Regime Shifts Implications for Asset Allocation Strategies

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Abstract

Regime Shifts present significant challenges for investors because they cause performance to depart significantly from the ranges implied by long-term average of means and covariances. However, regime shifts also present opportunities for gain. Involving market regimes in portfolio management and investment strategies has become trendy nowadays. This paper is going to explore portfolio management involving market regimes. Our main purposes are building sophisticated market regimes models, examining performances of multiple portfolio strategies with and without market regimes and optimizing portfolio strategies by adjusting certain variables. We mainly applied Markov-switching models to forecast market regimes by using multiple market indexes. Mean variance optimization, minimum variance optimization and maximum diversity optimization for portfolio were employed by asset allocation, and we improved buy and hold with periodically rebalancing the portfolio, according to market regime states. Examinations of our models were performed over an out-of-sample time period, presenting our results that regime-dependent portfolio management yields better performance dependending on the metrics pre decided. It was also verified that involving market regimes works extensively on portfolio strategies, further proving its values on investment, who seek to avoid large losses.

Keywords: Markov-Switching Models, Bayesian Information Criterion, Mean-Variance Optimization, Maximum Diversification Optimization, Portfolio Rebalancing

1 Introduction

It is well known that asset allocation behavior can vary over different economic scenarios. For example, business cycles tend to influence cyclical versus non-cyclical companies in markedly different ways, primarily due to sensitivities of consumers and producers to economic growth. Also, it is known that different assets respond differently to economic situations. For fixed income assets, its prices fall when the interest rates rise. For commodities prices, it can be influenced by inflation expectations, and can go up vastly when inflation goes higher.

Regime switching models have been with the scope of financial academic interest for quite some time. In the past most studies, they assumed linear decencies between regressors and dependent variables. More recently, it has been proven that such dependencies are not linear and they can vary across time. ((Ang & Bekaert 2002*a*, Ang & Bekaert 2002*b*, René & Pierre 1996, Guidolin & Timmermann 2005*a*, Guidolin & Timmermann 2005*b*, Guidolin & Timmermann 2006, Guidolin & Hyde 2012, Ang & Chen 2002, Ang & Bekaert 2004, Ang & Timmermann 2012) to cite a few).

In the past, many studies used a 2-regime model to solve an asset allocation problem and this model shows significant improvement compared to the non regime dependent model. Recently, authors tried to increase the number of regimes in the models for a better fit for the reality. Guidolin and (Guidolin & Timmermann 2005*a*) investigate the economic implication of 3 regimes - bear, bull, and normal market - on UK stocks and bonds. Guidolin and (Ang & Timmermann 2012) show that 4 regimes are required to capture the joint distribution of both equities and bonds. They are characterized as crash, slow growth, bull and recovery states.

We first performed a static asset allocation based on the model from the whole sample. Then we tested how the allocation performance changed along with the changes of the number of regimes in the market. It showed that it is not always optimal to increase the number of regimes. After that, we implemented different kinds of asset allocation strategies and compared the performances between them.

2 Literature Review

To implement our proposed models, we reviewed some methodologies of switching market regimes, asset allocation and investment strategy under regimes.

In the finance industry, HMMs have been used extensively to build regime-based models, since Hamilton proposed using a regime- switching model to identify economic cycles using the GNP series (Hamilton 1989). As stated in Ang & Timmermann work (2012), HMMs can simultaneously capture multiple char- acteristics from financial return series such as time-varying correlations, skewness and kurtosis, while also providing good approximations even in processes for which the underlying model is unknown (Ang & Bekaert 2004, Bulla, Mergner, Bulla, Sesboüé & Chesneau 2011, Bulla & Bulla 2006, Nystrup, Lindström & Madsen 2020). In addition, HMMs allow for good interpretability of results, as thinking in terms of regimes is a natural approach in finance. Examples of dynamic asset allocation are (Reus & Mulvey 2016) that use a HMM to build a dynamic portfolio using currency futures and (Bae, Kim & Mulvey 2014) that use a HMM to identify market regimes using different asset classes, with regime information helping portfolios to avoid risk during left-tail events.

Asset portfolio composition is also one of the main practices in the financial market. The most popular approach to asset allocation is that of (Markowitz 1952), who states that agents should diversify their investments in order to minimize risks. Investors want to invest in a variety of assets that have diversified risks and more attractive returns compared to investing in a smaller number of assets. Even outside the financial sector, commercial companies and manufacturers develop strategies based on asset allocation principles in order to obtain more advantageous client and supplier portfolios.

Asset allocation consists of the distribution of agents' wealth between a set of assets. Portfolio allocation depends on agents' preferences in terms of expected risks and returns, where the estimation of risks and returns is an important stage in asset allocation analysis. The allocations depend on the conditional expectations for the risks and returns of the assets. Asset allocation is a sequential process that depends on estimations of the conditional distribution of returns.

This article is organized in the following way: after the introduction, in the second section we characterize asset allocation dependent on the states. The third section addresses the statistical model for estimating the risks and returns for the allocation, presenting the time series models with Markovian regime switching. The next subsection establishes different ways of constructing portfolios. The fourth section empirically applies the portfolio with regime switching, comparing the result with other constructing methods.

