



深蓝学院
shenlanxueyuan.com

第三章作业分享



主讲人 潘光帅



代码分享

①

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 3 \\ 0 & 2 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 14 & 20 \\ 15 & 24 \end{bmatrix}$$
$$\begin{bmatrix} 0 & 2 & 1 \\ 0 & 3 & 2 \\ 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 24 \\ 17 & 26 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 3 & 3 & 2 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$$

②

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 0 & 1 & 3 \\ 1 & 1 & 0 & 2 \\ 1 & 3 & 2 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 2 \\ 1 & 1 \\ 0 & 1 \\ 1 & 2 \\ 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 & 3 \\ 2 & 1 & 3 & 2 \\ 0 & 3 & 1 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 1 & 1 & 3 \\ 0 & 1 & 3 & 3 \\ 1 & 3 & 3 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 2 \\ 1 & 1 \\ 0 & 1 \\ 1 & 2 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}$$

kernel

$$A = \begin{bmatrix} a_{11}, a_{12}, \dots, a_{1n} \\ a_{21}, a_{22}, \dots, a_{2n} \\ \vdots \\ a_{m1}, a_{m2}, \dots, a_{mn} \end{bmatrix}$$
$$B = \begin{bmatrix} b_{11}, b_{12}, \dots, b_{1n} \\ b_{21}, b_{22}, \dots, b_{2n} \\ \vdots \\ b_{m1}, b_{m2}, \dots, b_{mn} \end{bmatrix}$$
$$C = \begin{bmatrix} c_{11}, \dots, c_{1j} \\ \vdots \\ c_{m1}, \dots, c_{mj} \end{bmatrix}$$

1. 将图像转化为矩阵，便于卷积加速

图像 $\begin{cases} W \times \text{宽} \\ H \times \text{高} \\ C \times \text{深} \end{cases}$

卷积核: ksize, ksize \Rightarrow 特征图尺寸: feature_w = $(W \times - k\text{size}) // \text{step} + 1$ 应感受野中元素个数、
(卷积核元素个数)。

代码分享

```
for deep in range(cx):
    temp=np.zeros((feature_w * feature_w, ksize * ksize))  
    num=0;  
    for i in range(feature_w):  
        for j in range(feature_w):  
            temp[num]=x[i*step:i*step+ksize,j*step:j*step+ksize,deep].reshape(-1)
            num=num+1  
    image_col[:,deep*ksize * ksize:(deep+1)*ksize * ksize]=temp
return image_col
```

手写注释：
- \Rightarrow 单行代码块的尺寸
- \rightarrow 水平移动总次数
- \rightarrow 垂直移动总次数 \rightarrow 感受野中的元素重新排列
- $\stackrel{\text{张}}{\text{数}}$
 $\stackrel{\text{行排好}}{\text{}} \downarrow$

代码分享

训练过程



```
def train(batch=32, lr=0.01, epochs=10):
    # Mnist手写数字集
    dataset_path = "./datasets/mnist"
    train_data = torchvision.datasets.MNIST(root=dataset_path, train=True, download=False)
    train_data.data = train_data.data.numpy() # [60000, 28, 28]
    train_data.targets = train_data.targets.numpy() # [60000]
    train_data.data = train_data.data.reshape(60000, 28, 28, 1) / 255. # 输入向量处理
    train_data.targets = onehot(train_data.targets, 60000) # 标签one-hot处理 (60000, 10)

    # [28, 28] 卷积 6x{5, 5} -> 6x{24, 24}
    conv1 = Conv(kernel_shape=(5, 5, 1, 6))
    relu1 = Relu()
    # 6x{24, 24} -> 6x{12, 12}
    pool1 = Pool()
    # 6x{12, 12} 卷积 16x{6x{12, 12}} -> 16x{8, 8}
    conv2 = Conv(kernel_shape=(5, 5, 6, 16)) # 8x8x16
    relu2 = Relu()
    # 16x{8, 8} -> 16x{4, 4}
    pool2 = Pool()

    # 在这里可以尝试增加网络的深度，再实例化conv3和pool3，记得后面的前向传播过程
    # 和反向传播过程也要有对应的步骤
    nn = Linear(256, 10)
    softmax = Softmax()
    for epoch in range(epochs):
        for i in range(0, 60000, batch):
            X = train_data.data[i:i + batch]
            Y = train_data.targets[i:i + batch]
            # 前向传播过程
            predict = conv1.forward(X)
            predict = relu1.forward(predict)
            predict = pool1.forward(predict)
            predict = conv2.forward(predict)
            predict = relu2.forward(predict)
            predict = pool2.forward(predict)
            predict = predict.reshape(batch, -1)
            predict = nn.forward(predict)
            # 误差计算
            loss, delta = softmax.cal_loss(predict, Y)
            # 反向传播过程
            delta = nn.backward(delta, lr)
            delta = delta.reshape(batch, 4, 4, 16)
            delta = pool2.backward(delta)
            delta = relu2.backward(delta)
            delta = conv2.backward(delta, lr)
            delta = pool1.backward(delta)
            delta = relu1.backward(delta)
            conv1.backward(delta, lr)
```

