CSC384 Tutorial 3: Heuristics.

In this tutorial, you will evaluate various heuristics.

Exercise 1: Jug puzzle.

Recall the jug puzzle from the lecture. We have two jugs with capacity 4 and 3 gallons respectively. A state will be represented by a tuple of non negative integers (p, q) where p is the number of gallons in first jug and q number of gallons in second jug. Clearly $0 \leq p \leq 4, 0 \leq q \leq 3$. Below is the list of legal actions:

- $(p, q) \rightarrow (4, q)$ fill first jug from tap to capacity
- $(p, q) \rightarrow (p, 3)$ fill second jug from tap to capacity
- $(p, q) \rightarrow (0, q)$ drain first jug
- $(p, q) \rightarrow (p, 0)$ drain second jug
- $(p, q) \rightarrow (p + c, q - c)$ pour c gallons from second jug to the first possibly filling first to capacity. (Question: what are the bounds for c ?)
- $(p, q) \rightarrow (p - c, q + c)$ pour c gallons from first jug to the second possibly filling second to capacity. (Question: what are the bounds for c ?)

In what follows $g(p, q)$ will be defined as the optimal number of actions required to transition from $s = (p_0, q_0)$ to a goal (p, q). Define the following heuristic function:

$$h(p,q) = \begin{cases} 2, & \text{if } 0 < p < 4 \text{ AND } 0 < q < 3 \\ 4, & \text{if } 0 < p < 4 \text{ OR } 0 < q < 3 \\ 8, & \text{if } (p = 0 \text{ AND } q = 3) \text{ OR} (p = 4 \text{ AND } q = 0) \\ 10, & \text{if } (p = 0 \text{ AND } q = 0) \text{ OR} (p = 4 \text{ AND } q = 3) \end{cases} \tag{1}$$

Answer the following questions:

(i) Is h admissible? Why yes or why no?

(ii) Write down the expansion of state space by performing three steps of A^* starting from $(0, 0)$.

Now consider the following heuristics:

$$h'(p, q) = \sqrt{(p - p_0)^2 + (q - q_0)^2}$$

Assuming there exist a sequence of legal moves from (p_0, q_0) to (p, q), is h' admissible? Why yes or why no?

Exercise 2: 8-puzzle

Consider 8-puzzle and the Manahattan heuristic implemented as follows. For each state n, we have two functions $r_n(i), c_n(i)$ that give the row and column number of a tile i, with convention that the empty tile is $i = 9$. For example, if n is the following state:

1	2	3
	4	6
7	5	8

then r_n and c_n are given by the following table:

tile	$r_n(tile)$	$c_n(tile)$
1	1	1
2	1	2
3	1	3
4	2	2
5	3	2
6	2	3
7	3	1
8	3	2
9	2	1

Define

$$h(n) = \sum_{i=1}^{8}(|r_n(i) - r_g(i)| + |c_n(i) - c_g(i)|)$$

where g is the goal state. Solve the above puzzle using A^*. For example, for the above state, $h(n) = 3$. Answer the following question:

(i) Write an explicit formula for r_g and c_g.

(ii) Use this formula to prove h is admissible.

(iii) Solve the above puzzle by drawing the steps of A^* algorithm.

Now consider the following heuristic. Two tiles a and b are in a linear conflict if they are in the same row or column ,also their goal positions are in the same row or column and the goal position of one of the tiles is blocked by the other tile in that row. For example,

4	2	5
1		6
3	8	7

we see that 1 and 4 are in linear conflict, and so are 7 and 8 and they are the only linear conflicts. Let $L(n)$ be the number of linear conflicts of the state n. Consider $H(n) = h(n) + 2L(n)$. Is H admissible? Why yes of why not?

Exercise 3

(Please work on this on your own, at home, not during the tutorial.)

Write code to experiment with $h(n)$ and $H(n)$. Which one seems to do better?