

Mathematical models and an effective exact algorithm for unrelated parallel machine scheduling with family setup times and machine cost

Kai Li^{1,2}, Fulong Xie^{1*}, Jianfu Chen¹, Wei Xiao³, Tao Zhou^{1,2}

¹School of Management, Hefei University of Technology, Hefei, 230009,
P.R. China.

²Key Laboratory of Process Optimization and Intelligent
Decision-making, Ministry of Education, Hefei, 230009, P.R. China.

³Business School, Hohai University, Nanjing 211100, PR China.

*Corresponding author(s). E-mail(s): fulong.xie@outlook.com;
Contributing authors: hfutlk@163.com; chenjianfu@mail.hfut.edu.cn;
wei.xiao@hhu.edu.cn; tao-zhou@hfut.edu.cn;

1 Detailed computational results

This appendix provides detailed experimental results from the main text. For the reader's convenience, Table 1 provides the correspondence between the tables in the current text and the tables in the main text.

Table 1: the correspondence between the tables in the current text and the tables in the main text.

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Table 2: Results of the B&P algorithm on small-scale instances.

m	n	f	BP ₁					BP ₂					BP ₃					BP ₄				
			Opt	Gap(%)	Nodes	T _{opt}	Time(ms)	Opt	Gap(%)	Nodes	T _{opt}	Time(ms)	Opt	Gap(%)	Nodes	T _{opt}	Time(ms)	Opt	Gap(%)	Nodes	T _{opt}	Time(ms)
3	10	3	10	0.00	1.2	20	20	10	0.00	1.2	12	12	10	0.00	1.0	22	22	10	0.00	1.0	13	13
		5	10	0.00	1.0	22	22	10	0.00	1.0	14	14	10	0.00	1.0	25	25	10	0.00	1.0	16	16
		8	10	0.00	1.0	21	21	10	0.00	1.0	16	16	10	0.00	1.0	28	28	10	0.00	1.0	15	15
20	3	10	0.00	1.6	299	299	299	10	0.00	2.0	291	291	10	0.00	2.2	925	925	10	0.00	1.6	434	434
		5	10	0.00	1.0	216	216	10	0.00	1.0	183	183	10	0.00	1.0	368	368	10	0.00	1.0	273	273
		8	10	0.00	1.4	297	297	10	0.00	1.4	236	236	10	0.00	1.4	521	521	10	0.00	1.4	397	397
30	3	10	0.00	3.6	2565	2565	2565	10	0.00	3.8	2770	2770	10	0.00	4.2	4308	4308	9	<0.001	5.0	4026	4663
		5	10	0.00	1.4	1691	1691	10	0.00	1.2	1513	1513	10	0.00	1.2	2788	2788	10	0.00	1.2	2421	2421
		8	10	0.00	2.8	2925	2925	9	<0.001	1.6	1566	1744	10	0.00	1.4	3086	3086	10	0.00	1.2	2550	2550
40	3	10	0.00	2.6	9937	9937	9937	10	0.00	2.8	9989	9989	10	0.00	4.4	19705	19705	9	<0.001	37.2	14820	195459
		5	10	0.00	18.8	90140	90140	10	0.00	4.0	16798	16798	10	0.00	3.4	26047	26047	10	0.00	1.8	16284	16284
		8	10	0.00	6.2	19814	19814	10	0.00	4.8	17922	17922	10	0.00	3.8	23224	23224	9	0.00	24.8	25771	203877
50	3	10	0.00	3.0	42554	42554	42554	10	0.00	2.6	37479	37479	10	0.00	2.8	48146	48146	9	<0.001	23.8	170022	333686
		5	8	0.07	30.6	47892	400647	10	0.00	4.4	50418	50418	9	0.01	22.6	189857	353701	8	0.08	24.8	140275	475208
		8	8	0.08	36.0	76996	424992	10	0.00	10.6	72806	72806	8	0.06	50.0	260639	570732	7	0.14	42.0	48330	578314
4	10	3	10	0.00	1.2	12	12	10	0.00	1.2	9	9	10	0.00	1.2	12	12	10	0.00	1.2	10	10
		5	10	0.00	1.2	14	14	10	0.00	1.2	11	11	10	0.00	1.2	14	14	10	0.00	1.2	11	11
		8	10	0.00	1.0	13	13	10	0.00	1.0	10	10	10	0.00	1.0	15	15	10	0.00	1.0	10	10
20	3	10	0.00	1.4	138	138	138	10	0.00	1.4	100	100	10	0.00	1.4	213	213	10	0.00	1.4	154	154
		5	10	0.00	1.4	138	138	10	0.00	1.4	110	110	10	0.00	1.2	197	197	10	0.00	1.2	137	137
		8	10	0.00	1.2	140	140	10	0.00	1.2	122	122	10	0.00	1.0	207	207	10	0.00	1.0	129	129
30	3	10	0.00	1.2	591	591	591	10	0.00	1.2	510	510	10	0.00	1.2	949	949	10	0.00	1.2	785	785
		5	10	0.00	3.2	1255	1255	10	0.00	3.0	1151	1151	10	0.00	3.6	2380	2380	10	0.00	2.8	2036	2036
		8	10	0.00	1.8	1070	1070	10	0.00	2.4	1183	1183	10	0.00	1.6	1838	1838	10	0.00	1.4	1437	1437
40	3	10	0.00	15.4	23190	23190	23190	10	0.00	5.0	5414	5414	8	0.06	120.0	4545	364519	9	0.03	54.0	6423	186925
		5	9	0.00	5.2	10396	10052	9	<0.001	4.8	7278	7328	8	0.03	98.8	7104	366964	8	0.03	68.0	9519	368123
		8	7	0.02	65.0	38233	215132	9	<0.001	6.2	10103	9673	7	0.08	120.0	7008	366631	7	0.02	84.4	9730	375821
50	3	10	0.00	1.6	9110	9110	9110	10	0.00	1.4	9915	9915	10	0.00	1.8	16277	16277	10	0.00	1.6	15996	15996
		5	10	0.00	2.6	13733	13733	10	0.00	2.6	14369	14369	10	0.00	4.2	26914	26914	10	0.00	3.6	27764	27764
		8	10	0.00	4.8	29541	29541	9	<0.001	4.0	18476	21156	9	0.01	23.2	22258	200144	10	0.00	3.4	30003	30003
5	10	3	10	0.00	1.2	11	11	10	0.00	1.6	11	11	10	0.00	1.2	14	14	10	0.00	1.2	9	9
		5	10	0.00	1.0	9	9	10	0.00	1.0	9	9	10	0.00	1.0	11	11	10	0.00	1.0	10	10
		8	10	0.00	1.0	9	9	10	0.00	1.0	9	9	10	0.00	1.0	11	11	10	0.00	1.0	9	9
20	3	10	0.00	1.2	69	69	69	10	0.00	1.2	61	61	10	0.00	1.2	101	101	10	0.00	1.2	71	71
		5	10	0.00	3.6	187	187	10	0.00	2.6	118	118	10	0.00	2.4	198	198	10	0.00	1.8	98	98
		8	10	0.00	1.2	94	94	10	0.00	1.2	75	75	10	0.00	1.2	152	152	10	0.00	1.2	79	79
30	3	10	0.00	3.4	702	702	702	10	0.00	2.8	685	685	10	0.00	2.8	1222	1222	10	0.00	4.0	1267	1267
		5	10	0.00	4.4	856	856	10	0.00	4.6	1037	1037	10	0.00	5.6	3630	3630	10	0.00	3.4	1115	1115
		8	10	0.00	1.4	591	591	10	0.00	1.4	579	579	10	0.00	1.4	1007	1007	10	0.00	1.4	941	941
40	3	10	0.00	3.0	1946	1946	1946	10	0.00	2.4	1944	1944	10	0.00	3.2	3218	3218	10	0.00	2.4	3104	3104
		5	9	0.03	54.2	5412	185143	10	0.00	7.0	6202	6202	9	0.03	68.0	6181	186180	8	0.08	97.0	8245	367901
		8	9	0.00	6.4	4316	5460	10	0.00	4.2	3688	3688	10	0.00	2.6	4230	4230	10	0.00	7.8	9321	9321
50	3	9	0.04	47.0	25689	203640	10	0.00	11.2	21397	21397	10	0.00	16.0	49563	49563	7	0.10	100.8	25462	560263	
		5	9	0.04	61.0	17274	196483	10	0.00	6.6	14014	14014	10	0.00	8.4	34305	34305	7	0.12	92.2	10783	549764
		8	9	0.03	57.2	17261	198496	10	0.00	10.4	22279	22279	8	0.09	73.2	12961	375227	7	0.11	77.2	11593	550138
Sum/Avg.			437	0.01	10.4	11053	46533	446	0.00	3.1	7842	7897	436	0.01	14.9	17476	67999	424	0.02	17.6	13375	108156

Table 3: Results of the B&P algorithm on large-scale instances.

