Simulation of stochastic processes using statistical method tests

3.1. Basic provisions

If the operation is interfered with by accidental factors, then it represents a random process, and the performance indicator is the probability some event or mathematical expectation some random variable.

In cases where the construction of an analytical model of a phenomenon for one reason or another is difficult to implement, another modeling method is used, known as the statistical test method or, otherwise, the Monte Carlo method.

The essence of the method. Instead of describing a random phenomenon using analytical dependencies, a “draw” is performed - modeling a random phenomenon using some procedure that gives a random result.

Having performed such a “draw” a **very large number of times**, we will obtain statistical material - many realizations of a random phenomenon - which can be processed by conventional methods of mathematical statistics.

Example 3.1.

A problem is being solved: for a certain goal 4 independent shots are fired, each of which hits it with p=0.5. At least two hits are required to hit the target. Determine the probability of hitting the target.

Analytical method.
The probability of hitting a target is calculated through the probability of the opposite event.

Probability of not hitting the target = the sum of the probabilities of no hit and exactly one hit;

probability of no hits 0.5^4;

the probability of one hit is: $C_4^1 \cdot 0{,}5^1 \cdot 0{,}5^3 = 4 \cdot 0{,}5^4$

Consequently: P= 1-(0.5^4+ 4* 0.5^4)=0.688

Solve **Example 3.1** using the Monte Carlo method

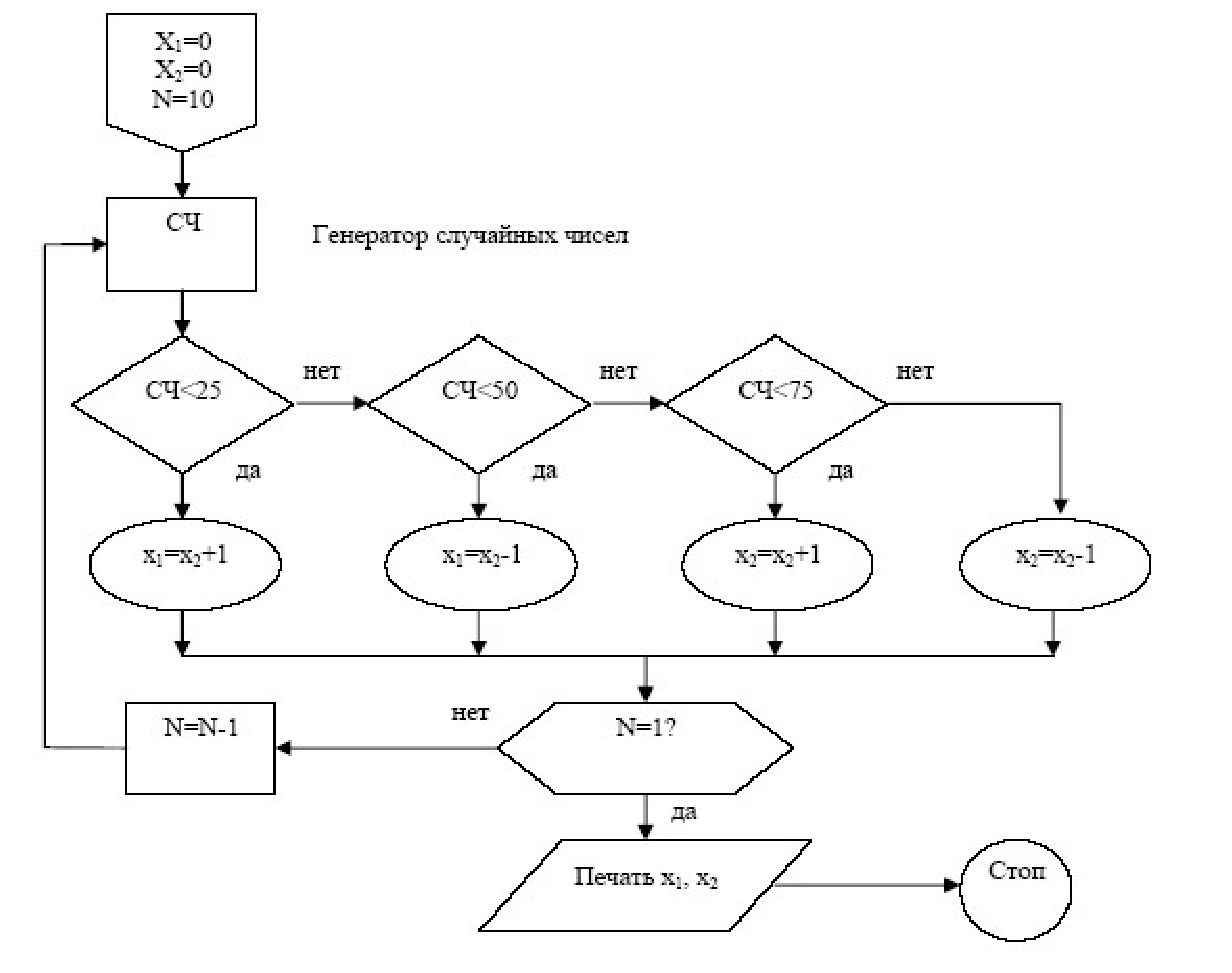
Example 3.2. Analysis of the behavior of a discrete object (discrete input and output variables) - “problem about drunken passerby or the random walk problem.”

