Finding Comfortable Settings of Chinese Checkers

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Abstract. Game refinement theory has been used as a reliable tool for measuring attractiveness and entertainment in various games. This research aims to explore the entertainment of a multiplayer game-Chinese checkers. This study investigated the entertainment of Chinese checkers by creating different battle modes of Chinese checkers. The results show that the three players battle mode has the highest game refinement value among 4 battle modes and the recommended number of players is three people. It is recommended that more master level players participate in the game will make the game more interesting. Consider the impact of board size on game experience, the study compares the playability among three sizes board of Chinese checkers which are 6 pieces, 10 pieces, and 15 pieces. The results found a phenomenon that 6 pieces version provides more entertainments for players and a new question which is proposed at the end of this paper.

1. Introduction

The game refinement (GR) theory had provide important understanding between the element of skill and chance which had been previously proposed by Iida *et al.* [1]. It had been utilized as an interesting tools to measure sophistication of board games, sport games, and and video games [2, 3, 4, 5, 6], which eventually converges towards $GR \in [0.07, 0.08]$. In addition, the game's revolution have revealed many facts on the change of attractiveness of games [1]. In this research, the GR theory is adopted for the multiplayer games in another interesting board game–Chinese checkers.

Chinese checkers is an example of multiplayer strategy board game which can be played by up to six people, playing individually or with partners. The game is a modern and simplified variant of the American game Halma [7]. The game objective is to be the first to race all of one's pieces across the hexagram-shaped board into "home"—the corner of the star opposite one's starting corner—using single-step moves or moves that jump over other pieces. In fact, Chinese checkers has multiple variants, depending on the board shapes and sizes. A typical Chinese checkers is the hexagonal board which can be played by up to six people and each player controls 10 pieces on the board. This study focuses on the analysis of the game entertainment with a focus on the performance level of the player, number of player, and the board sizes. Moreover, the simulation of Halma is also conducted to observe its historical development into the modern Chinese checkers. As such, this study attempts to answer the following questions:

- What is the most comfortable number of players in Chinese Checkers?

- What's the link between performance level and entertainment?
- What regulations (board size in this case) are more entertaining?

This research quantified the entertainment values of the Chinese checkers game using the GR theory in several simulation games. The game simulation was realized by implementing AI (five players mode was not included to avoid the unfairness of board space) to self-play the game. Since the GR measure is the point gained by the players, the mathematical model of the game refinement theory will be applied to this game and the refinement values will be calculated to determine the most suitable number of players. The impact of different board sizes to the game experience were also conducted.

This paper is organized as following. Section 2 introduces the related work about multiplayer games and Chinese checkers. In Section 3, an overview of the Chinese checkers and its implementation are given. Then, the fundamental idea of game refinement theory and the experimental setups were given in Section 4. Then, the obtained results was analyzed and discussed in the Section 5. Finally, Section 6 conclude this paper.

2. Related Works

2.1. Multiplayer games

Multiplayer game is one of the popular research focus in the game domains. Multiplayer game is a game which is played by more than two players. The players might be independent opponents, formed into teams or be just a single team pitted against the game. Many works in multiplayer game have been published, such as different approaches to the development of the multiplayer algorithms and its comparison [8, 9], multiplayer Go [10], decision algorithms for multiplayer non-cooperative games [11] and its beginner bound [12], and computing equilibrium in multiplayer stochastic games [13],

There are several research that had successfully applied GR theory in multiplayer games, like RoShamBo [14], UNO [15], and Snake games [16]. The research about RoShamBo introduces a family of RoShamBo games, denoted by $RSB(n, b, s, r)$ which means that n players simultaneously show a move among b possible moves with possible s winning regulations, at each round out of r round matches in total [14]. A game informatical analysis of $RSB(n, b, s, r)$ using game refinement measure is carried out. The experiment result of this study show that $RSB(n, 3, 1, 1)$ is best to play with $n = 8, 9$ and $RSB(2, 3, 1, r)$ is best to play with $r = 9, 10$.

Another recent work by Ramadhan *et al.* [15] has been proposed in multiplayer incomplete-information game, specifically UNO, in order to obtain the recommended number of players. Experiment was conducted by developing varying types of computer player to simulate UNO games. The research revealed that the second last and the last player get the most enjoyment out of the game.