3 Methodology

3.1 Data

Since there is a low, even negative correlation between equity and commodity markets and this correlation will vary in different economic scenarios, adding commodities in the pure equity portfolio seems to be an efficient way to diversify the risk, this paper mainly focuses on the equity and commodity markets. To maximize the time range we can model, the portfolio is composed of AMZN, JPM, BEN,C01, CL1, GOLDS, which relatively represents Amazon stock, JPMorgan Chase & Co stock, Franklin Resources, Inc., Castor seed, Crude Oil WTI and Gold. We set S&P500 and SPGSCI as the benchmark for regime analysis. Since we picked stocks and commodities to construct our portfolio, using these two indexes to differentiate the different market regime would be fair enough to catch different changes in different markets. In this case, our portfolio ranges from 07/23/2004 to 07/02/2021, which contains 3878 days in total. We divided our data into training and testing dataframes, which relatively ranges from 2004 to 2019 and 2019 to 2021. Figure 3-1 shows the price trend for assets and benchmark indexes. According to history, there were mainly two great recessions during the period. The first one is the great recession from 2007 to 2009. We can see from the chart that during this period, every asset prices dropped more or less with different scopes. The second great recession is related to the Covid-19 at the beginning of 2020. At that point, all asset prices decreased, especially assets or indexes with higher based prices. If our model can successfully detect these regime changes and modify portfolio weight, huge loss can be mitigated to some extent for investors.

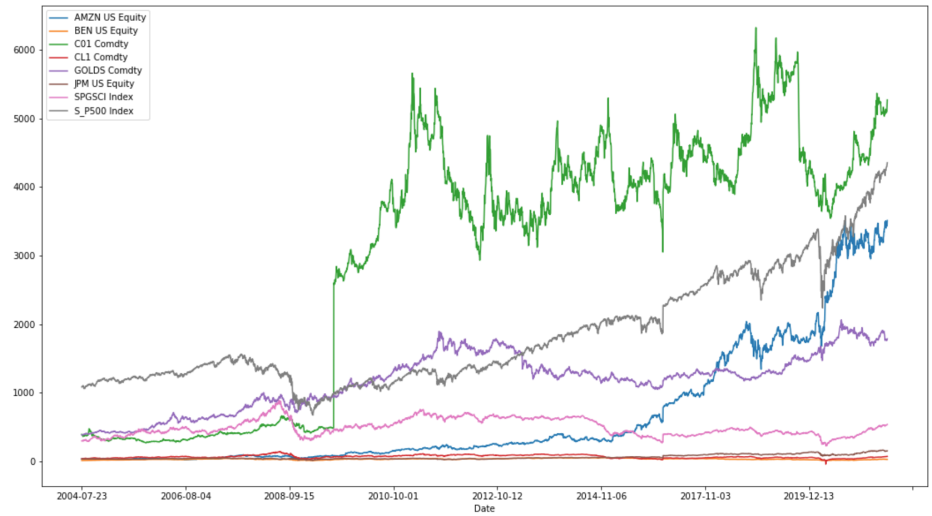

Figure 3-1: Prices of assets and indexes from 2004 to 2021

3.2 Decide Market Regimes

3.2.1 Market Regimes Switching Models

Based on our research, we applied the Hidden Markov models (HMM) to divide the training time frame into several states. In the case of detecting changes of jumps from one persistent state to another, many scholars applied HMM to determine the sequence of states (Nystrup et al. 2020). Briefly stated, given the model parameters and observed indexes price data, the optimality of hidden states can be estimated by a first-order Markov chain. Moreover, the model also can calculate likelihood and estimate the parameters, through applying multiple algorithms, including Viterbi algorithm, Forward-backward algorithm and Baum-Welch algorithm. In our research, we implemented the Python package hmm in sklearn to conduct the whole process.

To introduce the basic mathematical logic behind HMM, we begin with its parameters. According to (Rabiner 1989), HMM has following parameters:

1) N, the number of states in the model.

2) M, the number of distinct observation symbols per state, which means the corresponding observation symbols of each state to determine which state the time point is at.

3) A, the state transition probability matrix, which represents the probability that now at state i transfer to state j

$$a_{ij} = P\left[q_{t+1} = S_j \mid q_t = S_i\right], \quad 1 \leq i, j \leq N. \tag{1}$$

4) B, the observation symbol probability distribution in state j, $B = \{b_j(k)\}$, where

$$b_j(k) = P\left[v_k \text{ at } t \mid q_t = S_j\right], \quad 1 \leq j \leq N, 1 \leq k \leq M. \tag{2}$$

5) π, the initial state distribution.

$$\pi_i = P\left[q_1 = S_i\right], \quad 1 \leq i \leq N. \tag{3}$$

We start with the general case. The first step of deciding the state for each observation is to choose the initial state according to the initial state distribution, and then according to the symbol probability distribution we assign the relative symbol to the current observation (Rabiner 1989). We transit to the new state according to the state transition probability distribution for the current state, then we move to the next observation and repeat the above procedure (Rabiner 1989). Regarding deciding market regimes, daily return is a very important feature to decide the next market regime. A significant assumption for HMM to decide market regimes is that historical prices can fully predict future prices (Costa & Kwon 2020). In that case, if the predicted next day asset return differs significantly from the actual asset return, we may describe this change as a transition into a different market regime.