A passerby decided to take a walk while standing on a street corner.
Let the probability that, upon reaching the next intersection, he will go north, south, east and west, be the same.
What is the probability that after walking 10 blocks, a passerby will end up no further than 2 blocks from the place where he started his walk.

Let us denote its location at each intersection by a two-dimensional vector (x1, x2) (“exit”), where x1 is the direction from east to west and x2 is the direction from north to south. Each move is one block east (x1 + 1), and each move is one block west (x1 – 1) (x1 is a discrete variable). North x2 + 1, south x2 – 1. Initial position (0,0). If at the end of the walk the absolute values of x1 and x2 are greater than 2, then we will assume that he has gone further than two blocks at the end of a walk of 10 blocks. Because the probability of our passerby moving in any of the 4 directions according to the condition is the same and equal 0.25, then you can estimate its movement using random number generator.

Let us agree that if a random number (RN) lies in range from 0 to 24, the drunk will go east and we increase x1 by 1; if from 25 to 49, then he will go west and x1-1; if from 50 to 74, he will go north and x2 + 1; if from 75 to 99, then south and x2 – 1.

Flowchart of passerby behavior.



3.2 Ways to organize a single lots

Let us assume that during the simulated process the moment has come when its further development (and therefore the result) depends on whether event A appeared at this stage or did not appear (for example: whether the target was hit). Then you need to “cast lots” to decide the question: did event A appear or not?

Let us agree to call a single lot any elementary experiment in which one of the questions is solved:
1. Did event A happen or not?
2. Which of the possible events A1,A2,…Ak occurred?
3. What value did the random variable X take?

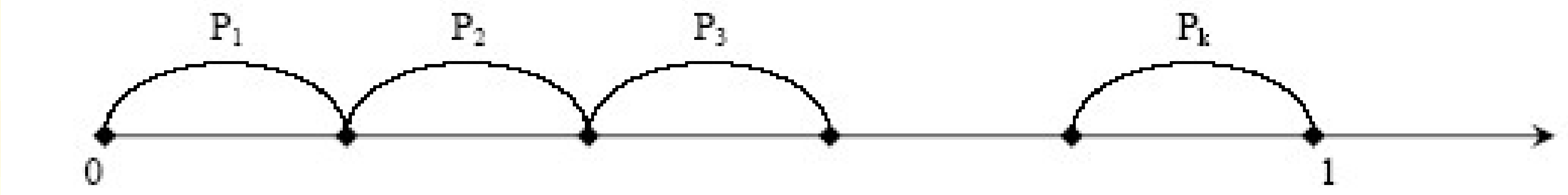
To solve this problem, we need to obtain a random variable distributed with a constant density from 0 to 1. Let us agree, for brevity, to call such a random variable a “random number from 0 to 1” and denote it by R.

3.2.1. Did event A appear or not?

Let the probability of event A be equal to p: P(A)=p. Using a standard mechanism, we will select a random number R and assume that if it is less than p, event A has occurred, if it is greater than p, it has not occurred.

3.2.2. Which of several possible events appeared?

Let there be a complete group of incompatible events: A1,A2,…Ak with probabilities p1,p2,…pk. Because events are inconsistent and form a complete group, then p1+p2+…+pk=1. Let us divide the entire interval from 0 to 1 into k sections of length p1, p2,…pk.

