

2.4

2.1 The Diffusion Equation

All programs are attached at the end of the report.

Question 1:

Consider the equation

$$\frac{\partial F}{\partial t}(x, t) = K \frac{\partial^2 F}{\partial x^2}(x, t), F(x, 0) = 0, F(0, t) = 1 \quad (1)$$

For simplicity, instead of (x, t) , we want to use one non-dimensional variable ξ that is a combination of quantities concerned in the equation (x , t , and K) to rewrite our equation as a single variable ODE. Let $\xi = x^a t^b K^c$. Since $[X] = M$, $[t] = S$ and $[K] = M^2 S^{-1}$,¹ by comparing indices we have $a + 2c = b - c = 0$. A reasonable choice is $a = 1, b = c = -\frac{1}{2}$, with $\xi = \frac{x}{(kt)^{1/2}}$.

Now with $F(x, t) = f(\xi) = f(\frac{x}{(Kt)^{1/2}})$, we use the chain rule to obtain:

$$\frac{\partial f}{\partial t} = \frac{\partial \xi}{\partial t} \frac{\partial f}{\partial \xi} = -\frac{x}{2(Kt^3)^{1/2}} \frac{\partial f}{\partial \xi} = -\frac{\xi}{2t} \frac{\partial f}{\partial \xi}$$

$$\frac{\partial f}{\partial x} = \frac{\partial \xi}{\partial x} \frac{\partial f}{\partial \xi} = \frac{1}{(kt)^{1/2}} \frac{\partial^2 f}{\partial \xi^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{Kt} \frac{\partial^2 f}{\partial \xi^2}$$

Equation (1) becomes

$$-\frac{\xi}{2} \frac{\partial f}{\partial \xi} = \frac{\partial^2 f}{\partial \xi^2} \quad (2)$$

Hence

$$\frac{\partial f}{\partial \xi} = A e^{-\xi^2/4}$$

$$f(\xi) = \int_{t_0}^{\xi} A e^{-x^2/4} dx \stackrel{u=\frac{x}{2}}{=} \int_t^{\xi/2} C e^{-u^2} du,$$

where C and t are determined by initial and boundary conditions.

¹Here square brackets denote dimension of a quantity

In terms of ξ , the initial and boundary conditions are $f(\infty) = 0$ and $f(0) = 1$, which gives $t = \infty$ and $C = -\frac{2}{\sqrt{\pi}}$. Therefore

$$f(\xi) = \frac{2}{\sqrt{\pi}} \int_{\xi/2}^{\infty} e^{-u^2} du = \operatorname{erfc}\left(\frac{1}{2}\xi\right) \quad (3)$$

Question 2:

Analytic Solutions:

Now consider the equation

$$U_T = U_{XX}, T > 0, 0 < X < 1 \quad (4)$$

with

$$U(X, 0) = 0 \text{ for } 0 < X < 1; U(0, T) = 1, U(1, T) = 0 \text{ for } T > 0$$

Note that boundary condition is not homogeneous. We identify a steady solution $U_s(X)$ and write $U(X, T) = U_s(X) + U_t(X, T)$ to settle the problem: The appropriate steady solution is $U_s(X) = 1 - X$.

Substitute $U(X, T) = U_s(X) + U_t(X, T)$ in (4) we find that

$$(U_t)_T = (U_t)_{XX}, T > 0, 0 < X < 1 \quad (5)$$

with initial and boundary conditions

$$U_t(X, 0) = X - 1 \text{ for } 0 < X < 1; U_t(0, T) = 0, U_t(1, T) = 0 \text{ for } T > 0 \quad (6)$$

Write $U_t(X, T) = F(X)G(T)$ we have

$$\frac{G'}{G} = \frac{F''}{F} = -\lambda, \text{ for some positive } \lambda \quad (7)$$

Hence

$$F(X) = A\cos(\sqrt{\lambda}X) + B\sin(\sqrt{\lambda}X).$$

Using the boundary condition $F(0) = F(1) = 0$ we have

$$A = 0, \lambda = n^2\pi^2$$

Thus

$$G(T) = e^{-\pi^2 n^2 T}, F(X) = B\sin(n\pi X) \quad (8)$$

The general solution is

$$U_t(X, T) = \sum_1^{\infty} B_n \sin(n\pi X) e^{-\pi^2 n^2 T}, \text{ for some } B_n \text{ to be determined.}$$

Using the initial condition and orthogonality of sine,

$$B_m = 2 \int_0^1 (x-1) \sin(m\pi x) dx = -\frac{2}{m\pi} \quad (9)$$

$$U = 1 - X - \frac{2}{\pi} \sum_1^{\infty} \frac{1}{n} e^{-\pi^2 n^2 T} \sin(n\pi X) \quad (10)$$

Now consider (5) with initial and boundary conditions

$$U_t(X, 0) = -1 \text{ for } 0 < X < 1; U_t(0, T) = 0, (U_t)_X(1, T) = 0 \text{ for } T > 0 \quad (11)$$