→ 固定



$$\text{前向: } z^{(l)} = w^{(l)}a^{(l-1)} + b^{(l)}$$

代码分享

全连接层（前向传播）



全连接层

```

class Linear(object):

    def __init__(self, inChannel, outChannel):
        scale = np.sqrt(inChannel / 2)
        self.W = np.random.standard_normal((inChannel, outChannel)) / scale
        self.b = np.random.standard_normal(outChannel) / scale
        self.W_gradient = np.zeros((inChannel, outChannel))
        self.b_gradient = np.zeros(outChannel)

    def forward(self, x):
        """前向过程"""
        ## 补全代码 ## (32x256) → (256x10)
        self.x=x
        return np.dot(self.x, self.W)+self.b

    def backward(self, delta, learning_rate):
        """反向过程"""
        ## 梯度计算
        batch_size = self.x.shape[0]
        ## 补全代码 ##
        self.W_gradient = np.dot(self.x.transpose(),delta)
        self.b_gradient = np.sum(delta, axis=0)
        delta_backward = np.dot(delta, self.W.transpose())
        ## 反向传播
        self.W -= learning_rate/batch_size*self.W_gradient
        self.b -= learning_rate/batch_size*self.b_gradient
        return delta_backward

```

```

def train(batch=32, lr=0.01, epochs=10):
    # Mnist手写数字集
    dataset_path = "./datasets/mnist"
    train_data = torchvision.datasets.MNIST(root=dataset_path, train=True)
    train_data.data = train_data.data.numpy() # [60000, 28, 28]
    train_data.targets = train_data.targets.numpy() # [60000]
    train_data.data = train_data.data.reshape(60000, 28, 28, 1) / 255.
    train_data.targets = onehot(train_data.targets, 60000) # 标签one-hot

    # [28, 28] 卷积 6x[5, 5] -> 6x[24, 24]
    conv1 = Conv(kernel_shape=(5, 5, 1, 6))
    relu1 = Relu()
    # 6x[24, 24] -> 6x[12, 12]
    pool1 = Pool()
    # 6x[12, 12] 卷积 16x(6x[12, 12]) -> 16x[8, 8]
    conv2 = Conv(kernel_shape=(5, 5, 6, 16)) # 8x8x16
    relu2 = Relu()
    # 16x[8, 8] -> 16x[4, 4]
    pool2 = Pool()

    # 在这里可以尝试增加网络的深度，再实例化conv3和pool3，记得后面的前向传播过程
    # 和反向传播过程也要有对应的过程
    nn = Linear(256, 10)
    softmax = Softmax()
    for epoch in range(epochs):
        for i in range(0, 60000, batch):
            X = train_data.data[i:i + batch]
            Y = train_data.targets[i:i + batch]
            # 前向传播过程
            predict = conv1.forward(X)
            predict = relu1.forward(predict)
            predict = pool1.forward(predict)
            predict = conv2.forward(predict)
            predict = relu2.forward(predict)
            predict = pool2.forward(predict)
            predict = predict.reshape(batch, -1)
            predict = nn.forward(predict)

```

(32x256)

(32x10)

