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When to use semi-classical GR instead of QFT to probe cosmological correlation functions: a quantitative view

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Introduction

The universe's inhomogeneities might have been quantum fluctuations during inflation.

- If so, can we tell when it stopped up to a precision criterion?
- Can we characterise the signatures of classical vs quantum inhomogeneities?

These are particularly important question given that

- classical simulations of inflation have become a norm [2]
- the quantumness of initial conditions has not been observed yet.

From quantum to classical inflation

- The quantification of perturbations in the inflaton and geometry via Bunch-Davies vacuum is standard

$$\mathcal{R}_k(\eta) = -\frac{H}{M_{Pl}}\sqrt{\frac{\pi}{8\varepsilon_1 k^3}}(-k\eta)^{3/2}H_\nu^{(1)}[-k\eta] \underset{\varepsilon_{1,2}\ll 1}{\simeq} \frac{iH}{M_{Pl}}\frac{1}{\sqrt{4\varepsilon_1 k^3}}(1+ik\eta)e^{-ik\eta}$$

- The **commutators and anti-commutators** are related

$$\begin{cases} \left\langle \left\{\hat{\mathcal{R}}(\mathbf{x},t),\hat{\mathcal{R}}(\mathbf{y},t')\right\}\right\rangle = 2\int d^3k e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\,\mathrm{Re}\left[\mathcal{R}_k(t)\mathcal{R}_k^*(t')\right] 2\int d^3k e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}F_k(t,t') \\ \left\langle \left[\hat{\mathcal{R}}(\mathbf{x},t),\hat{\mathcal{R}}(\mathbf{y},t')\right]\right\rangle = 2\int d^3k e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\,\mathrm{Im}\left[\mathcal{R}_k(t)\mathcal{R}_k^*(t')\right]\int d^3k e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}G_k(t,t') \end{cases}$$

- **SuperHubble limit** $F_k(t_f,t) \underset{k\ll aH(t)}{\gg} G_k(t_f,t)$

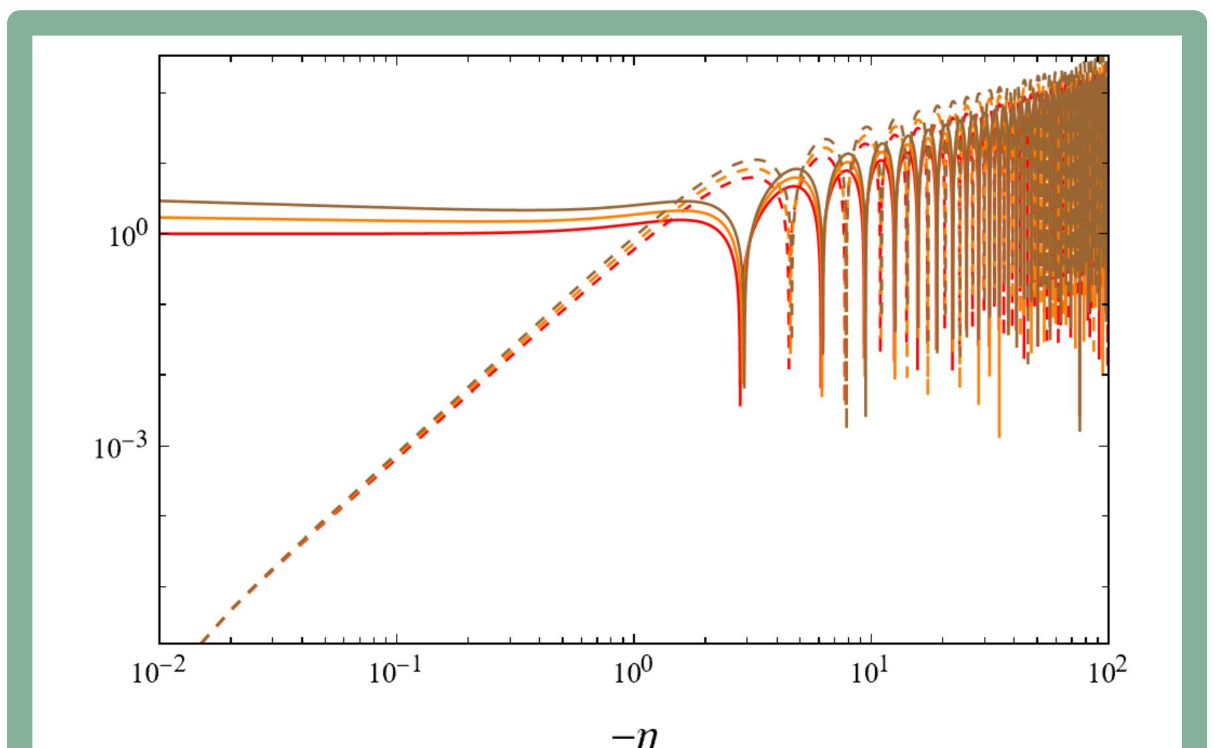


Figure 1: Absolute ($k=1$) F (solid lines) and G (dashed lines) in quasi-dS ($\nu = 3/2+0$ (red), $+0.05$ (orange), $+0.1$ (brown)) for η up to $\eta_f = -0.01$ and, up to constants. Oscillations amplify when $\eta = -\infty$.