m	n	f	BP ₁			BP ₂			BP ₃			BP ₄						
			Opt	Gap(%)	Nodes	T _{opt} (ms)	Time(ms)	Opt	Gap(%)	Nodes	T _{opt} (ms)	Time(ms)	Opt	Gap(%)	Nodes	T _{opt} (ms)	Time(ms)	
5	60	5	10	0.00	8.4	46814	46814	10	0.00	9.6	54174	54174	9	< 0.001	7.8	38923	69976	223422
		8	9	0.03	23.6	49957	228047	10	0.00	8.2	43644	43644	8	0.04	31.6	47430	224231	39085
		12	6	0.17	98.2	97063	782597	10	0.00	27.8	106427	106427	6	0.16	92.4	33891	744249	38.0
		70	5	7	0.23	47.8	53446	581949	10	0.00	25.4	153394	153394	9	0.08	17.0	85084	258330
80	80	8	6	0.07	59.0	150101	817499	10	0.00	15.4	179813	179813	7	0.36	30.8	121611	628340	148
		12	3	0.47	106.6	150159	1318056	10	0.00	38.8	354993	354993	4	0.62	65.0	191500	1168482	143823
		5	7	0.10	38.2	242631	719393	10	0.00	23.8	277951	277951	4	0.14	46.6	148570	1154402	815145
		8	4	1.48	56.4	417999	1261196	8	0.00	40.4	483840	621480	6	0.37	25.4	188791	118712	844825
90	90	12	2	1.76	55.2	50350	1468794	7	0.19	55.2	531061	918018	3	1.54	43.2	351769	937300	1379426
		5	6	0.59	23.2	240300	877096	9	0.18	13.6	394686	535505	6	0.56	23.6	670740	1132361	230537
		8	5	1.07	28.6	465965	1145708	7	0.10	24.2	494207	788401	4	1.57	22.2	356455	1233975	191922
		100	12	1	2.23	39.0	182097	1657263	4	0.09	53.0	904938	1463075	1	2.31	26.2	232891	1501409
100	100	5	3	2.26	29.6	184610	1335461	5	0.72	17.0	464495	921281	3	1.83	15.4	298742	1225447	503203
		8	3	1.34	25.2	862422	1534487	7	0.52	18.2	746392	1073082	4	2.07	13.8	985327	1486150	302137
		12	1	2.53	22.0	307076	1672477	5	1.21	23.8	809665	1319211	1	2.08	17.8	469505	1682804	1371076
		5	10	0.00	8.0	10001	10001	10	0.00	10.8	12123	12123	9	< 0.001	13.8	11824	21915	355801
80	80	8	10	0.00	27.0	49471	49471	10	0.00	17.0	25257	25257	9	0.07	48.6	137131	303989	14.8
		12	6	0.17	163.6	14780	730171	10	0.00	23.6	44324	44324	7	0.13	77.8	22273	557176	388019
		5	8	0.08	60.0	24176	385192	10	0.00	12.6	26951	26951	9	0.07	32.6	29794	207024	210075
		8	8	0.06	75.6	31699	386973	10	0.00	12.6	40634	40634	8	0.06	47.2	36174	390274	330016
80	80	12	7	0.06	71.6	21204	556466	10	0.00	11.0	37188	37188	6	0.07	66.4	23908	738093	35483
		5	6	0.45	93.2	28541	568221	10	0.00	53.4	181341	181341	7	0.39	61.8	100590	613804	149140
		8	3	0.54	140.8	55042	1288384	10	0.00	79.0	275710	275710	2	1.19	106.6	78968	1470152	1210264
		12	5	0.19	114.0	23231	916861	9	0.02	93.8	176909	339602	4	0.97	77.6	34608	1106938	41976
90	90	5	6	0.15	94.6	276998	890285	8	0.01	80.8	269018	419001	5	0.61	66.4	38167	927110	502
		8	4	0.33	100.0	139642	1147998	8	0.04	88.0	329284	625864	3	0.60	64.0	120244	1310877	248848
		12	2	1.08	103.4	131823	1479101	10	0.00	82.0	600755	600755	1	0.59	72.8	41055	1635074	1355207
		5	5	0.84	67.8	241046	1029709	10	0.00	58.2	541482	541482	6	0.61	35.6	182956	836620	1077294
100	100	8	0	3.46	105.0	-	1817330	6	0.66	105.8	659134	1123534	2	4.77	54.6	516447	1557871	37.6
		12	4	1.56	53.6	79786	1125608	9	0.01	74.2	753702	859833	3	1.91	46.8	245970	1346060	235724
		5	10	0.00	31.2	29551	29551	10	0.00	39.0	34857	34857	10	0.00	30.8	35398	191336	35393
		8	10	0.00	16.2	14543	14543	10	0.00	17.2	16607	16607	9	0.06	37.2	11422	191336	21.4
70	70	12	10	0.00	17.6	44080	44080	9	< 0.001	14.0	20221	20276	9	< 0.001	12.4	21835	23588	19.6
		5	10	0.00	25.2	39110	39110	10	0.00	28.0	36595	36595	9	0.03	48.6	29222	207750	10
		8	7	0.04	150.2	48769	575757	9	< 0.001	36.4	58993	56315	5	0.13	95.4	59627	584847	102278
		12	6	0.12	156.8	151450	813617	10	0.00	61.2	114229	114229	6	0.23	110.2	68715	768714	65486
80	80	5	8	0.31	125.4	214974	534748	9	< 0.001	74.6	183425	168569	3	0.41	133.8	49331	1288987	402167
		8	10	0.00	15.6	66874	66874	9	< 0.001	18.2	72767	74757	7	0.03	36.6	67437	414999	138799
		12	4	0.34	151.8	302566	1209028	10	0.00	95.2	243540	243540	5	0.24	113.6	237973	1025931	334059
		5	9	0.01	46.2	188727	354410	10	0.00	42.6	158608	158608	6	0.12	92.2	106432	959824	366142
90	90	8	2	1.98	150.6	79840	1467358	8	0.35	157.2	390586	679760	1	1.02	126.2	23317	1639527	801252
		12	4	0.71	102.0	89440	1126843	7	0.05	100.4	229164	548922	3	0.27	89.0	304848	1361016	112403
		5	5	0.84	97.6	31780	923724	8	0.25	124.8	290256	594112	5	1.45	69.8	52050	935493	49681
		8	4	1.11	104.6	359982	1235085	9	0.01	131.2	627700	745639	2	1.16	93.4	418540	1537424	185022
12	12	1	1.76	124.2	75057	1646057	7	0.16	133.0	524652	910199	3	2.48	64.8	372824	1381426	131985	
		257	0.68	72.3	144663	842431	397	0.11	50.4	288600	408136	239	0.83	55.8	172907	892896	170537	
Sum/Avg.			257	0.68	72.3	144663	842431	397	0.11	50.4	288600	408136	239	0.83	55.8	172907	892896	170537

Table 4: Comparison results between I3 and AR, AR-FP, and AF on small instances.

m	n	f	AR				AR-FP				AF				I3			
			Opt	feaSol	Gap(%)	Time(ms)	Opt	feaSol	Gap(%)	Time(ms)	Opt	feaSol	Gap(%)	Time(ms)	Opt	feaSol	Gap(%)	Time(ms)
	10	3	1	5	16.65	511448	1	5	15.01	503490	5	5	0.00	5345	5	5	0.00	522
		5	5	5	0.00	311610	5	5	0.00	241692	5	5	0.00	15034	5	5	0.00	834
		8	3	5	5.65	316413	4	5	2.48	156297	5	5	0.00	8879	5	5	0.00	2093
		3	0	5	87.62	600008	0	5	82.79	600007	0	5	4.59	600076	0	5	11.45	600034
		20	5	0	88.91	600008	0	5	74.52	600004	0	5	64.22	601071	0	5	20.66	600029
	3	8	0	5	85.35	600010	0	5	58.25	600009	0	5	76.04	601261	0	5	22.61	600050
		3	0	5	93.24	600018	0	5	90.61	600008	0	4	63.52	600719	0	5	19.91	600216
		30	5	0	94.27	600434	0	5	91.81	600113	0	5	85.57	604776	0	5	35.76	610706
		8	0	5	93.40	600036	0	5	87.10	600030	0	2	96.27	609696	0	5	42.71	600061
		3	0	5	94.03	601953	0	5	91.44	605573	0	0	100.00	602019	0	5	33.41	600082
	40	5	0	5	95.22	600102	0	5	92.09	601508	0	0	100.00	603657	0	5	44.71	600058
		8	0	5	95.71	602777	0	5	93.21	614062	0	0	100.00	609400	0	5	52.29	600066
		3	0	5	95.20	600510	0	5	93.10	608110	0	0	100.00	610834	0	5	42.34	600178
		50	5	0	96.21	611719	0	5	93.14	607138	0	0	100.00	604125	0	5	47.45	600141
		8	0	5	96.74	600375	0	5	94.51	601578	0	0	100.00	630987	0	5	56.50	600190
	10	3	0	5	33.87	634522	0	5	27.12	617207	5	5	0.00	6806	5	5	0.00	978
		5	3	5	6.84	391148	3	5	2.84	369888	5	5	0.00	8170	5	5	0.00	851
		8	5	5	0.00	314188	5	5	0.00	133564	5	5	0.00	6694	5	5	0.00	2016
		3	0	5	90.68	600010	0	5	86.13	600009	0	5	3.63	626763	0	5	8.92	600010
		20	5	0	90.50	600007	0	5	78.87	600010	0	5	22.72	607896	0	5	11.21	600019
	4	8	0	5	83.44	600007	0	5	68.64	600015	0	3	80.50	609903	0	5	18.18	600009
		3	0	5	93.21	600007	0	5	89.81	600012	0	0	100.00	602464	0	5	16.29	600043
		30	5	0	93.92	600011	0	5	89.85	600012	0	0	100.00	601204	0	5	23.81	600039
		8	0	5	94.02	600712	0	5	89.51	600014	0	0	100.00	600519	0	5	32.63	600059
		3	0	5	94.35	600074	0	5	92.67	600043	0	0	100.00	605341	0	5	24.50	600093
	40	5	0	5	95.02	603543	0	5	93.69	600239	0	0	100.00	606152	0	5	32.79	600108
		8	0	5	95.41	604977	0	5	93.71	600427	0	0	100.00	604316	0	5	42.93	600100
		3	0	5	95.79	607688	0	5	94.56	603557	0	0	100.00	611083	0	5	30.68	600186
		50	5	0	96.11	617268	0	5	95.40	609724	0	0	100.00	684460	0	5	40.35	600146
		8	0	5	96.29	607376	0	5	94.88	604984	0	0	100.00	634799	0	5	52.77	600164
	10	3	0	5	50.00	600018	0	5	37.59	600964	5	5	0.00	6133	5	5	0.00	482
		5	1	5	22.22	582435	4	5	2.05	448133	5	5	0.00	10051	5	5	0.00	590
		8	1	5	8.24	522723	4	5	2.24	228290	5	5	0.00	10685	5	5	0.00	1236
		3	0	5	91.31	600288	0	5	86.87	600620	5	5	0.00	329458	0	5	4.74	600018
		20	5	0	90.39	613912	0	5	78.18	605607	2	5	3.26	490155	2	5	3.34	457224
	5	8	0	5	82.52	611634	0	5	70.60	602185	0	5	33.28	603485	0	5	11.62	600021
		3	0	5	93.99	610733	0	5	92.60	601999	0	0	100.00	612352	0	5	15.77	600042
		30	5	0	93.82	606722	0	5	91.58	608705	0	0	100.00	601607	0	5	13.00	600036
		8	0	5	94.70	603329	0	5	91.01	629054	0	0	100.00	601423	0	5	28.92	600053
		3	0	5	94.68	624602	0	5	92.94	619477	0	0	100.00	600586	0	5	23.90	600127
	40	5	0	5	94.95	603889	0	5	93.25	606652	0	0	100.00	600693	0	5	25.90	600134
		8	0	5	95.71	604409	0	5	94.17	620813	0	0	100.00	618080	0	5	38.29	600178
		3	0	5	95.13	600534	0	5	94.07	613869	0	0	100.00	601038	0	5	28.78	600152
		50	5	0	96.04	600315	0	5	94.60	603393	0	0	100.00	601331	0	5	34.79	600192
		8	0	5	96.64	601671	0	5	95.09	601957	0	0	100.00	604540	0	5	42.44	600133
	Sum./Avg.	19	225	225	77.73	576137	26	225	72.79	557134	52	99	65.19	480370	47	225	23.03	477349

