Dynamic Allocation or Diversification: *A Regime-Based Approach to Multiple Assets*

PETER NYSTRUP, BO WILLIAM HANSEN, HENRIK OLEJASZ LARSEN, HENRIK MADSEN, AND ERIK LINDSTRÖM

PETER NYSTRUP
is an industrial Ph.D. student at Sampension in Hellerup, Denmark, and in the department of applied mathematics and computer science at Technical University of Denmark in Lyngby, Denmark.
pnys@dtu.dk

BO WILLIAM HANSEN
is the head of investment analysis at Sampension in Hellerup, Denmark.
bwh@sampension.dk

HENRIK OLEJASZ LARSEN
is the CIO at Sampension in Hellerup, Denmark, and an external associate professor in the department of economics at Copenhagen University in Copenhagen, Denmark.
hzl@sampension.dk

HENRIK MADSEN
is a professor and head of the section for dynamical systems in the department of applied mathematics and computer science at Technical University of Denmark in Lyngby, Denmark.
hmad@dtu.dk

ERIK LINDSTRÖM
is a professor at the Centre for Mathematical Sciences at Lund University in Lund, Sweden.
erikl@maths.lth.se

Regime changes present a big challenge to traditional strategic asset allocation (SAA) approaches seeking to develop static "all-weather" portfolios that optimize efficiency across a range of economic scenarios. If economic conditions are persistent and strongly linked to asset class performance, then a dynamic strategy should add value over rebalancing to static weights, as argued by Sheikh and Sun [2012].

Within the last 15 years, many studies have examined the profitability of regime-based asset allocation (RBAA). RBAA is distinct from tactical asset allocation, which relies on forecasting, in that it is based on reacting to observed changes in market conditions. The purpose of RBAA is not to predict regime changes or future market movements, but to identify when a regime change has occurred and then benefit from the persistence of equilibrium returns, volatilities, and correlations to take advantage of favorable regimes and reduce potential drawdowns.

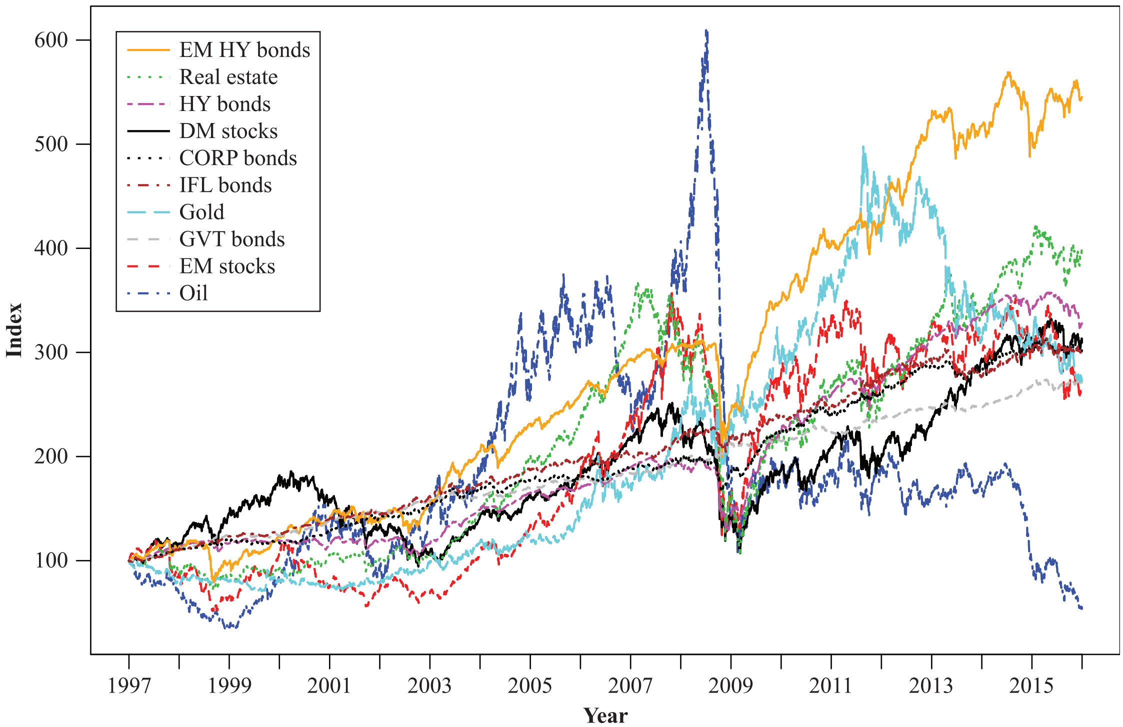
The hidden Markov model (HMM) is a popular choice for inferring the state of financial markets. Ang and Bekaert [2002, 2004], Guidolin and Timmermann [2007], Bulla et al. [2011], Kritzman et al. [2012], and Nystrup et al. [2015] have all found RBAA approaches based on HMMs to be profitable. Ang and Bekaert [2002, 2004] and Guidolin and Timmermann [2007], however, did not account for transaction costs, which is important because frequent rebalancing can offset a dynamic strategy's potential excess return.

All of the aforementioned studies considered dynamic allocation to stocks in combination with bonds and/or a risk-free asset, often involving larger changes in allocation than most investors are willing to or allowed to implement. The potential benefit from taking large positions in a few assets at a time comes at the cost of reduced diversification. The benefits of diversification include lower downside risk and higher risk-adjusted returns. To analyze this trade-off, one must compare the performance of RBAA to a static benchmark using a more comprehensive asset universe, because the potential for diversification is limited by the size of the asset universe.

