Regime-Based Versus Static Asset Allocation: *Letting the Data Speak*

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Financial markets' behavior changes abruptly. Although some changes may be transitory, the new behavior often persists for several periods after a change. The mean, volatility, and correlation patterns in stock returns, for example, changed dramatically at the start of, and continued through the global financial crisis of 2007 to 2008. Similar regime changes, some of which can be recurring (recessions versus expansions) and some of which can be permanent (structural breaks), are prevalent across a wide range of financial markets and in the behavior of many macro variables (Ang and Timmermann [2012]).

Observed regimes in financial markets are related to the phases of the business cycle (see Campbell [1998] and Cochrane [2005]). The link is complex and difficult to exploit for investment purposes, due to the large lag in the availability of data related to the business cycle. Our intention is to let the data speak by focusing on readily available market data, instead of attempting to establish the link to the business cycle.

Regime changes present a big challenge to traditional strategic asset allocation (SAA). In the presence of time-varying investment opportunities, portfolio weights should be adjusted as new information arrives. Traditional SAA approaches seek to develop static, "all-weather" portfolios that optimize efficiency across a range of economic scenarios. If economic conditions are persistent and strongly linked to asset class performance, however, then a dynamic strategy should add value over static weights (Sheikh and Sun [2012]). The purpose of a regime-based strategy is to take advantage of favorable economic regimes, withstand adverse economic regimes, and reduce potential drawdowns.

Regime-based investing is distinct from tactical asset allocation (TAA). While the latter is shorter term, higher frequency (i.e., weekly or monthly), and driven primarily by valuation considerations, regime-based investing targets a longer time horizon (i.e., a year or more) and is driven by changing economic fundamentals. A regime-based approach has the flexibility to adapt to changing economic conditions within a benchmark-based investment policy, which can involve more than one rebalancing within a year. It straddles a middle ground between strategic and tactical (Sheikh and Sun [2012]).

LETTING THE DATA SPEAK

This article examines whether regime-based asset allocation (RBAA) can effectively respond to financial regimes, in an effort to provide better long-term results when compared to static approaches. Dopfel [2010] showed the potential outperformance of a RBAA strategy, assuming complete information about the prevailing regime and future regime shifts is available. Dopfel [2010], however, concluded that an investor who does not possess exceptional forecasting skill is better off holding a static portfolio that is hedged against the uncertainty associated with regime shifts.

This conclusion contrasts with the large number of studies that have documented the profitability of dynamic asset allocation (DAA) strategies based on regime-switching models. See, for example, Ang and Bekaert [2002, 2004], Guidolin and Timmermann [2007], Bulla et al. [2011], and Kritzman, Page, and Turkington [2012]. The investor should accept this profitability with caution, as not all the studies account for transaction costs when comparing the performance of dynamic and static strategies. This is important, as frequent rebalancing can offset a dynamic strategy's potential excess return. Furthermore, the in-sample performance generally exceeds the out-of-sample performance—if the strategies are at all tested out of sample.

Inspired by the apparent profitability of regime-switching strategies, this article challenges Dopfel's [2010] conclusion by letting the data speak. In an investment universe consisting of a global stock index (MSCI ACWI)[1] and a global government bond index (JPM GBI),[2] we compare the performance of a RBAA strategy to that of a strategy based on rebalancing to static weights. Exhibit 1 shows the development in the two indices over the 20-year data period.

Our intention is to identify regimes in the stock returns using a regime-switching model and let the asset allocation depend on the identified regime. The focus on modeling the stock returns is natural, as portfolio risk is typically dominated by stock market risk. In addition, the stock markets generally lead the economy (see Siegel [1991], for example). The goal is not to predict regime shifts or future market movements, but to identify when a regime shift has occurred and then benefit from the persistence of equilibrium returns and volatilities.

The regime-switching process can be interpreted as a momentum process when it is more likely to continue in the same state than to transition to another state (see Ang and Bekaert [2002]).

Exhibit 2 shows the stock index's daily log-returns.[3] The volatility forms clusters, as large price movements tend to be followed by large price movements and vice versa, as Mandelbrot [1963] noted.[4] The RBAA strategy aims to exploit the volatility's persistence, as risk-adjusted returns, on average, are substantially lower during turbulent periods, irrespective of the source of turbulence, as Kritzman and Li [2010] showed. Our purpose is not to outline the optimal strategy, but rather to discuss the profitability of an RBAA approach.

