A compact setup for broadband polarization tomography

Yujia Yuan, Michael Li, Sema Kuru, Behrooz Semnani, Michal Bajcsy

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Abstract

Polarizatoin Tomography(PT) is used to reconstruct the quantum state. In this paper we first introduce the convention taken in our laboratory that is consistent with the electric field theory and Poincare Sphere(PS). Followed by is our compact design of the apparatus that takes only 20cm^2 area on the optics table that can do broadband tomography. Finally we demonstrate that the setup gives results with high purity and fidelity.

1 Introduction

PT is used to reconstruct the polarization state by the information of how much content is in each component[1]. Conventional PT setups[2] contain three beam splitters extracting specific polarizations and six power meters to measure how much power is in each polarization. The advantage of this setup is that it can continuously measuring polarization states at a fast speed because there is no mechanical part.

However, such a design is expensive and bulky because copies of the same devices are used. In this paper we discuss our implementation of a compact PT setup. With piezo-electric motors that rotate accurately, we only need one Half Waveplate(HWP), one Quarter Waveplate(QWP), one linear polarizer(LP) and one power meter.

Well-defined conventions are crucial in order to do so, such as the direction and angle wave plates should rotate. To visualize the waveplate transformation, we introduce the Poincare Sphere(PS) that represents polarization states in a simple way.

We implement the PT device according to the conventions created, and demonstrate that the reconstructed state is accurate, with both the fidelity and purity higher than 95%.

Further more, we show that the system can use a spectrometer instead of a power meter to measure the power spectrum, thus giving the polarization states for individual wavelengths. And because both the spectrometer and the optical elements are broadband, we achieve broadband tomography with a compact setup that is otherwise bulky and expensive.

2 Polarization tomography conventions

All waves propagating to +z direction can be decomposed into superposition of plane waves with form of $\vec{E} = (E_x\hat{x} + E_y\hat{y})e^{i(kz-\omega t)}$. We can also represent it in Jone's vector: (E_x, E_y). With this convention, wave retarders with fast axis aligning to horizontal(x) direction can be represented by $\vec{E} = (E_f\hat{f} + e^{i\phi}E_s\hat{s})e^{i(kz-\omega t)}$, and its the matrix formalism of the phasor is:

$$\hat{E} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} \quad \phi \in \mathbf{R}^+ \tag{1}$$

Where $\phi = \pi/2$ for half waveplate and $\phi = \pi/4$ for quater waveplate. For all the components, we choose the (fast) axis aligning to x-axis. Here, for example, when we equip a half waveplate, then E_x keeps the same while E_y shifts to the -z direction by half-wavelength. The effect of it on E_y is that y-component arrives at a point later than x-component, which is "retarded". This phenomenon is due to "Birefringing effect", which means the material has different phase(due to different speed of wave) for different polarized wave.

We set horizontally polarized wave to be (1,0) state. For circularly polarized wave, we will look from the direction of propagation. Which is, the direction of rotation is represented by that of four fingers and direction of propagation that of the thumb. So we have:

Ket notation	$\|H\rangle$	$\|V\rangle$	$\|D\rangle$	$\|A\rangle$	$\|L\rangle$	$\|R\rangle$
Matrix notation	$\begin{pmatrix}1\\0\end{pmatrix}$	$\begin{pmatrix}0\\1\end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$

Table 1: Corresponding ket and matrix notations

Here we use the density matrices $\hat{\rho}$ to represent the polarization state to accommodate to mixed states. For example, for a 50% horizontal polarization and another 50% vertical polarization, we have

$$\hat{\rho} = 0.5\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1 & 0\end{pmatrix} + 0.5\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0 & 1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{2}$$

3 Poincaré Sphere

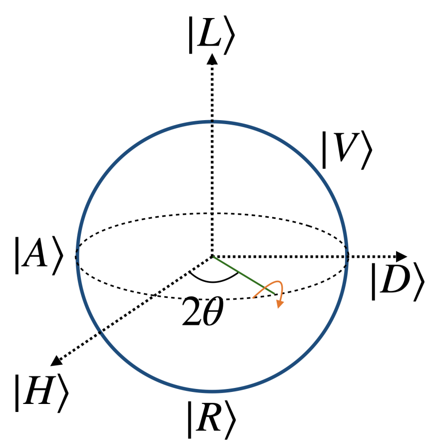

Figure 1: Poincare sphere representation of polarized light. It is a rotated Bloch Sphere. θ is the angle of the fast axis of waveplates with respect to horizontal, the base line for calibration.