Dynamic asset allocation is, by definition, more restricted than SAA in terms of the size of the investment opportunity set because it is difficult to invest dynamically in illiquid assets such as private real estate, private equity, infrastructure, timber, etc. This is worth mentioning, given that illiquid alternatives have become a larger part of institutional investors' portfolios in recent years. Although restricted to the universe of liquid assets, there are more opportunities than just stocks and government bonds.

Regime-based approaches are very popular, inter alia, because of the link to the

EXHIBIT 1
Development of the 10 Indexes over the 19-Year Data Period



Notes: The legends are sorted according to the index values at the end of 2015. A color version of this exhibit is available at www.iijpm.com.

phases of the business cycle. As argued in Nystrup et al. [2015], the link is complex and difficult to exploit for investment purposes because of the large lag in the availability of data related to the business cycle. Therefore, in this article, we focus on readily available market data, instead of attempting to establish the link to the business cycle.

The underlying two-state HMM with time-varying parameters is the same as in Nystrup et al. [2015]. However, this study includes more asset classes, and a new, more intuitive way of inferring the hidden market states based on an online version of the Viterbi algorithm is introduced.[1] We examine whether RBAA can effectively respond to financial regimes in an effort to provide better long-term results when compared to a diversified, fixed-weight benchmark, with emphasis on the trade-off between dynamic allocation and diversification.

ASSET UNIVERSE

In this article, we consider an asset universe of developed market (DM) and emerging market (EM) stocks, listed real estate, DM and EM high-yield bonds, gold, oil, corporate bonds, inflation-linked bonds, and government bonds.[2] All indexes measure the total net return in USD with a total of 4,944 daily closing prices per index covering the period from 1997 through 2015.

Exhibit 1 shows the development of the 10 indexes over the 19-year data period. As we can see, there are large differences in the asset classes' behavior, which is why diversification works over the long run. The Global Financial Crisis of 2007–2008 stands out, in that respect, because the majority of the indexes suffered large losses in this period.

Exhibit 2 summarizes the annualized return, standard deviation, Sharpe ratio, and maximum drawdown of the indexes.[3] To ensure that the performance comparison is not distorted by autocorrelation in the daily returns, we have adjusted the reported standard deviations for autocorrelation using the procedure outlined by Kinlaw, Kritzman, and Turkington [2015].[4] The differences in performance are substantial. Out of the 10 indexes, the oil price index has been at both the lowest and highest value during the 19-year period. It is the only index that has

EXHIBIT 2

Performance of the 10 Indexes over the 19-Year Data Period, 1997–2015

Index	Annualized Return	Standard Deviation	Sharpe Ratio	Maximum Drawdown
1. MSCI World (stocks)	0.061	0.18	0.34	0.57
2. MSCI EM (stocks)	0.052	0.29	0.18	0.65
3. FTSE/EPRA REIT (real estate)	0.075	0.22	0.34	0.72
4. High-Yield Bonds (credit)	0.064	0.12	0.56	0.35
5. EM High-Yield Bonds (credit)	0.093	0.12	0.75	0.36
6. S&P GSCI Crude Oil WTI (commodity)	–0.032	0.43	–0.07	0.91
7. S&P GSCI Gold (commodity)	0.054	0.16	0.34	0.46
8. Corporate Bonds Inv Grade (fixed income)	0.060	0.06	1.07	0.16
9. Inflation-Linked Bonds (fixed income)	0.059	0.04	1.40	0.10
10. JPM Global GBI (fixed income)	0.054	0.03	1.75	0.05

had a negative return. The EM high-yield bond index finished at the highest value, whereas the fixed-income indexes realized the highest Sharpe ratios while also suffering the smallest drawdowns. Fixed income benefited from falling interest rates over the considered period.

The differences in Sharpe ratios are too large for a diversified portfolio to be able to outperform a portfolio with an overweight of fixed income. There should not be a strong preference, ex ante, for portfolios that overweight fixed income, because the environment for bonds is unlikely to remain as favorable in the coming years. It is, therefore, important to ensure that portfolios have the same average allocation to fixed income before comparing their performance. Alternatively, the mean values of the indexes could be adjusted so that they all have the same Sharpe ratio.

Exhibit 3 shows the correlations between the 10 indexes estimated from the daily returns over the 19-year data period. The indexes are divided into two groups based on whether they are positively or negatively correlated with DM stocks. Gold stands out as having a very low correlation with the other assets. Investment-grade corporate bonds could have been labeled as credit rather than fixed income, but the index appears to be strongly correlated with inflation-linked bonds and government bonds. High-yield bonds, on the other hand, are more strongly correlated with stocks than government bonds.

Volatility Regimes

The regime detection will focus on the log returns of the MSCI World Index because portfolio risk is typically dominated by stock market risk (see, e.g., Goyal, Ilmanen, and Kabiller [2015]). In addition, the stock markets generally lead the economy (Siegel [1991]). Exhibit 4 shows the log returns of the MSCI World Index.[5] The volatility forms clusters, because large price movements tend to be followed by large price movements and vice versa, as noted by Mandelbrot [1963].[6]

RBAA aims to exploit this persistence of the volatility, because risk-adjusted returns, on average, are substantially lower during turbulent periods, irrespective of the source of turbulence, as shown by Kritzman and Li [2010]. The negative correlation between volatility and returns is sometimes explained by changes in attitudes toward risk; because high-volatility regimes are associated with increased risk aversion and reduced risk capacity,[7] a high-volatility environment is likely to be accompanied by falling asset prices.