代码分享

$$S_i^{(l)} = -(y_i - \hat{a}_i^{(l)}) f'(z_i^{(l)}) \quad \text{全连接层(反向传播)}$$

$$\delta_i^{(l)} = \left(\sum_{j=1}^{n_{l+1}} S_j^{(l+1)} \hat{o}_{ji}^{(l+1)} \right) f'(z_i^{(l)})$$



```

## 全连接层
class Linear(object):

    def __init__(self, inChannel, outChannel):
        scale = np.sqrt(inChannel / 2)
        self.W = np.random.standard_normal((inChannel, outChannel)) / scale
        self.b = np.random.standard_normal(outChannel) / scale
        self.W_gradient = np.zeros((inChannel, outChannel))
        self.b_gradient = np.zeros(outChannel)

    def forward(self, x):          10
        """前向过程"""
        ## 补全代码 ##
        self.x=x
        return np.dot(self.x, self.W)+self.b      input: 256

    def backward(self, delta, learning_rate):  32x10
        """反向过程"""
        ## 梯度计算 32x256
        batch_size = self.x.shape[0]      (256x32) (32x10)
        ## 补全代码 ##
        self.W_gradient = np.dot(self.x.transpose(), delta)
        self.b_gradient = np.sum(delta, axis=0)
        delta_backward = np.dot(delta, self.W.transpose())
        ## 反向传播
        self.W -= learning_rate/batch_size * self.W_gradient
        self.b -= learning_rate/batch_size * self.b_gradient
        return delta_backward      (32x256)
    
```

```

## Softmax 函数
class Softmax(object):
    def cal_loss(self, predict, label):      (32x10)
        batchsize, classes = predict.shape
        self.predict(predict)
        loss = 0
        delta = np.zeros(predict.shape)
        for i in range(batchsize):
            delta[i] = self.softmax[i] - label[i]
            loss -= np.sum(np.log(self.softmax[i]) * label[i])
        loss /= batchsize
        return loss, delta

    def predict(self, predict):
        batchsize, classes = predict.shape
        self.softmax = np.zeros(predict.shape)
        for i in range(batchsize):
            predict_tmp = predict[i] - np.max(predict[i])
            predict_tmp = np.exp(predict_tmp)
            self.softmax[i] = predict_tmp / np.sum(predict_tmp)
        return self.softmax
    
```

$$\frac{\partial E}{\partial \theta_{ij}^{(l)}} = \sum_{i=1}^m S_i^{(l+1)} \hat{o}_{ji}^{(l+1)}$$

$$\frac{\partial E}{\partial b_i^{(l)}} = \sum_{i=1}^m S_i^{(l+1)} = \sum_{j=1}^{n_{l+1}} \delta_j^{(l+1)} \hat{o}_{ji}^{(l+1)}$$

代码分享

卷积层(前向传播)



```
conv
class Conv(object):
    def __init__(self, kernel_shape, step=1, pad=0):
        # [w, h, d]
        width, height, in_channel, out_channel = kernel_shape
        self.step = step
        self.pad = pad
        scale = np.sqrt(3 * in_channel * width * height / out_channel)
        self.k = np.random.standard_normal(kernel_shape) / scale
        self.b = np.random.standard_normal(out_channel) / scale
        self.k_gradient = np.zeros(kernel_shape)
        self.b_gradient = np.zeros(out_channel)

    def forward(self, x):
        self.x = x
        if self.pad != 0:
            self.x = np.pad(self.x, ((0, 0), (self.pad, self.pad), ...
                                    (self.pad, self.pad), (0, 0)), 'constant')
        # x batch, width, height, channel
        bx, wx, hx, cx = self.x.shape
        # kernel的宽、高、通道数、个数
        wk, hk, ck, nk = self.k.shape
        feature_w = (wx - wk) // self.step + 1 # 返回的特征图尺寸
        feature = np.zeros((bx, feature_w, feature_w, nk))

        self.image_col = []
        # kernel也进行了reshape, 便于卷积加速, 只保留通道维度, 是个二维的矩阵
        kernel = self.k.reshape(-1, nk)
        ## 补全代码 ##
        for i in range(bx):
            image_col=img2col(self.x[i],wk,self.step)
            feature[i] =(np.dot(image_col,kernel)+self.b).reshape(feature_w, feature_w,nk)
            self.image_col.append(image_col)
        return feature
```

① 图像转化为矩阵

② $\text{Net}^l = \text{Conv}(W^l, a^{l-1}) + b^l$

kernel 重新排列

代码分享

池化层(前向传播)



```
## Max Pooling 层
class Pool(object):
    def forward(self, x):
        b, w, h, c = x.shape
        feature_w = w // 2
        feature = np.zeros((b, feature_w, feature_w, c))
        self.feature_mask = np.zeros((b, w, h, c)) # 记录最大池化时最大值的位置信息用于反向传播
        for bi in range(b):
            for ci in range(c):
                for i in range(feature_w):
                    for j in range(feature_w):
                        ## 补全代码
                        feature[bi, i, j, ci] = np.max(x[bi, i*2:i*2+2, j*2:j*2+2, ci])
                        index = np.argmax(x[bi, i * 2:i * 2 + 2, j * 2:j * 2 + 2, ci])
                        self.feature_mask[bi, i * 2 + index // 2, j * 2 + index % 2, ci] = 1
        return feature

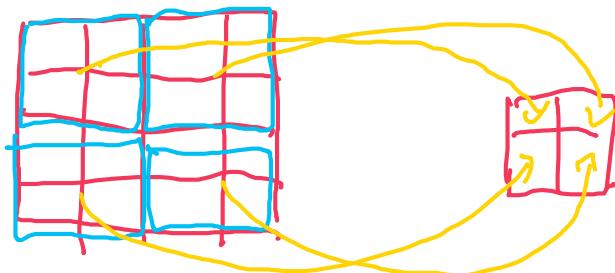
    def backward(self, delta):
        return np.repeat(np.repeat(delta, 2, axis=1), 2, axis=2) * self.feature_mask
```

```
>>> a
array([[-1.28095222, -1.78081359],
       [ 0.6110716 ,  0.33153531]])
>>> np.argmax(a)
2
```