3.2.2 Find Optimal Number of Regimes

After we build the model of market regimes, we need to decide the optimal number of regimes. Bayesian information criterion (BIC) was first introduced by (Schwarz 1978), and was applied to test performance of different numbers of regimes in many papers, including (Nystrup et al. 2020) and (Costa & Kwon 2020). BIC formula we applied in our research is as follows

$$\begin{aligned} a &= l \cdot b + l \cdot b^2 + l^2, \\ \text{BIC} &= -2\ln\widehat{L} + a\ln p, \end{aligned} \tag{4}$$

where l is the number of regimes, b is the number of factors used for regime detection ($b = 1$ in our case), $\bar{L}$ is the likelihood calculated from the Baum–Welch algorithm, and p is the total number of observations in the time series. A lower value of BIC is preferred.

3.3 Portfolio Strategies

To achieve our purposes at the beginning of the paper, we mainly focused on mean variance strategy and compared it with minimum variance strategy and maximum diversification strategy. Rebalancing was applied weekly and monthly based on different portfolio management approaches. We followed the step of finding the regime-dependent parameters, calculating the optimal weight for the portfolio for different strategies and rebalancing. In terms of rebalancing and prediction states vary along with the time, regime-dependent parameters will be dynamic and change along with the time.

Mean variance optimization is the basic strategy we chose to perform. It minimizes risk which is the portfolio variance while subject to a target return R and weight sum 1.

$$
\begin{aligned}
&\min_{x} \quad x^T \Sigma x \quad \text{s.t.} \quad \mu^T x \geq R, \mathbf{1}^T x = 1 \\
&R = \left(\frac{1+\kappa}{n}\right) \mathbf{1}^T \mu.
\end{aligned} \tag{5}
$$

Target return is set to be $1+k$ times return mean. k is set to be 10% when the mean return is positive and -10% when the mean return is negative. In this case, our target return becomes more reasonable in different situations.

The second strategy we performed is minimum variance optimization. It's the same with Mean variance optimization without subject to target return. We also performed a Maximum diversification strategy and its aim is to maximize diversification ratio. The optimal weight is expressed as following:

$$
W_{\text{MD}} = \frac{\Omega^{-1}\sigma}{\sigma'\Omega^{-1}\sigma} \tag{6}
$$

where Ω is the $n \times n$ variance covariance matrix of returns, and σ is the $n \times 1$ vector of asset volatilities. Rebalancing was also applied to portfolio construction on a weekly or monthly basis in our project.

4 Results

4.1 With or Without Market Regimes

We first want to see whether regime-switching models help improve portfolio performance. Here is monthly 4-year portfolio performance with and without market regime based on mean variance strategy assuming we have $100 at start.

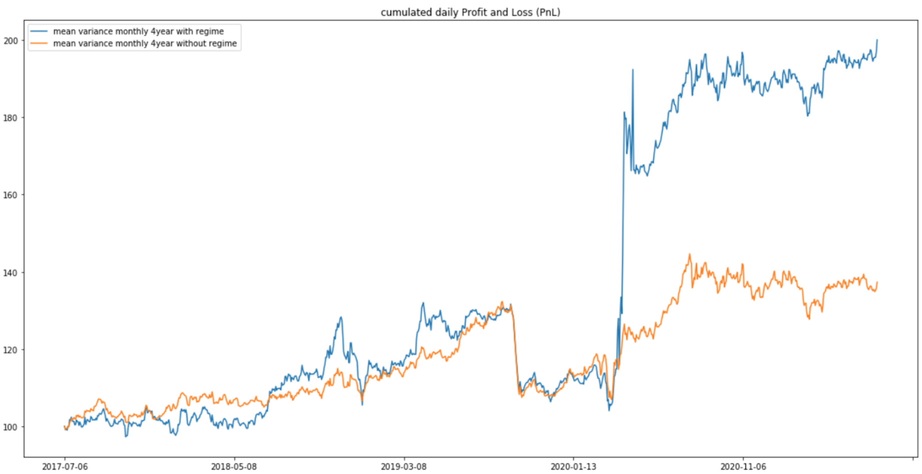

Figure 4-1: Portfolio value of Mean-variance optimization rebalanced monthly in 4 years with and without regimes

As the graph (Figure 4-1) above shows, there are two sudden increases in portfolio value with regime so that portfolio with regime outperforms portfolio without regime. One jump is near Dec 2018, and the other one is somewhere around March 2020.

Figure 4-2: S&P 500 index 4-year trend

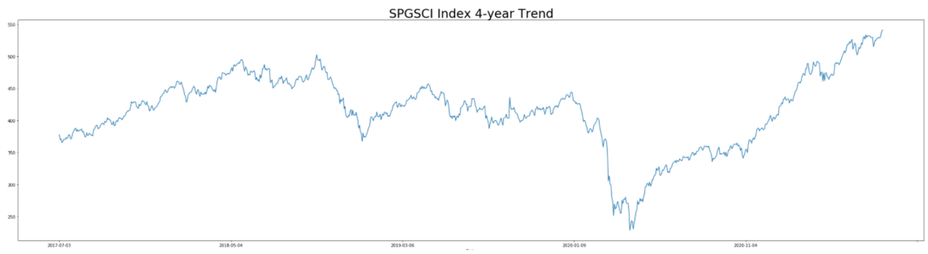

Figure 4-3: SPGSCI index 4-year trend

Figure 4-2 and Figure 4-3 show a 4-year trend for benchmark index S&P 500 and SPGSCI. Both benchmark index drops in late 2018 due to interest rate hike and political instability of the US government. In addition, it drops dramatically during March 2020 due to coronavirus.