Table 5: Comparison results between I3 and AR, and AR-FP on small instances.

m	n	f	RA					RA-FP					I3				
			Opt	feaSol	Gap(%)	Time(ms)	Opt	feaSol	Gap(%)	Time(ms)	Opt	feaSol	Gap(%)	Time(ms)	Opt	feaSol	Time(ms)
5	60	5	0	5	97.21	625455	0	5	96.32	606695	0	5	36.78	600156	0	5	600156
		8	0	5	97.55	661022	0	5	96.36	607111	0	5	44.97	600203	0	5	600203
		12	0	5	97.68	620841	0	5	97.39	606585	0	5	56.08	600191	0	5	600191
		5	0	0	100.00	601831	0	5	97.40	645264	0	5	42.86	600227	0	5	600227
	70	8	0	2	99.39	602148	0	5	98.12	674149	0	5	51.56	600126	0	5	600126
		12	0	4	98.66	619248	0	5	98.27	602324	0	5	53.85	600255	0	5	600255
		5	0	3	99.07	606760	0	5	97.91	603368	0	5	49.78	600566	0	5	600566
		8	0	2	99.46	602400	0	5	98.07	600419	0	5	56.49	600512	0	5	600512
	90	12	0	1	99.78	600033	0	5	98.29	605447	0	5	62.49	600653	0	5	600653
		5	0	2	99.71	612055	0	5	98.10	608332	0	3	83.73	600276	0	3	600276
		8	0	1	99.79	600016	0	5	98.40	604814	0	3	87.53	600761	0	3	600761
		12	0	1	99.81	619418	0	5	98.73	600361	0	5	73.44	600314	0	5	600314
8	60	5	0	0	100.00	600425	0	5	98.31	600120	0	0	100.00	600233	0	0	600233
		8	0	0	100.00	603448	0	5	98.69	600979	0	0	100.00	600216	0	0	600216
		12	0	0	100.00	606078	0	5	98.83	602721	0	2	93.37	600240	0	2	600240
		5	0	2	99.08	610811	0	5	98.01	605070	0	5	21.46	600161	0	5	600161
	70	8	0	1	99.61	613346	0	5	98.49	603824	0	5	27.67	600161	0	5	600161
		12	0	5	97.72	600231	0	5	98.56	628670	0	5	35.79	600113	0	5	600113
		5	0	1	99.47	689337	0	5	98.47	601972	0	5	34.96	601472	0	5	601472
		8	0	1	99.68	606379	0	5	98.38	600056	0	5	37.31	602717	0	5	602717
	80	12	0	0	100.00	602773	0	5	99.03	705264	0	5	50.54	600835	0	5	600835
		5	0	0	100.00	607145	0	5	98.64	606585	0	2	91.10	600285	0	2	600285
		8	0	1	99.70	612166	0	5	98.91	603697	0	4	77.91	600302	0	4	600302
		12	0	0	100.00	611861	0	5	98.89	607642	0	4	82.09	600301	0	4	600301
10	60	5	0	0	100.00	624135	0	5	98.87	612997	0	0	100.00	600219	0	0	600219
		8	0	0	100.00	600388	0	5	98.80	612337	0	0	100.00	600238	0	0	600238
		12	0	0	100.00	674465	0	5	99.30	630552	0	0	100.00	600285	0	0	600285
		5	0	0	100.00	613581	0	5	98.75	602056	0	0	100.00	600264	0	0	600264
	70	8	0	0	100.00	602105	0	5	99.20	675540	0	0	100.00	600280	0	0	600280
		12	0	0	100.00	600042	0	5	99.23	613847	0	0	100.00	600284	0	0	600284
		5	0	1	99.46	612734	0	5	98.65	602898	0	5	18.09	600204	0	5	600204
		8	0	1	99.47	600185	0	5	98.53	600036	0	5	20.24	601877	0	5	601877
	80	12	0	3	98.79	602284	0	5	98.73	616567	0	5	29.00	602189	0	5	602189
		5	0	0	100.00	600777	0	5	98.72	602184	0	5	72.01	600333	0	5	600333
		8	0	1	99.59	601634	0	5	98.87	601634	0	5	52.29	601085	0	5	601085
		12	0	0	100.00	600126	0	5	99.09	600682	0	5	58.20	600815	0	5	600815
100	60	5	0	0	100.00	600098	0	5	98.95	606301	0	0	100.00	600189	0	0	600189
		8	0	0	100.00	600023	0	5	99.06	602888	0	0	100.00	600215	0	0	600215
		12	0	0	100.00	600030	0	5	99.26	602430	0	0	100.00	600252	0	0	600252
		5	0	0	100.00	600085	0	5	99.33	600084	0	1	87.40	600252	0	1	600252
	70	8	0	0	100.00	600798	0	5	99.31	603932	0	0	100.00	600276	0	0	600276
		12	0	0	100.00	624203	0	5	99.32	600516	0	0	100.00	600228	0	0	600228
		5	0	0	100.00	645110	0	5	99.39	603148	0	0	100.00	600313	0	0	600313
		8	0	0	100.00	613383	0	5	99.43	618505	0	0	100.00	600276	0	0	600276
	80	12	0	0	100.00	603135	0	5	99.46	646148	0	0	100.00	600307	0	0	600307
		5	0	48	99.57	612323	0	225	98.60	613039	0	129	70.87	600481	0	129	600481

Table 6: Comparison results between BP_{sc} and BP_2 on small instances.

