Factors in Time: *Fine-Tuning Hedge Fund Replication*

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The primary utility of factor models is that they can help simplify the analysis of portfolios consisting of many assets by explaining their behavior using a parsimonious set of drivers. This is true of linear regression–based factor models and those constructed using principal component analysis, as well as nonlinear factor models, including those based on machine learning algorithms.[1] By providing a more transparent view of the systemic risks to which a portfolio is exposed, factor models can be employed in both risk management and alpha generation. Asset allocators, for example, use factor models both to track the amount and type of systemic risk carried by different asset classes and to build factor-tracking portfolios in which specific factor exposures (loadings) are targeted during portfolio construction to more precisely express their investment views or to replicate strategies with specific factor profiles.

One application of factor-tracking portfolio construction is hedge fund replication, where liquid investment vehicles are used to build portfolios that are expected to mimic the investment behavior of different hedge fund styles. In regression-based frameworks, the construction of factor-tracking portfolios entails a number of risks. Perhaps the major risk is overfitting, in which the calculation of beta coefficients may depend so precisely on the training data that a given model fails to generalize on test data, producing poor predictive results. This issue has historically been somewhat of a secondary concern for those engaged in building hedge fund replication models because they have been primarily interested in building frameworks that maximize in-sample explanatory power. Nevertheless, predictive power, our focus in this article, is necessarily the central concern of practitioners who want to apply hedge fund replication *ex ante* in portfolio management. Another potential risk with replication strategies is multicollinearity, where predictors are linearly correlated to a degree that renders them problematic for use in an ordinary least squares (OLS) regression. The latter risk is ever-present in hedge fund replication, in which practitioners generally use publicly available vehicles, such as exchange-traded funds (ETFs), as factor proxies. A final risk is that, because of the dynamic nature of hedge funds, models built using a single sample may fail to capture the evolution of hedge fund exposures over time.

As a remedy to the foregoing shortcomings of the standard approach to factor-based hedge fund replication, in this article the authors demonstrate how to use the regularization technique known as *ridge regression* (RR) to both mitigate the impact of factor

[1] See Simonian et al. (2019) for an example of the latter approach.

interdependence and enhance the predictive power of OLS-type models through time. Accordingly, the present article can be considered a contribution to the regression-based hedge fund replication approach previously explored by Agarwal and Naik (2000), Fung and Hsieh (1997, 2004), Ennis and Sebastian (2003), Markov et al. (2006), Hasanhodzic and Lo (2007), Jaeger (2008), and Amenc et al. (2010), among others.

BUILDING FACTOR-MATCHING PORTFOLIOS

In practice, the standard approach to factor portfolio construction using a linear factor model is generally based on a straightforward application of relatively basic optimization methods such as linear programming. As an illustration of the standard approach to factor matching, consider the following simple example, in which we have a portfolio of three assets—1, 2, and 3—whose behavior is explained by the following two-factor model:

$$R_i = \beta_1 r_1 + \beta_2 r_2 \quad (1)$$

where R_i represents the return on asset i, β_1 represents the sensitivity of asset i to the rate of change in the price level of the broad equity market, r_1 represents the rate of change in the price level of the broad equity market, β_2 represents the sensitivity of asset i to the rate of change in 10-year Treasury yields, and r_2 represents the rate of change in 10-year Treasury yields. We list hypothetical individual betas for each asset in Equation 2:

Asset	β_1	β_2
1	0.75	1.00
2	0.50	0.25
3	1.50	0.75

(2)

Now, suppose we wish to allocate our assets so that our portfolio has a beta of 1.0 to β_1, and a beta of 0.50 to β_2. As previously mentioned, a standard approach to solving the factor-matching problem is to use a linear program; for example:

$$\text{Min}|1.00 - \beta_1^T x_i| + |0.50 - \beta_2^T x_i| \quad (3)$$

subject to
$\Sigma_{i=1}^{n} x_i = 1$ (No leverage constraint)
$x_1, x_2, x_3 \geq 0$ (No shorting constraint)

The solution to Equation 3 is given by the following portfolio weights: $x_1 = 0\%$, $x_2 = 50\%$, $x_3 = 50\%$.

