Online Appendix for

“A coordinated optimization research on timetable and skip-stop pattern for urban rail lines”

Jiacheng Yao[1], Fan Liu[1]

[1] School of Management & Engineering, Nanjing University, China
502022150046@smail.nju.edu.cn
liufan@nju.edu.cn

Appendix A: Comparisons of modeling scenarios in the urban rail optimization

Table A.1 provides a comparison between our work and several major related researches.

Table A.1. Comparisons of modeling scenarios in the urban rail optimization.

Objective	Input demand	Strategy	Fairness	Transfer	Key references
min TWT	TDODD	Timetabling	No	No	Niu et al.(2013)
min OC & TWT	TDSD	Skip-stopping +timetabling	No	No	Wang et al.(2014)
min TTT	TDODD	Skip-stopping	No	No	Jamili et al.(2015)
min TWT	TDODD	Timetabling	No	No	Niu et al. (2015)
min max TWT	TDODD	Timetabling	Yes	No	Wu et al.(2015)
min OC	TDSD	Skip-stopping +timetabling	No	No	Shi et al.(2017)
min OC	TDSD	Skip-stopping	No	No	Yang et al.(2019)
min OC	TDODD	Timetabling	Yes	No	Li et al.(2019)
min longest WT	GROUP	Skip-stopping +timetabling	Yes	No	Yang et al.(2021)
min OC & TWT	TDODD	Skip-stopping +timetabling	No	No	Yang et al.(2021)
min longest WT	ITDODP	Skip-stopping +timetabling	Yes	Yes	This paper

Note: WT: waiting time, TWT: total waiting time, TTT: total travel time, OC: Operational cost, TDODD: time-dependent O-D demand, ITDODP: individual time-dependent O-D passenger, TDSD: time-dependent section demand, Group: group of same O-D passengers.

Appendix B: Notations and definition

Fig. B.1 illustrates the physical structure of the rail transit line, which including two directions, namely "up" and "down", and a total of N platforms. Each passenger has an independent arrival time and O-D demand. Passengers follow a strict FIFO rule: passengers arriving at the same station with the same direction should follow the order to get on the underground train. The passenger who arrives earlier should achieve railway service first as long as the following train satisfies his strategy demand.

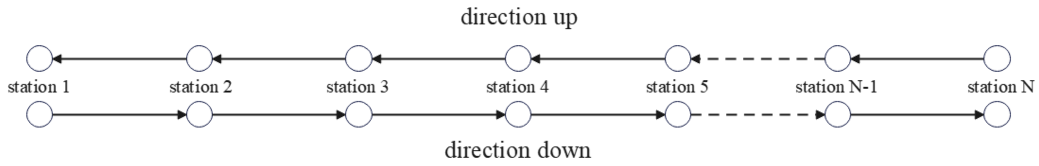


Fig. B.1. The physical structure of our rail transit line

We introduce the notations and their definition in Table B.1. The notations used in our paper are mainly divided into three categories: sets, parameters, and variables.

Set notations refer to the basic subjects of our model. In the urban rail system, there are mainly three types of subjects: underground trains, stations, and passengers.

Parameter notations refer to various input parameters of the model. For example, T refers to the length of the operating period. Different from many studies researching daily operation, we mainly focus on peak hours. Therefore, our T may only cover 2-3 hours during the rush hours; *S*, *SS*, *Skip*, *Skip-Skip* are all parameters relevant to the skip-stop strategy.

Variable notations refer to decision variables and other variables in our model. $x_{i^d j}$ is the indicator of the skip-stop decision of train i^d in station j. All stations' skip-stop decisions compose X, which refers to the skip-stop strategy of train i^d. The departure timetable's and passenger travelling strategy's notations are in common with X.