m	n	f	BP _{sc}					BP ₂						
			Opt	GapIp(%)	Gap(%)	Nodes	T _{opt} (ms)	Time(ms)	Opt	GapIp(%)	Gap(%)	Nodes	T _{opt} (ms)	Time(ms)
3	10	3	5	0.00	0.00	1.0	17	17	5	0.00	0.00	1.0	9	9
		5	5	0.00	0.00	1.0	23	23	5	0.00	0.00	1.0	10	10
		8	5	0.00	0.00	1.0	28	28	5	0.00	0.00	1.0	13	13
	20	3	5	0.00	0.00	1.0	167	167	5	0.00	0.00	1.0	82	82
		5	5	1.77	0.00	4.2	633	633	5	1.49	0.00	1.8	133	133
		8	5	0.49	0.00	1.8	362	362	5	0.49	0.00	1.4	146	146
	30	3	5	0.00	0.00	1.0	888	888	5	0.00	0.00	1.0	345	345
		5	5	0.87	0.00	5.8	4039	4039	5	0.01	0.00	1.4	518	518
		8	5	8.33	0.00	5.8	5767	5767	5	8.33	0.00	15.0	3311	3311
	40	3	5	0.73	0.00	1.8	4468	4468	5	0.00	0.00	1.0	1422	1422
4	50	5	5	5.85	0.00	24.6	57425	57425	5	0.72	0.00	9.0	3855	3855
		8	5	4.68	0.00	3.8	12997	12997	5	4.22	0.00	6.6	5478	5478
		3	5	5.19	0.00	3.4	29482	29482	5	2.66	0.00	1.4	6666	6666
	10	5	5	2.12	0.00	6.2	57898	57898	5	0.00	0.00	1.0	5106	5106
		8	5	4.60	0.00	13.0	119927	119927	5	2.63	0.00	18.2	43551	43551
		3	5	0.01	0.00	1.4	15	15	5	0.01	0.00	1.4	9	9
	20	5	5	0.00	0.00	1.0	16	16	5	0.00	0.00	1.0	9	9
		8	5	0.00	0.00	1.0	23	23	5	0.00	0.00	1.0	12	12
		3	2	2.09	0.11	10.2	962	527	5	2.09	0.00	9.4	173	173
	30	5	5	0.20	0.00	1.8	223	223	5	0.00	0.00	1.0	63	63
5	40	8	4	0.96	0.04	6.2	315	522	5	0.54	0.00	1.8	111	111
		3	5	6.11	0.00	719.8	147820	147820	5	5.83	0.00	44.6	3256	3256
		5	5	1.50	0.00	5.4	3396	3396	5	1.27	0.00	2.6	591	591
	10	8	5	4.57	0.00	8.2	6077	6077	5	4.55	0.00	4.2	938	938
		3	4	2.70	0.00	111.0	123184	100902	5	2.63	0.00	53.4	13417	13417
		5	5	2.84	0.00	8.6	15188	15188	5	1.11	0.00	1.8	1165	1165
	50	8	5	3.18	0.00	7.8	17289	17289	5	1.04	0.00	1.4	1503	1503
		3	3	7.69	0.02	117.8	111385	194585	4	7.51	0.11	359.0	233572	233572
		5	5	2.16	0.00	3.8	18560	18560	5	2.01	0.00	2.2	7445	7445
	8	5	0.44	0.00	1.8	13943	13943	5	0.44	0.00	1.8	4385	4385	
Sum./Avg.	10	3	5	0.00	0.00	1.0	14	14	5	0.00	0.00	1.0	5	5
		5	5	0.16	0.00	1.4	23	23	5	0.16	0.00	1.4	7	7
		8	5	0.00	0.00	1.0	17	17	5	0.00	0.00	1.0	5	5
	20	3	5	1.74	0.00	3.4	211	211	5	1.74	0.00	3.0	64	64
		5	5	0.00	0.00	1.0	104	104	5	0.00	0.00	1.0	45	45
		8	5	1.31	0.00	2.6	349	349	5	0.00	0.00	1.0	54	54
	30	3	5	2.90	0.00	24.6	7022	7022	5	2.82	0.00	82.6	3207	3207
		5	5	0.00	0.00	1.0	442	442	5	0.00	0.00	1.0	237	237
		8	5	6.10	0.00	12.2	4398	4398	5	3.29	0.00	3.4	514	514
	40	3	5	6.24	0.00	14.2	13154	13154	5	4.93	0.00	81.8	13869	13869
5	5	0.00	0.00	1.0	1307	1307	5	0.00	0.00	1.0	907	907		
8	5	2.07	0.00	5.8	8510	8510	5	2.00	0.00	1.8	1038	1038		
50	3	3	7.30	0.02	200.6	87783	293189	4	7.14	0.12	405.4	55723	164644	
	5	5	3.18	0.00	3.0	11410	11410	5	1.94	0.00	12.6	8366	8366	
	8	5	1.87	0.00	1.4	6041	6041	5	0.00	0.00	1.0	2191	2191	
	Avg.	216	2.27	0.00	30.1	19851	25764	223	1.64	0.01	25.5	7372	11832	

Table 7: Comparison results between BP_{sc} and BP_2 on large instances.

		BP _{sc}										BP2									
m	n	f	Opt	GapIp(%)	Gap(%)	Nodes	T _{opt} (ms)	Time(ms)	Opt	GapIp(%)	Gap(%)	Nodes	T _{opt} (ms)	Time(ms)							
60	5	5	5	5.79	0.00	5.8	43965	43965	5	4.14	0.00	7.0	10748	10748							
	8	4	5	5.12	0.00	40.2	99925	200193	5	4.82	0.00	11.0	18387	18387							
	12	5	4	7.71	0.00	16.6	143374	143374	5	6.33	0.00	15.8	29697	29697							
	5	2	2	8.20	1.13	24.2	61673	387525	5	1.10	0.00	2.2	13118	13118							
	70	8	1	11.62	2.51	32.2	245451	533996	5	1.29	0.00	2.2	14128	14128							
	12	2	2	10.74	2.88	23.4	112912	409975	5	6.25	0.00	4.6	34879	34879							
	5	2	2	13.11	3.19	16.6	471509	555940	5	7.70	0.00	8.6	83943	83943							
	80	8	0	9.01	2.24	24.2	-	605914	5	7.08	0.00	9.0	75087	75087							
	12	4	4	10.50	0.04	6.6	169352	257051	5	5.51	0.00	4.2	51342	51342							
	90	5	1	16.66	9.90	12.2	94147	507227	5	3.92	0.00	4.6	94390	94390							
100	8	1	12.69	8.87	9.4	48673	496065	4	5.50	0.20	10.6	48280	166726								
	12	0	14.00	13.48	10.6	-	607282	5	13.18	0.00	6.6	149942	149942								
	5	2	6.37	4.29	6.6	216662	452195	4	3.76	0.02	9.4	108802	207363								
	8	0	12.50	7.43	9.8	-	608197	3	5.54	1.71	15.0	198421	370838								
	12	0	16.76	16.20	9.0	-	607677	3	10.06	4.55	12.6	103404	315272								
	5	4	4.30	0.00	161.0	154439	243813	4	4.24	0.04	715.0	177714	262286								
	60	8	4	6.95	0.00	90.6	157888	246378	5	6.92	0.00	185.0	91608	91608							
	12	4	6.47	0.00	45.4	39621	152326	5	2.46	0.00	1.8	4618	4618								
	5	2	10.60	0.09	112.6	532087	575310	5	10.58	0.00	96.2	102441	102441								
	70	8	5	6.09	0.00	9.0	62507	62507	4	5.97	0.02	108.6	8704	127245							
80	12	5	3.96	0.00	21.4	177077	177077	5	0.00	0.00	1.0	5751	5751								
	5	1	11.44	4.79	70.2	58526	496581	1	11.31	1.50	477.4	22611	486142								
	8	5	5.19	0.00	9.0	139481	139481	5	3.48	0.00	1.8	10983	10983								
	12	3	11.64	5.27	30.2	234772	383319	5	9.09	0.00	19.4	46205	46205								
	5	0	11.13	6.43	43.4	-	607507	0	11.00	1.98	301.8	-	602338								
	90	8	2	5.52	1.80	18.2	75959	394366	5	2.60	0.00	5.4	22629	22629							
	12	1	11.07	3.05	25.0	501025	584535	5	8.86	0.00	25.4	125227	125227								
	5	0	12.42	9.63	26.2	-	606496	1	12.48	3.52	131.8	475224	578807								
	100	8	0	14.40	12.24	13.4	-	605864	4	7.89	0.06	17.0	62315	172367							
	12	1	11.14	7.69	13.0	40576	492926	4	10.92	0.02	27.4	120924	217018								
60	5	4	5.63	0.00	117.0	62219	170179	5	5.66	0.00	231.4	59482	59482								
	8	2	8.19	0.07	232.2	180147	432924	5	8.07	0.00	417.4	134318	134318								
	12	5	6.69	0.00	17.0	51330	51330	5	0.00	0.00	1.0	4432	4432								
	5	2	6.56	0.07	124.6	195979	439054	4	6.40	0.00	399.0	137032	229818								
	70	8	3	10.67	0.04	118.2	451245	512372	4	10.42	0.19	273.4	125194	220211							
	12	5	8.02	0.00	16.6	116761	116761	5	8.02	0.00	9.4	23452	23452								
	5	0	10.19	4.53	91.0	-	606621	4	10.01	0.03	174.2	43350	154827								
	80	8	0	11.64	3.94	82.2	-	604520	3	11.50	0.06	259.4	133202	321643							
	12	4	5.21	0.01	35.8	233007	307926	5	2.10	0.00	1.4	10186	10186								
	5	1	6.48	3.57	45.8	14151	487766	3	6.58	0.76	215.4	216419	371079								
90	8	0	11.68	7.47	45.8	-	607230	1	11.52	1.39	254.6	77776	497351								
	12	5	5.63	0.00	5.0	100432	100432	5	5.62	0.00	7.4	33098	33098								
	5	0	6.69	4.18	42.2	-	606679	3	6.44	0.63	341.816	446080	446080								
	100	8	0	13.80	10.81	29.4	-	606594	1	13.69	1.95	203.4	494109	579299							
	12	4	6.12	0.08	7.4	92111	195002	5	1.80	0.00	16.2	78505	78505								
	Sum./Avg.		101	9.25	3.51	43.2	162999	400677	185	6.71	0.41	109.7	95998	170118							

Table 8: Results for small-scale instances with different dominance relationship.