THE HIDDEN MARKOV MODEL

Imagine knowing a person's heart rate. While the person sleeps, we see a low average heart rate with low volatility. When the person wakes up, we see a sudden rise in the heart rate's average level and volatility. Without actually seeing the person, we can reasonably conclude whether he or she is awake or sleeping, that is, which state the person is in.

EXHIBIT 1
The investment universe

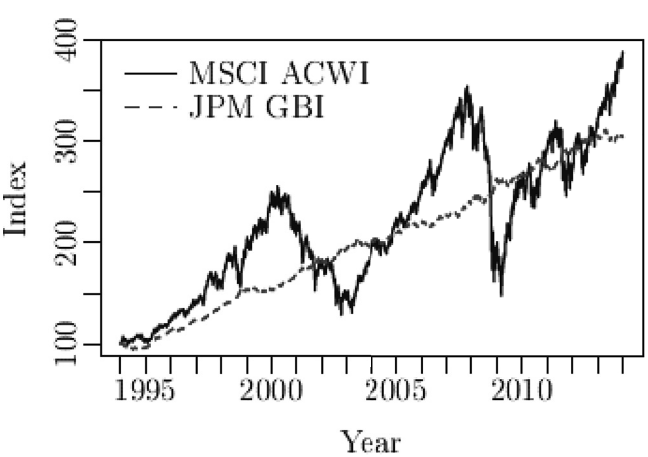
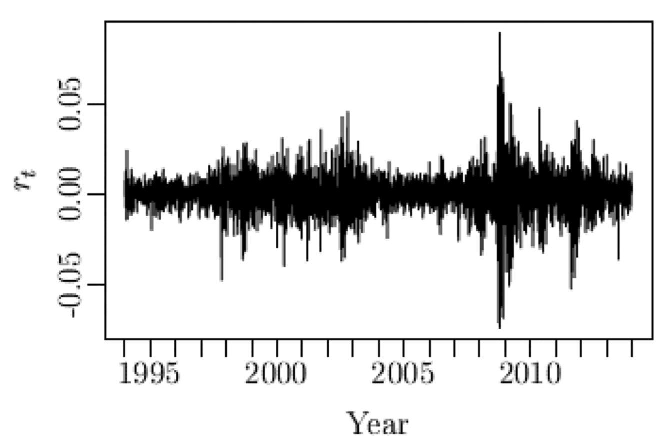


EXHIBIT 2
Volatility clustering in the daily log-returns



Returns are the heart rate of a financial market. The use of hidden Markov models (HMMs) to infer the state of financial markets has gained popularity over the last decade. The HMM is a black-box model, but the inferred states can often be linked to business cycle phases (see Guidolin and Timmermann [2007], for instance). The possibility of interpreting the states, combined with the model's ability to reproduce stylized facts of financial returns, is part of the reason that HMMs have become increasingly popular.

In a hidden Markov model, the probability distribution that generates an observation depends on the state of an unobserved Markov chain. A sequence of discrete random variables $\{X_t : t \in \mathbb{N}\}$ is said to be a first-order Markov chain if, for all $t \in \mathbb{N}$, it satisfies the Markov property:

$$\Pr\left(X_{t+1} \middle| X_t, \ldots, X_1\right) = \Pr\left(X_{t+1} \middle| X_t\right) \tag{1}$$

The conditional probabilities $\Pr(X_{t+1} = j | X_t = i) = \gamma_{ij}$ are called transition probabilities.

As an example, consider the two-state model with Gaussian conditional distributions:

$$Y_t \sim N\left(\mu_{X_t}, \sigma^2_{X_t}\right)$$

where

$$\mu_{X_t} = \begin{cases} \mu_1, & \text{if } X_t = 1, \\ \mu_2, & \text{if } X_t = 2, \end{cases} \quad \sigma^2_{X_t} = \begin{cases} \sigma^2_1, & \text{if } X_t = 1, \\ \sigma^2_2, & \text{if } X_t = 2, \end{cases} \quad \text{and}$$

$$\Gamma = \begin{bmatrix} 1-\gamma_{12} & \gamma_{12} \\ \gamma_{21} & 1-\gamma_{21} \end{bmatrix}$$

When the current state X_t is known, the distribution of Y_t depends only on X_t.

The sojourn times are implicitly assumed to be geometrically distributed:

$$\Pr\left(\text{'staying } t \text{ time steps in state } i\text{'}\right) = \gamma_{ii}^{t-1}\left(1-\gamma_{ii}\right) \tag{2}$$

The geometric distribution is without memory, implying that the time until the next transition from the current state is independent of the time spent in the state.