- Interactions are much more complicated and much more imprinted by quantumness. We focus on the **bispectrum**.

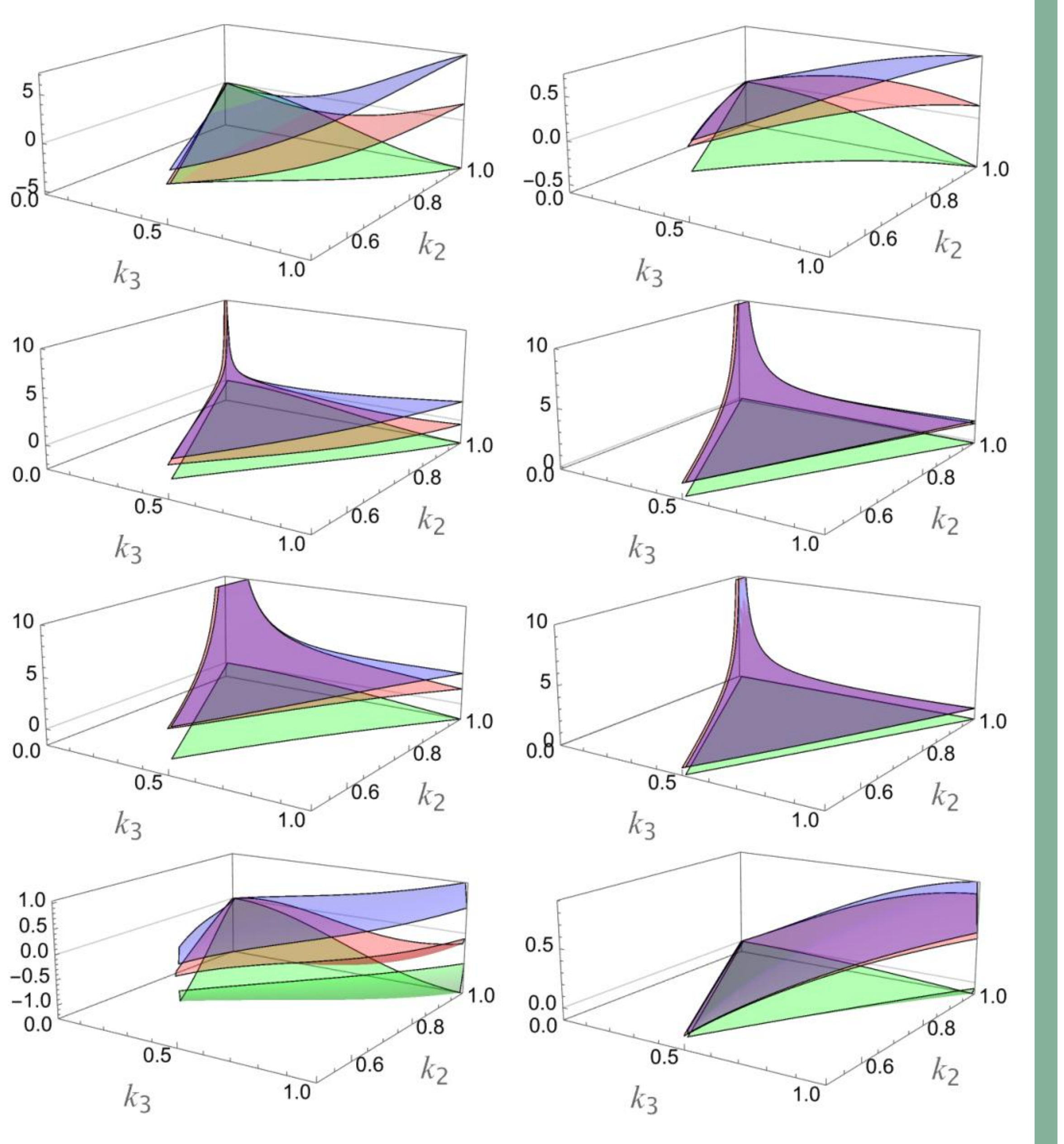


Figure 2: Blabla

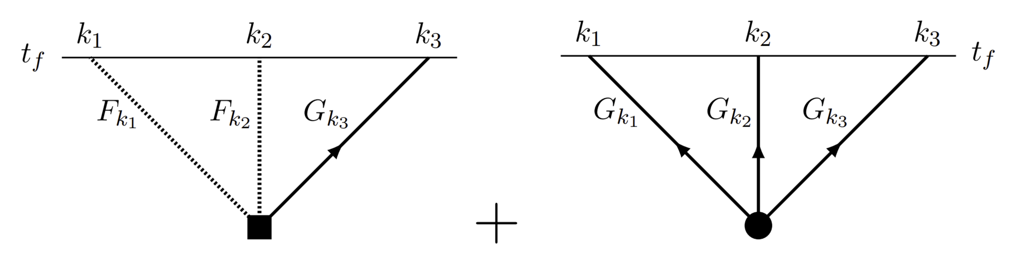
Semi-classical correlation function

- The bispectrum is the 3pt correlation function and is the smallest-n correlator with a leading order contribution, next-in line wrt data.
- The Keldysh diagrammatics dissociate on-shell dynamics from off-shell ones in interacting theories (eg $\lambda\phi^3$ here), when propagating from past infinity

$$\langle\hat{\mathcal{R}}_{k_1}\hat{\mathcal{R}}_{k_2}\hat{\mathcal{R}}_{k_3}|_{-\infty}^{t_f}\rangle \underset{\vec{k}_1+\vec{k}_2+\vec{k}_3=0}{=} -2\lambda\int_{-\infty}^{t_f} dt\, a(t)F_{k_1}(t,t_f)F_{k_2}(t,t_f)G_{k_3}(t_f,t) \Big\} \text{ on-shell}$$

$$\text{off-shell } \Big\{ +\frac{1}{4}\lambda\int_{-\infty}^{t_f} dta(t)G_{k_1}(t_f,t)G_{k_2}(t_f,t)G_{k_3}(t_f,t) + k - perm. + O(\lambda^2)$$

$=$



- When including leading terms of general relativity, we need to look at

$$[g_1(t)\dot{\mathcal{R}}^3] + g_2(t)\mathcal{R}\dot{\mathcal{R}}^2 + g_3(t)\mathcal{R}(\partial\mathcal{R})^2 + g_4(t)\dot{\mathcal{R}}\partial\mathcal{R}\partial\chi \subset \mathcal{H}^{(3)}$$