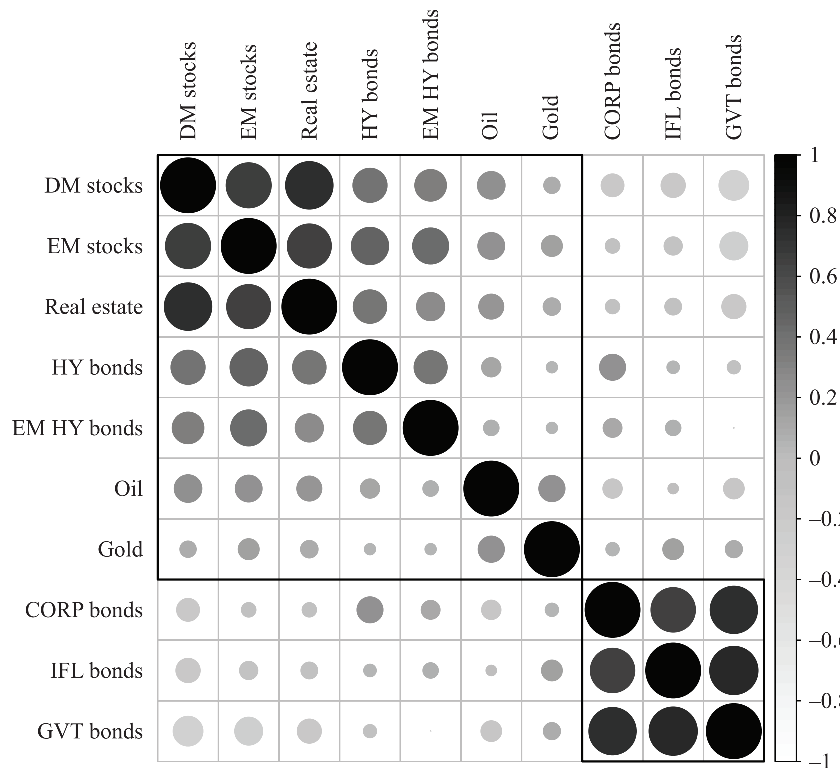
Our intention is to identify high- and low-volatility regimes in the stock returns using a regime-switching model and let the asset allocation depend on the identified regime. Our purpose is not to outline the *optimal* strategy, but rather to discuss the potential profitability of RBAA in a comprehensive asset universe. Exhibit 1 appears to show that the turning points are not exactly the same for all asset classes; however, we leave for future research to show whether the results presented in this article can be improved by including information from the other asset classes in the regime-detection process.

A 60/40 Benchmark

The first column in Exhibit 5 outlines the weight of the indexes in a 60/40 portfolio. The weight of stocks, real estate, credit, and commodities sum to 60%, and the

Exhibit 3

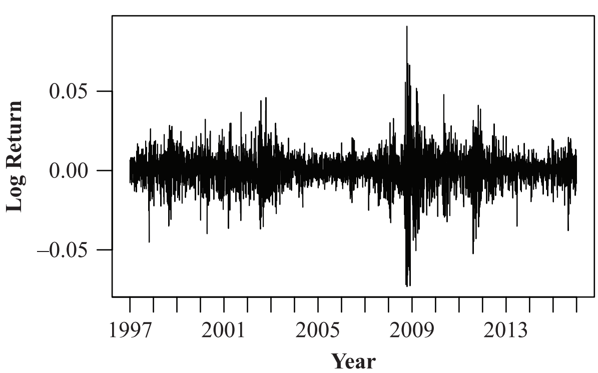
Correlations between the 10 Indexes over the 19-Year Data Period, 1997–2015



Notes: The size of each circle illustrates the absolute value and the shading indicates the numerical value of the correlation. The indexes are divided into two groups based on their correlation with DM stocks.

Exhibit 4

Volatility Clustering in the Log Returns of the MSCI World Index



weight of corporate, inflation-linked, and government bonds sum to 40%. This portfolio will serve as a benchmark, though the results are not sensitive to the specific choice of benchmark allocation.[8] The weights have not been optimized, but are chosen to mimic a 60/40 long-only SAA portfolio of an institutional investor, in order to make the study as realistic as possible.[9] The allocation to high-yield bonds is considered part of the 60% allocation to stocks because they are more strongly correlated with stocks than government bonds (see Exhibit 3).

In Exhibit 5, we compare the 10 indexes' weights in the "risk-on" and "risk-off" RBAA portfolios to their weights in the 60/40 benchmark portfolio when a fraction of the portfolio, $p = 0.5$, is allocated to the RBAA strategy. *Risk-on* means that in low-volatility regimes the weight of the risky assets (everything that is positively correlated with DM stocks) is increased above 60% and the weight of fixed income is decreased below 40%. *Risk-off* is the opposite.[10]

The indexes' weights are increased and decreased in proportion to their relative weights in the benchmark portfolio. The larger the percentage of the portfolio allocated to the RBAA strategy, the more the weights are increased and decreased. When $p = 0.5$, the weight of

EXHIBIT 5

The 10 Indexes' Weights in the Risk–On and Risk–Off RBAA Portfolios When p = 0.5, Compared to Their Weights in the Benchmark SAA Portfolio

Index	SAA		Risk–on		Risk–off	
1. MSCI World (stocks)	25.0%	60%	33.3%	80%	12.5%	30%
2. MSCI EM (stocks)	5.0%		6.7%		2.5%	
3. FTSE/EPRA REIT (real estate)	10.0%		13.3%		5.0%	
4. High-Yield Bonds (credit)	5.0%		6.7%		2.5%	
5. EM High-Yield Bonds (credit)	5.0%		6.7%		2.5%	
6. S&P GSCI Crude Oil WTI (commodity)	5.0%		6.7%		2.5%	
7. S&P GSCI Gold (commodity)	5.0%		6.7%		2.5%	
8. Corporate Bonds Inv Grade (fixed income)	10.0%	40%	5.0%	20%	17.5%	70%
9. Inflation-Linked Bonds (fixed income)	10.0%		5.0%		17.5%	
10. JPM Global GBI (fixed income)	20.0%		10.0%		35.0%	

Note: p is the percentage of the portfolio that is allocated to the RBAA strategy.

government bonds, for example, is 10% in the risk–on portfolio and 35% in the risk–off portfolio, compared to 20% in the SAA portfolio. Adjusting the weight of the risky assets relative to fixed income, rather than just adjusting the weights of stocks and government bonds, ensures a minimum level of diversification even when $p = 1$.