	AMZN US Equity	BEN US Equity	C01 Comdty	CL1 Comdty	GOLDS Comdty	JPM US Equity
2019-11-22	0.208037	0.166648	0.152326	0.151218	0.156043	0.165728
2019-12-24	0.208802	0.165685	0.151956	0.151496	0.155909	0.166152
2020-01-27	0.184710	0.129591	0.130394	-0.029696	0.384385	0.200616
2020-02-26	0.176788	0.124583	0.131628	-0.030062	0.398357	0.198707
2020-03-27	-7.808774	3.160421	-0.262039	-0.250548	6.088161	0.072779
2020-05-05	0.246007	0.045723	0.292977	0.009474	0.489545	-0.083726
2020-06-04	0.130783	0.161085	0.082009	0.006739	0.319571	0.299814
2020-07-06	0.130783	0.161085	0.082009	0.006739	0.319571	0.299814
2020-08-04	0.128491	0.160431	0.083307	0.008860	0.321409	0.297503

Figure 4-4: Portfolio weights with regimes from 12/24/2019 to 08/04/2020

2019-12-24	0.145743	0.010221	0.056947	-0.032356	0.735545	0.083901
2020-01-27	0.123942	0.018770	0.051554	-0.019277	0.735990	0.089022
2020-02-26	0.117661	0.017177	0.049728	-0.015773	0.737663	0.093545
2020-03-27	0.122975	0.016626	0.056646	-0.030479	0.738946	0.095286
2020-05-05	0.117001	0.035147	0.052744	-0.049211	0.736364	0.107953
2020-06-04	0.118111	0.038789	0.055291	-0.052879	0.732834	0.107853
2020-07-06	0.119043	0.036753	0.057108	-0.057729	0.734104	0.110720
2020-08-04	0.113136	0.041575	0.057513	-0.051559	0.723353	0.115982

Figure 4-5: Portfolio weights without regimes from 12/24/2019 to 08/04/2020

Figure 4-4 suggests the portfolio weight with regime and Figure 4-5 suggests the portfolio weight without regime. This implies that regime-switching models can detect extreme cases and make corresponding changes according to different states. Therefore, portfolios with regime have better performance than portfolios without regime.

However, we also examine weekly 1-year portfolio performance with and without regime under mean variance strategy (Figure 4-6 – 4-8). The accumulated profit trend is similar and portfolios without regime seem to perform well in terms of sharpe ratio, VaR and CVaR. This implies if there is no sudden jump, the benefit of regime-switching models cannot display.

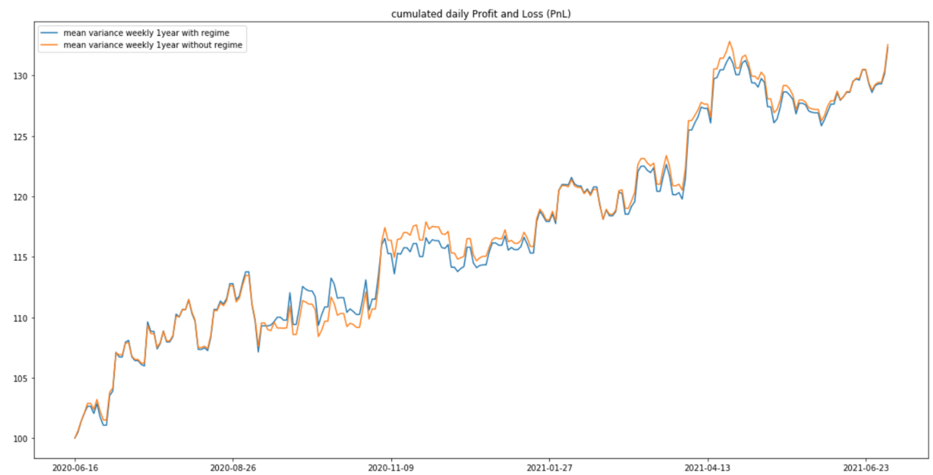

Figure 4-6: Cumulated daily P&L of Mean-variance strategy rebalanced monthly in 1 year with and without regimes

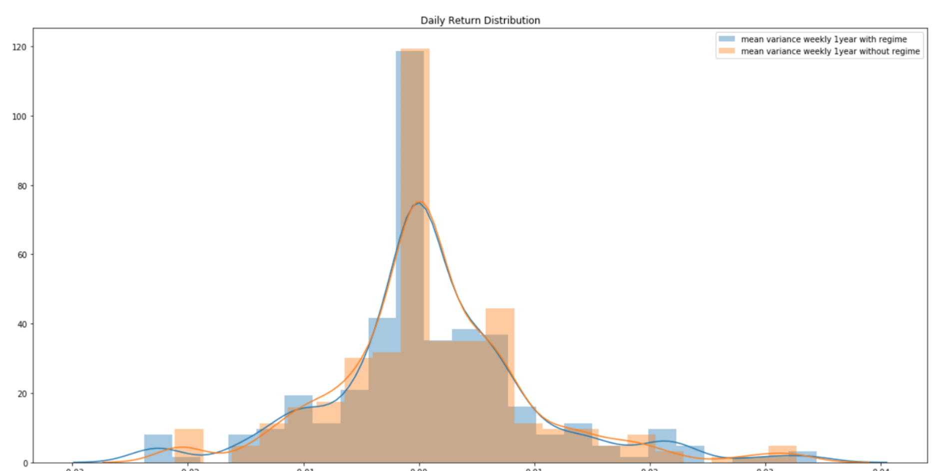

Figure 4-7: Daily return distributions of Mean-variance strategy rebalanced monthly in 1 year with and without regimes

