

Linguagens de Programação

Semântica Denotacional

– Soluções –

1. A. $y:=2*x+2; z:=x*2; x:=2*x+1$

$$\begin{aligned} C[[y := 2 * x + 2; z := x * 2; x := 2 * x + 1]](s_0) &= \\ C[[z := x * 2; x := 2 * x + 1]](C[[y := 2 * x + 2]](s_0)) &= \\ C[[z := x * 2; x := 2 * x + 1]](s_1) &= \\ C[[x := 2 * x + 1]](C[[z := x * 2]](s_1)) &= \\ C[[x := 2 * x + 1]](s_2) &= \\ s_3 \end{aligned}$$

onde

$$\begin{aligned} s_0 &= \{(x, x_0), (y, y_0), (z, z_0)\} \\ s_1 &= \text{modificar}(s_0, y, E[[2 * x + 2]](s_0)) \\ &= \text{modificar}(s_0, y, s_0(2 * x + 2)) = \\ &= \{(x, x_0), (y, 2 * x_0 + 2), (z, z_0)\} \\ s_2 &= \text{modificar}(s_1, z, E[[x * 2]](s_1)) \\ &= \text{modificar}(s_1, z, s_1(x * 2)) = \\ &= \{(x, x_0), (y, 2 * x_0 + 2), (z, x_0 * 2)\} \\ s_3 &= \text{modificar}(s_2, x, E[[2 * x + 1]](s_2)) \\ &= \text{modificar}(s_2, x, s_2(2 * x + 1)) = \\ &= \{(x, 2 * x_0 + 1), (y, 2 * x_0 + 2), (z, x_0 * 2)\} \end{aligned}$$

B. $x:=2*x+1; y:=x+1; z:=x-1$

$$\begin{aligned} C[[x := 2 * x + 1; y := x + 1; z := x - 1]](s_0) &= \\ C[[y := x + 1; z := x - 1]](C[[x := 2 * x + 1]](s_0)) &= \\ C[[y := x + 1; z := x - 1]](s_1) &= \end{aligned}$$

$$\begin{aligned}
& C[[z := x - 1]](C[[y := x + 1]])(s_1) = \\
& C[[z := x - 1]](s_2) = \\
& s_3
\end{aligned}$$

onde

$$\begin{aligned}
s_0 &= \{(x, x_0), (y, y_0), (z, z_0)\} \\
s_1 &= \text{modificar}(s_0, x, E[[2 * x + 1]](s_0)) \\
&= \text{modificar}(s_0, x, s_0(2 * x + 1)) \\
&= \{(x, 2 * x_0 + 1), (y, y_0), (z, z_0)\} \\
s_2 &= \text{modificar}(s_1, y, E[[x + 1]](s_1)) \\
&= \text{modificar}(s_1, y, s_1(x + 1)) \\
&= \{(x, 2 * x_0 + 1), (y, (2 * x_0 + 1) + 1), (z, z_0)\} \\
&= \{(x, 2 * x_0 + 1), (y, 2 * x_0 + 2), (z, z_0)\} \\
s_3 &= \text{modificar}(s_2, z, E[[x - 1]](s_2)) \\
&= \text{modificar}(s_2, z, s_2(x - 1)) \\
&= \{(x, 2 * x_0 + 1), (y, 2 * x_0 + 2), (z, (2 * x_0 + 1) - 1)\} \\
&= \{(x, 2 * x_0 + 1), (y, 2 * x_0 + 2), (z, 2 * x_0)\}
\end{aligned}$$

2. if x=0 then (y:=z^2; x:=2*y) else (y:=(z+x)^2; z:=z+x; x:=x+2*y)

(a) P = {if x=0 then (y:=z^2; x:=2*y) else (y:=(z+x)^2; z:=z+x; x:=x+2*y) }

$$\begin{aligned}
& C[[P]](s_0) = \\
& \text{if } E[[x = 0]](s_0) \text{ then} \\
& \quad C[[y := z^2; x := 2 * y]](s_0) \\
& \text{else} \\
& \quad C[[y := (z + x)^2; z := z + x; x := x + 2 * y]](s_0)
\end{aligned}$$

Seja $s_0 = \{(x, x_0), (y, y_0), (z, z_0)\}$.