→ 构建 feature map

→ 记录最大值的位置

→ 将最大值位置处置为1



代码分享.

卷积层(反向传播)



```
def backward(self, delta, learning_rate):
    bx, wx, hx, cx = self.x.shape # batch, 14, 12, inchannel 32, 12, 12, 6
    wk, hk, ck, nk = self.w.shape # 5, 5, inChannel, outChannel 5, 5, 6, 16
    bd, wd, hd, cd = delta.shape # batch, 10, 10, outChannel 32, 8, 8, 16
    # 计算self.k_gradient, self.b_gradient
    # 参数更新过程
    ## 补全代码 ##
    delta_col = delta.reshape(bd, -1, cd)
    for i in range(bx):
        self.k_gradient += np.dot(self.image_col[i].transpose(), ...
            delta_col[i]).reshape(self.k .shape)
    self.k_gradient /= bx
    self.b_gradient += np.sum(delta_col, axis=(0,1))
    self.b_gradient /= bx (1, 16)
    # 计算delta_backward
    # 误差的反向传递
    delta_backward = np.zeros(self.x.shape)
    # numpy矩阵(对应kernel) 旋转180度
    ## 补全代码 ##
    ① k_180 = self.k
    k_180 = k_180[::-1, ::-1] (保留深度通道)
    k_180_col = k_180.reshape(-1,ck)
    if hd - hk + 1 != hx:
        pad = (hx - hd + hk - 1) // 2
        pad_delta = np.pad(delta, ((0, 0), ...
            (pad, pad), (pad, pad), (0, 0)), 'constant')
    else:
        pad_delta = delta
    for i in range(bx):
        pad_delta_col = img2col(pad_delta[i], wk, self.step)
        delta_backward[i] = np.dot(pad_delta_col, k_180_col).reshape(wx, hx, ck)
    # 反向传播
    self.k -= self.k_gradient * learning_rate
    self.b -= self.b_gradient * learning_rate
    return delta_backward
```

以 COV2 为例：

COV2(5, 5, 6, 16)

k_shape : 5x 6

inchannel : 6

outchannel : 16

step=1 Depth=6 Filter=16

$$\begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \xrightarrow{\text{flipped}} \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix}$$

```
>>> a  
array([[0, 1],  
       [2, 3]])  
②>>> a[::-1, ::-1]  
array([[3, 2],  
       [1, 0]])
```

(旋转180°)

代码分享.

池化层(反向传播)

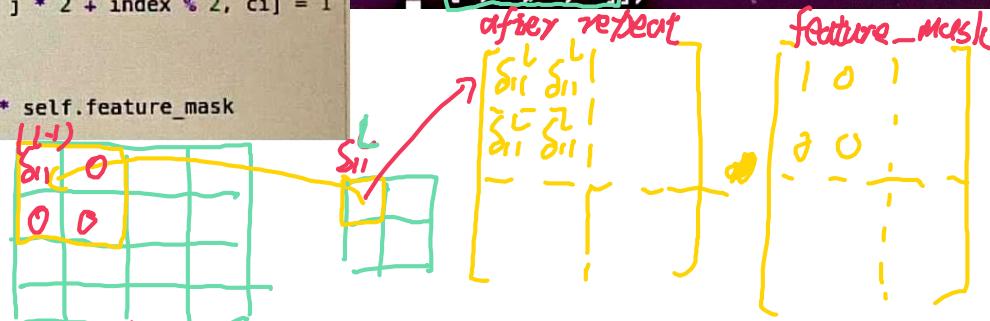
```
## Max Pooling 层
class Pool(object):
    def forward(self, x):
        b, w, h, c = x.shape
        feature_w = w // 2
        feature = np.zeros((b, feature_w, feature_w, c))
        self.feature_mask = np.zeros((b, w, h, c)) # 记录最大池化时最大值的位置信息用于反向传播
        for bi in range(b):
            for ci in range(c):
                for i in range(feature_w):
                    for j in range(feature_w):
                        ## 补全代码
                        feature[bi, i, j, ci] = np.max(x[bi, i*2:i*2+2, j*2:j*2+2, ci])
                        index = np.argmax(x[bi, i * 2:i * 2 + 2, j * 2:j * 2 + 2, ci])
                        self.feature_mask[bi, i * 2 + index // 2, j * 2 + index % 2, ci] = 1
        return feature
    def backward(self, delta):
        return np.repeat(np.repeat(delta, 2, axis=1), 2, axis=2) * self.feature_mask
```

