Homework 4

Quantum computation

Task 1.

Suppose that we have the following (mixed) quantum state: $\frac{1}{3}\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} + \frac{2}{3}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

a) Write density matrix expression for the mentioned quantum system.

b) Write expression with trace and projection and estimate the probabilities for state 0 and state 1.

Task 2.

Consider the decoherence operator D that we discussed in lecture 19. Apply D to the following quantum state represented by density matrix:

$$\begin{pmatrix} \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

Please write each step of D affecting the quantum system.

Task 3.

Consider the topic of the lecture 21. Apply measuring operator $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ to the state $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. Remember that we start by joining the system with $|0\rangle\langle 0|$. What is the outcome?

Task 4.

Write down step-by-step application of Shor's algorithm to the number 15. When you describe steps, use $a = 7$ when random number is picked in step 3.