m	n	f	$\bar{b} \in [0.1, 0.2]$										$\bar{b} \in [0.2, 0.6]$										$\bar{b} \in [0.6, 1.2]$									
			Opt	Gap _{nb}	Gap	TWC	TUC	TC	Nodes	T_{opt}	Time	Opt	Gap _{nb}	Gap	TWC	TUC	TC	Nodes	T_{opt}	Time	Opt	Gap _{nb}	Gap	TWC	TUC	TC	Nodes	T_{opt}	Time			
3	3	5	5	5.00	0.00	35162	13112	30641	4.2	46	46	5	0.00	0.00	32157	26400	35221	1.0	11	11	5	0.00	0.00	35750	51385	56610	1.0	10	10			
	10	5	5	1.68	0.00	38486	12521	32522	3.4	33	33	5	0.00	0.00	28468	23731	34234	1.0	11	11	5	0.09	0.00	35649	48161	52541	1.4	17	17			
	8	5	5	1.78	0.00	34941	10263	36673	9.8	72	72	5	0.00	0.00	40192	23728	34720	1.0	14	14	5	1.01	0.00	33591	49241	53855	1.8	22	22			
	3	5	5	3.45	0.00	67068	43319	80503	22.6	1043	1043	5	0.46	0.00	67273	77510	98082	1.4	102	102	5	0.00	0.00	77868	193153	201985	1.0	107	107			
	20	5	5	1.74	0.00	74212	39925	87680	24.2	1024	1024	5	0.00	0.00	78369	83124	101891	1.0	126	126	5	0.00	0.00	84732	179252	187155	1.0	118	118			
	8	5	5	0.31	0.00	78318	36764	84853	3.8	256	256	5	0.00	0.00	86420	81792	101617	1.0	114	114	5	0.22	0.00	106127	193140	202135	2.2	245	245			
	3	5	5	0.35	0.00	112862	81351	148188	7.8	1502	1502	5	0.00	0.00	120891	198558	222270	1.0	518	518	5	0.00	0.00	112985	417944	429081	1.0	506	506			
	30	5	5	0.67	0.00	105076	78301	142986	10.2	3208	3208	5	0.00	0.00	134371	200502	223200	1.0	468	468	5	0.00	0.00	147953	410522	422586	1.0	693	693			
	8	5	5	1.19	0.00	127152	79109	153561	3.0	1183	1183	5	0.00	0.00	143666	150417	186831	1.0	689	689	5	0.19	0.00	165217	437637	448184	1.4	915	915			
	40	5	5	1.45	0.00	161748	145396	229838	5.4	5888	5888	5	0.22	0.00	193488	293205	339003	1.4	3766	3766	5	0.00	0.00	217868	669644	685705	1.0	2393	2393			
5	8	5	5	0.18	0.00	206218	151482	248027	1.8	3701	3701	5	0.68	0.00	229147	333368	372998	4.6	7132	7132	5	0.64	0.00	253639	784714	802304	3.4	5942	5942			
	3	5	5	0.66	0.00	199833	191514	325473	2.6	10746	10746	5	0.00	0.00	226511	432843	484873	1.0	8524	8524	5	0.00	0.00	238077	1116651	1136036	1.0	5822	5822			
	50	5	5	0.00	0.00	252649	210940	326184	1.0	6300	6300	5	0.92	0.00	304225	543086	585022	5.0	38525	38525	5	0.00	0.00	274997	1129627	1149945	1.0	9719	9719			
	8	5	5	2.07	0.00	280698	228136	350604	8.6	28322	28322	5	0.16	0.00	325232	473717	521134	3.0	22320	22320	5	0.23	0.00	339318	992768	1012748	2.2	15644	15644			
	3	5	5	5.80	0.00	25924	10010	21877	3.0	30	30	5	0.00	0.00	31284	14225	24232	1.0	10	10	5	0.00	0.00	30044	34182	38556	1.0	8	8			
	10	5	5	4.74	0.00	28721	8581	22899	11.4	91	91	5	0.00	0.00	23753	16455	25521	1.0	11	11	5	0.00	0.00	27971	32565	37309	1.0	10	10			
	8	5	5	5.72	0.00	31432	9212	26652	2.6	29	29	5	0.98	0.00	28012	14817	25801	1.8	17	17	5	0.00	0.00	30581	36645	41400	1.0	11	11			
	3	5	5	1.86	0.00	55478	30379	68799	33.4	1181	1181	5	0.00	0.00	58572	57523	72891	1.0	58	58	5	0.00	0.00	57708	111481	118815	1.0	68	68			
	20	5	5	0.46	0.00	60484	27556	61312	1.4	163	163	5	0.10	0.00	64425	60580	78177	1.4	102	102	5	0.00	0.00	63979	106174	114168	1.0	77	77			
	8	5	5	0.10	0.00	69041	23115	60760	1.4	153	153	5	0.00	0.00	74298	51454	73626	1.0	92	92	5	0.00	0.00	81107	102965	111459	1.0	97	97			
3	5	5	2.11	0.00	99644	61726	115696	14.2	2644	2644	5	0.00	0.00	108484	123837	154162	1.0	395	395	5	0.00	0.00	116597	244751	257585	1.0	361	361				
4	30	5	5	0.82	0.00	119001	56225	122961	3.8	1060	1060	5	0.83	0.00	128737	117434	151290	5.8	1404	1404	5	0.07	0.00	119060	271332	283641	1.4	594	594			
	8	5	5	1.37	0.00	111942	54514	128604	2.6	960	960	5	0.00	0.00	128667	124470	152778	1.0	595	595	5	0.00	0.00	126767	264008	276513	1.0	573	573			
	3	5	5	1.54	0.00	135516	80549	161611	127.0	52025	52025	5	0.00	0.00	150153	217211	252908	1.0	1552	1552	5	0.08	0.00	148295	442939	458045	1.4	1969	1969			
	40	5	5	1.65	0.00	155296	85466	177296	27.0	14109	14109	5	0.54	0.00	166939	183731	224222	3.4	2871	2871	5	0.31	0.00	192863	498616	513377	1.4	1925	1925			
	8	5	5	0.88	0.00	182045	102071	183328	15.4	8007	8007	5	0.47	0.00	174923	223176	258181	2.2	3112	3112	5	0.19	0.00	202286	434238	451530	2.6	3738	3738			
	3	4	2.97	0.09	181192	133623	243608	194.6	129257	223450	223450	5	0.00	0.00	210995	305800	352519	1.0	4870	4870	5	0.00	0.00	205983	752872	772932	1.0	3498	3498			
	50	5	4	4.07	0.27	213998	149722	269135	91.8	89871	193229	5	0.07	0.00	223584	384810	425908	2.6	7722	7722	5	0.00	0.00	239457	688505	709829	1.0	4446	4446			
	8	5	5	2.00	0.00	206448	136104	272875	81.4	146459	146459	5	0.05	0.00	255551	329084	379091	1.8	7996	7996	5	1.04	0.00	278696	674810	696502	14.6	50830	50830			
	3	5	5	3.50	0.00	21393	6097	15623	3.0	28	28	5	0.00	0.00	27528	8337	16385	1.0	9	9	5	0.00	0.00	22781	17640	21482	1.0	8	8			
	10	5	5	2.02	0.00	28468	5948	19006	2.6	18	18	5	0.00	0.00	21555	8478	17657	1.0	9	9	5	0.00	0.00	30976	22232	26349	1.0	9	9			
8	5	5	2.21	0.00	30085	5659	19576	3.0	20	20	5	0.00	0.00	23129	11028	17981	1.0	9	9	5	0.00	0.00	24755	21933	25380	1.0	10	10				
5	3	5	5	3.25	0.00	52211	26801	52348	4.6	204	204	5	0.00	0.00	49578	40624	56853	1.0	56	56	5	0.00	0.00	58345	74257	82172	1.0	50	50			
	20	5	5	8.94	0.00	59936	23544	53204	4.2	228	228	5	0.00	0.00	58721	36401	54741	1.0	62	62	5	0.00	0.00	59475	78675	86949	1.0	61	61			
	8	5	5	2.12	0.00	61023	22483	51281	2.2	186	186	5	0.05	0.00	53783	37879	54590	1.4	80	80	5	0.00	0.00	64590	77881	85936	1.0	78	78			
	3	5	5	5.01	0.00	90211	46194	88115	23.4	3281	3281	5	0.56	0.00	100196	83397	109006	11.4	999	999	5	0.00	0.00	98325	171327	182887	1.0	247	247			
	5	30	5	5.29	0.00	92154	37659	95763	17.8	2782	2782	5	0.33	0.00	95893	76211	106589	1.8	425	425	5	0.31	0.00	115966	181639	193431	2.2	480	480			
	8	5	5	1.45	0.00	102383	39702	93958	1.8	876	876	5	1.40	0.00	112524	86342	114052	8.2	1435	1435	5	0.35	0.00	118730	175606	188156	1.8	569	569			
	3	4	3.02	0.06	114046	65868	136526	395.8	6239	125096	125096	5	0.01	0.00	127719	148477	182995	1.8	1486	1486	5	0.00	0.00	147043	315466	331318	1.0	1232	1232			
	40	5	5	3.18	0.00	138306	70682	153015	23.0	12902	12902	5	1.36	0.00	144298	153934	187908	1.8	2556	2556	5	0.10	0.00	165518	310505	326051	1.8	2227	2227			
	8	5	5	1.92	0.00	1391879																										