One popular factor-matching approach among practitioners is *returns-based style analysis* (RBSA), a method introduced by Sharpe (1988, 1992). As a species of factor portfolio construction, it can be considered a framework tailored to long-only investors who seek to replicate the performance of particular strategies using publicly available investment vehicles. RBSA is regression based and is expressed formally as

$$R_t^m = \alpha + \sum_{i=1}^{I} \beta_i R_t^i + \epsilon_t \quad (4)$$

where R_t^m is the return stream for the investment strategy to be replicated, R_t^i represents the return streams of a set of investable factor proxies, I is the number of investable proxies, and ϵ_t is the error term. Formally, RBSA differs from the factor-matching optimization described above in that it seeks to match the return of a target portfolio rather than its specific profile of systemic exposures. In the standard case, two important constraints are applied to Equation 4, constraints that are put in place to produce a combination of investable proxies suitable for a long-only implementation. First, each beta coefficient is constrained to be greater than zero; that is, $\beta_i > 0$, $\forall i$. Second, the sum of the betas is constrained to sum to unity; that is, $\Sigma_{i=1}^{I} \beta_i = 1$. As such, each beta is interpreted as a weight assigned to a particular investable proxy in a replication portfolio.

In hedge fund replication research (such as the articles cited earlier), it has been customary to relax the no-shorting and sum-to-unity constraints, which transform their frameworks to standard OLS regressions that are differentiated, more or less, by their use of investable vehicles as factors. We follow that convention here as well. However, it is important to note that, whereas in the past the relaxation of these constraints may have posed implementation challenges to some investors, given the breadth and depth of modern derivatives markets, including the advent of options on ETFs, it is relatively easy for long-only investors to create net exposures deviating from unity. Thus, the difference between factor portfolio construction using standard RBSA and constraint-relaxed RBSA is minimal today, given the set of investment tools available to portfolio managers.

APPLYING RIDGE REGRESSION TO HEDGE FUND REPLICATION

Because hedge fund replication is generally executed using publicly available investment vehicles, the factors in hedge fund replication models are generally correlated to a sufficient degree to disqualify them as true factor models, which by definition need to consist of a (parsimonious) set of uncorrelated predictor variables. From a statistical standpoint, multicollinearity negatively affects a regression's output by inducing inaccurate estimates of regression coefficients, inflating standard errors, generating false p-values, and significantly reducing the predictive utility of a given model. Accordingly, to counter the risk of heightened variance between training and test samples, it may be desirable to inject some bias into an OLS regression to generalize the model to a greater degree.

RR[2] is a regression framework that provides a way of formally addressing multicollinearity in a standard OLS regression by adjusting the OLS regression coefficient estimates using a penalty parameter (so called because it penalizes less influential predictors), that injects varying degrees of bias into the regression. When the penalty parameter is applied to the diagonal of the regression matrix, it forces the columns of the matrix to be linearly independent. The expectation is that by applying the penalty parameter, the regression standard errors will be reduced and lead to the generation of more predictively useful models. Formally, an RR is described as follows:

$$\hat{\beta} = \operatorname{argmin}_{\beta \in \mathbb{R}^p} \sum_{i=1}^{n} (y_i - x_i^T \beta)^2 + \lambda \sum_{j=1}^{p} \beta_j^2 = \operatorname{argmin}_{\beta \in \mathbb{R}^p} \underbrace{\| y - X\beta \|_2^2}_{\text{Loss}} + \lambda \underbrace{\| \beta \|_2^2}_{\text{Penalty}} \quad (5)$$

where $y \in \mathbb{R}^n$ is a vector of target observations, $X \in \mathbb{R}^{n \times p}$ is a predictor matrix, and $\lambda \geq 0$ is the penalty parameter.

HEDGE FUND REPLICATION

To test the efficacy of RR in hedge fund replication, we set up an investable[3] four-factor model and test it on a set of seven hedge fund strategies. Our factors are the MSCI USA Value and Momentum indices, the Bloomberg High Yield Total Return Index, and the ICE Core Treasury Bond Index. The hedge fund strategies we test are merger arbitrage, convertible arbitrage, long/short equity, global macro, equity market neutral, S&P 500 PutWrite, and S&P 500 BuyWrite.[4] We show the full-sample correlations of our factors in Exhibit 1.