- Analytic expressions computed for the first time in this formalism, e.g.

$$\begin{cases} \langle\hat{\mathcal{R}}_{k_1}\hat{\mathcal{R}}_{k_2}\hat{\mathcal{R}}_{k_3}|_{-\infty}^{\eta_f}\rangle_{\mathcal{R}\dot{\mathcal{R}}^2}^{\text{on-shell}} \propto \dfrac{e_1^2\left(3\eta_f^2e_2-2\right)e_3+e_2\left(1-\eta_f^2e_2\right)e_3+e_1e_2\left(-e_2^2\eta_f^4-e_2^2\eta_f^2+e_2\right)}{2e_1^2e_3^3} \\ \langle\hat{\mathcal{R}}_{k_1}\hat{\mathcal{R}}_{k_2}\hat{\mathcal{R}}_{k_3}|_{-\infty}^{\eta_f}\rangle_{\mathcal{R}\dot{\mathcal{R}}^2}^{\text{off-shell}} \propto \dfrac{\left(e_1^2-2e_2\right)\left(\eta_f^2\left(8e_2+e_1\left(e_1\left(\eta_f^2\left(e_1^2-4e_2\right)-4\right)+8\eta_f^2e_3\right)\right)-8\right)}{2e_1^2\left(e_1^3-4e_1e_2+8e_3\right)^2} \end{cases}$$

where $e_1 = k_1+k_2+k_3,\ e_2 = k_1k_2+k_2k_3+k_1k_3,\ e_3 = k_1k_2k_3$

- None of the studied interaction show that we can neglect quantum dynamics.
- Some scales are special though:
  - Folded limit: maximally comparable
  - Squezed limit: classical
- All on-shell functions have physical poles.