THE HIDDEN MARKOV MODEL

Imagine a market that is either in a bullish or a bearish regime. When the market is in the bullish regime, the average return is positive and the volatility is low. When the market is in the bearish regime, the average return is negative and the volatility is high. Although the market regime can never be observed, we can reasonably conclude based on the returns whether it is a bull or a bear market—that is, which state the market is in.

The use of HMMs to infer the state of financial markets has gained popularity over the last decade. The HMM is a black-box model, but the inferred states can often be linked to phases of the business cycle (see, e.g., Guidolin and Timmermann [2007]). The possibility of interpreting the states, combined with the model's ability to reproduce stylized facts of financial returns, is part of the reason that HMMs have become increasingly popular.

In a hidden Markov model, the probability distribution that generates an observation depends on the state of an unobserved Markov chain. A sequence of discrete random variables $\{X_t : t \in \mathbb{N}\}$ is said to be a first-order Markov chain if, for all $t \in \mathbb{N}$, it satisfies the Markov property

$$\Pr(X_{t+1} \mid X_t, \ldots, X_1) = \Pr(X_{t+1} \mid X_t). \tag{1}$$

The conditional probabilities $\Pr(X_{t+1} = j \mid X_t = i) = \gamma_{ij}$ are called *transition probabilities*.

As an example, consider the two-state model with Gaussian conditional distributions:

$$Y_t \sim N(\mu_{X_t}, \sigma^2_{X_t}),$$

where

$$\mu_{X_t} = \begin{cases} \mu_1, & \text{if } X_t = 1, \\ \mu_2, & \text{if } X_t = 2, \end{cases} \quad \sigma^2_{X_t} = \begin{cases} \sigma^2_1, & \text{if } X_t = 1, \\ \sigma^2_2, & \text{if } X_t = 2, \end{cases} \quad \text{and}$$

$$\Gamma = \begin{bmatrix} 1-\gamma_{12} & \gamma_{12} \\ \gamma_{21} & 1-\gamma_{21} \end{bmatrix}.$$

When the current state X_t is known, the distribution of Y_t is given—that is, the distribution of Y_t depends only on X_t.

The sojourn times are implicitly assumed to be geometrically distributed:

$$\Pr(\text{'staying } t \text{ time steps in state } i\text{'}) = \gamma_{ii}^{t-1}(1-\gamma_{ii}). \tag{2}$$

The geometric distribution is memoryless, implying that the time until the next transition out of the current state is independent of the time spent in the state.

HMMs can match the tendency of financial markets to change their behavior abruptly and the phenomenon that the new behavior often persists for several periods after a change (Ang and Timmermann [2012]). They are well suited to capture the stylized behavior of many financial series including volatility clustering and leptokurtosis, as shown by Rydén, Teräsvirta, and Åsbrink [1998].

Subsequent articles have extended the classical Gaussian HMM by considering other sojourn-time distributions than the memoryless geometric distribution (Bulla and Bulla [2006]), other conditional distributions than the Gaussian distribution (Bulla [2011]), and a continuous-time formulation as an alternative to the dominating discrete-time models (Nystrup, Madsen, and Lindström [2015]). In Nystrup, Madsen, and Lindström [2017], it was found that the need to consider other sojourn-time distributions and other conditional distributions can be eliminated by adapting to the time-varying behavior of the underlying data process.

Parameter Estimation

The parameters of an HMM are typically estimated using the maximum-likelihood method. Every observation is assumed to be of equal importance, no matter the length of the sample period. This approach works well when the sample period is short and the underlying process does not change over time. The time-varying behavior of the parameters documented in previous studies (Rydén, Teräsvirta, and Åsbrink [1998], Bulla [2011], and Nystrup, Madsen, and Lindström [2017]) calls for an adaptive approach that assigns more weight to the most recent observations while keeping in mind the past patterns at a reduced confidence.

In Nystrup, Madsen, and Lindström [2017], an adaptive estimation approach based on weighting the observations with exponentially decreasing weights—in other words, using exponential forgetting—was outlined. The same estimation approach was used in Nystrup et al. [2015] and will be used in this article. The regime-switching model is still a two-state HMM with Gaussian conditional distributions, but one that adapts to the time-varying behavior of the underlying process in an effort to produce more robust state estimates.

EXHIBIT 6
Online Step Algorithm

$t = T = 1$
while $T \leq$ the number of observations
    calculate the probabilities $\Pr(X_t = i | Y_1, Y_2, \ldots, Y_T)$
    for all states i at time $t \leq T$ based on knowledge of the
    first T observations
    if $\max_i \Pr(X_t = i | Y_1, Y_2, \ldots, Y_T) >$ threshold
        classify state $X_t = \arg\max_i \Pr(X_t = i | Y_1, Y_2, \ldots, Y_T)$
        $t = t + 1$
        $T = \max(t, T)$
    else
        $T = T + 1$
    endif
endwhile

STATE INFERENCE

Once the parameters of the hidden Markov model have been estimated, the hidden states can be inferred. The most likely sequence of states can be computed efficiently using the algorithm of Viterbi [1967]. The entire output sequence $\{Y_t: t \in 1, 2, \ldots, T\}$ must be observed before the state for any time step can be generated. A widely used approach is to break the input sequence into fixed-size windows and apply the Viterbi algorithm to each window. Larger windows lead to higher accuracy but result in higher latency.