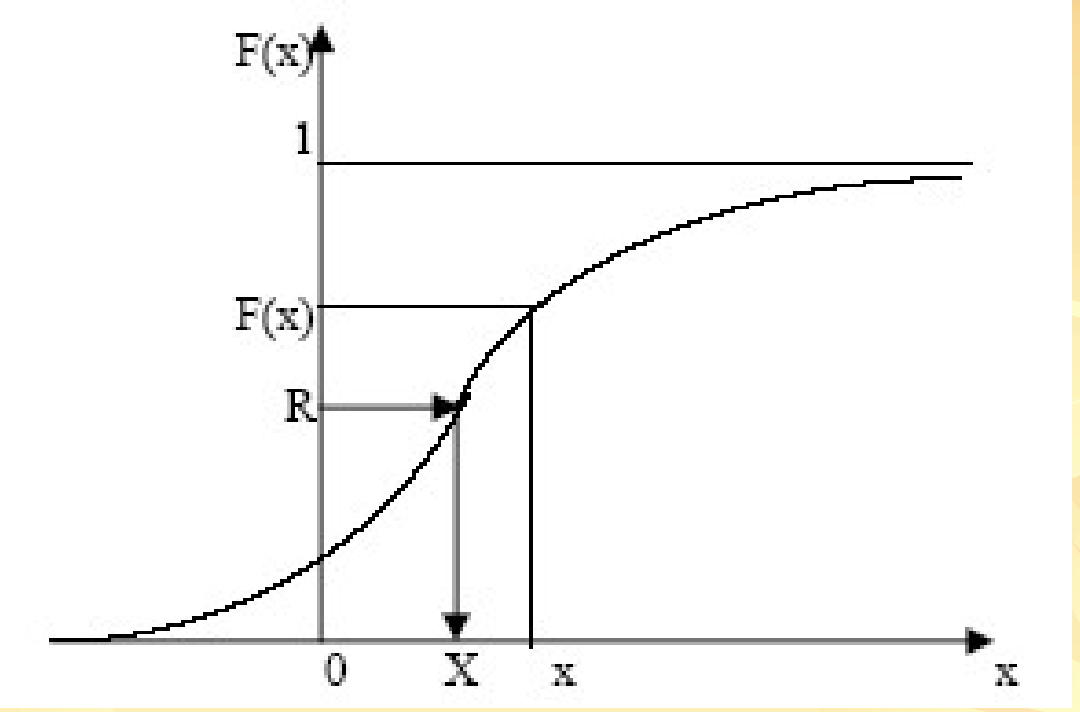


If the random number R generated by the standard mechanism falls, for example, on section p3, this means that event A3 has occurred.

3.2.3 What value did the random variable X take?

Suppose we need to “play” the value of a random quantity X, which has a known distribution law.

Consider the case when the random variable X is continuous and has a given continuous probability distribution function F(x)



If we take a random number R (from 0 to 1) on the ordinate axis and find the value of X at which F(x) = R, then the resulting random variable X will be the desired value.

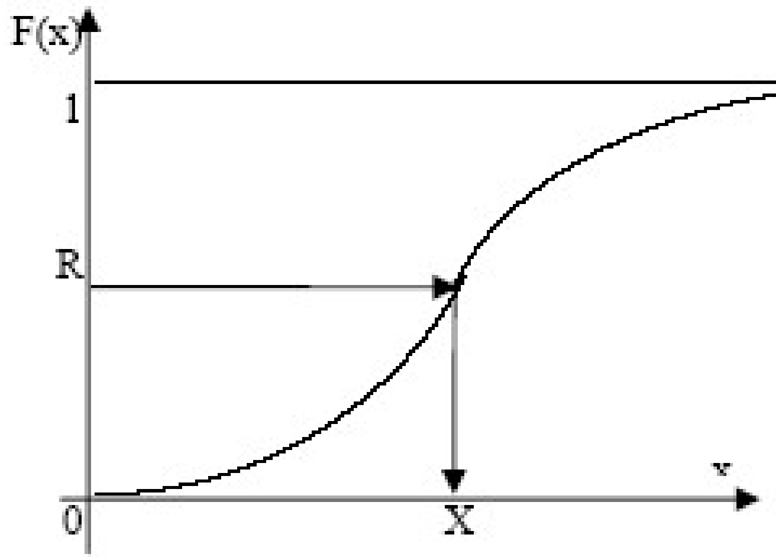
Thus, drawing the value of a random variable X with a given distribution function F(x) comes down to the following procedure: obtain a random number **R from 0 to 1** and take the value for which **R=F(X)** as the value of **X**.

Example 3.3. Random variable X is distributed over exponential law with density $f(x)=\lambda e^{-\lambda x}$ $(x>0)$. Construct a single lot procedure to obtain the value of X.

Given the density f(x), we find the function probability distributions:

$$F(x)=\int_{0}^{x} f(x)dx=\int_{0}^{x} \lambda e^{-\lambda x}dx=1-e^{-\lambda x}$$

Graphically, the value of the random variable X can be play like this: take a random number from 0 to 1 on the axis ordinate and find the corresponding abscissa value X.



The same can be done by calculation if you write:

$R=1-e^{-\lambda X}$ and solve this equation for **X** (i.e. find the function inverse to **F**).

In the end we will get

$$e^{-\lambda X}=1-R,\ -\lambda X=\ln(1-R) \Rightarrow X=-\frac{1}{\lambda}\ln(1-R).$$