MML - Homework 4

Monday, March 6, 2023 5:03 PM

1. $\frac{\partial trace(A^T W)}{\partial W} = \frac{\partial}{\partial W} trace(A^T W) = \frac{\partial}{\partial W} trace(A W^T) = trace(A(\cancel{\frac{\partial}{\partial W} W^T}))$

$= trace(A)$

$= \boxed{A}$

$\frac{\partial trace(W^T A W B)}{\partial W} = \frac{\partial}{\partial W} trace(W^T A W B) = \frac{\partial}{\partial W} trace(A W B^T W) = \frac{\partial}{\partial W} trace(B^T A^T W^T W)$

$= \frac{\partial}{\partial W} trace(A^T W^T W (B^T))$

$= trace(A^T W B^T) = \boxed{AWB + A^T W B^T}$

$\frac{\partial trace[(Y-XW)^T(Y-XW)]}{\partial W} = \frac{\partial}{\partial W} trace[(Y-XW)^T(Y-XW)] = \frac{\partial}{\partial W} trace[\cancel{Y^T Y} - 2Y^T X W + W^T X^T X W]$

$= -2trace(Y^T X)$

$= -2X^T Y + 2X^T X W = \boxed{2X^T(Y-XW)}$

$\frac{\partial W^{-1}}{\partial w_{ij}}$ where each entry of matrix $W_{ij} = w_{ij}$ $= -W^{-1} \frac{\partial W}{\partial w_{ij}} W^{-1} = -W^{-1} C_{ij} W^{-1} + \frac{\partial W^{-1}}{\partial w_{ij}} W = 0$

i,jth cofactor of W

$= \boxed{(W^{-1})(W^{-1}) C_{ij}}$ (when W is invertible) $|W| \neq 0$

2. $E(W) = \sum_{i=1}^{N} ||y_i - W^T x_i||_2^2 + \lambda \sum_{k=1}^{K_1} \sum_{l=1}^{K_2} w_{kl}^2$; $(\{x_i, y_i\}_{i=1}^{N}$ is the set of training patterns (instances) w/ $x_i \in \mathbb{R}^{K_1}$ and $y_i \in \mathbb{R}^{K_2}$

$= \sum_{i=1}^{N} (y_i - W^T x_i)^T (y_i - W^T x_i)$

$= (Y - XW)^T (Y - XW)$

$= trace[(Y-XW)^T(Y-XW)]$

(This occurs through X being the $NK_1$ matrix with i-th row $x_i^T$ and Y being the $NK_2$ matrix with i-th row $y_i^T$.)

$= \lambda \sum_{k=1}^{K_1} \sum_{l=1}^{K_2} (W^T W)_{kl}$

$= \lambda trace(W^T W)$

(This occurs due to $(W^T W)_{kl}$ being the entry in the k-th row and l-th column of matrix $W^T W$ and $trace(W^T W)$ is the sum of its diagonal entries)

Combining these terms results in

$E(W) = trace[(Y-XW)^T(Y-XW)] + \lambda trace(W^T W)$

3. https://colab.research.google.com/drive/15VPnh3eba3k1NN4N1aNtu2Fz0NduykA_?usp=sharing

4. https://colab.research.google.com/drive/1fAigI9aWUJ9qhYh5QuKk9Moc1eCSkDVa?usp=sharing