Stochastic Modelling and Random Processes

Yiming MA

December 13, 2020

Contents

Chapter 1

Discrete-Time Markov Chains

1.1 Countable Discrete-Time Markov Chains

One can extend much of what we have done for finite discrete-time Markov chains to the countably infinite case, e.g. the **simple random walk** on $\mathbb{Z}$, but some results become more subtle. For example, the simple random walk is *not SP-ergodic*, despite being *irreducible*. Actually, it even *fails to have a stationary probability*; also it is *not aperiodic*, and it has a *period* 2.

Example 1.1.1. Using definition of the simple random walk:

$$Y_n = \sum_{i=0}^{n-1} X_i,$$

where X_i's are independent and identically distributed, with

$$X_i = \begin{cases} +1 & \text{with probability } p \\ -1 & \text{with probability } 1-p \end{cases},$$

Compute the $\mathrm{E}[Y_n]$ and $\mathrm{Var}[Y_n]$.

One has to refine various concepts.

Chapter 2

Continuous-Time Markov Chains