1.2: $c \neq 0$, $v \in V$ } Prove that if $cv = 0$, then $v = 0$.

Proof:
$$cv + c = vc + 1c = (v+1)c$$
$$-c \qquad -c$$
$$cv = (v+1)c - c$$
$$cv \Rightarrow 0 = c(v+1-1)$$
$$0 = c(c \neq 0)v$$
$$0 = v \longrightarrow \boxed{v = 0}$$

Alternative Proof:
$$c^{-1} \cdot \underset{(0)}{cv} = 0$$
$$1v = 0$$
$$\boxed{v = 0}$$

1.7: $A_1, \ldots, A_r$ are vecs. in $\mathbb{R}^n$
W is set of vecs. B in $\mathbb{R}^n$ such that $B \cdot A_i = 0$ for every $i = 1, \ldots, r$ } Prove that W is subspace of $\mathbb{R}^n$.

Proof: B and A_i are linearly independent, because $B \cdot A_i = 0$.
From this, we could also say $0 \cdot A_{1,\ldots,r} = 0$, as 0 is in W. (1)
Suppose c is a number in $\mathbb{R}^n$:
$cB \cdot A_i = 0 \rightarrow c(B \cdot A_i) = 0$, so cB is orthogonal to A_i. (2)
X is an arbitrary vector in $\mathbb{R}^n$:
$(B+X)A_i = 0 \longrightarrow BA_i + XA_i = 0 \longrightarrow 0 + XA_i = 0 \longrightarrow XA_i = 0$ (3)

$\boxed{W \text{ is a subspace of } \mathbb{R}^n.}$

2.4: (a,b) and (c,d) are 2 vectors in the plane. If $ad - bc = 0$, show linear dependence.
If $ad - bc \neq 0$, show linear independence.

Proof for linear dependence: $x_1(a,b) + x_2(c,d) = 0$ $\begin{cases} x_1 a + x_2 c = 0 \\ x_1 b + x_2 d = 0 \end{cases}$ $>$ $\begin{array}{l} \cancel{x_1 ab} + x_2 bc = 0 \\ -\ \cancel{x_1 ab} + x_2 ad = 0 \end{array}$
$$x_2(bc - ad) = 0$$
$$x_2(0) = 0$$
$\boxed{x_2 \neq 0, \text{ or may not be, proving linear dependence by contradiction.}}$

Proof for linear independence: $x_1(a,b) + x_2(c,d) = 0$ $\begin{cases} x_1 a + x_2 c = 0 \\ x_1 b + x_2 d = 0 \end{cases}$ $>$ $\begin{array}{l} \cancel{x_1 ab} + x_2 bc = 0 \\ -\ \cancel{x_1 ab} + x_2 ad = 0 \end{array}$
$$x_2(bc - ad) = 0$$
$$x_2(\neq 0) = 0$$
$\boxed{x_2 = 0, \text{ proving linear independence.}}$

2.9: $A_1, \ldots, A_r$ are vectors in $\mathbb{R}^n$ and are mutually perp.
No A_i is equal to 0. $>$ Prove that A_i (they) are linearly independent.

Proof for linear independence: If dot product is 0, then linearly independent.
$$x_1 A_1 + x_2 A_2 = 0$$
$$x_1 A_{r-1} + x_2 A_{r-2} = 0$$
$$\Longrightarrow (x_1 A_1)(A_1) + (x_2 A_2)A_1 = 0$$
$$(x_1 A_{r-1})(A_1) + (x_2 A_{r-2})A_1 = 0$$
$$\Downarrow$$
$$x_1(A_1 \cdot A_1) + x_2(\cancel{A_2 A_1}) = 0$$
$$x_1(\cancel{A_{r-1} A_1}) + x_2(\cancel{A_{r-2} A_1}) = 0$$
$$\Downarrow$$
$$x_1(0) + x_2(0) = 0$$
$$x_1(0) + x_2(0) = 0$$
$\boxed{x_1 \text{ and/or } x_2 \text{ may not equal to } 0 \text{ when trying to prove linear dependence (contradiction).}}$

$$\Downarrow$$
$$x_1(c \neq 0) + x_2(\neq 0) = 0$$
$$x_1(\neq 0) + x_2(\neq 0) = 0$$
$\boxed{x_1 + x_2 = 0, \text{ proving linear independence.}}$

(Note: All $A \cdot A_i = 0$ except $A_i \cdot A_i$)
($c_i \|A_i\| = 0$ and $A_i = 0$, then $c_i = 0$ and thus all $c_i, \ldots, c_r$ must be 0)

2.10: v, w are elements of vector space
$v \neq 0$
v, w are linearly dependent
$>$ Prove that there is num. a such that $w = av$.

Proof: To assume v, ω are linearly dependent is to say that there are n numbers $x_1, \ldots, x_n$ not all equal to zero.

$$x_1 v + x_2 \omega = 0$$
$$\omega = -\frac{x_1}{x_2} v$$
$$\omega = a v$$

$\boxed{\text{This multiple that is evaluated with element } v \neq 0 \text{ would produce a linearly dependent element } \omega.}$ ✓