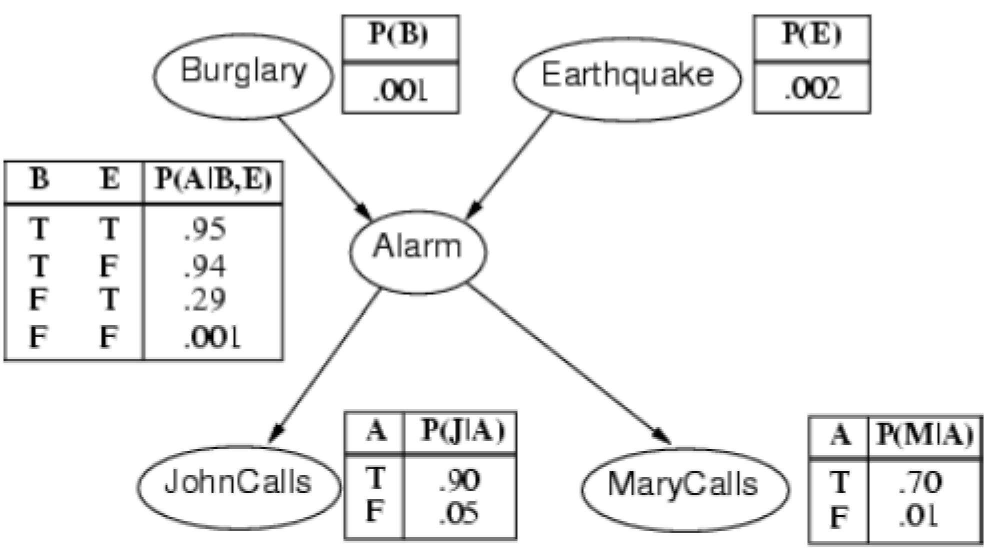
Following the last lecture, consider $P(J)$ where $J = JohnCalls$. Write a formal proof that M is irrelevant to the computation of $P(J)$.

Solution.

$P(J)$
$= \sum_{M,A,B,E} P(J, M, A, B, E)$
$= \sum_{M,A,B,E} P(J|A)P(B)P(A|B, E)P(E)P(M|A)$
$= \sum_A P(J|A) \sum_B P(B) \sum_E P(A|B, E)P(E) \sum_M P(M|A)$ # Let $f_1(A) = \sum_M P(M|A)$
$= \sum_A P(J|A) \sum_B (P(B) \sum_E P(A|B, E) f_1(A)$ # Let $f_2(A, B) = \sum_E (P(A|B, E)P(E) f_1(A)$
$= \sum_A P(J|A) \sum_B f_2(A, B)$ # Let $f_3(A) = \sum_B f_2(A, B)$
$= \sum_A P(J|A) f_3(A)$
$= f_4(J)$

Problem 4.

Back again to the burglary example.



Use variable elimination to calculate $P(Burglary|JohnCalls = true, MaryCalls = false)$. Elimination order is E, A. Show all computations.

Solution Use lecture convention x denotes$X = true$, $\neg x$ denotes $X = false$.
$P(b|j, \neg m) = P(b, j, \neg m)/P(j, \neg m)$.
$P(b, j, \neg m) = \sum_{E,A} P(b, E, A, j, \neg m)$
$= P(b, e, a, j, \neg m) + P(b, \neg e, a, j, \neg m) + P(b, e, \neg a, j, \neg m) + P(b, \neg e, \neg a, j, \neg m)$
$= P(b)P(e)P(a|b, e)P(j|a)P(\neg m|a)$
$+P(b)p(e)p(\neg a|b, e)P(j|\neg a)P(\neg m|\neg a)$
$+P(b)P(\neg e)P(a|b, \neg e)P(j|a)P(\neg m|a)$
$+P(b)P(\neg e)P(\neg a|b, \neg e)P(j|\neg a)P(\neg m|\neg a)$.
Do the same for $P(\neg b, j, \neg m)$ and normalize.