	mean variance weekly 1year with regime	mean variance weekly 1year without regime
Cumulated Return (%)	132.335	132.568
Daily Mean Return (%)	0.113151	0.113423
Daily Min Return (%)	-2.37839	-2.1108
Max 10 Days Drawdown (%)	6.49412	7.0374
Volatility Daily Price	8.08734	8.31711
Volatility Daily Return (%)	0.906765	0.859805
Sharpe Ratio	0.124786	0.131917
Daily Return Skewness	0.50301	0.625671
Daily Return Kurtosis	2.1055	2.27037
Modified VaR (%)	-1.83634	-1.76162
CVaR (%)	-2.2557	-2.05618

Figure 4-8: Performance metrics of Mean-variance strategy rebalanced monthly in 1 year with and without regimes

4.2 With or Without Rebalance

In this section, we want to compare portfolio performance with rebalance strategy and buy-and-hold strategy.

As for the 2-year investment period (Figure 4-9 – 4-11), portfolios with rebalance perform better than portfolios simply buy and hold for all three strategies. However, as we mentioned in section 4.1 that the optimal weight for individual assets changes a lot in March 2020, the portfolio value increases due to the rebalance in that month. So this large jump in portfolio value should be mainly attributed to the introduction of the regime-switching model.

In the previous section, we also conclude that the market regime has little impact on the 1-year portfolio. So by analyzing weekly 1-year portfolio performance with or without rebalance, we can understand the effect of rebalance on the portfolio performance.

Figure 4-12 – 4-14 shows accumulated profit and loss trend for portfolio in recent one year with or without rebalance for each asset allocation strategy. It suggests that performance of portfolios with rebalance has a more steady increasing trend. This result can be verified by “Max 10 Days Drawdown” in Figure 4-15 – 4-17 that portfolios with rebalance have lower max 10 days drawdown.

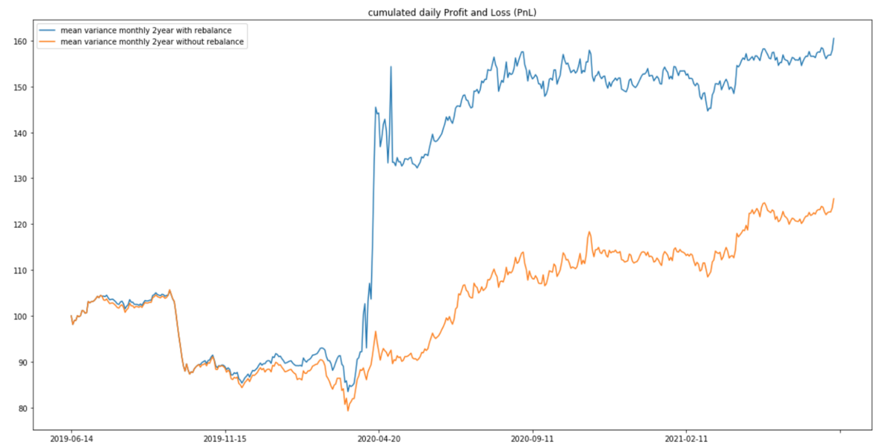

Figure 4-9: Cumulated daily P&L of Mean-variance strategy rebalanced monthly in 2 years with and without rebalance

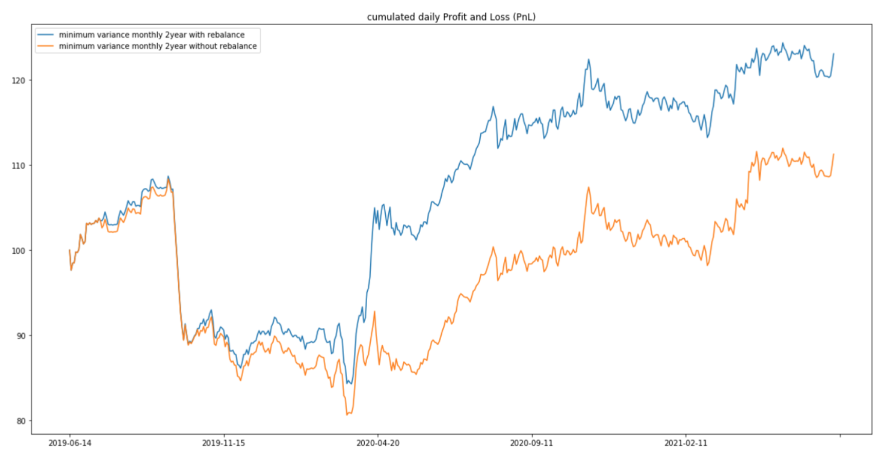

Figure 4-10: Cumulated daily P&L of Min variance strategy rebalanced monthly in 2 years with and without rebalance