Similarly, another research in multiplayer game analysis is the Snake game [16]. The study used GR theory to explore the attractiveness and sophistication of Snake game. A basic AI player was created and the experimental results showed the Snake game had enough sophistication for it to be popular on mobile phones which makes people feel entertaining and exciting. Referring to those previous work on multiplayer games, to continually expand the field of multiplayer game, this research use GR theory to study another interesting board game—Chinese Checkers.

2.2. Artificial intelligence (AI) for multiplayer games

There are lots of AI algorithms that have been developed for the multiplayers games, such as alpha beta algorithm [17], max^n algorithm [18], paranoid algorithm [19], MCTS algorithm [20], best reply algorithm [21], and so on. It is known that max^n algorithm for playing multiplayer games is flexible, but there are very limited techniques for pruning the max^n game trees.

The research on the comparison of algorithms presents other theoretical limitations of the max^n algorithm and that zero-window search is not possible in max^n game trees [9]. The study also present quantitative results derived from playing max^n and the paranoid algorithm against each other on various multiplayer game domains, showing that paranoid widely outperforms max^n in Chinese Checkers, by a lesser amount in Hearts and that they are evenly matched in Spades [9]. In this study, an AI that able to play Chinese checkers was implemented by adopting recursive algorithm and different sorting methods.

3. Chinese Checkers

Chinese checkers is a modern and simplified variant of the American game Halma (Figure 1). The name "Chinese Checkers" originated in the United States as a marketing scheme by Bill and Jack Pressman in 1928. The Pressman company's game was originally called "Hop Ching Checkers" [22]. It was invented in Germany in 1892 under the name "Stern-Halma" as a variation of the older American game Halma. The "Stern" (German for star) refers to the board's star shape (in contrast to the square board used in Halma). Halma is a strategy board game invented in 1883 by George Howard Monks, a US thoracic surgeon at Harvard Medical School. The game board is checkered and divided into 8×8, 10×10 or 16×16 squares board, either of which is adequate for two players and they have 10, 15 and 19 pieces per player, respectively. In Chinese checkers, the board is hexagram-shaped board where the corner of the star is the player's "home" (Figure 2). The game is won by being first to transfer all of one's pieces from one's own camp (or home) into the the opposing corner and can be played by two or four players (or up to six player for Chinese checkers).

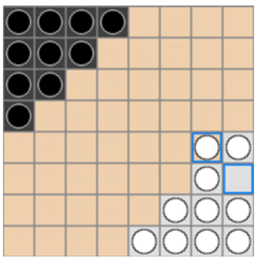

Figure 1. Halma games.

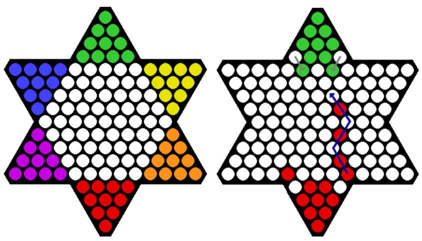

Figure 2. Chinese Checkers.

3.1. Basic rules

In Chinese checkers, the objective is to be first to race all of one's pieces across the hexagram-shaped board into "home" —the corner of the star opposite one's starting corner—using single step moves or moves that jump over other pieces (Figure 2). The remaining players continue the game to establish second-, third-, fourth-, fifth-, and last-place finishers [23]. This game aims to race all one's pieces into the star corner on the opposite side of the board before opponents do the same. Each player has 10 pieces. In "hop across", the most popular variation, each player starts with their colored pieces on one of the six points or corners of the star and attempts to race them all home into the opposite corner. Players take turns moving a single piece, either by moving one step in any direction to an adjacent empty space, or by jumping in one hop over other single pieces. A player may not combine hopping with a single-step move—a move consists of one or the other. There is no capturing in Chinese Checkers, so hopped pieces remain active and in play. Turns proceed clockwise around the board.

A basic strategy is to create or find the longest hopping path that leads closest to home, or immediately into it. (Multiple-jump moves are faster to advance pieces than step-by-step moves.)

Since either player can make use of any hopping 'ladder' or 'chain' created, a more advanced strategy involves hindering an opposing player in addition to helping oneself make jumps across the board. The players' strategies for emptying and filling their starting and home corners are equally important. Games between top players are rarely decided by more than a couple of moves [3]. Differing numbers of players result in different starting layouts, in turn imposing different best-game strategies. For example, if a player's home destination corner starts empty (i.e. is not an opponent's starting corner), the player can freely build a 'ladder' or 'bridge' with their pieces between the two opposite ends. But if a player's opponent occupies the home corner, the player may need to wait for opponent pieces to clear before filling the home vacancies. All of these basic rules and strategy were considered in the simulation of Chinese checkers.