Narasimhan, Viola, and Shilman [2006] proposed an online step algorithm that makes it possible to dynamically trade off latency for expected accuracy, without having to choose a fixed window size up front. The essence of their algorithm is that the initial state becomes increasingly certain as more observations are included in the sequence and the latency increases. Once the certainty estimate reaches a dynamically computed threshold, the identified initial state is outputted and the algorithm proceeds to estimate the next state in the sequence. The algorithm is shown in pseudo code in Exhibit 6. Despite being very intuitive, the algorithm has never been applied in studies of RBAA.

Choice of Threshold

Using the online step algorithm, the trade-off between accuracy and latency can be made explicit by letting the threshold be a decreasing function of the latency. It can be argued, however, that this is not desirable in the present application; if the delay in classifying an observation is large, then part of the reallocation premium has been missed, and incurring unnecessary transaction costs would only make it worse.

Instead, we chose a constant confidence threshold of $1 - 1/T \approx 0.9998$, where T is the number of observations. This threshold is not comparable to the 95% threshold applied in Nystrup et al. [2015], who classify an observation as belonging to the current regime unless it immediately and with 95% confidence could be classified as belonging to the other regime. In the algorithm proposed by Narasimhan, Viola, and Shilman [2006], an observation is not classified until enough observations have been gathered that the confidence requirement is met.

With a constant threshold of 0.9998, the median delay in classifying an observation is 7 days (not trading days). The median delay in detecting regime changes is 25 days. By lowering the confidence threshold, the delay can be reduced, but this also leads to detection of more (spurious) regime changes and increased transaction costs.

For a given level of transaction costs, there is an optimal threshold that balances the cost of rebalancing with the cost of not reacting to regime changes or delaying the reaction. A confidence threshold that corresponds to an expectation of one misclassification for the entire sample may be too conservative. Attempting to find the optimal threshold, however, would introduce a backtesting bias.

EMPIRICAL RESULTS

The testing is done one day at a time in a live-sample setting to make it as realistic as possible. The model is fitted to the first t observations of the MSCI World index, assigning most weight to the most recent observations.[11] Based on the estimated parameters, we estimate the probability that on day t the market was in the high- and low-volatility states, respectively.[12] The asset allocation remains unchanged if the certainty of the estimate does not exceed the threshold of 0.9998. The closing price on day $t + 1$ is then included in the sample; the model is re-estimated, and the state probabilities for day t are estimated based on knowledge of the closing price on day $t + 1$. This procedure is repeated sequentially by including the observations, one at a time, until the certainty of the estimate exceeds the threshold and the state on day t can be classified.

Once the certainty exceeds the threshold and the state on day t has been classified based on knowledge of the first T observations, the asset allocation can be updated. If the estimated state on day t is different from the state that the current asset allocation is based on, then the allocation is changed based on the closing price at day $T + 1$—that is, we assume a one-day delay in the implementation. Otherwise the asset allocation remains unchanged. If, based on knowledge of the first T observations, the states on day t and $t + 1$ can be classified simultaneously, then it is the state on day $t + 1$ that determines whether the asset allocation is changed.

When as many observations as possible have been classified based on knowledge of the first T observations, then the closing price on day $T + 1$ is included in the sample and the model is re-estimated. The next day in the sequence must then be classified—for instance, $t + 2$, if, based on knowledge of the first T observations, the states on day t and $t + 1$ were classified. This procedure is repeated sequentially by including the observations, one at a time, from January 1, 1998, all the way through the sample.[13] The portfolio is rebalanced only when the allocation changes from risk-on to risk-off or vice versa.

