Artificial Intelligence – Spring 2021

Homework 4

Issued: April 19th, 2021 Due: May 3rd, 2021

Problem 1:

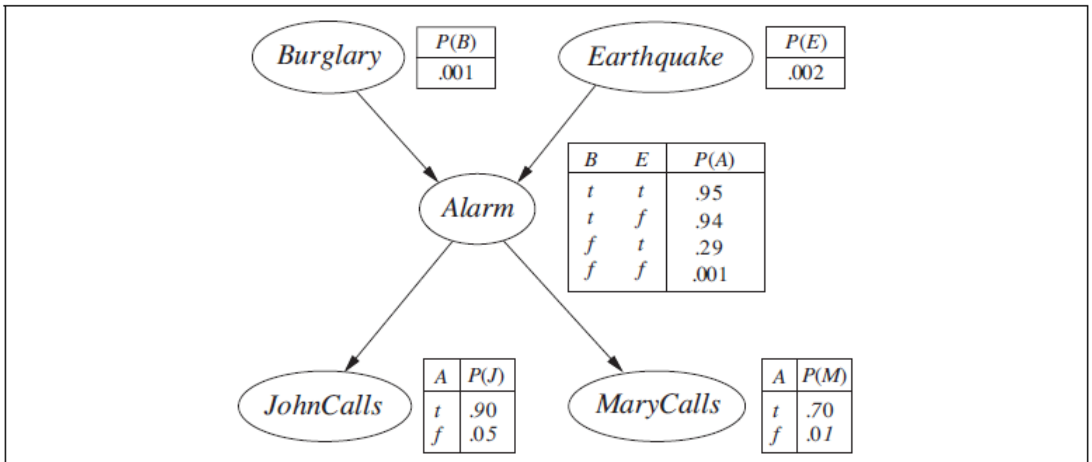

Figure 14.2 A typical Bayesian network, showing both the topology and the conditional probability tables (CPTs). In the CPTs, the letters B, E, A, J, and M stand for *Burglary*, *Earthquake*, *Alarm*, *JohnCalls*, and *MaryCalls*, respectively.

14.4 Consider the Bayesian network in Figure 14.2.

a. If no evidence is observed, are *Burglary* and *Earthquake* independent? Prove this from the numerical semantics and from the topological semantics.

b. If we observe $Alarm = true$, are *Burglary* and *Earthquake* independent? Justify your answer by calculating whether the probabilities involved satisfy the definition of conditional independence.

Problem 2:

14.6 Let H_x be a random variable denoting the handedness of an individual x, with possible values l or r. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps there is a gene G_x, also with values l or r, and perhaps actual handedness turns out mostly the same (with some probability s) as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability m of a random mutation flipping the handedness.

a. Which of the three networks in Figure 14.20 claim that $\mathbf{P}(G_{father}, G_{mother}, G_{child}) = \mathbf{P}(G_{father})\mathbf{P}(G_{mother})\mathbf{P}(G_{child})$?

b. Which of the three networks make independence claims that are consistent with the hypothesis about the inheritance of handedness?

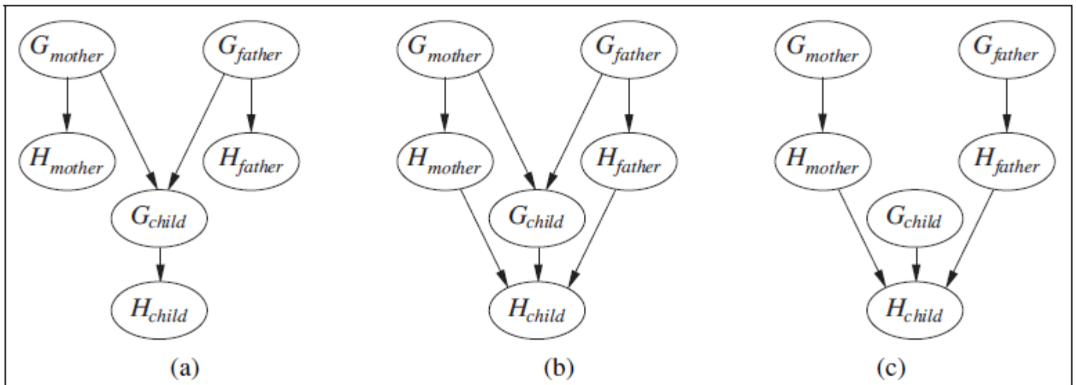

Figure 14.20 Three possible structures for a Bayesian network describing genetic inheritance of handedness.

c. Which of the three networks is the best description of the hypothesis?

d. Write down the CPT for the G_{child} node in network (a), in terms of s and m.

e. Suppose that $P(G_{father}=l) = P(G_{mother}=l) = q$. In network (a), derive an expression for $P(G_{child}=l)$ in terms of m and q only, by conditioning on its parent nodes.

f. Under conditions of genetic equilibrium, we expect the distribution of genes to be the same across generations. Use this to calculate the value of q, and, given what you know about handedness in humans, explain why the hypothesis described at the beginning of this question must be wrong.

Problem 3:

15.13 A professor wants to know if students are getting enough sleep. Each day, the professor observes whether the students sleep in class, and whether they have red eyes. The professor has the following domain theory:

- The prior probability of getting enough sleep, with no observations, is 0.7.
- The probability of getting enough sleep on night t is 0.8 given that the student got enough sleep the previous night, and 0.3 if not.
- The probability of having red eyes is 0.2 if the student got enough sleep, and 0.7 if not.
- The probability of sleeping in class is 0.1 if the student got enough sleep, and 0.3 if not.

Formulate this information as a dynamic Bayesian network that the professor could use to filter or predict from a sequence of observations. Then reformulate it as a hidden Markov model that has only a single observation variable. Give the complete probability tables for the model.

Problem 4:

15.14 For the DBN specified in Exercise 15.13 and for the evidence values

$\mathbf{e}_1 =$ not red eyes, not sleeping in class
$\mathbf{e}_2 =$ red eyes, not sleeping in class
$\mathbf{e}_3 =$ red eyes, sleeping in class

perform the following computations:

a. State estimation: Compute $P(EnoughSleep_t | \mathbf{e}_{1:t})$ for each of $t = 1, 2, 3$.

b. Smoothing: Compute $P(EnoughSleep_t | \mathbf{e}_{1:3})$ for each of $t = 1, 2, 3$.

c. Compare the filtered and smoothed probabilities for $t = 1$ and $t = 2$.