3.2. Developing AI player for Halma and Chinese Checkers

In this research, a program was implemented using C# compiler to simulate the player of the Halma and Chinese checkers. This involves self-play simulation where each player's activity was recorded. Although the rule is fairly easy, it has many options to play and different number of solution at each turn. Halma and Chinese checkers are mainly reflected in the different shape of the board. Thus, considered this point, Halma program adjusted the values in the checkerboard index array which was used in Chinese checkers. Different players were distinguished by setting four different values zone in the checkerboard index array. Let these numerical areas represent two or four players, respectively. Then, the AI player can be allotted to these four different areas in order to increase (or decrease) the player. The AI player and the possible moving function used in Halma are the same in Chinese checkers. The program will set two playing modes which are two players and four players to collect the specific data based the GR theory.

The AI player choose and move the piece in the game is designated by *Intelligent-high* (correspond to the "AI instruction" in Figure 3). This process starts by selecting the move piece and decide its movable reach. This is realized by calculating the distance from all the pieces in the moving area to the opposite area. Then, after sorting all the distances, the longest one is chosen. As such, the AI player will choose the piece that can get as far from the opposite area as possible. These process was realized by implementing a construct that can obtain the reachable range, designated as *PossibleMove* (correspond to the "Search for Movable Range and Sort" in Figure 3). A recursion was used to identify the eligible falling places and stored all the possible pieces in a set. Then, all the possible pieces in the set is sorted and the piece with the farthest distance from the opposite end is selected. A basic insertion sort and quick sort was implemented to sort the possible pieces. Also, to reduce the probability of the same action, random element was added to the selection mechanism.

Furthermore, different level of players is determined by adjusting the distance of selecting the reachable piece. For example, high level AI player (*Intelligent-high*) selects the one which is the farthest piece and beginner level AI player selects any of the second (or third) farthest piece from the set, respectively (the set save all the sorted possible movable pieces in descending order). Sometimes, the possible move has only one choice for the move, in this case, the program will continually select the farthest one. As such, the beginner level AI player is designated as *Intelligent-beginner.* To compare the game experiences of different board sizes for Chinese checkers, we also adjust the length and width of the board. As such, three board sizes were constructed (6 pieces, 10 pieces and 15 pieces) for each player.

4. Analysis of Chinese Checkers using Game Refinement Theory

In the experiments, we set four battle modes which are two, three, four, or six players, respectively. Meanwhile, in order to observe the historical development of the Chinese checkers, the simulation was also conducted on the ancestor of Chinese checkers—Halma. The battle modes were realized by self-play AI simulation. In order to avoid the unfairness of board space,

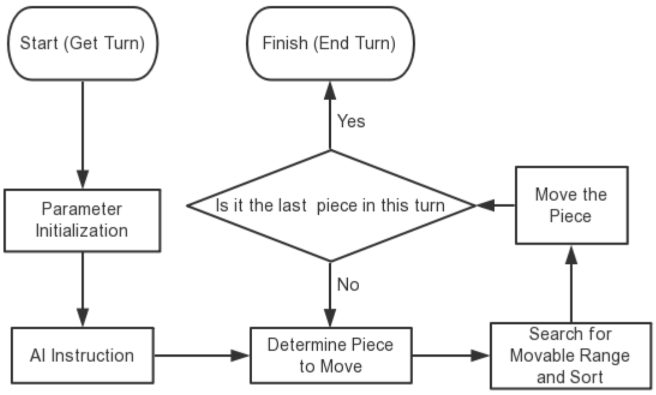

Figure 3. Flowchart of each player turn process.

five players mode was excluded. Since the GR measure involves utilizing the point gained by the players, the GR theory will be adapted to Chinese checkers and Halma game, where the refinement values were calculated. These values will be used to analyze the play-ability of each battle mode. Then, the most suitable number of players is inferred supported by the analysis of the results. Concerning the impact of different board sizes and player levels to the game experience of the Chinese checkers, three board sizes (6 pieces, 10 pieces and 15 pieces) and two different level of AI players (high and beginner level AI player) were also analyzed.