HMMs can match financial markets' tendency to change their behavior abruptly, as well as the new behavior's tendency to persist for several periods after a change. They are well suited to capture the stylized behavior of many financial series, including volatility clustering and leptokurtosis, as shown by Rydén, Teräsvirta, and Åsbrink [1998].

Subsequent articles have extended the classical Gaussian HMM by considering sojourn time distributions other than the memory-less geometric distribution (Bulla and Bulla [2006]), conditional distributions other than the Gaussian distribution (Bulla [2011]), and a continuous-time formulation as an alternative to the dominating discrete-time models (Nystrup, Madsen, and Lindström [2015a]). In Nystrup, Madsen, and Lindström [2015b], the authors found that the need to consider other sojourn time distributions and other conditional distributions can be eliminated by adapting to the data process's time-varying behavior.

An HMM's parameters are typically estimated using the maximum-likelihood method. Every observation is assumed to be of equal importance, no matter the sample period's length. This approach works well when the sample period is short and the underlying process does not change over time. The parameters' time-varying behavior, documented in previous studies (Rydén, Teräsvirta, and Åsbrink [1998], Bulla [2011], and Nystrup, Madsen, and Lindström [2015b]), calls for an adaptive approach that assigns more weight to the most recent observations while keeping in mind the past patterns at a reduced confidence.

Nystrup, Madsen, and Lindström [2015b] outlined an adaptive estimation approach based on weighting observations with exponentially decreasing weights—in other words, using exponential forgetting. This article pursues the same approach. The regime-switching model is still a two-state HMM with Gaussian conditional distributions, but one that adapts to the time-varying behavior of the underlying process in an effort to produce more robust state estimates.

EMPIRICAL RESULTS

We perform testing one day at a time in a live-sample setting, to make it as realistic as possible. The model is fitted to the first t observations, assigning the most weight to the most recent observations. Based on the estimated parameters, we calculate the probability that on day t the market was in the high- or low-volatility state, respectively, along with the predicted state probabilities for day $t+1$. As the states are highly persistent ($\gamma_{ii} \gg 0.5$), the state that is predicted to be most likely on day $t+1$ will be the same state identified to be most likely on day t. If the state predicted to be most likely on day $t+1$ is different than the state on which the asset allocation on day t is based and the confidence in the prediction is above 95%[5], then we change the allocation based on the closing price at day $t+1$, i.e., we assume a one-day delay in the implementation. Otherwise the asset allocation remains unchanged. We then include the log-return on day $t+1$ in the sample, re-estimate the model, and calculate the state probabilities based on the new parameters. We repeat this procedure sequentially by including the observations, one at a time, from January 1, 1996 all the way through the sample.[6]

We compare the performance of two regime-based strategies (Stocks–Bonds and Long–Short) to the performance of the two indices and that of a static portfolio with a fixed allocation of 49% to stocks, as shown in Exhibit 3. The first strategy is fully invested in the stock index in the low-volatility state and the bond index in the high-volatility state. The average allocation to the stock index over the period was 49%. The second strategy is long the stock index in the low-volatility state and short the stock index in the high-volatility state. Exhibit 4 shows the development of the strategies and the indices. In the shaded periods, the allocation was based on being in the high-volatility state.

The identified regimes seem intuitive when we look at the log-returns at the bottom of Exhibit 4. There have been a total of 16 regime changes over the 17-year period. The length of the identified regimes varies considerably, from a few weeks up to six years. This is different from what we would expect if the regimes were based on a business cycle indicator. There appear to be large differences in the volatility level within the six-year, high-volatility regime that begins in 1998 and includes both the build-up and burst of the dot-com bubble. This suggests that the market states were less persistent around this time.

The bond index has realized the highest Sharpe ratio (SR), with an annualized return (AR) of 6.0% and an adjusted annualized standard deviation (SD) of only 3%. We have adjusted the reported SDs for autocorrelation using the procedure outlined by Kinlaw, Kritzman, and Turkington [2015].[7] The data period was characterized by falling interest rates and low inflation, leading to a strong performance for bonds. It is unlikely that the environment for bonds will be as favorable going forward.

