**Homework 1**

**Classical computation**

**Task 1.** Suppose that we have given a binary input (input alphabet is {0,1}). Construct a Turing machine that checks whether the numbers of 0's and 1's in the input are the same. If number of 0's is equal to number of 1's, then machine should write 1 in first symbol of input, otherwise - first symbol on the tape should be 0.

Example:

0011001101 - here machine should write 1 in first symbol after the work.

0111111 - here machine should write 0 in first symbol after the work.

**Task 2.** Construct a Boolean circuit that outputs the multiplication of 2 2-bit numbers. Input is represented as 4 bits, first 2 bits represent the first number, last 2 bits represent the second number. Output will be of 4 bits. Examples:

Input 0110 means 01*10=1*2, output should be 2=0010

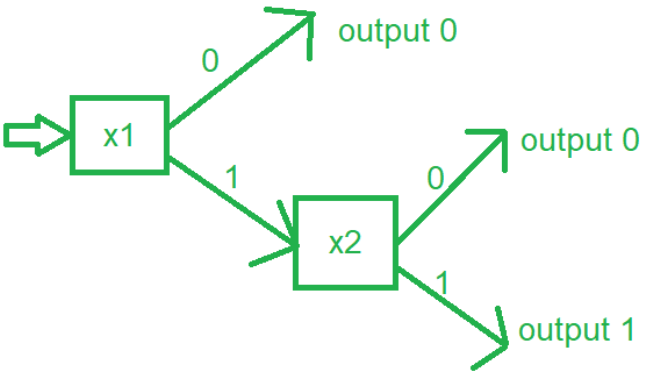
Input 1111 means 11*11=3*3, output should be 9=1001

**Task 3.** Reduce the following Boolean function to the 3-SAT formula (remember the 3-CNF form of 3-SAT):

$$(x_1 \wedge x_2) \vee \neg (x_3 \wedge \neg x_4)$$

**Task 4.** We are given query type of algorithm. Suppose that our task is to compute the given Boolean function output by querying values of the variables. We are allowed to do fixed number of queries.

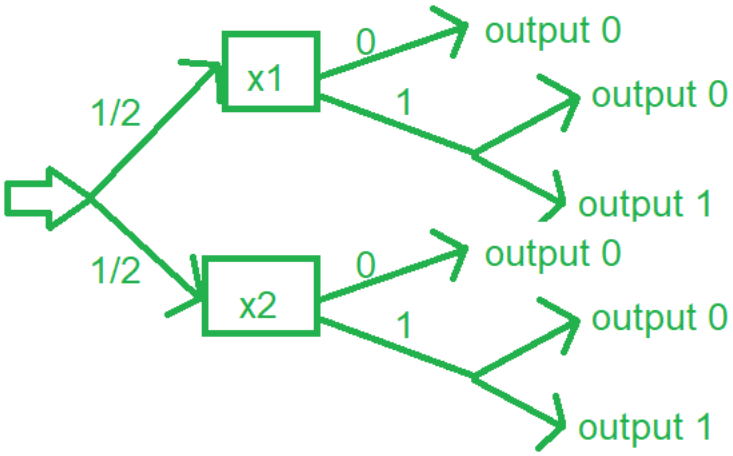
Example: $x_1 \wedge x_2$. We have to compute $x_1 \wedge x_2$ by querying variables. In case of deterministic algorithm, suppose that we can query 2 variables at most. Then we can design the following algorithm:




Here we first query x1. If x1=0, then algorithm will output 0. If x1=1, then algorithm will query the value of x2. If x2=0, then output 0, if x2=1, then output 1.

Design a probabilistic query algorithm to solve $x_1 \wedge x_2$ with one query, such that probability of correct answer is more than 1/2.

Example of probability 1/2:




Here we pick with probability 1/2 x1 or x2, ask its value. If value is 0, we output 0, but if value is 1, we output 0 or 1 with equal probabilities. So, if value of variable is 1, we have 50% of error, because we do not know the value of the second variable.