Se $x_0 = 0$ tem-se

$$\begin{aligned}
& C[[y := z^2; x := 2 * y]](s_0) = \\
& C[[x := 2 * y]](C[[y := z^2]](s_0)) = \\
& C[[x := 2 * y]](s_1) = \\
& s_2
\end{aligned}$$

com

$$\begin{aligned}
s_1 &= \text{modificar}(s_0, y, E[[z^2]](s_0)) \\
&= \text{modificar}(s_0, y, s_0(z^2)) \\
&= \{(x, 0), (y, z_0^2), (z, z_0)\} \\
s_2 &= \text{modificar}(s_1, x, E[[2 * y]](s_1)) \\
&= \text{modificar}(s_1, x, s_1(2 * y)) \\
&= \{(x, 2 * (z_0^2)), (y, z_0^2), (z, z_0)\}
\end{aligned}$$

Se $x_0 \neq 0$ tem-se

$$\begin{aligned}
C[[y := (z + x)^2; z := z + x; x := x + 2 * y]](s_0) &= \\
C[[z := z + x; x := x + 2 * y]](C[[y := (z + x)^2]](s_0)) &= \\
C[[z := z + x; x := x + 2 * y]](s_1) &= \\
C[[x := x + 2 * y]](C[[z := z + x]](s_1)) &= \\
C[[x := x + 2 * y]](s_2) &= \\
s_3
\end{aligned}$$

com

$$\begin{aligned}
s_1 &= \text{modificar}(s_0, y, E[(z + x)^2]](s_0)) \\
&= \text{modificar}(s_0, y, s_0((z + x)^2)) \\
&= \{(x, x_0), (y, (z_0 + x_0)^2), (z, z_0)\} \\
s_2 &= \text{modificar}(s_1, z, E[[z + x]](s_1)) \\
&= \text{modificar}(s_1, z, s_1(z + x)) \\
&= \{(x, x_0), (y, (z_0 + x_0)^2), (z, z_0 + x_0)\} \\
s_3 &= \text{modificar}(s_2, x, E[[x + 2 * y]](s_2)) \\
&= \text{modificar}(s_2, x, s_2(x + 2 * y)) \\
&= \{(x, x_0 + 2 * (z_0 + x_0)^2), (y, (z_0 + x_0)^2), (z, z_0 + x_0)\}
\end{aligned}$$

(b) $y := (z + x)^2; z := z + x; x := x + 2 * y$

3. $\mathbf{x}_1, \mathbf{x}_2 := \mathbf{e}_1, \mathbf{e}_2$

(a) $C[[x := e]](s) = \text{modificar}(s, x, E[[e]](s))$

$$C[[x_1, x_2 := e_1, e_2]](s) = \text{modificar}(\text{modificar}(s, x_1, E[[e_1]](s)), x_2, E[[e_2]](s))$$

(b)

$$\begin{aligned}
C[[x, y := y, x]](s_0) &= \text{modificar}(\text{modificar}(s_0, x, E[[y]](s_0)), y, E[[x]](s_0)) \\
&= \text{modificar}(\text{modificar}(s_0, x, s_0(y)), y, s_0(x)) \\
&= s_1
\end{aligned}$$

onde

$$\begin{aligned}
s_0 &= \{(x, x_0), (y, y_0)\} \\
s_1 &= \text{modificar}(\text{modificar}(s_0, x, s_0(y)), y, s_0(x)) \\
&\quad \{(x, y_0), (y, x_0)\}
\end{aligned}$$

4. `i:=0; q:=0; while i<n do (q:=q+2*i+1; i:=i+1)`

Seja

$$\begin{aligned}
A_1 &= (i:=0) \\
A_2 &= (q:=0) \\
A_3 &= (\text{while } i < n \text{ do } (q:=q+2*i+1; i:=i+1)) \\
A_4 &= (q:=q+2*i+1) \\
A_5 &= (i:=i+1)
\end{aligned}$$

Assim

$$\begin{aligned}
C[[A_1; A_2; A_3]](s_0) &= \\
C[[A_2; A_3]](C[[i := 0]](s_0)) &= \\
C[[A_2; A_3]](s_1) &= \\
C[[A_3]](C[[q := 0]](s_1)) &= \\
C[[A_3]](s_2) &=
\end{aligned}$$

com

$$\begin{aligned}
s_0 &= \{(i, i_0), (q, q_0), (n, n_0), \dots\} \\
s_1 &= \text{modificar}(s_0, i, E[[0]](s_0)) = \text{modificar}(s_0, i, s_0(0)) \\
&= \{(i, 0), (q, q_0), (n, n_0), \dots\} \\
s_2 &= \text{modificar}(s_1, q, E[[0]](s_1)) = \text{modificar}(s_1, q, s_1(0)) \\
&= \{(i, 0), (q, 0), (n, n_0), \dots\}
\end{aligned}$$