The Stocks–Bonds strategy has realized the second highest SR of 1.23. The annualized SD is the same as for the static portfolio, which has the same average exposure to the stock index, but the realized return is higher as long as transaction costs do not exceed 239 basis points per one-way transaction. This is when we ignore the costs associated with rebalancing to static weights, so the break-even transaction cost is higher than 239 basis points. In addition, the maximum drawdown[8] (MDD) for the Stocks–Bonds strategy is much smaller than that of the static strategy.

The Long–Short strategy has been less profitable, but it still outperforms the stock index when transaction costs are less than 130 basis points per one-way transaction. The outperformance essentially happened during the financial crisis in 2008. The Long–Short strategy has a lower tail risk than that of the stock index, but the risk of the strategy underperforming the index going forward seems real.

This observation is confirmed in Exhibit 5, which summarizes the performance of the indices and the regime-based strategies, when we exclude 2008 from the sample. Although it still has a lower tail risk than that of the stock index, the Long–Short strategy underperforms the stock index. The performance of the bond index and the Stocks–Bonds strategy barely changes, whereas the AR and the SR of the stock index and the static portfolio increase. The average allocation to the stock index when we exclude 2008 is 52%, which equals the fixed allocation to stocks in the static portfolio (Exhibit 5). The Stocks–Bonds strategy's realized return still exceeds that of the static portfolio as long as transaction costs do not exceed 140 basis points per one-way transaction.

EXHIBIT 3

The performance of the indices and the strategies from 1996–2014

Index/Strategy	AR	SD	SR	MDD
Bond index (JPM GBI)	0.060	0.03	1.90	0.05
Stock index (MSCI ACWI)	0.069	0.18	0.38	0.58
Static Portfolio	0.064	0.09	0.72	0.32
Stocks–Bonds	0.114	0.09	1.23	0.13
Long–Short	0.096	0.18	0.52	0.44

SUMMARY AND DISCUSSION

Our results indicate that no level of forecasting skill is required for a RBAA strategy to be more profitable