Table 9: Results for large-scale instances with different dominance relationship.

m	n	f	$\bar{b} \in [0.1, 0.2]$										$\bar{b} \in [0.2, 0.6]$										$\bar{b} \in [0.6, 1.2]$														
			Opt	Gap _{lb}	Gap	TWC	TUC	TC	Nodes	T_{opt}	Time	Opt	Gap _{lb}	Gap	TWC	TUC	TC	Nodes	T_{opt}	Time	Opt	Gap _{lb}	Gap	TWC	TUC	TC	Nodes	T_{opt}	Time								
5	60	5	3	2.86	1.23	232108	140969	265023	144.2	45882	268369	5	0.00	0.00	255403	326819	379222	1.0	7423	7423	5	0.34	0.00	287909	680424	706338	2.2	14164	7423	5	0.34	0.00	287909	680424	706338	2.2	14164
		8	5	1.76	0.00	239011	150676	288611	89.0	144055	144055	5	2.35	0.00	274387	32076	387302	37.8	80603	80603	5	0.91	0.00	298281	690770	713717	17.0	62355	80603	5	0.91	0.00	298281	690770	713717	17.0	62355
		12	5	2.71	0.00	266573	155047	296051	50.2	105766	105766	5	2.12	0.00	341151	339536	394868	11.8	48182	48182	5	0.00	0.00	374740	71073	754678	1.0	13828	48182	5	0.00	0.00	374740	71073	754678	1.0	13828
		5	4	2.45	0.05	293043	196154	352708	45.4	78896	183885	5	0.76	0.00	327268	441250	500161	9.8	76314	76314	5	0.14	0.00	337870	96536	986528	1.8	25504	76314	5	0.14	0.00	337870	96536	986528	1.8	25504
	70	8	4	3.55	0.01	294387	197887	358992	67.0	161730	250363	4	1.79	0.07	369178	422669	488476	37.0	52523	62258	5	0.48	0.00	414486	1026817	1055139	1.4	20836	52523	4	0.48	0.00	414486	1026817	1055139	1.4	20836
		12	2	3.50	0.24	343016	201902	370614	81.4	69058	389038	5	3.76	0.00	399664	478968	537199	6.6	59147	59147	4	1.44	0.03	414485	916368	944748	26.2	186401	59147	4	1.44	0.03	414485	916368	944748	26.2	186401
		5	4	2.43	0.14	339726	240065	436900	21.8	29987	144025	4	2.34	0.14	371582	512483	591201	26.2	150080	247157	5	0.00	0.00	428519	1244768	1276191	1.0	42712	150080	5	0.00	0.00	428519	1244768	1276191	1.0	42712
		8	3	3.66	1.84	399007	263049	462774	46.2	235178	387295	5	2.20	0.00	430012	541768	616751	2.6	55889	55889	4	0.80	0.00	447401	1141158	1174848	10.2	243900	55889	4	0.80	0.00	447401	1141158	1174848	10.2	243900
	90	12	4	5.37	0.17	413754	259897	476819	36.2	189111	274272	3	3.49	0.18	466229	591530	661919	33.0	251123	397523	4	1.00	0.10	581338	1275142	1306109	16.2	226439	251123	4	1.00	0.10	581338	1275142	1306109	16.2	226439
		5	5	3.97	0.00	383609	319960	506224	12.6	189563	189563	5	1.03	0.00	454951	716393	790404	2.2	128553	128553	5	0.66	0.00	457973	1474683	1509449	2.6	103572	128553	5	0.66	0.00	457973	1474683	1509449	2.6	103572
		8	3	3.20	1.74	437640	310531	507818	30.6	311958	435983	4	2.67	0.42	471860	681317	774066	14.6	193112	279705	5	0.18	0.00	591512	1523768	1561199	1.4	77906	193112	5	0.18	0.00	591512	1523768	1561199	1.4	77906
		12	4	4.67	0.52	478193	309392	540994	17.8	218640	298018	4	4.44	0.30	563246	637866	724672	15.8	325352	386917	1	2.37	0.19	709745	1580929	1620164	25.0	560609	325352	1	2.37	0.19	709745	1580929	1620164	25.0	560609
100	8	5	5	0.94	0.00	434721	360122	587611	6.2	152111	152111	3	2.23	1.59	551205	832018	928359	8.6	222892	390298	5	0.00	0.00	565434	1884897	1922307	1.0	163923	222892	5	0.00	0.00	565434	1884897	1922307	1.0	163923
		8	1	5.23	4.48	528892	386776	610724	21.0	67104	620872	1	5.27	3.06	650150	923451	1019297	9.0	468145	615563	2	1.96	0.63	781313	1888573	1927728	6.6	109304	468145	2	1.96	0.63	781313	1888573	1927728	6.6	109304
		12	0	7.91	5.44	631279	392044	622432	25.8	-	620872	1	5.27	3.06	650150	923451	1019297	9.0	468145	615563	2	1.96	0.63	781313	1888573	1927728	6.6	109304	468145	2	1.96	0.63	781313	1888573	1927728	6.6	109304
		5	2	8.46	0.26	195191	97087	199823	367.8	177210	432184	5	1.98	0.00	189470	147348	214908	22.6	30333	30333	5	0.19	0.00	210891	344405	370657	2.2	8165	30333	5	0.19	0.00	210891	344405	370657	2.2	8165
	60	8	3	7.07	0.65	199535	95434	207148	304.6	130617	319021	5	2.87	0.00	225378	181943	233782	85.8	107083	107083	5	0.53	0.00	249584	361949	385959	4.2	12135	107083	5	0.53	0.00	249584	361949	385959	4.2	12135
		12	4	5.56	0.01	216533	94323	219310	118.2	93803	195235	5	0.73	0.00	229014	176852	233722	12.6	23399	23399	5	0.93	0.00	262984	327319	352191	11.4	19616	23399	5	0.93	0.00	262984	327319	352191	11.4	19616
		5	4	5.65	0.38	235096	107655	233901	132.2	90577	192812	5	1.24	0.00	257259	208235	279877	10.2	36563	36563	5	0.14	0.00	268128	458143	486610	3.4	25473	36563	5	0.14	0.00	268128	458143	486610	3.4	25473
		8	2	7.30	1.34	244581	122250	256506	274.2	299417	481025	5	1.68	0.00	252471	244443	304272	8.2	36793	36793	5	1.27	0.00	289588	442803	473426	7.0	40323	36793	5	1.27	0.00	289588	442803	473426	7.0	40323
	70	12	5	4.93	0.00	254341	119752	266641	146.6	250789	250789	5	2.10	0.00	283197	247579	316947	37.8	86806	86806	5	0.60	0.00	307331	471458	501115	9.8	48532	86806	5	0.60	0.00	307331	471458	501115	9.8	48532
		5	3	5.67	0.24	264675	148305	295586	140.2	125009	315658	4	2.15	0.19	285351	256358	338550	26.6	73351	178920	5	0.45	0.00	331816	611772	645865	3.8	42236	73351	5	0.45	0.00	331816	611772	645865	3.8	42236
		8	1	5.20	1.28	293464	140782	307605	169.8	84382	500127	3	3.22	0.14	312574	304255	377704	67.0	320154	433592	5	1.07	0.00	348312	565391	598923	11.4	95426	320154	5	1.07	0.00	348312	565391	598923	11.4	95426
		12	4	6.46	0.11	303231	148061	312698	122.6	317258	605724	5	2.80	0.00	340214	299527	371365	17.4	118938	118938	5	2.13	0.00	376311	500862	621301	7.0	95151	320154	5	2.13	0.00	376311	500862	621301	7.0	95151
8	80	5	0	6.66	4.09	311311	169319	374295	102.2	-	605724	5	0.36	0.00	327350	355127	440913	1.4	56582	56582	5	1.46	0.00	386742	787375	824593	12.2	139854	56582	5	1.46	0.00	386742	787375	824593	12.2	139854
		8	2	6.37	0.73	331604	181720	374499	93.4	157948	424937	5	1.81	0.00	375871	379072	466369	14.2	181742	181742	4	1.42	0.00	429816	834230	871270	15.8	17615	424937	5	1.81	0.00	429816	834230	871270	15.8	17615
		12	0	5.00	3.03	368351	186072	397666	129.8	-	603300	5	3.85	0.00	407536	378751	476258	17.4	142909	142909	5	1.99	0.00	453494	757341	797673	11.0	230350	603300	5	1.99	0.00	453494	757341	797673	11.0	230350
		5	1	5.15	2.47	363334	215093	426570	87.8	484470	578820	3	3.23	0.65	385281	414100	515131	24.6	323321	449497	5	1.22	0.00	416675	879654	921547	2.6	201125	323321	5	1.22	0.00	416675	879654	921547	2.6	201125
	60	8	0	5.62	5.42	410882	213337	469729	82.6	-	60587	3	3.14	0.60	450130	471454	571516	28.6	254850	396785	5	1.02	0.00	502053	991958	1030536	5.0	225212	254850	5	1.02	0.00	502053	991958	1030536	5.0	225212
		12	2	6.23	2.40	415369	222998	456969	61.4	370440	512694	1	4.30	1.41	480996	483312	575598	33.4	333746	563661	5	2.71	0.00	501939	933465	975196	8.2	273132	333746	5	2.71	0.00	501939	933465	975196	8.2	273132
		5	5	7.40	0.00	171060	70584	161007	59.0	59716	59716</																										