Table B.1. Notations and definition

Notation	Definition
Sets	
direction	Rail's direction. $direction \in \{``up", ''down"\}$
I^{up}	Set of rails which *direction* is *"up"*
I^{down}	Set of rails which *direction* is *"down"*
$I^{direction}$	Set of rails, $i^{up} \in \{1^{up}, 2^{up}, \ldots, \lvert I^{up} \rvert^{up}\}$, $i^{down} \in \left\{1^{down}, 3^{down}, \ldots, \lvert I^{down} \rvert^{down}\right\}$
J	Set of stations
j, k	Number of stations, $j, k \in \{1,2,3,\ldots,\lvert J \rvert\}$
P	Set of passengers
p, l	Number of passengers, $p, l \in \{1,2,3,\ldots,\lvert P \rvert\}$
Parameters	
T	Length of the operating period
$t_{direction}^{first}$	The first rail's departure time in *direction*
t_{min}^{seq}	The minimum interval of sequent rails
C	Train capacity

o_p	Origin station of passenger p
d_p	Destination station of passenger p
t_p	Arrival time of passenger at origin station
s_{i^dj}	Dwelling time of Rail i^d in station j
$\tau_{j,j+1}$, $\tau_{j,j-1}$	Running time between two adjacent stations
t_h	Minimum headway between two adjacent stations
S	Maximum skipping times of one rail
SS	Maximum number of continuous skipping for one rail
$Skip$	Maximum times of one station to be skipped
$Skip\text{-}Skip$	Maximum times of two stations when at least one being skipped
Variables	
X	Indicator of the stop-skipping strategy of rails, $\lvert I\rvert \times \lvert J\rvert$ matrix
x_{i^dj}	Indicator of the stop-skipping decision of rail i^d: x_{i^dj} = 0 if station j is skipped, and x_{i^dj} = 1, otherwise
T_d	The timetable of rails, $\lvert I\rvert$-length vector
t_{i^d}	The departure time of rail i^d
Y_p	passenger p's travelling strategy when they can transfer, including y_{pi^d}, s_{pi^d}, t_{pi^d}
y_{pi^d}	Indicator of the travelling decision of passenger p: y_{pi^d} = 0 if passenger p choose rail i^d, and y_{pi^d} = 1, otherwise
s_{pi^d}	Station where passenger p gets on the rail in the strategy
t_{pi^d}	Passenger p's arriving time of each station where i^d stops in the strategy
a_{i^dj}	Arrival time of rail i^d at station j
d_{i^dj}	Departure time of rail i^d from station j
c_{i^dj}	The number of passengers remained when rails depart from stations
W_p	Passenger's waiting time

Table B.2 classifies passengers into different groups.

Table B.2. Groups of passengers

Group	definition
$\hat{P}_{j^{up}}$	Group of passengers departing from station j with direction "up"
$\hat{P}_{j^{down}}$	Group of passengers departing from station j with direction "down"
$\check{P}_{j^{up}}$	Group of passengers going to station j with direction "up"
$\check{P}_{j^{down}}$	Group of passengers going to station j with direction "down"

Appendix C: Our algorithms

Algorithm 1 shows how passengers choose their transfer strategies. Considering passenger p's choice, before the earliest train both stopping at p's origin and destination, p may have several transfer strategies to choose from. We have proved that the number of transfer strategies for one passenger is limited. According to p's travelling time, he will choose one strategy which brings him the shortest travelling time. The choosing process is shown in algorithm 1.

Algorithm 1: passenger choose

Input: passenger information; timetable and skip-stop strategy

Output: passenger travel choice

for $p \in P,\ i \in I$ **do**

    calculate the latest direct subway i^*'s arrival time to o_p: $a_{i^* o_p}$

    set $i_j \in I_j$ where $t_p < a_{i_j o_p} < a_{i^* o_p}$ and i_j stops at station o_p meanwhile skips d_p

    set $i_k \in I_k$ where $t_p < a_{i_k o_p} < a_{i^* o_p}$ and i_k skips station o_p meanwhile stops at d_p

    #forward transfer

    for $i_j \in I_j,\ i_k \in I_k$ **do**

        if $a_{i_j o_p} < a_{i_k o_p}$ and there exists at least one station $l \in (j, k)$ where i_j, i_k stop **then**

            passenger *p* can choose forward transfer, keep it as s_1

        end if

    end for

    #O-turn transfer

    for $i_j \in I_j$ **do**

        if there exists at least one rail i_k meeting $a_{i_k o_p} \in (a_{i_j o_p}, a_{i^* o_p})$ **then**

            if there exists at least one station *l* where i_j, i_k stop **then**

                passenger *p* can choose O-turn transfer, keep it as s_2

            end if

        end if

    end for

    #D-turn transfer

    for $i_k \in I_k$ **do**

        if there exists at least one rail *l* which stops at o_p **then**

            if passenger *p* can get i_k by *l* **then**

                passenger *p* can choose D-turn transfer, keep it as s_3

            end if

        end if

    end for

    Compare travel time of s_1, s_2, s_3 together with $a_{i^* o_p}$

    choose the minimum as *p*'s strategy

end for

Algorithm 2 shows the main structure of our genetic algorithm, including encoding/decoding, initialization, selection, crossover and mutation.