EXHIBIT 4

The development of the strategies and the indices across the inferred regimes

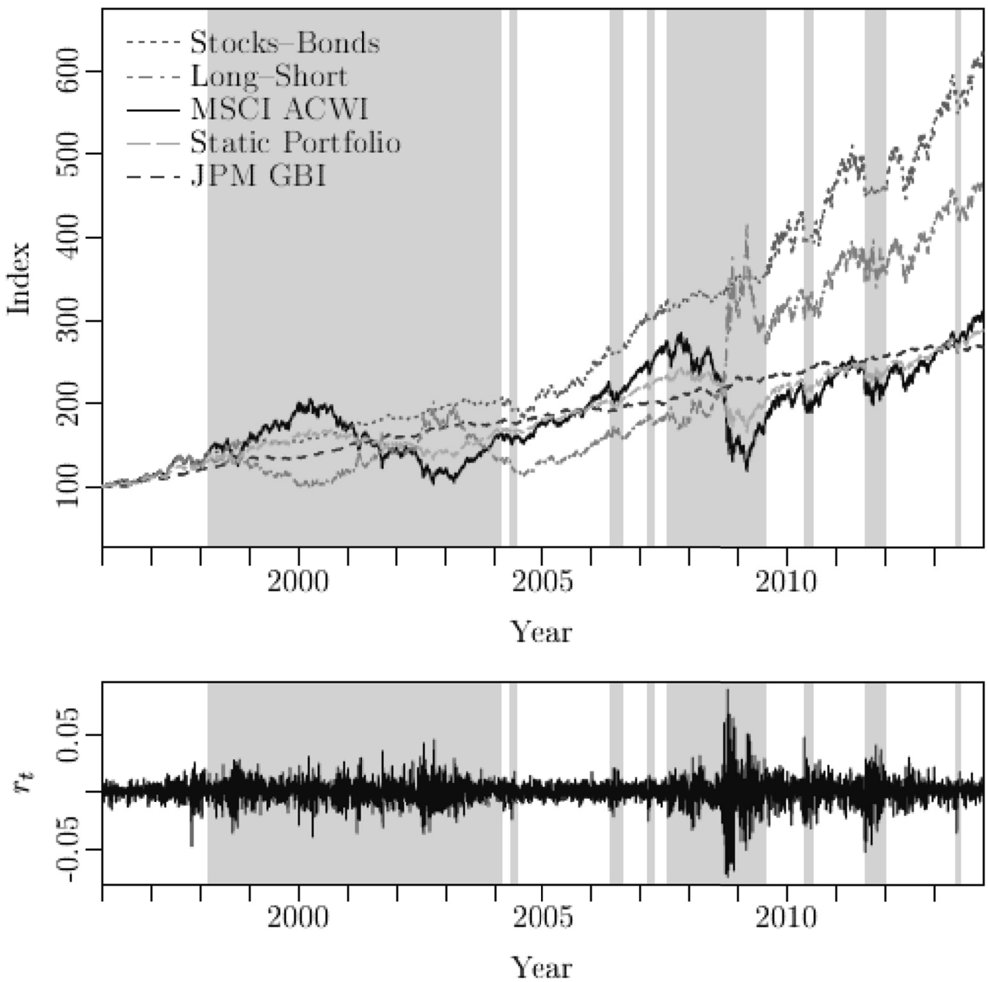


EXHIBIT 5

The performance of the indices and the strategies from 1996–2014

Index/Strategy	AR	SD	SR	MDD
Bond index (JPM GBI)	0.058	0.03	1.89	0.05
Stock index (MSCI ACWI)	0.111	0.17	0.66	0.50
Static Portfolio	0.084	0.08	1.04	0.21
Stocks–Bonds	0.115	0.09	1.22	0.13
Long–Short	0.063	0.17	0.38	0.44

than a static strategy. We did the testing one day at a time in a live-sample setting to make it as realistic as possible. Our approach was based on identifying regimes in market returns, using a regime-switching model with time-varying parameters. As the parameters are updated every day, the same approach should work in other time periods as well. The results are robust, as they are based on available market data with no assumptions about equilibrium returns, volatilities, or correlations. Additionally, it might be possible to improve performance by including economic variables, interest rates, investor sentiment surveys, or other indicators.

The outperformance of the strategy that switched between stocks and bonds appeared to be most reliable, as it did not just happen during one year, as was the case with the Long–Short strategy. Although the break-even transaction cost was more than 239 basis points (140 basis points when excluding 2008), the strategies will only remain profitable if the market states' persistence remains high. Volatility clustering is not a new phenomenon, but it can occur at different levels of market persistence.

The tested strategies may be based on larger changes in allocation than most investors are willing to and/or allowed to implement. Suppose your benchmark is a 50–50 allocation to stocks and bonds and that you are allowed to vary between a 60–40 and a 40–60 allocation. For the Stocks–Bonds strategy, this is equivalent to allocating 80% of the portfolio to a static 50–50 portfolio and the remaining 20% to the regime-based strategy. The excess return that you can obtain is then 20% of the excess return that could be obtained by allocating the entire portfolio to the regime-based strategy.

Our results have important implications for portfolio managers with medium to long-term investment horizons. Even without any level of forecasting skill, holding a static portfolio may not be optimal. With some level of forecasting skill, an RBAA strategy's potential outperformance could be substantial. It is definitely worth considering a more dynamic approach to asset allocation, if not only to reduce the tail risk.

ENDNOTES

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[1]The MSCI All Country World Index, denominated in USD, captures large- and mid-cap representation across 23 developed market and 21 emerging market countries. The difference compared to the more well-known MSCI World Index is the weight on EM countries. The data before 1999, where the total return index began, has been reconstructed based on the price index by adding the average daily net dividend return over the period from 1999 to 2013—0.007%—to the price returns.

[2]The global J.P. Morgan Government Bond Index measures the performance across 13 developed, fixed-income bond markets hedged to USD. The constituents are selected from all government bonds, excluding floating rate notes, perpetuals, bonds targeted at the domestic market for tax purposes, and bonds with less than one year to maturity. The index had a modified duration of 6.8 at the end of 2013.

[3]The log-returns are calculated using, $r_t = \log(P_t) - \log(P_{t-1})$, where P_t is the closing price of the index on day t and log is the natural logarithm.

[4]A quantitative manifestation of this fact is that while returns themselves are uncorrelated, absolute and squared returns display a positive, significant, and slowly decaying autocorrelation function.

[5]If the confidence threshold is changed to 85% or 90%, there will be more regime changes, but the results will only change somewhat. For a given level of transaction costs, there exists an optimal threshold that balances the cost of rebalancing with the cost of not reacting to regime shifts or delaying the reaction.

[6]The log-returns from the two years before 1996 are used for initialization.

[7]The adjustment for autocorrelation leads to slightly higher standard deviations and correspondingly lower Sharpe ratios. The adjustment is not important to the conclusions drawn, as the indices and strategies all displayed fairly similar, low amounts of autocorrelation.

[8]The maximum drawdown is the largest relative decline from a historical peak in the index value.

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