Table 10: Results for BP_2 on small instances with triangular distributions

m		n		$\omega = 0.1$					$\omega = 0.3$					$\omega = 0.5$					$\omega = 0.7$					$\omega = 0.9$				
	f	Opt	Gap(%)	Nodes	$T_{opt}(ms)$	Time(ms)	Opt	Gap(%)	Nodes	$T_{opt}(ms)$	Time(ms)	Opt	Gap(%)	Nodes	$T_{opt}(ms)$	Time(ms)	Opt	Gap(%)	Nodes	$T_{opt}(ms)$	Time(ms)	Opt	Gap(%)	Nodes	$T_{opt}(ms)$	Time(ms)		
3	10	3	5	0	1.0	11	11	5	0	1.0	14	14	5	0	1.0	28	28	5	0	1.4	29	29	5	0	1.0	15	15	
3	10	5	5	0	1.4	18	15	5	0	1.0	23	23	5	0	1.0	15	15	5	0	1.4	24	24	5	0	1.4	23	23	
3	10	8	5	0	1.0	13	13	5	0	1.0	15	15	5	0	1.0	46	46	5	0	1.8	30	30	5	0	1.0	19	19	
3	20	3	5	0	1.8	190	190	5	0	1.8	268	268	5	0	1.8	308	308	5	0	1.4	218	218	5	0	1.4	298	298	
3	20	5	5	0	2.6	267	267	5	0	2.6	301	301	5	0	1.4	313	313	5	0	3.8	588	588	5	0	1.4	397	397	
3	20	8	5	0	1.8	353	353	5	0	1.8	354	354	5	0	1.4	1541	1541	5	0	1.4	1803	1803	5	0	1.0	2066	2066	
3	30	3	5	0	1.8	1357	1357	5	0	3.4	1858	1858	5	0	1.4	6171	6171	5	0	7.0	4767	4767	5	0	2.2	2974	2974	
3	30	5	5	0	2.2	1442	1442	5	0	1.4	1790	1790	5	0	9.0	6172	6172	5	0	4.2	4009	4009	5	0	5.8	5208	5208	
3	30	8	5	0	1.4	1655	1655	5	0	3.4	2407	2407	5	0	1.8	2171	2171	5	0	3.0	10387	10387	5	0	3.0	14974	14974	
3	40	3	5	0	3.4	6663	6663	5	0	1.4	6324	6324	5	0	2.2	8800	8800	5	0	3.0	10387	10387	5	0	4.6	17605	17605	
3	40	5	5	0	1.0	4974	4974	5	0	3.0	10152	10152	5	0	1.4	11455	11455	5	0	5.0	21496	21496	5	0	9.4	125116	125116	
3	40	8	5	0	15.8	27738	27738	5	0	1.4	7229	7229	5	0	1.8	31440	31440	5	0	2.2	45197	45197	5	0	1.4	50169	50169	
3	50	3	5	0	7.0	26781	26781	5	0	6.6	4831	4831	5	0	4.6	54320	54320	5	0	4.6	70949	70949	5	0	1.0	41415	41415	
3	50	5	5	0	3.4	35677	35677	5	0	3.8	39422	39422	5	0	19.8	139522	139522	5	0	2.2	45197	45197	5	0	1.4	50169	50169	
3	50	8	5	0	5.8	39834	39834	5	0	1.0	10	10	5	0	1.0	12	12	5	0	2.2	45197	45197	5	0	1.4	50169	50169	
4	10	3	5	0	1.0	17	17	5	0	1.0	10	10	5	0	1.0	71	71	5	0	1.0	18	18	5	0	1.0	21	21	
4	10	5	5	0	1.0	19	19	5	0	1.0	10	10	5	0	1.0	12	12	5	0	1.0	20	20	5	0	1.0	40	40	
4	10	8	5	0	1.8	18	18	5	0	1.4	15	15	5	0	2.6	313	313	5	0	3.0	255	255	5	0	1.4	158	158	
4	20	3	5	0	5.8	269	269	5	0	1.4	100	100	5	0	2.6	313	313	5	0	4.2	1848	1848	5	0	1.0	196	196	
4	20	5	5	0	1.8	431	431	5	0	1.4	155	155	5	0	2.2	218	218	5	0	2.6	283	283	5	0	1.0	196	196	
4	20	8	5	0	1.4	438	438	5	0	4.6	1526	1526	5	0	4.6	1753	1753	5	0	4.2	1848	1848	5	0	1.8	1225	1225	
4	30	3	5	0	3.4	1118	1118	5	0	3.8	1183	1183	5	0	3.4	1563	1563	5	0	2.6	1442	1442	5	0	1.0	137	137	
4	30	5	5	0	4.2	1515	1515	5	0	3.0	1377	1377	5	0	8.2	3114	3114	5	0	5.0	2230	2230	5	0	2.2	1774	1774	
4	30	8	5	0	5.8	4463	4463	5	0	1.8	3367	3367	5	0	2.6	4009	4009	5	0	4.2	7179	7179	5	0	1.8	5931	5931	
4	40	3	5	0	9.8	6933	6933	5	0	5.4	6520	6520	5	0	3.8	6237	6237	5	0	6.2	10354	10354	5	0	2.6	7488	7488	
4	40	5	5	0	1.8	3460	3460	5	0	12.2	12371	12371	5	0	2.2	16778	16778	5	0	3.4	21531	21531	5	0	2.6	14975	14975	
4	40	8	5	0	2.2	6331	6331	5	0	3.8	13963	13963	5	0	5.8	24727	24727	5	0	3.4	21531	21531	5	0	5.8	33364	33364	
4	50	3	5	0	4.6	15763	15763	5	0	30.6	76575	76575	5	0	2.2	16258	16258	5	0	3.8	22276	22276	5	0	35.0	107898	107898	
5	10	3	5	0	1.0	17	17	5	0	1.0	20	20	5	0	1.0	31	31	5	0	1.4	21	21	5	0	1.4	17	17	
5	10	5	5	0	1.0	16	16	5	0	1.0	29	29	5	0	1.0	53	53	5	0	1.0	35	35	5	0	1.0	20	20	
5	10	8	5	0	1.0	19	19	5	0	1.0	31	31	5	0	2.6	47	47	5	0	1.4	23	23	5	0	1.0	30	30	
5	20	3	5	0	3.0	106	106	5	0	1.8	92	92	5	0	1.4	105	105	5	0	2.2	128	128	5	0	2.2	111	111	
5	20	5	5	0	3.4	139	139	5	0	1.0	90	90	5	0	1.0	65	65	5	0	2.6	129	129	5	0	3.8	238	238	
5	20	8	5	0	1.0	78	78	5	0	1.0	87	87	5	0	5.8	347	347	5	0	1.0	107	107	5	0	1.8	150	150	
5	30	3	5	0	3.4	588	588	5	0	12.2	1548	1548	5	0	4.2	947	947	5	0	1.0	556	556	5	0	2.2	665	665	
5	30	5	5	0	3.8	741	741	5	0	3.8	960	960	5	0	3.0	926	926	5	0	1.4	670	670	5	0	3.8	1502	1502	
5	30	8	5	0	6.2	1383	1383	5	0	5.0	1185	1185	5	0	6.6	1679	1679	5	0	9.4	2250	2250	5	0	9.0	2730	2730	
5	40	3	5	0	5.8	2000	2000	5	0	2.2	2071	2071	5	0	3.0	2512	2512	5	0	5.0	4474	4474	5	0	1.8	3010	3010	
5	40	5	5	0	5.4	3643	3643	5	0	23.0	11404	11404	5	0	3.4	4048	4048	5	0	3.8	4361	4361	5	0	4.6	5007	5007	
5	40	8	5	0	1.0	1629	1629	5	0	1.8	2687	2687	5	0	5.4	5013	5013	5	0	3.4	4451	4451	5	0	5.4	6963	6963	
5	50	3	5	0	12.6	11723	11723	5	0	7.8	20388	20388	5	0	2.2	6819	6819	5	0	3.8	11588	11588	5	0	4.0	6629	6629	
5	50	5	5	0	2.2	5200	5200	5	0	4.6	3363	3363	5	0	3.8	12797	12797	5	0	4.2	2974	2974	5	0	8.6	24967	24967	
5	50	8	5	0	47.4	4941.3	4941.3	5	0	10.6	18501	18501	5	0	8.2	10548	10548	5	0	1.4	8379	8379	5	0	38.2	38177	38177	
Sum./Avg.		225	0		5.4	7603	7603	225	0	4.4	7566	7566	225	0	3.8	9466	9466	225	0	3.0	8126	8126	225	0	3.6	12068	12068	

