Homework 3

Quantum computation

Task 1.

a) Calculate the probability to measure the first qubit is state 1 when we have the following 2-qubit system:

$$\begin{pmatrix} \frac{1}{2} \\ \frac{-i}{2} \\ \frac{i}{2} \\ \frac{-1}{2} \end{pmatrix}$$

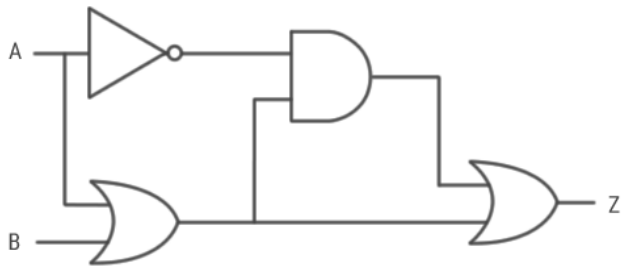
b) Calculate the probability to measure the second qubit is state 0 when we have the following 2-qubit system:

$$\begin{pmatrix} \frac{1}{3} \\ \frac{i\sqrt{2}}{3} \\ \frac{2-i}{3} \\ \frac{-1}{3} \end{pmatrix}$$

c) Calculate the probability to measure the state 11 when we have the following 2-qubit system:

$$\begin{pmatrix} 0 \\ \frac{1}{3} \\ \frac{-i}{3} \\ \frac{2-i\sqrt{3}}{3} \end{pmatrix}$$

Task 2. a) Implement the following classical circuit reversibly:



This circuit can be expressed as ((NOT A) AND (A OR B)) OR (A OR B)

b) Implement this circuit as a quantum circuit by using additional qubits, Toffoli gates and NOT gates.

Remark: in task b) you can consider transforming OR operator into combination of AND and NOT operators by using de-Morgan laws.

Task 3. Implement the quantum circuit that takes values of 8 qubits as input and puts the result of their multiplication in output qubit (which means output is equal to 1 only if all 8 qubits are in state 1; otherwise output is equal to 0). For this task use only Toffoli gates. For this implementation you will need ancilla qubits. Please try to make a circuit that has 15 qubits (8 for input, 6 ancillas and 1 for output).

Task 4. Analyze behavior of Grover's Search when we have 4 elements and 2 of them are marked. What will be the outcome if we do the measurement after

a) 1 iteration of Grover's Search for such setting?

b) 2 iterations of Grover's Search for such setting?

c) 3 iterations of Grover's Search for such setting?

d) 4 iterations of Grover's Search for such setting?

Bonus points if you explain behavior of Grover's Search for cases where exactly half of elements in search space is marked.