Models and Methods of Decision Making Support

General remarks

- Decisions are made on the basis of some model or the results of some calculations.
- A mathematical model is a set of relationships and various kinds of equations.
- Unfortunately, there is no universal way to create a model of any object.

In order to create a model of an object, it is necessary to study in detail all its features.

Establish major relationships.

Currently, modeling is used very widely.

The main goal of modeling is to study all possible operating modes of an object and find optimal conditions or parameters.

Determining optimal conditions or parameters is the solution to the decision-making problem.

There are no general or universal modeling methods, but we can explore modeling tools.

Mathematical models often contain differential equations.

Therefore, as a necessary modeling tool, we consider the method of solving differential equations.

The best modeling results are achieved using numerical methods for solving differential equations.

1.1 Euler method.

Let us find an approximate solution to the equation

$$\frac{dy}{dx} = f(x, y) \quad (1.1)$$

on the segment $[a, b]$,

satisfying the initial condition at

$$x = x_0, \quad y = y_0.$$

Divide the segment $[a, b]$ By **points** $x_0, x_1, \ldots, x_n = b$

into n equal parts $(x_0 < x_1 < \ldots < x_n)$

Let's denote $h = \Delta x = x_1 - x_0 = x_2 - x_1 = \\ = \ldots = b - x_{n-1},$

those $h = \dfrac{b-a}{n}.$

Let's denote $\Delta y_0 = y_1 - y_0, \quad \Delta y_1 = y_2 - y_1 ,\ldots, \quad \Delta y_{n-1} = y_n - y_{n-1}$

At each point $x_0, x_1, \ldots, x_n$

in equation (1.1) we replace the derivative with the ratio of finite differences:

$$\frac{\Delta y}{\Delta x} = f(x, y) \quad (1.2$$

$$\Delta y = f(x, y) \cdot \Delta x \quad (1.3)$$

At $x = x_0$ will have

$$\frac{\Delta y_0}{\Delta x} = f(x_0, y_0), \quad \Delta y_0 = f(x_0, y_0)\Delta x$$

or

$$y_1 - y_0 = f(x_0, y_0)h$$

In this equation x_0, y_0, h are known, therefore, we find:

$$y_1 = y_0 + f(x_0, y_0)h.$$

Similarly we find:

$$y_2 = y_1 + f(x_1, y_1)h$$

$$y_3 = y_2 + f(x_2, y_2)h$$

$$y_n = y_{n-1} + f(x_{n-1}, y_{n-1})h$$

Thus, the approximate values of the solution at the points $x_0, x_1, \dots, x_n$ found.

The solution is obtained in the form of a table

$$(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$$

Solving equations using the Euler method is easy to program. However, it is currently used only for rough and approximate estimates due to low accuracy.

Higher solution accuracy is provided by the Runge-Kutta method

1.2 Runge-Kutta method.

Let us find an approximate solution to the equation $\frac{dy}{dx} = f(x, y)$

on the segment $[a, b]$, satisfying the initial condition at $x = x_0,$ $y = y_0.$

In this method the values y_{i+1} at a given value

y_0 sequentially calculated using formulas

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), \quad (1.4)$$

$$k_1 = hf(x_i, y_i) \qquad k_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right) \qquad (1.5)$$

$$k_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right) \qquad k_4 = hf\left(x_i + h, y_i + k_3\right)$$

Thus, the Runge-Kutta method requires at each step a fourfold calculation of the right side of the equation $f(x, y).$

For practical error assessment, Runge's rule is used:

If y_h And y_{2h} - numerical solutions to the problem,

found by formulas (1.4) with steps h and 2h, respectively, then the error of the solution at a smaller step

$$\varepsilon_i = \frac{1}{15}(y_h(x_i) - y_{2h}(x_i))$$

If the value calculated using this formula ε_i does not provide the specified accuracy, the grid spacing should be reduced.

Example **1.**

Solve the Cauchy problem $$\frac{dy}{dx} = 2(x^2 + y),$$

If $$y(0) = 1,\ 0 \le x \le 1, h = 0.1.$$

Solution.

We carry out calculations using the following algorithm:

parameters k_1, k_2, k_3, k_4

calculated using formulas**(1.5).**

Let's write down the results of the calculations

$$k_1 = hf(x_0, y_0) = 0.1 \cdot 2(x_0^2 + y_0) = 0.2$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1^0}{2}\right) = 0.1 \cdot 2(0.05^2 + 1.1) = 0.2205$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2^0}{2}\right) = 0.2(0.05^2 + 1.1103) = 0.2226$$

$$k_4 = hf\left(x_0 + h, y_0 + k_3^0\right) = 0.2(0.1^2 + 1.2226) = 0.2465$$

$$y_1 = y_0 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) =$$

$$= 1 + \frac{1}{6}\left(0.2 + 2\cdot 0.2205 + 2\cdot 0.2226 + 0.2465\right) =$$

$$= 1 + 0.2221 = 1.2221$$

Let's look at the computer implementation of the Euler and Runge-Kutta methods and compare them with the exact solution