

1. (5 pts each, 30 pts total) Fill in each blank below with Y (= Yes) or N (= No) depending on whether or not the first order predicate logic sentence correctly expresses the English sentence.

a.     N     “All cats are mammals.”  $\forall x \text{ Cat}(x) \ \& \ \text{Mammal}(x)$

$\forall x \text{ Cat}(x) \ \& \ \text{Mammal}(x)$  means “Everything is a cat and also is a mammal.”

$\forall x \text{ Cat}(x) \Rightarrow \text{Mammal}(x)$  means “All cats are mammals.”

*Remember that implication is the natural connective to use with  $\forall$ .*

b.     Y     “Spot has a sister who is a cat.”  $\exists x \text{ Sister}(x, \text{Spot}) \ \& \ \text{Cat}(x)$

*Remember that conjunction is the natural connective to use with  $\exists$ .*

c.     N     “For every person, there is someone that that person likes.”  $\exists x \forall y \text{ Likes}(x, y)$

$\exists x \forall y \text{ Likes}(x, y)$  means “There is someone who likes everyone.”

$\forall x \exists y \text{ Likes}(x, y)$  means “For every person, there is someone that that person likes.”

*Remember that the second quantifier is inside the scope of the first quantifier.*

d.     N     “There is someone who is liked by everyone.”  $\forall x \exists y \text{ Likes}(x, y)$

$\forall x \exists y \text{ Likes}(x, y)$  means “For every person, there is someone that that person likes.”

$\exists x \forall y \text{ Likes}(y, x)$  means “There is someone who is liked by everyone.”

e.     Y     “Everyone likes ice cream.”  $\neg \exists x \neg \text{ Likes}(x, \text{IceCream})$

f.     Y     “All men are mortal.”  $\forall x \text{ Man}(x) \Rightarrow \text{Mortal}(x)$

2. (5 pts each, 40 pts total) Let  $PKF(x, y)$  mean “Person  $x$  Knows Fact  $y$ ”. For purposes of this question only, you may assume that the first argument is a person and the second is a fact. For each English sentence below, write the first order predicate logic sentence that best expresses it. Use “ $\neg$ ” to mean “not.” The first one is done for you as an example.

a. Every person knows every fact.  $\forall x \forall y PKF(x, y)$ .

b. Every person knows at least one fact.  $\forall x \exists y PKF(x, y)$ .

c. There is a person who knows at least one fact.  $\exists x \exists y PKF(x, y)$ .

d. There is a person who knows every fact.  $\exists x \forall y PKF(x, y)$ .

e. No person knows every fact.  $\forall x \exists y \neg PKF(x, y)$ .

alternatively  $\forall x \neg \forall y PKF(x, y)$  or  $\neg \exists x \forall y PKF(x, y)$ .

f. There is a person who knows no fact.  $\exists x \forall y \neg PKF(x, y)$ .

alternatively  $\exists x \neg \exists y PKF(x, y)$  or  $\neg \forall x \exists y PKF(x, y)$ .

g. No person knows any fact.  $\forall x \forall y \neg PKF(x, y)$ .

alternatively  $\forall x \neg \exists y PKF(x, y)$  or  $\neg \exists x \exists y PKF(x, y)$ .

h. There is a fact that is known by every person.  $\exists y \forall x PKF(x, y)$ .

i. There is a fact that no person knows.  $\exists y \forall x \neg PKF(x, y)$ .

alternatively  $\exists y \neg \exists x PKF(x, y)$  or  $\neg \forall y \exists x PKF(x, y)$ .

*Godel’s Incompleteness Theorem shows that (i) is valid in any world with arithmetic (p. 352, 3<sup>rd</sup> ed.; p. 302, 2<sup>nd</sup> ed.).*

3. (5 points) Write down a logical sentence such that every world in which it is true contains exactly one object. You may use “ $A = B$ ” to mean “ $A$  equals  $B$ .”

$$\forall x \forall y (x=y)$$

4. (5 points each, 10 points total) Let  $P(x)$  mean  $x$  is a politician, and let  $F(x, y, t)$  mean  $x$  fools  $y$  at time  $t$ . Consider this English sentence:

“Politicians can fool some of the people all of the time,  
and they can fool all of the people some of the time,  
but they can’t fool all of the people all of the time.”