4.1. Game Refinement Theory

We show a basic mathematical model of Game refinement theory. In the game theory, "game progress" is an important concept. A logistic model about game progress was constructed in the framework of Game refinement theory and applied to many board games. The measure of game refinement will be derived from a game progress model. The game progress is twofold [24]. First, it can represent game speed or scoring rate. Second, it can represent game information progress that focuses on the game outcome. In general, game progress presents the degree of certainty of a game's result in time or in steps. Having full information of the game progress, i.e. after its conclusion, game progress $x(t)$ is a linear function of time t with $0 \leq t \leq t_k$ and $0 \leq x(t) \leq x(t_k)$, as given by (1).

$$x(t) = \frac{x(t_k)}{t_k} t \tag{1}$$

However, the game information progress given by (1) is unknown during the in-game period. The presence of uncertainty during the game, often until the final moments of a game, reasonably renders game progress as exponential. Hence, a realistic model of game information progress is given by (2).

$$x(t) = x(t_k)(\frac{t}{t_k})^n \tag{2}$$

Here n stands for a constant parameter which is given based on the perspective of an observer of the game considered. Only a very boring game would progress in a linear function, and most of course do not. Therefore, it is reasonable to assume the parameter $n = 2$ which is based on the perception of game progress prior to completion. If the information of the game is completely known, the value of $n = 1$ which makes the game progress appears as a straight line. In most games, especially in competitive ones, much of the information is incomplete, the value of n cannot be assumed, and therefore game progress is a steep curve until its completion, along with $x(t_k)$, t_k, $x(t)$ and t, just prior to game's end. Then, acceleration of game information progress given by (3) is obtained by deriving (2) twice and solving it at $t = t_k$.

$$x''(t_k) = \frac{x(t_k)}{(t_k)^n}(t_k)^{n-2}n(n-1) = \frac{x(t_k)}{(t_k)^2}n(n-1) \tag{3}$$

It is assumed in the current model that game information progress in any type of game is encoded and transported in our brains. We do not yet know about the information physics of the brain, but it is likely that the acceleration of information progress is subject to the forces and laws of physics. Too little game information acceleration may be easy for human observers and players to compute, and becomes boring. In contrast, too much game information acceleration surpasses the entertaining range and causes frustration, and at some point beyond that could become overwhelming and incomprehensible. Therefore, it is reasonable to assume larger $\frac{x(t_k)}{(t_k)^2}$ is more exciting, due in part to the uncertainty of game outcome. Thus, utilizing its root square form, $\frac{\sqrt{x(t_k)}}{t_k}$, the game refinement measure for the game under consideration was given by (**??**) and called GR value for short.

$$GR = \frac{\sqrt{x(t_k)}}{t_k} \tag{4}$$

The effectiveness of proposed GR measure have been justified by many previous works [2, 3, 4, 5, 6] which was applied to popular and sophisticated board games (such as Chess and Go) and sports games (Basketball and Soccer) were compared (Table 1). It can be observed that these sophisticated games have a $GR \in [0.07, 0.08]$. In the context of entertainment in games, this range is currently considered as a good zone for games. If the GR value is in this zone, a game will be considered as sophisticated with appropriate entertainment and attractiveness. This is due to the information acceleration as the factor that impact the engagement or excitement of all game types.

Table 1. Measures of game refinement for board games and sports games [3]

Game	B or G	D or T	GR
Chess	35	80	0.074
Go	250	208	0.076
Basketball	36.38	82.01	0.073
Soccer	2.64	22	0.073

B/G : branching factor/goal or score;
D/T : total game length/attempted goal or score;

4.2. Experiment and Result

In GR theory, branching factors and game length are the main factors to determine game information outcome [1] where the average number of possibilities and turns were proposed in board games [3]. Because Chinese checkers is a kind of classical board game, the important parameters were determined based on to the previous work (e.g, [24, 3]). The parameters were the total average moves of each round as the game length D and the average selection step numbers of each move in one round as the branching factors B; thus corresponds to (5). Thus, the sophistication of the Chinese checker and Halma game can be determined.

$$x(t) = B(\frac{t}{D})^n \tag{5}$$

4.2.1. Number of players It is known that Chinese checkers is a modern and simplified variant of the American game Halma. To observe the GR measure of Halma, two battle modes (two and four players modes) were conducted. 100 games for each modes were collected and the GR value can be computed. The result is shown in Table 2(a). Based on the Game refinement theory, it can be said that the entertainment between the two modes in Halma is not much different since the refined values are 0.024 and 0.022, respectively. Meanwhile, based on the available players, four battle modes for the Chinese checkers were conducted. For each mode, the simulation was conducted in 100 games and the GR measure of the four battle modes. The result is shown in Table 2(b). From the Table 2(b), three players battle mode has the highest refined value among these four battle modes which implies the biggest enjoyment.