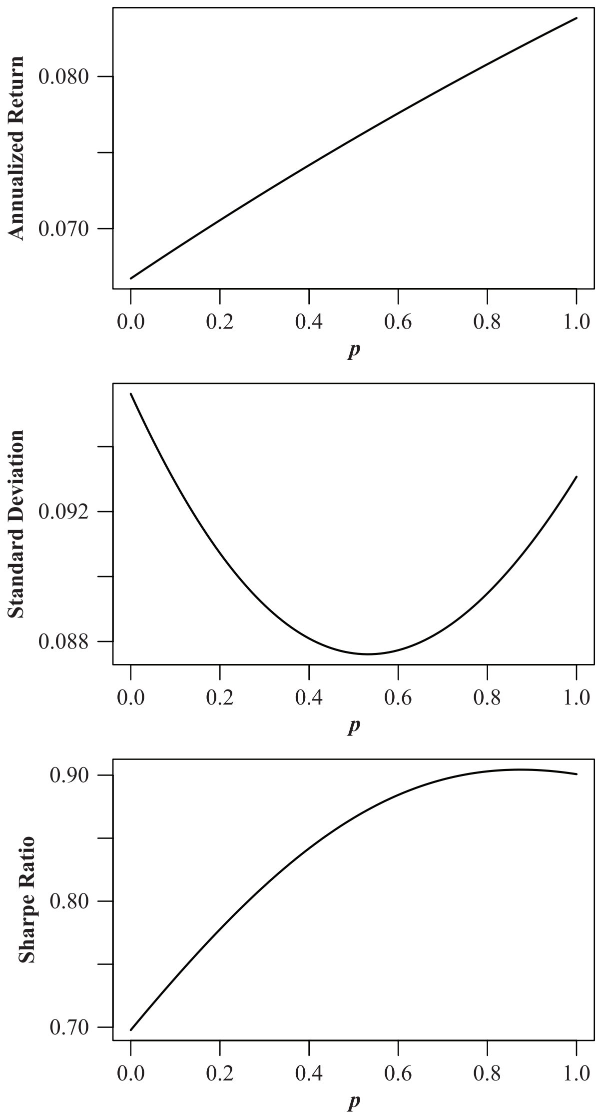
Dynamic Allocation or Diversification

Exhibit 7 shows the annualized return, standard deviation, and Sharpe ratio as a function of the percentage p of the portfolio that is allocated to the RBAA strategy; $p = 0$ corresponds to the benchmark 60/40 SAA portfolio rebalanced at the change points. In terms of the rebalancing frequency of the benchmark portfolio, it turns out that annual rebalancing would have been better than quarterly or monthly. This is a lower frequency than institutional investors typically use and indicates the presence of momentum in the asset returns. Rebalancing at the change points, however, performed almost as well as annual rebalancing. By rebalancing the benchmark portfolio at the same points in time as the allocation of the RBAA portfolios changes, the timing has no impact on the relative performance.

The annualized return of the RBAA portfolio increases linearly as a function of p. The annualized standard deviation—adjusted for autocorrelation—is minimized when p is around 0.5, and the Sharpe ratio

Exhibit 7

Annualized Return, Standard Deviation, and Sharpe Ratio as a Function of the Percentage of the Portfolio Allocated to the Regime-Based Strategy, 1998–2015



is maximized when p is approximately 0.8. Although this is when there are no transaction costs, adding 10 basis points of transaction costs does not change the shape of the graphs. The value of p that maximizes the Sharpe ratio would be much smaller if it was only the weight of DM stocks and government bonds that was adjusted dynamically, because the decline in diversification would be steeper. In summary, RBAA increases portfolio return, decreases portfolio risk, and, thus, leads to increased risk-adjusted returns.

Performance across the Inferred Regimes

In Exhibit 8, the development of the RBAA strategy with $p = 0.5$ is compared to the 60/40 SAA portfolio and the MSCI World Index over the period from 1998 to 2015. In the shaded periods, the allocation was risk–off. The 60/40 portfolio is, in fact, a suitable benchmark for this period, because it turns out that the RBAA strategy was risk–on 61% of the time—that is, the two portfolios had almost the same average allocation to fixed income.

The inferred regimes seem intuitive when looking at the log returns of the MSCI World Index in the lower panel of Exhibit 8. A total of 34 regime changes are detected over the 18-year period. The length of the identified regimes varies considerably from a few weeks up to four years, which is different from what would be expected if the regimes were based on a business cycle indicator.

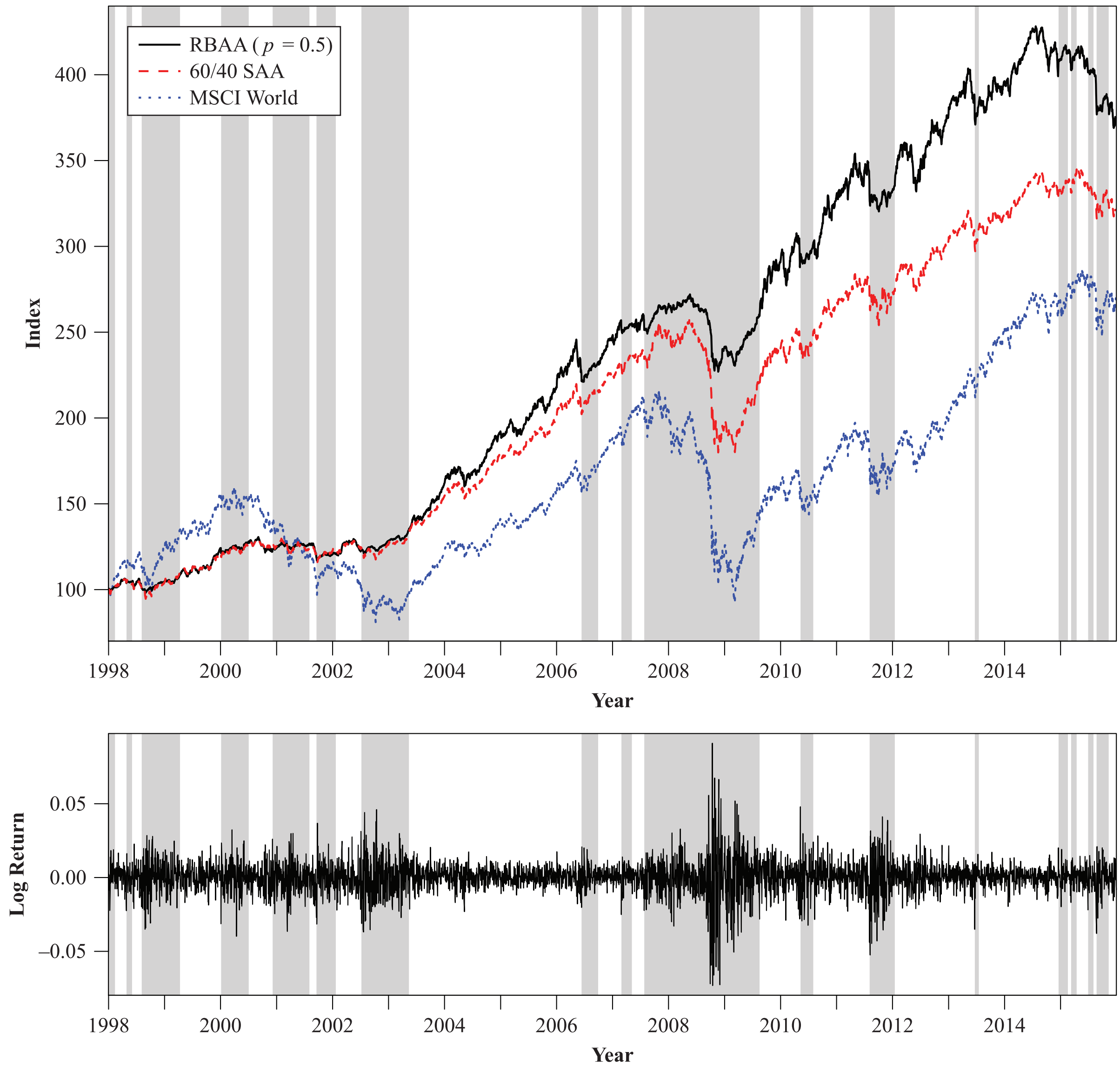
The RBAA strategy with $p = 0.5$ outperforms both the 60/40 benchmark and the MSCI World Index, when there are no transaction costs. The outperformance relative to the benchmark portfolio begins in 2003 and then slowly accumulates all the way through the crisis in 2008. Part of the outperformance is lost in the first half of 2009 when the market rebounds and the RBAA portfolio is still risk–off. Once the allocation is changed to risk–on in the second half of 2009, the RBAA strategy again starts to outperform the benchmark. In 2015, however, the RBAA strategy underperformed the SAA portfolio.