Timing semi-classicality

- Starting previous integrals (and equivalently simulations) at a well-chosen time t_0 can make a difference for on vs off-shellness.
- We defined and studied the quantum interactivity

$$\mathcal{QI}(t_0,t_f,\{k_j\}) = \frac{|\langle\hat{\mathcal{R}}_{k_1}\hat{\mathcal{R}}_{k_2}\hat{\mathcal{R}}_{k_3}|_{t_0}^{t_f}\rangle^{\text{off-shell}}|}{|\langle\hat{\mathcal{R}}_{k_1}\hat{\mathcal{R}}_{k_2}\hat{\mathcal{R}}_{k_3}|_{t_0}^{t_f}\rangle^{\text{off-shell}}|+|\langle\hat{\mathcal{R}}_{k_1}\hat{\mathcal{R}}_{k_2}\hat{\mathcal{R}}_{k_3}|_{t_0}^{t_f}\rangle^{\text{on-shell}}|}$$

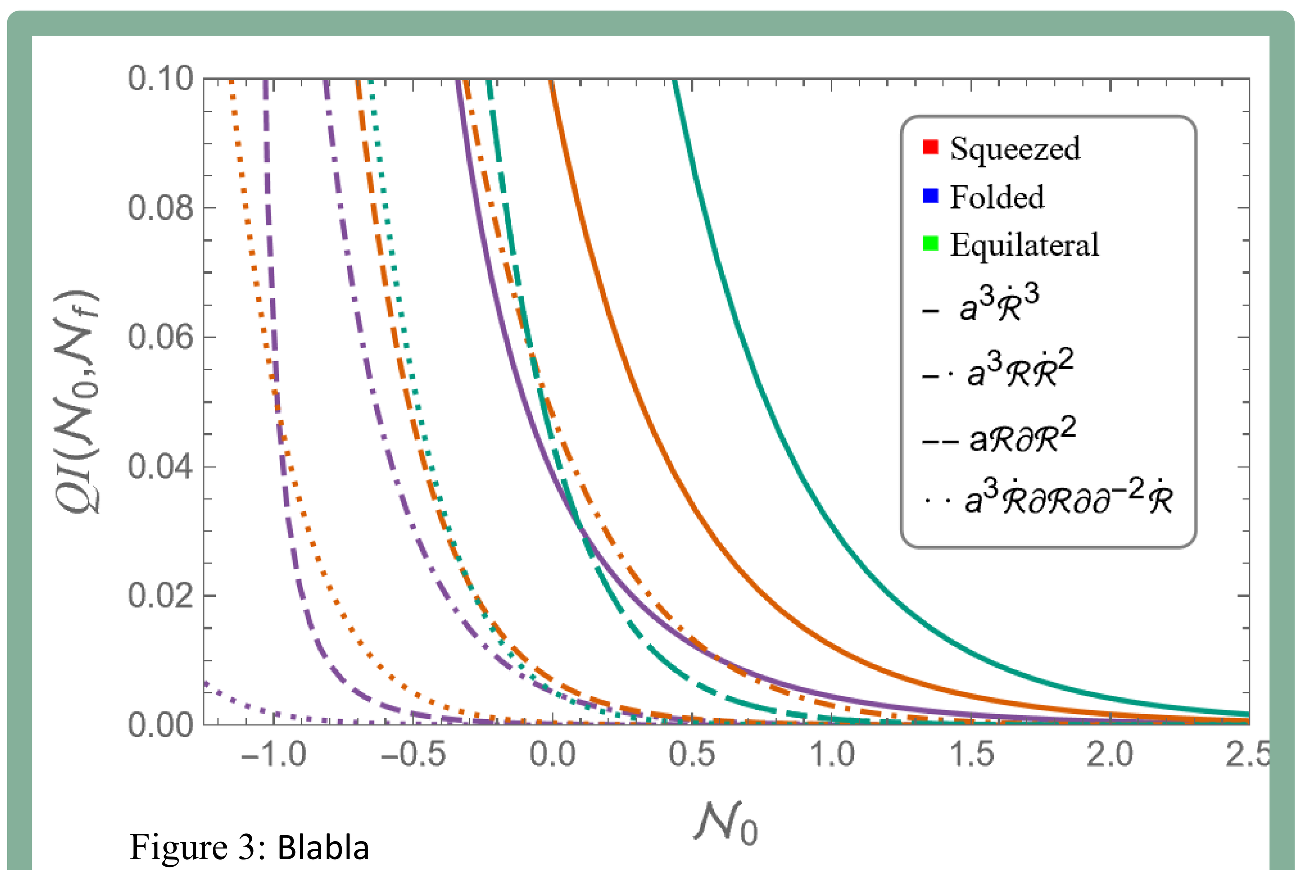


Figure 3: Blabla

- The classicality time of the bispectrum depends on:
  - The precision we have
  - The interactions: ~more time derivatives → later
  - The scales: squeezed is classical early, then comes folded and equilateral

Take-away

- On-shell dynamics have special signatures
- Off and on-shell dynamics compete sufficiently deep in the horizon
- Classicality of the 3pt function can be reached much before what the commutators say, depending on the scales and interactions.

References:

[1] XXX
[2] YYY
[3] ZZZ

SCAN ME

← **Related publication**: Yoann L. Launay et al JCAP05(2025)071.

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