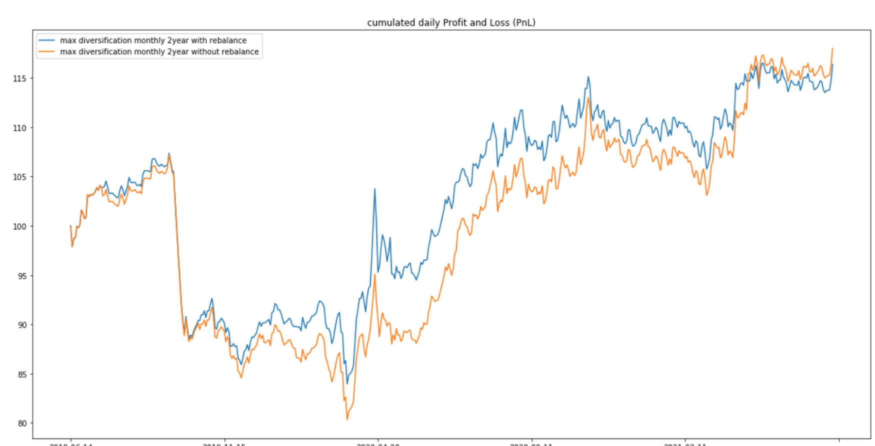

Figure 4-11: Cumulated daily P&L of Max diversification strategy rebalanced monthly in 2 years with and without rebalance

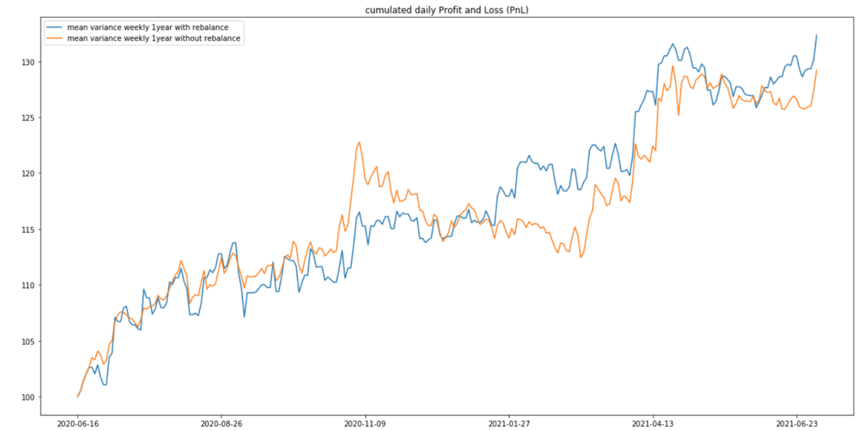

Figure 4-12: Cumulated daily P&L of Mean-variance strategy rebalanced weekly in 1 year with and without rebalance

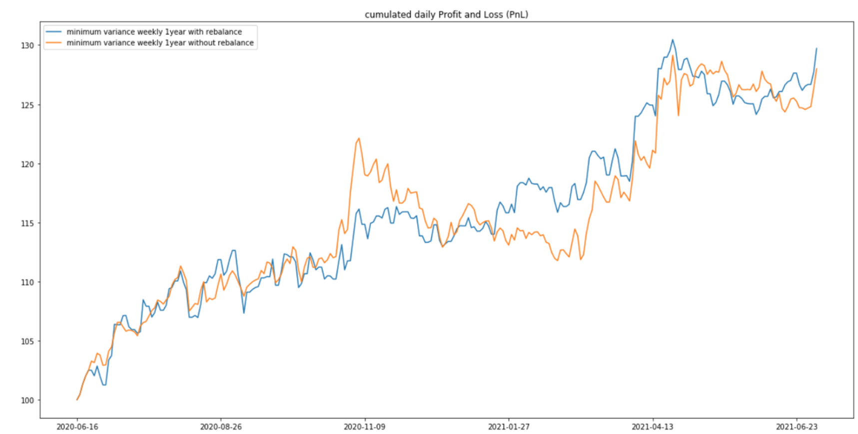

Figure 4-13: Cumulated daily P&L of Min variance strategy rebalanced weekly in 1 year with and without rebalance

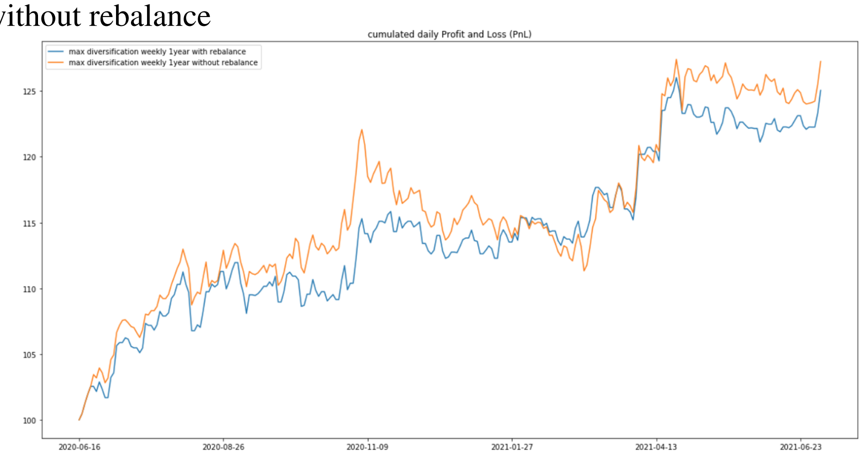

Figure 4-14: Cumulated daily P&L of Max diversification strategy rebalanced weekly in 1 year with and without rebalance

	mean variance weekly 1year with rebalance	mean variance weekly 1year without rebalance
Cumulated Return (%)	132.335	129.217
Daily Mean Return (%)	0.113151	0.10296
Daily Min Return (%)	-2.37839	-2.28666
Max 10 Days Drawdown (%)	6.49412	8.07084
Volatility Daily Price	8.08734	7.01323
Volatility Daily Return (%)	0.906765	0.799708
Sharpe Ratio	0.124786	0.128748
Daily Return Skewness	0.50301	0.496021
Daily Return Kurtosis	2.1055	2.28112
Modified VaR (%)	-1.83634	-1.64483
CVaR (%)	-2.2557	-1.97378