It turns out that English is ambiguous! The sentence can mean several different things.

a. Write down a logical sentence that means, “There are some people who can be fooled by politicians every single time, and any person can be fooled by politicians at some point in time or another, but politicians can’t fool all of the people all of the time.”

*This answer is sufficient to get credit on this problem:*

$$\forall x P(x) \Rightarrow [(\exists y \forall t F(x, y, t)) \wedge (\forall y \exists t F(x, y, t)) \wedge (\exists y \exists t \neg F(x, y, t))]$$

*It is also OK to move the negation sign outside the last quantifiers, provided they are changed:*

$$\forall x P(x) \Rightarrow [(\exists y \forall t F(x, y, t)) \wedge (\forall y \exists t F(x, y, t)) \wedge (\neg \forall y \forall t F(x, y, t))]$$

**Five (5) Extra Credit points if you noticed the need to say that  $y$  is a person and  $t$  is a time:**

$$\begin{aligned} \forall x P(x) \Rightarrow & [(\exists y \text{ Person}(y) \wedge (\forall t \text{ Time}(t) \Rightarrow F(x, y, t))) \wedge \\ & (\forall y \text{ Person}(y) \Rightarrow (\exists t \text{ Time}(t) \wedge F(x, y, t))) \wedge \\ & (\exists y \exists t \text{ Person}(y) \wedge \text{Time}(t) \wedge \neg F(x, y, t))] \end{aligned}$$

b. Write down a logical sentence that means, “At every point in time there will be some people who can be fooled by politicians at that time, and sometimes politicians can fool every single person all at once at that time, but politicians can’t fool all of the people all of the time.”

*This answer is sufficient to get credit on this problem:*

$$\forall x P(x) \Rightarrow [(\forall t \exists y F(x, y, t)) \wedge (\exists t \forall y F(x, y, t)) \wedge (\exists y \exists t \neg F(x, y, t))]$$

*It is also OK to move the negation sign outside the last quantifiers, provided they are changed:*

$$\forall x P(x) \Rightarrow [(\forall t \exists y F(x, y, t)) \wedge (\exists t \forall y F(x, y, t)) \wedge (\neg \forall y \forall t F(x, y, t))]$$

**Five (5) Extra Credit points if you noticed the need to say that  $y$  is a person and  $t$  is a time:**

$$\begin{aligned} \forall x P(x) \Rightarrow & [(\forall t \text{ Time}(t) \Rightarrow (\exists y \text{ Person}(y) \wedge F(x, y, t))) \wedge \\ & (\exists t \text{ Time}(t) \wedge (\forall y \text{ Person}(y) \Rightarrow F(x, y, t))) \wedge \\ & (\exists y \exists t \text{ Person}(y) \wedge \text{Time}(t) \wedge \neg F(x, y, t))] \end{aligned}$$

5. (5 pts each, 20 pts total) For each of the following pairs of sentences, give the most general unifier, or write NONE if none exists. The first one is done for you as an example.

*See Figure 9.1, p. 328 (3<sup>rd</sup> ed.; p. 278, 2<sup>nd</sup> ed.).*

a.  $P(A, B, B), P(x, y, z)$ .  $x/$  A ,  $y/$  B ,  $z/$  B .

b.  $Q(x, G(A, B)), Q(y, G(y, y))$ .  $x/$  NONE ,  $y/$  \_\_\_\_\_.

*y cannot unify with A and with B simultaneously.*

c.  $\text{Older}(\text{Father}(y), y), \text{Older}(\text{Father}(x), \text{John})$ .  $x/$  John ,  $y/$  x .

d.  $\text{Knows}(\text{Father}(y), y), \text{Knows}(x, x)$ .  $x/$  NONE ,  $y/$  \_\_\_\_\_.

*x cannot unify with y and with Father(y) simultaneously.*

e  $\text{Knows}(\text{John}, x), \text{Knows}(y, z)$ .  $x/$  z ,  $y/$  John ,  $z/$  \_\_\_\_\_.