$C[[A_3]](s_2) =$
if *not* $E[[i < n]](s_2)$ **then** s_2 **else** $C[[A_3]](C[[A_4; A_5]](s_2))$

$C[[A_4; A_5]](s_2) =$
 $C[[A_5]](C[[q := q + 2 * i + 1]](s_2)) =$
 $C[[i := i + 1]](s_3) =$
 s_4

com

$s_3 = \text{modificar}(s_2, q, E[[q + 2 * i + 1]](s_2))$
 $= \text{modificar}(s_2, q, s_2(q + 2 * i + 1))$
 $= \{(i, 0), (q, 0 + 1), (n, n_0), \dots\}$
 $s_4 = \text{modificar}(s_3, i, E[[i + 1]](s_3)) = \text{modificar}(s_3, i, s_3(i + 1))$
 $= \{(i, 1), (q, 0 + 1), (n, n_0), \dots\}$

$C[[A_3]](s_4) =$
if *not* $E[[i < n]](s_4)$ **then** s_2 **else** $C[[A_3]](C[[A_4; A_5]](s_4))$

$C[[A_4; A_5]](s_4) =$
 $C[[A_5]](C[[q := q + 2 * i + 1]](s_4)) =$
 $C[[i := i + 1]](s_5) =$
 s_6

com

$s_5 = \text{modificar}(s_4, q, E[[q + 2 * i + 1]](s_4))$
 $= \text{modificar}(s_4, q, s_4(q + 2 * i + 1))$
 $= \{(i, 1), (q, 0 + 1 + 3), (n, n_0), \dots\}$
 $s_6 = \text{modificar}(s_5, i, E[[i + 1]](s_5)) = \text{modificar}(s_5, i, s_5(i + 1))$
 $= \{(i, 2), (q, 0 + 1 + 3), (n, n_0), \dots\}$

$C[[A_3]](s_6) =$
if *not* $E[[i < n]](s_6)$ **then** s_6 **else** $C[[A_3]](C[[A_4; A_5]](s_6))$

$C[[A_4; A_5]](s_6) =$
 $C[[A_5]](C[[q := q + 2 * i + 1]](s_6)) =$
 $C[[i := i + 1]](s_7) =$
 s_8

com

$s_7 = \text{modificar}(s_6, q, E[[q + 2 * i + 1]](s_6))$
 $= \text{modificar}(s_6, q, s_6(q + 2 * i + 1))$
 $= \{(i, 2), (q, 0 + 1 + 3 + 5), (n, n_0), \dots\}$
 $s_8 = \text{modificar}(s_7, i, E[[i + 1]](s_7)) = \text{modificar}(s_7, i, s_7(i + 1))$
 $= \{(i, 3), (q, 0 + 1 + 3 + 5), (n, n_0), \dots\}$
 \dots
 $s_{2*(n_0-1)+2} = \{(i, n_0 - 1), (q, 0 + 1 + 3 + \dots + 2 * (n_0 - 2) + 1), (n, n_0), \dots\}$

$C[[A_3]](s_{2*(n_0-1)+2}) =$
if *not* $E[[i < n]](s_{2*(n_0-1)+2})$ **then** $s_{2*(n_0-1)+2}$ **else** $C[[A_3]](C[[A_4; A_5]](s_{2*(n_0-1)+2}))$

$C[[A_4; A_5]](s_{2*(n_0-1)+2}) =$
 $C[[A_5]](C[[q := q + 2 * i + 1]](s_{2*(n_0-1)+2})) =$
 $C[[i := i + 1]](s_{2*(n_0-1)+2+1}) =$
 s_{2*n_0+2}

com

$s_{2*n_0+2} = \{(i, n_0), (q, 0 + 1 + 3 + \dots + 2 * (n_0 - 1) + 1), (n, n_0), \dots\}$

$$\begin{aligned}
& C[[A_3]](s_{2*n_0+2}) = \\
& \text{if } \text{not } E[[i < n]](s_{2*n_0+2}) \text{ then } s_{2*n_0+2} \text{ else } C[[A_3]](C[[A_4; A_5]](s_{2*n_0+2})) = \\
& s_{2*n_0+2}
\end{aligned}$$

$$\begin{aligned}
0 + 1 + 3 + \dots + 2 * (n_0 - 1) + 1 &= \sum_{i=0}^{n_0-1} 2 * i + 1 \\
&= 2 \left(\sum_{i=0}^{n_0-1} i \right) + n_0 \\
&= 2 \frac{(n_0 - 1)n_0}{2} + n_0 \\
&= n_0^2
\end{aligned}$$

Assim, o programa é equivalente a $(i := n; q := n^2)$.