Although the matrix in Exhibit 1 is full rank, and thus valid for a regression, three of the factors show fairly high correlations to each other (value, momentum, and high yield). Given this, RR can still play a useful role in mitigating some of the potentially deleterious modeling impact of the positive co-movement of these factors.

The central consideration in any application of RR is what value the penalty parameter λ should take. Here we calibrate ("fine-tune") the value of λ by testing a range of values through time and choosing the λ value that exhibits the best predictive power over a set of rolling windows. By evaluating different values for the penalty parameters across evolving sample sets, we ultimately arrive at the penalty value that minimizes prediction error on test data. Given that our primary goal is to correct for overfitting, it is more prudent to calibrate λ over a multitude of samples to avoid tying the value of λ to a single set of data; doing the latter would risk the same type of overfitting we are trying to avoid with the regression coefficients.[5] This approach is also adaptive to new information because the optimal λ may change over time as additional rolling samples are evaluated.

In Exhibit 2, we show the values of λ that provide the best predictive results over two different test horizons. The first is a "tactical" horizon, where we derive

[2] The ideas underlying ridge regression were independently discovered by a number of individuals. See Larson (1931), Tikhonov (1943, 1963), Foster (1961), and Hoerl and Kennard (1970).

[3] ETFs tracking the performance of each of these indexes are available.

[4] For merger arbitrage, convertible arbitrage, long/short equity, global macro, and equity market neutral, we use the corresponding Barclay Hedge index for each. For S&P 500 PutWrite and S&P 500 BuyWrite, we use the indexes published by the Chicago Board Options Exchange (CBOE).

[5] Our evaluation methodology can be considered a species of cross-validation. However our approach is arguably more suited to time series analysis as we exclude the possibility of using data in a testing (validation) set that chronologically precedes the data in a training set, as is the case with many standard cross-validation methodologies (for a discussion of cross-validation see Hastie et al. 2009).

Exhibit 1

Full-Sample Correlations of Factors (monthly data, 1/2006–12/2018)

	MSCI USA Value Net Total Return USD Index	MSCI USA Momentum USD Net Total Return Index	ICE US Treasury Core Bond TR Index	Bloomberg Barclays US Corporate High Yield Total Return Index Value Unhedged USD
MSCI USA Value Net Total Return USD Index	1	0.81	–0.28	0.68
MSCI USA Momentum USD Net Total Return Index	0.81	1	–0.27	0.62
ICE US Treasury Core Bond TR Index	–0.28	–0.27	1	–0.25
Bloomberg Barclays US Corporate High Yield Total Return Index Value Unhedged USD	0.68	0.62	–0.25	1

Source: Natixis Investment Managers.

Exhibit 2

Hedge Fund Replication—OLS versus Ridge Regression (monthly data, 1/2006–12/2018)

Strategy	Optimal λ	Average SSE	Batting Average	In-Sample R^2	OLS Average SSE	OLS In-Sample R^2
Rolling 12-Month Windows; 1-Month-Ahead Replication Horizon						
Merger Arbitrage	1.4	6.52E-05	77%	0.35	1.01E-04	0.51
Convertible Arbitrage	0.1	1.95E-04	78%	0.70	2.22E-04	0.73
Long/Short Equity	0.5	7.88E-05	73%	0.70	1.03E-04	0.77
Global Macro	1.4	1.47E-04	90%	0.40	2.51E-04	0.58
Equity Market Neutral	0.2	3.89E-05	85%	0.54	5.21E-05	0.59
S&P 500 PutWrite	0.4	4.26E-04	84%	0.71	6.32E-04	0.76
S&P 500 BuyWrite	0.3	2.65E-04	75%	0.86	4.22E-04	0.89
Rolling 36-Month Windows; 12-Month-Ahead Replication Horizon						
Merger Arbitrage	2.4	5.08E-04	71%	0.27	6.03E-04	0.41
Convertible Arbitrage	0.1	7.84E-04	52%	0.68	8.11E-04	0.70
Long/Short Equity	0.2	6.34E-04	63%	0.75	6.81E-04	0.77
Global Macro	0.3	1.21E-03	73%	0.42	1.31E-03	0.45
Equity Market Neutral	1.5	3.62E-04	67%	0.25	4.40E-04	0.46
S&P 500 PutWrite	0.3	2.48E-03	65%	0.73	2.70E-03	0.74
S&P 500 BuyWrite	0.1	1.70E-03	50%	0.87	1.73E-03	0.88