In Exhibit 9, the performance of the RBAA portfolio with $p = 0.5$ and $p = 0.8$, respectively, is compared to the 60/40 SAA portfolio rebalanced at the change points over the period 1998–2015. Recall from Exhibit 7 that $p = 0.5$ was the percentage that minimized the standard deviation and the Sharpe ratio was maximized around $p = 0.8$. Although the annual turnover of the RBAA strategies is much higher than that of the benchmark portfolio, the Sharpe ratios exceed that of the SAA portfolio as long as transaction costs do not exceed 79 and 60 basis points per one-way transaction, respectively.

In addition, the maximum drawdown of the RBAA portfolios is much smaller than that of the SAA portfolio. This means that the Calmar ratio, which is the annualized return divided by the maximum drawdown, is more than twice as high for the RBAA portfolio than for the SAA portfolio.

EXHIBIT 8

Development of RBAA Strategy with $p = 0.5$ Compared to 60/40 SAA Portfolio and the MSCI World Index across the Inferred Regimes



Notes: The legends are sorted according to the index values at the end of 2015; p is the percentage of the portfolio that is allocated to the RBAA strategy. In the shaded, high-volatility periods, the allocation was risk–off.

Comparison with Other Approaches

In Exhibit 10, we compare the performance of the RBAA portfolio with $p = 0.5$ and the SAA portfolio with two other RBAA approaches. The first is based on the median filter that Bulla et al. [2011] applied to filter the state probabilities instead of the online step algorithm. The inferred state on day t is the median of the most likely states over the last 21 days.[14] The Sharpe ratio is almost the same regardless of whether the number of days is 5 or 21 (one week or one month), but the annual turnover is much higher when fewer days are considered.

The average allocation of the median-filter approach is 71% risk–on and 29% risk–off, meaning that the 60/40 SAA portfolio is not a suitable benchmark

Exhibit 9

Performance of RBAA Portfolios Compared to 60/40 SAA Portfolio with Rebalancing at the Change Points, 1998–2015

	RBAA ($p = 0.5$)	RBAA ($p = 0.8$)	SAA
Annualized Return	0.076	0.081	0.067
Standard Deviation	0.088	0.089	0.096
Sharpe Ratio	0.87	0.90	0.70
Maximum Drawdown	0.17	0.18	0.30
Calmar Ratio	0.46	0.46	0.22
Annual Turnover	0.92	1.47	0.05

Note: p is the percentage of the portfolio that is allocated to the RBAA strategy.

Exhibit 10

Performance of RBAA Portfolio and 60/40 SAA Portfolio with Rebalancing at the Change Points Compared to a Median Filter and an EWMA Approach, 1998–2015

	RBAA ($p = 0.5$)	SAA	Median Filter	EWMA
Annualized Return	0.076	0.067	0.079	0.055
Standard Deviation	0.088	0.096	0.103	0.094
Sharpe Ratio	0.87	0.70	0.76	0.59
Maximum Drawdown	0.17	0.30	0.21	0.23
Calmar Ratio	0.46	0.22	0.38	0.25
Annual Turnover	0.92	0.05	1.06	3.02

for this approach. Despite the lower average exposure to fixed income, the median-filter approach realizes a higher Sharpe ratio than that of the SAA portfolio. It relies on the same HMM as the RBAA portfolio to distinguish between market regimes, but realizes a lower Sharpe ratio with a higher turnover.

The second strategy is based on an exponentially weighted moving average (EWMA) of the standard deviation of the MSCI World Index. When the EWMA rises above its 60% quantile, the allocation is changed to risk–off; and when it falls below its 60% quantile, the allocation is changed to risk–on. By design, the average allocation of the RBAA strategy based on the EWMA is 60/40. The annual turnover is highly dependent on the memory length of the EWMA. Exhibit 10 shows the result when using an effective memory length of 21 days. This value yields a reasonable trade-off between Sharpe ratio (before transaction costs) and turnover.