We represent all the polarizations in Pauli matricies instead of kets. This is also a paradigm approach for quantum information processing where transformation between states are important[3].

$$\sigma_0 = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \quad \sigma_1 = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} \quad \sigma_2 = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix} \quad \sigma_3 = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$$

Any single-qubit density matrix $\hat{\rho}$ can be uniquely represented by four parameters $\{S_0, S_1, S_2, S_3\}$:[3]

$$\hat{\rho} = \frac{1}{2S_0}\sum_{i=0}^{3} S_i\hat{\sigma}_i \tag{3}$$

and the S_i values are given by

$$S_i \equiv \mathrm{Tr}\{\hat{\sigma}_i\hat{\rho}\} \tag{4}$$

For this project, S parameters are normalize by dividing each of them by S_0, so $S_0 = 1$ and polarization states are power-independent. For all pure states, $\sum_{i=1}^{3} S_i^2 = S_0$; for mixed states, $\sum_{i=1}^{3} S_i^2 < S_0$; for the completely mixed state, $\sum_{i=1}^{3} S_i^2 = 0$.

Physically, each of these parameters can be measured by specific pair of projective measurements[1]:

$$\begin{aligned} P_0 &= P_{|H\rangle} + P_{|V\rangle} \\ P_1 &= P_{|H\rangle} - P_{|V\rangle} \\ P_2 &= P_{|D\rangle} - P_{|A\rangle} \\ P_3 &= P_{|L\rangle} - P_{|R\rangle} \end{aligned} \tag{5}$$

where $P_{|\psi\rangle}$ is the power of polarization state $|\psi\rangle$. S parameters are just the powers normalized by P_0:

$$S_i = P_i/P_0 \tag{6}$$

and thus S_0=1, representing the normalized total power.

Again, here we define left hand circularly polarized light to be positive S_3 direction on the sphere because for $(\sigma_0 + \sigma_y)/2$, eigenvector whose eigenvalue is not zero is (1,i), which is LCP.

Waveplates rotate the Poincare Sphere along the line 2θ from $|H\rangle$ on the $|H\rangle|D\rangle$ plane by their retardances, as shown by Figure 1. For example, if a quarter waveplate has its fast axis 45 degrees from horizontal, it rotates the sphere by the retardance, $\pi/2$, around the line 90^o to $|H\rangle$, which is $|H\rangle$. Thus $|L\rangle$ before the quarter waveplate becomes $|H\rangle$ after it.

S_0 is also the total power passing through the device. Normalizing the other parameters by dividing them by S_0 allows us to have power-independent terms.

4 Polarization tomography setup

It is proven[4] that a half waveplate(HWP) together with a quarter waveplate(QWP) can rotate any point on the surface of the Poincare Sphere to $|H\rangle$ by different θs. We setup the tomography apparatus like Figure 2.