Algorithm 2: genetic algorithm

Input: population size: *n*; maximum number of iteration: *max_iter*

Output: global best timetable and skip-stop strategy

generate initial population of *n* strategy chromosomes

set iteration counter $c = 0$
compute the fitness value of each chromosome
while $c < max\_iter$ **do**
  calculate each chromosome's fitness according to **Algorithm 3**
  select a pair of chromosomes from initial population by fitness
  apply crossover on selected pair to generate offspring
  apply mutation on each chromosome
  increment c by 1
end while
return the best timetable and skip-stop strategy

Algorithm 3 shows the method of calculating the fitness of one chromosome. The fitness is in accordance with the objective of our model.

Algorithm 3: fitness

Input: passenger information; one chromosome
Output: chromosome's fitness

decode chromosome to get the timetable and skip-stop strategy
for $p \in P$ **do**
  passenger p chooses travel strategy according to **Algorithm 1**
end for
set iteration $t = 0$
while $t < T$ **do**
  subway enters the station
  passenger get off the rail according to the travel strategy
  passenger get on the rail according to the travel strategy
  accumulate the waiting time of each passenger and add it to W_p
  rail leaves the station
  increment t by 1
end while
return $max\ W_p$

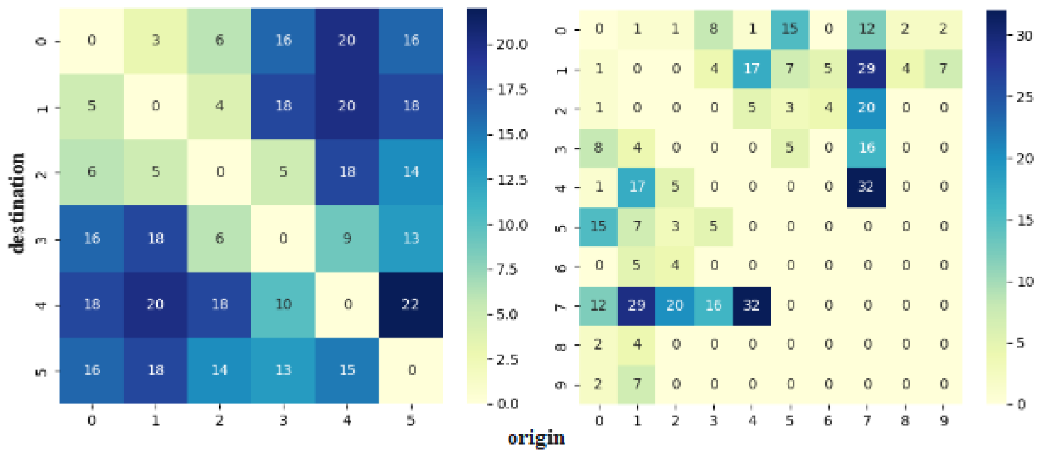
Fig. C.1 shows an example of one chromosome for the GA algorithm. Considering a subway system with 3 stations and 3 trains, our encoded chromosome consists of two segments. One of the segments stores the information of skip-stop strategy; the other segment stores the information of the departure timetable. In the segment of the skip-stop strategy, two directions of the skip-stop decisions are encoded by 0-1 variables, with the sequence of "up" first and "down" second. An element valued 1 indicates that the corresponding underground train stops at that station, while 0 indicates that the station will be skipped.

1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1	1	-1	0	1	-1	0	0

Fig. C.1. An example of strategy encoding

The strategy indicates that the second trip in the "up" direction skips the second station, and the third trip in the "down" direction skips the third station. The first train in both directions depart 1 minute earlier. The third trip in the "up" direction departs 1 minute later. The decoding process simply follow the encoding steps.