Figure 4-15: Performance metrics of Mean-variance strategy rebalanced weekly in 1 year with and without regimes

	minimum variance weekly 1year with rebalance	minimum variance weekly 1year without rebalance
Cumulated Return (%)	129.711	127.998
Daily Mean Return (%)	0.104393	0.0993188
Daily Min Return (%)	-2.12039	-2.63816
Max 10 Days Drawdown (%)	6.11361	8.28154
Volatility Daily Price	7.3672	7.08627
Volatility Daily Return (%)	0.793024	0.806033
Sharpe Ratio	0.131639	0.123219
Daily Return Skewness	0.516915	0.54357
Daily Return Kurtosis	2.23708	3.05474
Modified VaR (%)	-1.62149	-1.77628
CVaR (%)	-1.94308	-2.47954

Figure 4-16: Performance metrics of Min-variance strategy rebalanced weekly in 1 year with and without regimes

	max diversification weekly 1year with rebalance	max diversification weekly 1year without rebalance
Cumulated Return (%)	125.04	127.235
Daily Mean Return (%)	0.0894578	0.0968233
Daily Min Return (%)	-2.64468	-2.49173
Max 10 Days Drawdown (%)	5.35405	7.53931
Volatility Daily Price	5.97604	6.33423
Volatility Daily Return (%)	0.704999	0.78458
Sharpe Ratio	0.126891	0.123408
Daily Return Skewness	0.377787	0.316548
Daily Return Kurtosis	2.94141	2.02098
Modified VaR (%)	-1.53968	-1.57922
CVaR (%)	-1.96776	-1.90934

Figure 4-17: Performance metrics of Max diversification strategy rebalanced weekly in 1 year with and without regimes

4.3 Monthly or Weekly Rebalance

In this section, we want to analyze how rebalance frequency affects portfolio performance assuming there is no transaction cost.

For all three strategies, that is, mean variance (Figure 4-18), minimum variance (Figure 4-19) and max diversification (Figure 4-20), we compare both weekly and monthly 1-year portfolio performance and find that higher rebalance frequency helps improve portfolio performance.

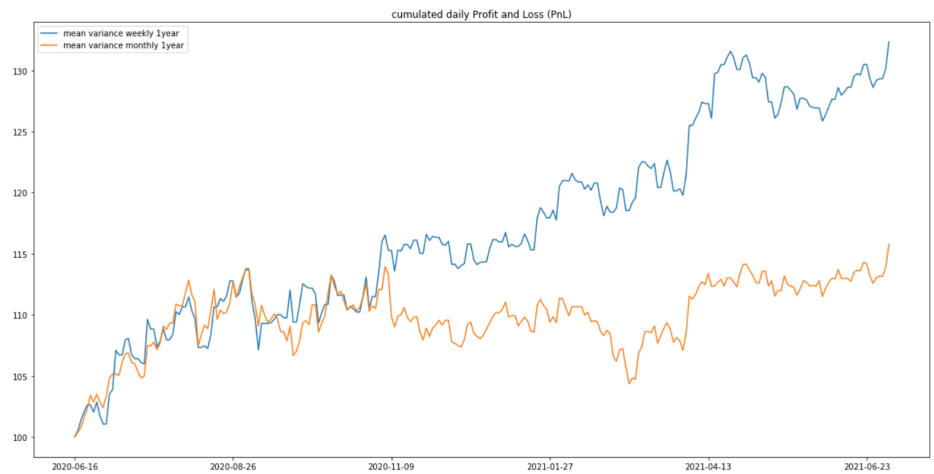

Figure 4-18: Cumulated daily P&L of Mean-variance strategy rebalanced weekly and monthly in 1 year

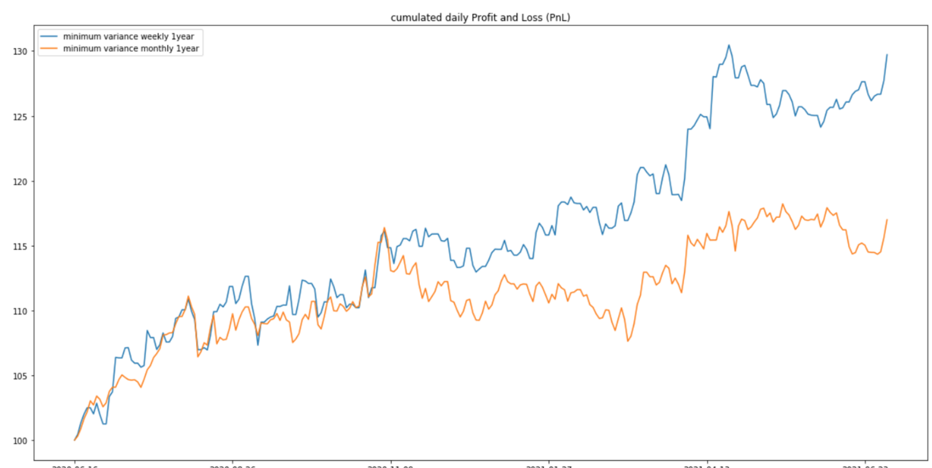

Figure 4-19: Cumulated daily P&L of Min-variance strategy rebalanced weekly and monthly in 1 year