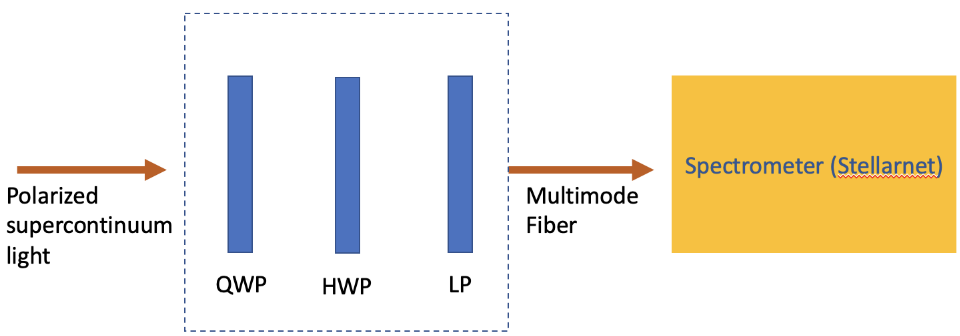

Figure 2

To demonstrate that our system works well for a broad range of wavelength, we choose the polarized super-continuum laser as our input. The wavelength range of the laser is 760-940nm. We select AQWP05M-980 as the QWP and AHWP10M-980 as the HWP. Both of them have the range from 690nm to 1200nm. We select LPNIR050 as the linear polarizer(LP), whose range is from 650nm to 2000nm and its extinction ratio is > 1000.

The base line is set to horizontal. That is, the polarizer is aligned to pass TE mode only and the "zero degree" position of two waveplates is when their fast axis align horizontally. The incident light from the left passes through waveplates at certain angles, thus having the polarization intended to be measured rotated to $|H\rangle$. By passing through the horizontally aligned polarizer, only $|H\rangle$ is left and thus we can measure the power. The angles of the waveplates to rotate each composition to $|H\rangle$ are in the Table 2. Notice that those are not the only combinations. For example, when the HWP is aligned to 22.5^o, it has the same effect as when it is aligned to 67.5^o.

Light coming out from the LP is then coupled to the spectrometer BLK-CXR-SR with a multi-mode fibre M91L01 with the core diameter[5] 200±2% and supported wavelength 250-1200nm. The

Power of	QWP	HWP
$\|H\rangle$	0	0
$\|V\rangle$	0	45^o
$\|D\rangle$	45^o	22.5^o
$\|A\rangle$	45^o	67.5^o
$\|L\rangle$	45^o	0
$\|R\rangle$	45^o	45^o

Table 2: Caption

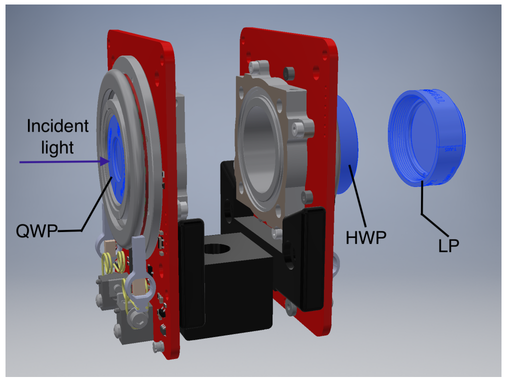

Figure 3: Caption

spectrometer allows us to measure the power of each frequency individually, so does the polarizatino tomography.

Components in the dashed line in Figure 2 are shown in Figure 3. All optical elements are manufactured by Thorlabs. Two red devices are ELL14K piezo motors that rotates precisely: with built-in encoders, they can be controlled to rotate to arbitrary positions with the error less than 0.1 degree. QWP and HWP are mounted on the motor in opposite directions. We self-designed the dual-motor holder(black) that allows us to mount the motors in a compact fashion with readiness. The footprint of the two motors is less than 50cm^2.

Motors communicate with the controlling computer through serial port by their communication protocols that can be found in [6]. The spectrometer uses the drive provided by StellarNet to connect to the computer. Note that the drive only support Python 2 environment and the USB port connectivity can be finicky at times.

5 Experimental results

We can choose an arbitrary wavelength in the range of values and plot the Poincare Sphere accordingly. Figure 5 is one example. The radius of the sphere is 1, with totally mixed states residing at the center of the sphere and pure polarization on its surface.

In order to prove that the waveplates rotates the Poincare Sphere in the right way, we add a HWP before our setup to turn $|H\rangle$ into $|A\rangle$ and do the measurement.

Notice that when the purity is greater than 1, spherical representation will have the coordinate outside the sphere, which is physically unrealistic. Normally this happens when the total laser power fluctuates, causing S0 lower than its time-averaged value. If this happens, we divide all the three S parameters by the purity to ensure that the point is on the surface of the sphere. It is also the most likely state because any line from the center to the point on the sphere is perpendicular to the surface.