Appendix D: More information about the results



(a) Instance 1 with 6 stations, 400 passengers (b) Instance 3 with 10 stations, 400 passengers

Fig. D.1. Passengers' O-D demand matrix

Fig. D.1 shows the O-D demand matrix of our small-size cases. The row represents the destination and the column represents the origin. In the instance of 6 stations, 400 passengers, for example, we can see that the number of passengers departing from station 0 to station 1 is 5; the number of passengers departing from station 1 to station 5 is 18. In the instance of 10 stations, 400 passengers, there exists a climax that the number of passengers whose destination is station 7 is much more than others.

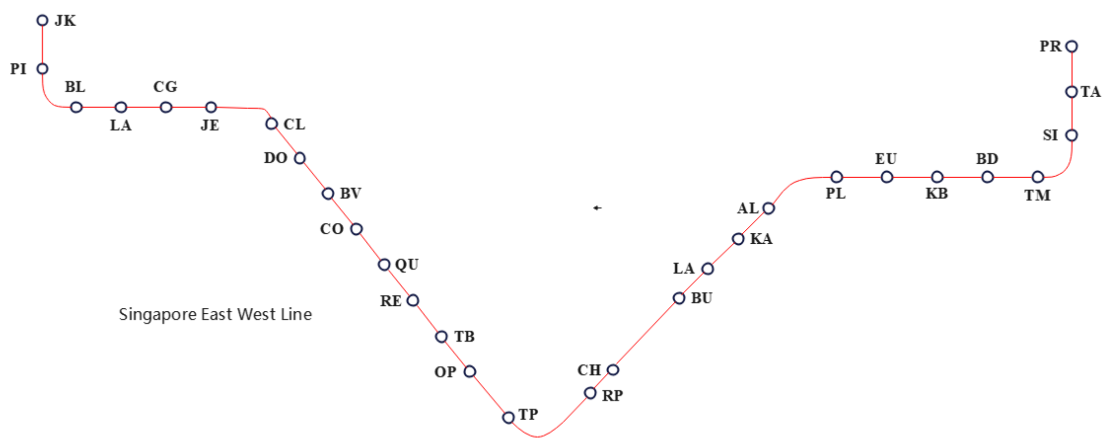


Fig. D.2. Illustration of the East West line in Singapore

The East West line is one of the busiest lines in the Singapore subway network, which consists of 29 stations as shown in Fig. D.2.

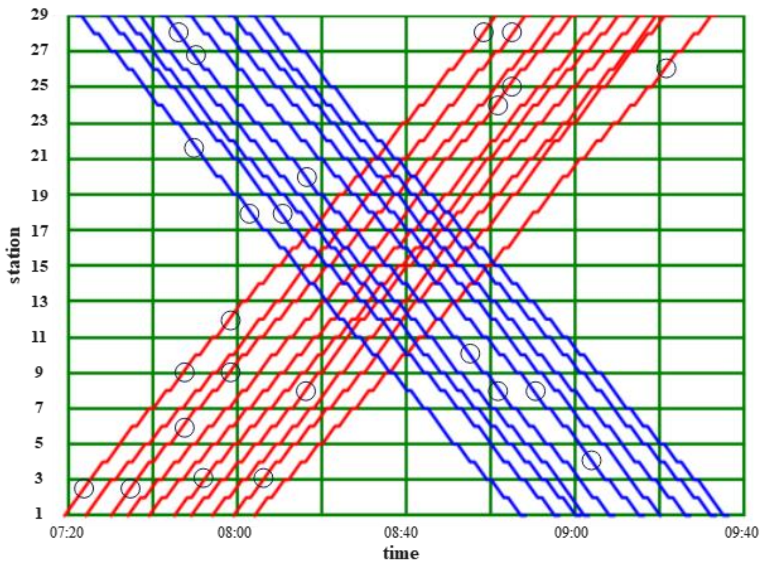


Fig. D.3. The timetable with optimized skip-stop pattern in the Singapore Metro Line 1

Fig. D.3 shows a part of the timetable with optimized skip-stop pattern in the Singapore Metro Line 1. The black circles indicate the corresponding skipped stations by underground trains.