Table 2. Game refinement measure of different battle modes

	(a) Halma game			(b) Chinese checkers		
Battle mode	B	D	GR	B	D	GR
2 players	13.63	154.79	0.024	17.56	79.27	0.053
3 players	–	–	–	20.18	75.04	0.060
4 players	12.01	157.92	0.022	15.56	82.62	0.048
6 players	–	–	–	12.29	92.72	0.038

B : average selection step of each move in one round;
D : total average moves of each round;
$--$: not available/not applicable;

4.2.2. Performance quality To further analyze the three players battle mode, the experiment sets two AI player level (simply expressed as high and beginner level) for the players in three players battle mode. After arranging a combination of these levels, three new battle groups were formed. For each group, the experiment was conducted for 100 games and the GR of different level was calculated. The result is shown in Table 3 (h stands for high level, l stands for beginner level). It can be observed that the GR values increases as the player level rises. This initial observations showed that the level of players decisively affect the feelings of enjoyment by the players.

4.2.3. Board size Besides the mainstream board size (10 pieces), Chinese checkers can be played in the other board sizes as mentioned in the Section 3. To investigate the relationship between board sizes and its entertainment values, the experiment was resets with different board sizes and collects the data from other two sizes which are 6 pieces and 15 pieces using two level AIs. The simulation reserved the other parameter settings as the experiment in Section **??**, except for the board size. Then, each board size has four battle modes and the experiment conducted 100 games for each mode. Thus, a total of 800 sets of data from two new versions of Chinese checkers were collected and the GR values are calculated, which summarized and given in Table 4.

Finally, to better analyze and discuss the relationship between entertainment and the board sizes, the value of D versus B were depicted (see the Figure 4 and Figure 5) with respect to

Table 3. Measures of game refinement for different player's levels in three players battle mode.

Player's level	B	D	GR
(l,l,l)	18.47	84.84	0.051
(l,l,h)	19.48	83.27	0.053
(h,h,l)	19.62	79.61	0.056
(h,h,h)	20.18	75.04	0.060

h : high level AI player;
l : beginner level AI player;

Table 4. Measures of game refinement for various player numbers and board sizes using higher and beginner level AIs

	high level AI player			beginner level AI player		
Numbers of player	6 pieces	10 pieces	15 pieces	6 pieces	10 pieces	15 pieces
2 players	0.098	0.053	0.0210	0.086	0.044	0.020
3 players	0.105	0.060	0.0230	0.095	0.051	0.021
4 players	0.102	0.048	0.0190	0.880	0.037	0.019
6 players	0.094	0.038	0.0160	0.062	0.019	0.017

the known zone value of the GR measure. According to the experiment result (Table 4), the study found that as the board size become smaller, the more enjoyment player will get. However, change in the player number only affects the GR value in small way. Therefore, the 6 pieces board is the most entertaining in this case.

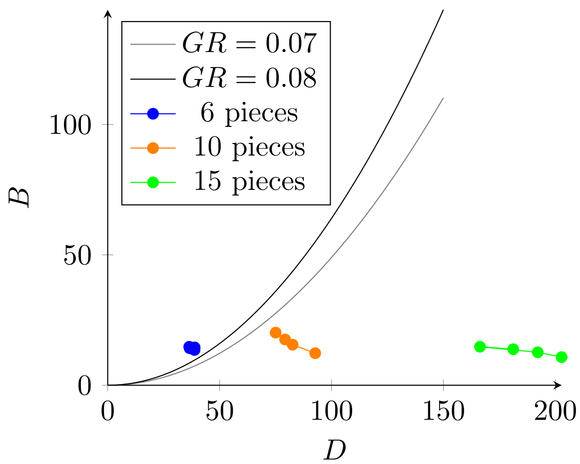

Figure 4. B and D for different board sizes and player numbers in Chinese checkers using high level of AI player.