Following the same procedure as above we find that

$$U_t = \sum_1^{\infty} C_n \sin((n - \frac{1}{2})\pi X) e^{-\pi^2 (n - \frac{1}{2})^2 T}$$

Note that

$$\begin{aligned} \int_0^1 \sin((n - \frac{1}{2})\pi x) \sin((m - \frac{1}{2})\pi x) dx &= \int_0^1 \frac{1}{2} (\cos((n-m)\pi x) - \cos((m+n-1)\pi x)) dx \\ &= \begin{cases} \frac{1}{2} & m = n \\ 0 & m \neq n \end{cases}. \end{aligned}$$

Exploiting this orthogonality relation we have

$$C_m = -2 \int_0^1 \sin((m - \frac{1}{2})\pi x) dx = -\frac{4}{(2m-1)\pi}$$

$$U(X, T) = 1 - \frac{4}{\pi} \sum_1^{\infty} \frac{1}{(2n-1)} e^{-\pi^2 (n - \frac{1}{2})^2 T} \sin((n - \frac{1}{2})\pi X) \quad (12)$$

Truncation error:

The truncation error in (10) is

$$E_N = -\frac{2}{\pi} \sum_{n=N}^{\infty} \frac{1}{n} e^{-\pi^2 n^2 T} \sin(n\pi X)$$

We have the inequality

$$|E_N| = \left| \frac{2}{\pi} \sum_{n=N}^{\infty} \frac{1}{n} e^{-\pi^2 n^2 T} \sin(n\pi X) \right| \leq \frac{2}{\pi} \sum_{n=N}^{\infty} \left| \frac{1}{n} e^{-\pi^2 n^2 T} \sin(n\pi X) \right| \leq \frac{2}{\pi} \sum_{n=N}^{\infty} |e^{-\pi^2 n^2 T}|$$

$$= \frac{2}{\pi} \sum_{n=0}^{\infty} e^{-\pi^2(N+n)^2T} \leq \frac{2}{\pi} \sum_{n=0}^{\infty} e^{-\pi^2(N^2+nN)T} = \frac{2e^{-\pi^2N^2T}}{\pi(1 - e^{-\pi^2NT})} \quad (13)$$

The truncation error in (12) is

$$E_N = -\frac{4}{\pi} \sum_{n=N}^{\infty} \frac{1}{2n-1} e^{-\pi^2(n-\frac{1}{2})^2T} \sin((n-\frac{1}{2})\pi X)$$

We have the inequality

$$\begin{aligned} |E_N| &\leq \frac{4}{\pi} \sum_{n=N}^{\infty} \left| \frac{1}{2n-1} e^{-\pi^2(n-\frac{1}{2})^2T} \sin((n-\frac{1}{2})\pi X) \right| \leq \frac{4}{\pi} \sum_{n=N}^{\infty} |e^{-\pi^2(n-\frac{1}{2})^2T}| \\ &\leq \frac{4}{\pi} \sum_{n=0}^{\infty} e^{-\pi^2(N^2+nN)T} = \frac{4e^{-\pi^2N^2T}}{\pi(1 - e^{-\pi^2NT})} \end{aligned} \quad (14)$$

For $N = 4$, the above estimation gives $|E_N| \leq 10^{-5}$ for all values of T concerned in the question.

Table 1: Values of U with Different Values of X

Values of X	0.1250	0.2500	0.3750	0.5000	0.6250	0.7500	0.8750	1.0000
Value of U in (10)	0.8543	0.7118	0.5751	0.4460	0.3251	0.2118	0.1044	0.0000
Value of U in (12)	0.8650	0.7355	0.6167	0.5130	0.4284	0.3658	0.3275	0.3146
Semi-infinite Solution	0.8597	0.7237	0.5959	0.4795	0.3768	0.2888	0.2159	0.1573

Graphs:

Now we plot U against X for different values of T :

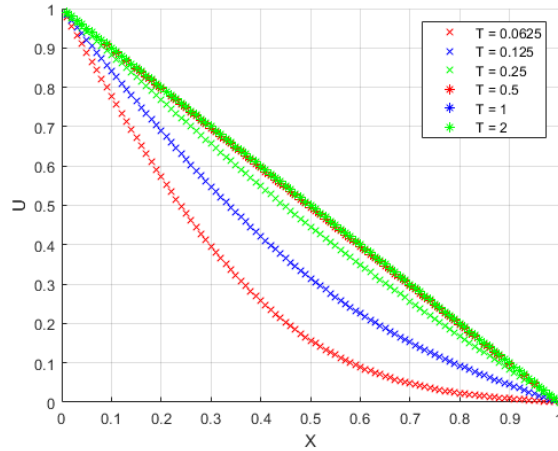


Figure 1: U against X for (10)

