Homework 2

Classical computation

Task 1. Prove that a 4-tape Turing machine working in time $T(n)$ for inputs of length n can be simulated by an ordinary Turing machine working in time $O(T^4(n))$. Bonus points if you design more efficient simulation that requires less than O(T^4(n)) time.

Task 2. Recall Task 2.7. from the Book that stated the following: Show that any function can be computed by a circuit of depth ≤3 with gates of type NOT, AND, and OR, if we allow AND- and OR-gates with arbitrary fan-in and fan-out.

Suppose now that we are limited to bases consisting of 2 logic gates. Show that that any function can be computed by a circuit:

a) of depth ≤5 with gates of type NOT and AND if we allow AND-gates with arbitrary fan-in and fan-out.

b) of depth ≤5 with gates of type NOT and OR if we allow OR-gates with arbitrary fan-in and fan-out.

Task 3. Consider the Subset Sum Problem. Given a set of non-negative integers, and a value sum, determine if there is a subset of the given set with sum equal to given sum.

Example: {3,4,5,6,8} sum=20. Here answer should be positive, since 3+4+5+8=20

Example: {2,3,7,8} sum=14. Here answer is negative, we cannot make 14 from numbers that we have.

Show that Subset Sum Problem is in NP.

Task 4. Consider TQBF (True quantified Boolean formula) problem that we discussed in lecture 7. For each formula, verify whether it is true or false:

a) $\forall x \exists y \exists z((x \vee y) \wedge z)$
b) $\forall x \exists y((x \vee y) \wedge (\neg x \vee \neg y))$
c) $\forall x(x)$
d) $\forall x \forall y \exists z((x \wedge z) \vee y)$