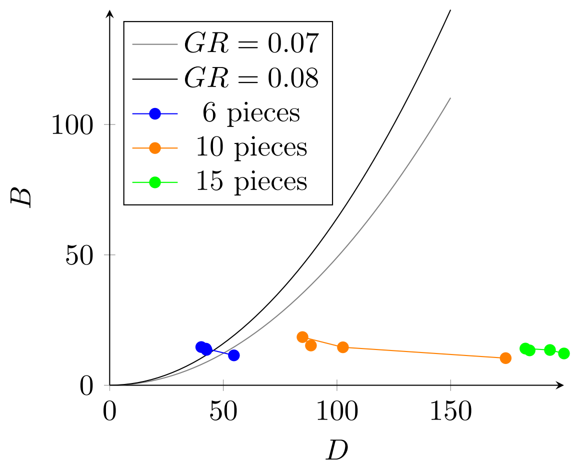

Figure 5. B and D for different board sizes and player numbers in Chinese checkers using beginner level of AI player.

5. Discussion

The experiment conducted on the different player numbers found that three players mode has the highest refinement value ($GR = 0.06$) among these four battle modes which is closest to what is currently considered as the most appropriate region based on GR theory. Thus, it is reasonable to infer that three players mode in Chinese checker provide the players with the biggest enjoyment. In addition, the initial experiment on Halma game found that the GR measure of Halma are far beginner the appropriate range ($GR = 0.02$), even when compared to the Chinese checkers ($GR = 0.053$ for 2 players, $GR = 0.048$ for 4 players). Referring to the previous work (e.g [15],[24]), this result implies that the change of Chinese checkers in the history development makes the game more interesting and attractive.

The link between performance level and entertainment can be analyzed from Table 3 where $GR \in [0.051 \to 0.06]$. When the number of high-level AI players increases, the GR value is increases towards the zone values. These findings indicate that as the player level rises, the game will become more enjoyable to play. In addition, the regulation of Chinese checkers (in this study context, the board size) shows that the $GR \in [0.094 \to 0.10]$ in 6 pieces version and $GR \in [0.016 \to 0.023]$ in 15 pieces version. Based on Figure 4 and Figure 5, it is found that the refinement values of 6 pieces version are closest to the zone value in both high and beginner AI levels, followed by the 10 pieces version (mainstream version) and 15 pieces version, respectively.

Referred to GR theory in board games [3] and sports games [24], the 6 pieces version are the most entertaining to player, especially to novice players. While the results had justified this situation, it is curious to question as the 10 pieces version is the mainstream Chinese checkers. One possible conjecture is that a novice level player may not find the mainstream Chinese checkers very interesting and attractive but are more entertaining to seasoned player. This situation is particularly true for highly skilled player (grandmaster), where more solutions and interesting routes can be discovered as well as obstacles can be set to interfere with opponents, all for the reason to increase the chance of winning in the game. Then, the expected game length will become longer and appropriate challenges will arouse their excitement.

6. Concluding remarks

The game refinement theory has been applied to measure entertainment and sophistication of the board games, video games and sports. In this study, we extended this theory to multiplayer game—Chinese checkers. It was found that the evolution of Chinese checkers is reasonable because the game became more interesting and attractive, while sophisticated enough with the development of history. Also, the recommended number of players to play Chinese checkers is three people. In the three players mode of Chinese checkers (mainstream case using 10 pieces), high level (or master) players may enjoy the game even more. The results from the experiments also revealed the link between performance level and entertainment where higher number of high level AI players gave much more entertainment compared to beginner level AI players. As such, novice player may not feel the same way as the master player in Chinese checkers. Therefore, analyzing the Chinese checkers based on game refinement theory offers a new perspective for determining the player number and the level of the player itself.

Meanwhile, comparing the game refinement measure of three board size (6 pieces, 10 pieces and 10 pieces), the 6 pieces version are the most entertaining to player even though the mainstream way in current social is 10 pieces version. Thus, it can be said that the 6 pieces version of the board is the recommended version for novice player to master the Chinese checkers game. Nevertheless, this study still found some room for improvement. Chinese checkers is a subtle board games where it have simple rules to play. However, the game length and the selection of moves will be changed totally different depend on the player level. In fact, the AI player can be further improved by considering broader and more refined abilities that may varies differently in terms of skill (such as the skill by human player). Therefore, it makes sense to try other algorithm and make the AI stronger in the future work.

To this end, a new question was found in this study—why 10 pieces version is the mainstream in current social? It is worth thinking about what caused this phenomenon, whether it is complexity or rules of Chinese checkers. These hypotheses need to be verified by experiments using data by human player as well as combination of simulation data from AI player in order to analyze this phenomenon in the future. Moreover, measures of advantage or disadvantage (typically known as advantage of initiative) in Chinese checkers also a good direction to explore since each one of the player should played the game fairly.

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