We prepare the polarization states by controlling horizontally polarized light with another HWP AHWP10M-980. The HWP introduces some incoherence: without the HWP the purity is uniformly 0.962.

In addition to the purity, we also care about the fidelity, which is the measurement of the "realness", or how much the reconstructed state overlaps with the actual state, is the fidelity[7]:

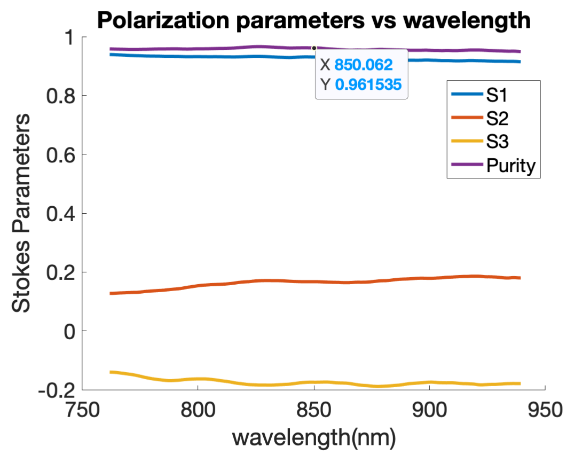

Figure 4: S parameters and purity versus wavelength plot. Input is a horizontally polarized supercontinuum laser beam with spectrum 500-900nm. The beam is then tested with our setup. The integration time of the spectrometer is 0.5 seconds.

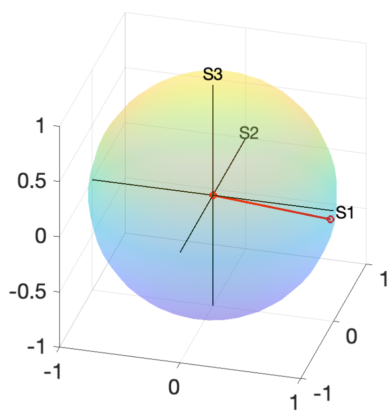

Figure 5: Poincare Sphere representation for 850nm light with the setup same as that of Figure 4. S1, S2 and S3 are normalized with respect to S0 for a relative value. Length of the red line is the purity. Position of the red hollow dot is the polarization state

$$F = \frac{1}{2}\left(\vec{S}_{prep} \cdot \vec{S}_{recon}\right) \tag{7}$$

Both of the S vectors are normalized sphere coordinates so that each of them has the length of 1, thus separating from the effect of purity.

For 850nm, we tested the linear input states as the illustration, and the result is in Table 3.

State	H	V	D	A
Fidelity	96.8%	96.2%	99.7%	99.8%

Table 3: Fidelity of linear polarization states. Polarization are controlled by a broadband HWP AHWP10M-980 after a horizontally aligned LP.

6 Conclusion

We demonstrate the compact design of the polarization tomography and its capability of measuring spectral polarization state.

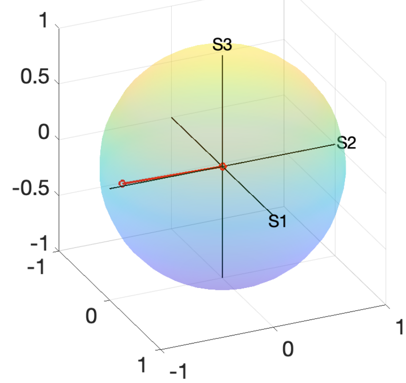

Figure 6: Poincare Sphere for 850nm light with a HWP having its fast axis 67.5^o to the horizontal axis before the device and after the polarized source.

There are some future works that may improve the existing design. First of all, the waveplates are not ideal. For example, slow-axis retardance is close to 0.26λ for the QWP instead of the ideal 0.25λ. As a result, the states but $|H\rangle$ we measure are not exactly accurate ones. There is a bit deviation from the theoretical states. In addition, the spectrometer has a limited resolution. Even with an input pulse whose linewidth is $\approx 50kHz$, it displays a Gaussian with 6nm FWHM. As a result, it is difficult to use the spectrometer for broadband polarization tomography that require fine linewidth to resolve thin spectral lines.

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