- ① 无学习的参数 $\text{axis}=1 \text{ axis}=2$
- ② 将误差反传到上一层, 没有梯度计算
- ③ 对于 Max Pooling, 下一层的误差只将值传递到上一层对应区中最大值对应的神经元, 其它神经元误差为0.

以 pool2 为例:

$\delta_{l-1}(32, 4, 4, 16)$

```
>>> a
array([[0, 1],
       [2, 3]])
>>> np.repeat(np.repeat(a, 2, axis=0), 2, axis=1)
array([[0, 0, 1, 1],
       [0, 0, 1, 1],
       [2, 2, 3, 3],
       [2, 2, 3, 3]])
```





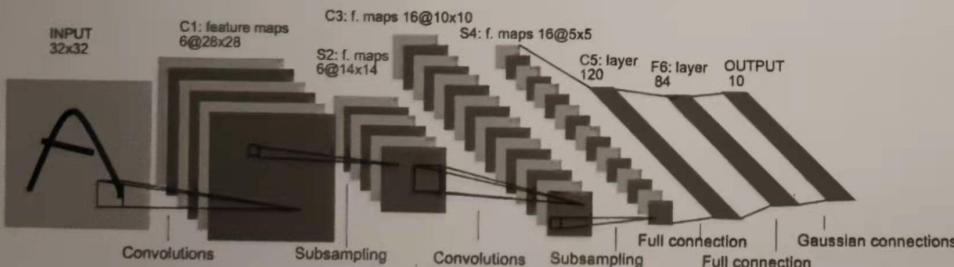
感谢各位聆听 !
Thanks for Listening !



作业讲解



1. 对比卷积神经网络与全连接神经网络，在图像分类任务中，原始图像大小的变化将会怎样影响模型可训练参数个数？
 2. 卷积神经网络是通过什么方式来完成可训练参数的减少？
 3. 如图是LeNet-5的示意图，试着写出每一层的参数个数。



作业讲解

Input: 32×32

\Rightarrow kernel_size: 5×5

C₁: feature_map: $6 \times (28 \times 28)$

C₁层参数: $k: 6 \times (1 \times 25) = 150$ $b: 6 \times (1 \times 1) = 6$ $k+b = 156$

S₂: pooling_size: 2×2

{若样本采用 max, mean 方法不需要参数训练}

{若采用一个训练参数，则需要: $k: 6 \times 1 = 6$ $b: 6 \times 1 = 6$ $k+b = 12$ }

作业讲解

C_3 : 根据右图的输入方式: 共需要:

$$\begin{aligned}
 & \xrightarrow{\text{相邻3个神经元}} 6 \times (3 \times 25 + 1) + \xrightarrow{\text{相邻4个神经元}} 6 \times (4 \times 25 + 1) + \xrightarrow{\text{间隔两个}} 3 \times (2 \times 25 + 1) + 3 \times (2 \times 25 + 1) \\
 & + 1 \times (\xrightarrow{\text{所有神经元}} 6 \times 25 + 1) = 1516
 \end{aligned}$$

S_4 层: { 若采用 max, mean 则不需要参数

$$\text{若采用可训练参数, 共需要 } 16 \times (1 + 1) = 32$$

C_5 层: 共有 120 个特征图, 每个特征图与 S_4 层 16 个单元相连.
由于 S_4 层的 16 个单元都为 5×5 , 特征图也为 5×5

$$\text{故 } C_5 \text{ 特征图大小为 } (5 - 3 + 1) = 1$$

作业讲解

C_5 层参数为: $120 \times (16 \times 5 \times 5 + 1) = 48120$

F_6 层: 与 C_5 层全相连.

$$84 \times (\underbrace{120 \times (1 \times 1)}_{\text{特征图}} + 1) = 10164$$

↑
↓
 C_5 特征图