Table 11: Results for BP_2 on large instances with triangular distributions

m	n	f	$\omega = 0.1$					$\omega = 0.3$					$\omega = 0.5$					$\omega = 0.7$					$\omega = 0.9$					
			Opt	Gap(%)	Nodes	$T_{opt}(ms)$	Time(ms)	Opt	Gap(%)	Nodes	$T_{opt}(ms)$	Time(ms)	Opt	Gap(%)	Nodes	$T_{opt}(ms)$	Time(ms)	Opt	Gap(%)	Nodes	$T_{opt}(ms)$	Time(ms)	Opt	Gap(%)	Nodes	$T_{opt}(ms)$	Time(ms)	
5	60	5	5	0.00	13.4	34353	34353	5	0.00	30.2	94005	94005	5	0.00	6.6	33080	33080	5	0.00	17.4	70680	70680	5	0.00	6.2	30516	30516	
5	60	8	5	0.00	13.8	41135	41135	5	0.00	5.8	23800	23800	5	0.00	13.4	65848	65848	5	0.00	17.8	80536	80536	5	0.00	7.0	60381	60381	
5	60	12	5	0.00	21.4	71540	71540	5	0.00	11.4	47929	47929	5	0.00	13.4	65848	65848	5	0.00	26.2	131399	131399	5	0.00	17.8	103704	103704	
5	70	5	4	0.23	61.4	131334	226016	5	0.00	18.6	147333	114723	3	0.00	29.8	225432	300811	5	0.00	6.2	92101	92101	5	0.00	7.9	102973	102973	
5	70	8	4	0.23	61.4	131334	226016	4	0.11	51.0	231009	305271	3	0.05	41.0	50633	273661	5	0.00	5.8	116168	116168	2	0.17	37.4	129782	416131	
5	70	12	4	0.23	47.8	203929	284881	2	0.56	21.4	172876	172876	5	0.00	16.6	211752	211752	5	0.00	30.6	286110	286110	4	0.04	15.4	119186	218170	
5	80	5	3	0.32	32.2	68216	284671	2	0.56	40.6	84616	409066	4	0.00	7.8	98032	109200	4	0.24	11.0	152190	243567	3	0.08	11.4	148368	333225	
5	80	8	5	0.00	15.8	185834	185834	2	0.16	41.8	444477	542694	4	0.12	12.2	86390	102848	4	0.28	13.0	107344	281394	4	0.34	12.2	227480	303147	
5	80	12	2	0.68	42.2	276529	478543	2	0.46	32.6	123900	454092	3	0.24	25.8	274010	409758	4	0.08	15.4	230819	308139	3	0.39	12.2	226522	383928	
5	90	5	5	0.00	4.2	13.8	217951	296618	4	0.19	10.6	234715	327700	2	0.38	17.8	166868	437507	4	0.49	8.6	384811	431829	4	0.78	5.8	265339	334056
5	90	8	2	3.07	27.4	293045	481585	2	0.50	19.8	297249	486102	2	0.74	13.8	177975	439895	4	0.49	8.2	346254	400885	0	1.51	15.4	-	-	
5	90	12	2	1.10	19.8	160545	436937	3	0.08	14.6	275836	460166	1	0.72	13.8	179959	547478	2	1.54	10.2	316910	494171	1	1.87	14.2	586007	606154	
5	100	5	3	2.04	12.2	202848	372065	1	0.89	15.4	347212	559318	0	1.26	10.2	-	621478	3	0.57	3.8	315543	439083	1	1.25	7.8	586665	614180	
5	100	8	1	4.50	19.8	172480	523314	1	2.49	13.8	264790	550727	2	1.07	7.4	424492	542842	1	0.98	7.4	212082	536243	1	2.68	7.8	620380	617069	
5	100	12	1	5.16	14.2	216023	530892	1	4.20	11.8	232111	548394	0	3.89	13.0	-	617087	0	3.83	9.4	-	617552	0	3.15	9.8	-	617740	
8	60	5	5	0.00	33.8	27850	27850	5	0.00	35.4	32932	32932	5	0.00	29.0	29.0	32.47	32.47	5	0.00	32.2	30315	30315	5	0.00	10.6	18241	18241
8	60	8	5	0.00	39.4	35084	35084	5	0.00	39.0	44110	44110	5	0.00	25.4	31585	31585	5	0.00	28.6	47340	47340	5	0.00	44.2	62395	62395	
8	60	12	5	0.00	118.2	111453	111453	5	0.00	89.4	142832	142832	5	0.00	35.8	83125	83125	5	0.00	27.0	71611	71611	5	0.00	25.0	66362	66362	
8	70	5	5	0.00	9.4	21514	21514	5	0.00	64.6	135435	135435	5	0.00	19.0	45431	45431	5	0.00	19.4	60199	60199	5	0.00	10.6	43152	43152	
8	70	8	4	0.07	95.0	42713	134191	5	0.00	14.6	47983	47983	5	0.00	28.6	79431	79431	5	0.00	17.0	52885	52885	5	0.00	27.4	89298	89298	
8	70	12	5	0.00	133.4	246592	246592	5	0.00	16.6	148333	212511	5	0.00	28.6	106313	106313	5	0.00	18.6	119069	119069	5	0.00	44.6	198390	198390	
8	80	5	4	0.00	167.0	302991	367740	4	0.02	81.4	114833	212511	3	0.09	89.0	93751	208226	5	0.00	40.2	182424	182424	5	0.00	62.6	309415	309415	
8	80	8	5	0.00	118.6	252111	252111	3	0.42	121.0	217743	371950	3	0.09	89.0	93751	208226	5	0.00	40.2	182424	182424	5	0.00	70.2	309415	309415	
8	80	12	4	0.01	84.2	113387	316525	4	0.26	77.4	147438	371950	5	0.00	74.6	105223	405290	4	0.02	79.8	131684	372118	4	0.02	30.2	232028	309083	
8	90	5	3	1.14	89.2	113387	316525	2	0.97	113.4	319093	445738	2	0.50	77.0	105223	405290	4	0.12	32.6	54067	291810	4	0.02	30.2	232028	309083	
8	90	8	3	0.54	90.2	203904	302704	1	0.98	104.6	429825	506543	5	0.00	11.4	98507	98507	5	0.00	31.4	104116	204669	5	0.00	24.6	260339	260339	
8	90	12	3	0.39	57.0	102723	302704	2	0.75	76.2	214201	406231	3	0.14	58.2	340038	447708	3	0.46	25.4	154454	333868	3	0.68	31.4	210360	370840	
8	100	5	1	2.43	95.8	-	607287	1	1.14	58.6	61312	494885	3	1.30	40.6	338831	441768	2	0.82	24.6	155454	333868	3	1.58	22.2	223519	370840	
8	100	8	2	0.49	79.0	547398	596287	2	1.13	58.6	330735	498750	4	0.77	47.8	110833	531363	3	0.52	24.6	200162	384206	4	0.27	17.4	267024	334721	
8	100	12	2	0.56	55.8	169063	435292	2	0.59	49.4	337596	506890	4	0.34	20.6	228340	305433	3	1.18	20.6	248945	384552	0	0.51	32.6	-	607148	
10	60	5	5	0.00	53.0	29464	29464	5	0.00	26.6	20100	20100	5	0.00	10.2	12083	12083	5	0.00	5.8	7702	7702	5	0.00	25.4	25009	25009	
10	60	8	5	0.00	33.0	23377	23377	5	0.00	13.8	16929	19929	5	0.00	12.6	16658	16658	5	0.00	15.8	20359	20359	5	0.00	5.0	11488	11488	
10	60	12	5	0.00	39.0	31193	31193	5	0.00	28.6	29929	29929	5	0.00	23.8	29218	29218	5	0.00	1.8	6283	6283	5	0.00	23.0	30251	30251	
10	70	5	5	0.00	140.2	135899	135899	4	0.13	143.4	33664	149009	5	0.00	67.8	86191	86191	5	0.00	8.2	20215	20215	5	0.00	30.6	57596	57596	
10	70	8	5	0.00	110.2	119681	119681	5	0.00	67.0	89515	89515	5	0.00	51.8	85548	85548	5	0.00	30.6	65824	65824	5	0.00	40.6	74509	74509	
10	70	12	5	0.00	113.8	144066	144066	5	0.00	127.4	176425	176425	4	0.01	97.8	59296	107743	5	0.00	37.4	79832	79832	5	0.00	42.2	84033	84033	
10	80	5	5	0.00	117.8	184786	184786	4	0.25	179.0	263315	320567	5	0.00	92.6	211460	211460	5	0.00	21.4	68959	68959	5	0.00	18.2	65491	65491	
10	80	8	4	0.00	242.2	344983	396708	5	0.00	111.0	218661	218661	4	0.05	90.6	112754	210285	5	0.00	60.6	170462	170462	5	0.00	21.8	80376	80376	
10	90	5	3	1.11	185.4	304776	429063	5	0.00	66.6	166680	166680	5	0.00	31.4	84675	84675	4	0.00	58.6	71651	177537	5	0.00	51.8	188377	188377	
10	90	8	2	1.48	157.8	222247	435855	2	0.90	122.2	138201	417355	4	0.15	49.4	101348	201505	4	0.07	104.2	335561	389441	5	0.00	47.8	240935	240935	
10	90	12	3	0.49	143.4	315182	428066	3	0.14	128.6	331099	431735	4	0.25	69.0	170236	257900	2	0.20	85.8	192154	440448	4	0.22	57.0	264855	333896	
10	100	5	2	0.33	87.8	229341	352243	2	1.07	108.6	339192	484871	4	0.29	60.0	233263	307302	5	0.00	32.2	265109	265109	3	0.10	62.6	207200	363327	
10	100	8	4	0.08	87.8	229341	352243	2	0.66	89.0	33992	498373	2	0.74	57.8	85769	390024	3	1.76	77.0	180603	521382	3	0.40	26.2	108363	303183	
10	100	12	1	1.07	94.2	89945	503320	1	1.20	109.4	501592	695593	1	1.00	74.2	147438	371950	3	0.46	77.0	147438	371950	3	0.46	77.0	147438	3719503	