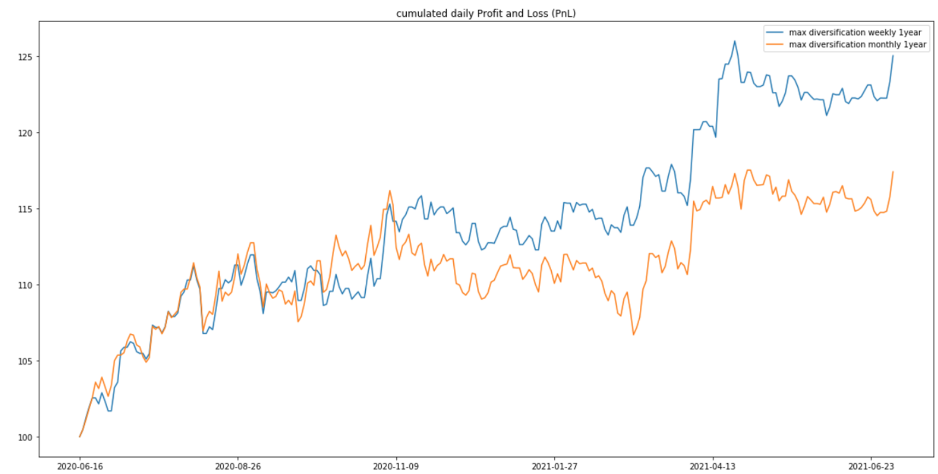

Figure 4-20: Cumulated daily P&L of Max diversification strategy rebalanced weekly and monthly in 1 year

4.4 Comparison for Three Strategies

In the last section, we want to compare the portfolio performance for mean variance, minimum variance and max diversification strategies. Figure 4-21 shows the accumulated return with monthly 2-year rebalance for all three strategies. Mean variance strategy has a higher yield than minimum variance strategy than max diversification strategy. The results seem to be reasonable because it is based on the definition of these three strategies. Both mean variance strategy and minimum variance strategy is trying to minimize portfolio variance except that mean variance strategy has one more constraint that the expected return should be greater or equal to target return. So, the result portfolio volatility should be higher for mean variance strategy and higher risk corresponds to higher return. This result can be verified by the table in Figure 4-22. Therefore, mean variance strategy has a higher accumulated return than minimum variance strategy. As for the max diversification strategy, it behaves poorly among these strategies because the purpose of this strategy is trying to allocate risk among many asset classes. So if there are more types of asset class in our portfolio, it might behave better.

Figure 4-21: Cumulated P&L of three strategies rebalanced monthly in 2 years

	max diversification monthly 2year	minimum variance monthly 2year	mean variance monthly 2year
Cumulated Return (%)	116.353	123.093	160.5
Daily Mean Return (%)	0.0348363	0.0455943	0.110107
Daily Min Return (%)	-5.24093	-3.07256	-13.5342
Max 10 Days Drawdown (%)	16.402	17.1715	36.6582
Volatility Daily Price	9.09346	11.8307	28.001
Volatility Daily Return (%)	0.926012	0.860203	1.73012
Sharpe Ratio	0.0376196	0.0530041	0.0636414
Daily Return Skewness	-0.620255	-0.251633	1.84998
Daily Return Kurtosis	4.34727	2.79402	30.755
Modified VaR (%)	-2.31552	-1.93074	12.9409
CVaR (%)	-3.05058	-2.46073	0.0808907

Figure 4-22: Performance metrics of three strategies rebalanced monthly in 2 years

5 Conclusion and Discussion

To sum up everything that stated so far, we implemented HMM and BIC to detect market regimes and optimal number of regimes. We applied three different strategies based on different frequencies, different time periods, with or without market regimes and with rebalancing and without rebalancing. Here comes our conclusions. First, implementing market-regimes with asset allocation, the portfolio performs better than without considering market-regimes. Second, the regime-switching model performs better in longer time, as there will be more extreme stages appeared in history. Third, the regime-switching model was verified to be efficient in detecting financial crises and preventing big loss during that period. Fourth, after comparing three strategies, we found the Mean-Variance strategy was the best one, from the perspective of Sharpe Ratio.

Then potential improvements and further considerations can be as follows. Considering more indexes to decide states, for example some economic indexes, can be a potential improvement. With more indexes involved rather than just S&P 500, we can detect market regimes more accurately and thus react to it quicker. We can also consider some other models to decide regimes, such as LPPLS, and add more asset classes and financial instruments to see if there's any other patterns regarding them. In terms of strategies, we found that our market regime model performs better when there is a huge financial crisis. In other words, it's good at decreasing risk under extreme periods, but it does not perform well when the market is bullish. This is mainly because our strategies generally aim to minimize risk. To improve the performance of a portfolio, we may consider different strategies under different market regimes. For example, we can apply aggressive strategy in a bullish market and conservative strategy in a bearish market. This may further improve portfolio performance. Another improvement regarding codes is to optimize it so we can run rebalancing over a longer period, where there are more historical crises for our model to function.

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