Source: Barclay Hedge, CBOE, and Natixis Investment Managers.

beta coefficients[6] over a 12-month window and attempt to match the performance of a strategy over the following

[6] We note that it is important to re-scale the raw ridge regression betas so that they are comparable to the original OLS betas. Given a training set $\{x_1, \ldots x_k\}$ and normalized returns $\frac{x_i - \bar{x}_i}{\sqrt{x_i^T x_i}}$ we derive re-scaled beta coefficients from the original ridge regression betas as follows:

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_{i,normalized} = \beta_0 + \sum_{i=1}^{k} \beta_i \frac{x_i - \bar{x}_i}{\sqrt{x_i^{\mathrm{T}} x_i}} = \beta_0 - \sum_{i=1}^{k} \frac{\beta_i \bar{x}_i}{\sqrt{x_i^{\mathrm{T}} x_i}}$$
$$+ \sum_{i=1}^{k} \frac{\beta_i}{\sqrt{x_i^{\mathrm{T}} x_i}} x_i = \beta_{0,re-scaled} + \sum_{i=1}^{k} \beta_{i,re-scaled} x_i .$$

month. The second test horizon is a "strategic" horizon, where we derive beta coefficients over a 36-month window and attempt to match the performance of a strategy over the following 12 months. Our measure of predictive error is the sum of squared errors (SSE) over 1-month and 12-month horizons, respectively. For example, in the tactical study, given the generation of beta coefficients at time t (e.g., end-of-month 12/2005), we multiply the latter coefficients by the factor returns at the end of time $t + 1$ (e.g., end-of-month 1/2006) and compare the weighted sum of the latter values to the actual hedge fund strategy returns for the month.

The first statistic we show in Exhibit 2 is the average SSE over the entire set of rolling windows. Next, we look at what we call the batting average of the RR replication framework versus a standard OLS approach. The batting average is simply the percentage of rolling windows in which the RR SSE is lower than the OLS SSE. This is an important statistic because it indicates the generalizability of the ridge framework over evolving sets of data.

As Exhibit 2 shows, in both the tactical and strategic studies, the RR-based replication models improve significantly upon the standard OLS models in terms of their average SSEs over the set of rolling windows. Perhaps the most important result, however, is that the batting average of the ridge approach is generally very high (the results for the convertible arbitrage and BuyWrite replication models in the strategic study being the sole exceptions), indicating that the predictive superiority of the RR models is consistent through time. Finally, it is notable that the deterioration in the in-sample R^2 when using RR is generally minimal, indicating that the significant improvement in predictive power gained through the use of RR-based models is not made at the expense of a commensurate reduction in *ex post* explanatory power.

CONCLUSION

Regression-based factor models are often employed in hedge fund replication because of their simplicity and familiarity. In practice, however, these models are often misapplied because the investment vehicles used to replicate a given strategy are typically correlated to a sufficient degree to disqualify them as true factor models. More importantly, OLS-based models are often overfitted to test data, resulting in poor predictive performance when tested on new observations. As a remedy to the latter maladies, the authors employ ridge regression, a technique that is designed to alleviate multicollinearity and produce more reliable estimates of predictor variables. Ridge regression uses a penalty parameter that penalizes less influential predictors, resulting in an increase in the bias and a decrease in the variance of the regression. The decrease in variance typically results in enhanced generalizability of a given model and, accordingly, an increase in its predictive power. An outstanding question in any application of ridge regression is what the value of the penalty parameter should be. The authors show how to calibrate the value of the penalty parameter dynamically by choosing the value that exhibits the strongest performance across a set of rolling samples. By testing different values for the penalty parameter across the training data, they are able to arrive at the parameter value that most consistently minimizes the prediction error on different sets of test data. The authors moreover show that the method described produces superior predictive results without significantly sacrificing a model's backward-looking explanatory power. Thus, ridge regression shows itself to be an enhancement over the standard OLS-based approach to hedge fund replication because it can be reliably employed in both *ex post* risk decomposition and *ex ante* portfolio management.

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