The basic idea of the EWMA approach—to reduce risk exposure when higher volatility has been observed for a while—is the same as that for the other RBAA approaches; however, the EWMA approach realizes a lower Sharpe ratio than does the SAA portfolio. The comparison emphasizes the value of the HMM for distinguishing between market regimes and the online step algorithm for filtering the regime probabilities.

CONCLUSION

The empirical results showed that regime-based asset allocation is profitable, even when compared to a diversified benchmark portfolio in a multi-asset universe. Our proposed strategy was based on adjusting the weight of risky assets relative to safe assets (fixed income) to maintain a minimum level of diversification in all regimes.

The results are robust because they are based on available market data with no assumptions about equilibrium returns, volatilities, correlations, or the ability to forecast their future values. Because the parameters of the hidden Markov model used to identify the regimes were updated every day, the same approach should work in other time periods as well. It will remain a possibility for future research to try to improve the performance by including information from other asset classes, economic variables, interest rates, investor sentiment surveys, or other possible indicators.

The benchmark portfolio was chosen to mimic a 60/40 long-only SAA portfolio of an institutional investor to make the comparison as realistic as possible. The performance of RBAA was analyzed as a function of the percentage of the portfolio that was allocated dynamically. To minimize the portfolio standard deviation, we had to allocate 50% to the regime-based strategy. This corresponded to an 80/20 allocation in the low-volatility regimes and a 30/70 allocation in the high-volatility regimes. The lower standard deviation combined with a higher return led to an improved Sharpe ratio, compared to the static fixed-weight benchmark. Even more remarkable was the improvement in the ratio of average return to maximum drawdown, as this ratio more than doubled when allocating half of the portfolio dynamically.

The results have important implications for portfolio managers with a medium- to long-term investment horizon. The percentage of a multi-asset portfolio that, with advantage, can be allocated dynamically is strongly dependent on the effectiveness of the regime-detection process. Rebalancing to a static benchmark is not optimal, however, when market regimes are persistent. It is definitely worth considering a more dynamic approach to asset allocation, if only to reduce the tail risk.

ENDNOTES

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[1]An online algorithm processes its input observation-by-observation in a sequential fashion, without having the entire input sequence available from the start.

[2]The 10 indexes are MSCI World, MSCI Emerging Markets, FTSE EPRA/NAREIT Developed Real Estate, BofA Merrill Lynch US High Yield, Barclays Emerging Markets High Yield, S&P GSCI Crude Oil (funded futures roll), S&P GSCI Gold (funded futures roll), Barclays US A Barclays World Inflation-Linked Bonds (hedged to USD), and J.P. Morgan Global Government Bonds (hedged to USD).

[3]The maximum drawdown is the largest relative decline from a historical peak in the index value.

[4]The adjustment leads to the reported standard deviations being higher than had they been annualized under the assumption of independence, because most of the indexes display positive autocorrelation. The adjustment had a large impact on the standard deviation of the high-yield bond index that went from 0.04 to 0.12.

[5]The log returns are calculated using $r_t = \log(P_t) - \log(P_{t-1})$, where P_t is the closing price of the index on day t and log is the natural logarithm.

[6]A quantitative manifestation of this fact is that while returns themselves are uncorrelated, absolute and squared returns display a positive, significant, and slowly decaying autocorrelation function.

[7]Increased risk aversion (a behavioral explanation) and reduced risk capacity (an institutional explanation) are difficult to distinguish in data. Both effects have support (e.g., Cohn et al. [2015] and Brunnermeier and Pedersen [2009]).

[8]Another possible benchmark portfolio is the $1/N$ portfolio that assigns equal weight to all assets. This agnostic portfolio has in many cases proved to be a difficult benchmark to beat (see DeMiguel, Garlappi, and Uppal [2009]), but it has realized a slightly lower Sharpe ratio than the proposed SAA portfolio over the considered period.

[9]The proposed benchmark allocation is within the range of variation of the average asset allocation of pension plans across the major OECD countries, according to OECD Global Pension Statistics.

[10]The weights in the RBAA portfolio depend on the percentage p of the portfolio that is allocated to the RBAA strategy and the regime X: $w^{\text{RBAA}}(p, X) = p \cdot w^{\text{risk-on/off}}(X) + (1-p) \cdot w^{\text{SAA}}$. The regime-dependent portfolio weights are $w^{\text{risk-on/off}}(X) = (25, 5, 10, 5, 5, 5, 5, 0, 0, 0)^{\text{T}} = 60 \cdot 1_{\text{risk-on}}(X) + (0, 0, 0, 0, 0, 0, 0, 10, 10, 20)^{\text{T}}/40 \cdot 1_{\text{risk-off}}(X)$, where the indicator function $1_{\text{risk-on}}(X) \equiv 1$ if X = risk-on and $1_{\text{risk-on}}(X) \equiv 0$ if X = risk-off

[11]An asymptotic memory length of 520 days is used when estimating the parameters (see Nystrup, Madsen, and Lindström [2017]).

[12]As the states are highly persistent ($\gamma_{ii} \gg 0.5$), the most likely state on day $t+1$ will be the same state that was estimated to be most likely on day t.

[13]The log returns from 1997 are used for initialization.

[14]The inferred state on day t is $\hat{X}_t^f = [\text{median}(\hat{X}_{t-20}, \hat{X}_{t-19}, \ldots, \hat{X}_t)]$, where $[\cdot]$ maps every number to its integer part and $\hat{X}_t = \arg\max_i \Pr(X_t = i | Y_1, Y_2, \ldots, Y_t)$.

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