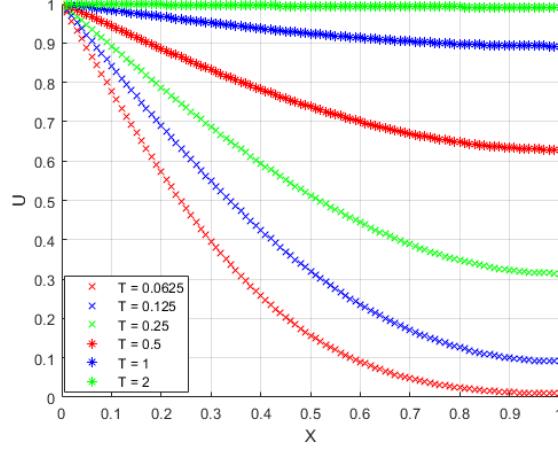


Figure 2: U against X for (12)

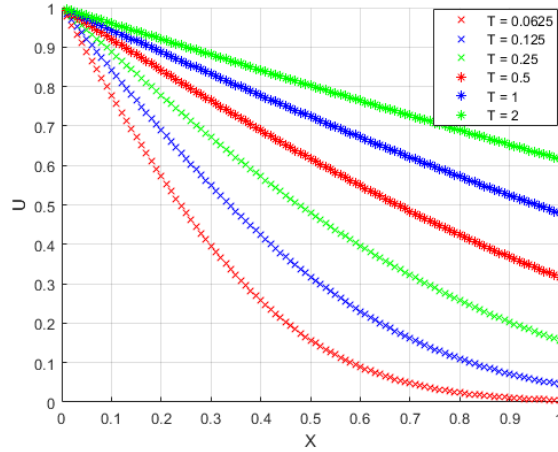


Figure 3: U against X for (3)

From Figure 1 we deduce that with initial condition (6) the solution converges rapidly to the line $U(X, T) = 1 - X$ as T becomes large. This is expected: From (10) and (13) with $N = 1$ we see that as T gets large the series tend to $1 - X$.

From Figure 2 we see that with initial condition (11) the solution converges to the line $U(X, T) = 1$ as T gets large but with a slower rate of convergence than Figure 1. This is expected: From (12) and (14) with $N = 1$ we see that as T gets large the series tend to 1. Also from (12) we see that the exponent of e is proportional to $(n - \frac{1}{2})^2$, smaller than n^2 , leading to a slower rate of convergence.

From Figure 3 we see that, for selected range of T , the graph moves upwards as T becomes large (but rather slowly). This is expected: $f(\xi)$ increases much slower since any increment in T will only result in a decrement proportional to $\frac{1}{\sqrt{T}}$ in the lower limit of the integral. Also, from (3) we deduce that in the limit as T tends to infinity the graph will converge to the line $U(X, T) = 1$. Note also that the analytic solution(3) is a very good approximation to (10).

X-Derivatives:

Differentiate (3) with respect to X :

$$-\frac{\partial f}{\partial X} = -\frac{2}{\sqrt{\pi}} \frac{\partial}{\partial \xi} \int_{\xi/2}^{\infty} e^{-u^2} du \frac{\partial \xi}{\partial x} = \frac{1}{\sqrt{\pi T}} e^{-\frac{X^2}{4T}}$$

Substitute $X = 0$,

$$-U_X(0, T) = \frac{1}{\sqrt{\pi T}} \quad (15)$$

Differentiate (term-by-term) (10) with respect to X :

$$-U_X = 1 + 2 \sum_{n=1}^{\infty} e^{-\pi^2 n^2 T} \cos(n\pi X)$$

Substitute $X = 0$,

$$-U_X(0, T) = 1 + 2 \sum_{n=1}^{\infty} e^{-\pi^2 n^2 T} \quad (16)$$

Consider the truncation error $E_N = 2 \sum_{n=N}^{\infty} e^{-\pi^2 n^2 T}$:

Similar to the above we have

$$|E_N| \leq \frac{2e^{-\pi^2 N^2 T}}{1 - e^{-\pi^2 N T}}$$

Differentiate (term-by-term) (12) with respect to X :

$$-U_X = 2 \sum_{n=1}^{\infty} e^{-\pi^2 n^2 T} \cos((n - \frac{1}{2})\pi X)$$

Substitute $X = 0$,

$$-U_X(0, T) = 2 \sum_{n=1}^{\infty} e^{-\pi^2 n^2 T} \quad (17)$$

The truncation error also satisfies the inequality

$$|E_N| \leq \frac{2e^{-\pi^2 N^2 T}}{1 - e^{-\pi^2 N T}}$$

Based on previous calculation we see that $N = 4$ will suffice for all values of $T \geq 0.0625$, since $\frac{2e^{-\pi^2 N^2 T}}{1 - e^{-\pi^2 N T}}$ is decreasing in T .

We use the first 4 terms in series (13) and (14) to produce the following graphs of $-U_X$ against T in $[0.0625, 1]$:

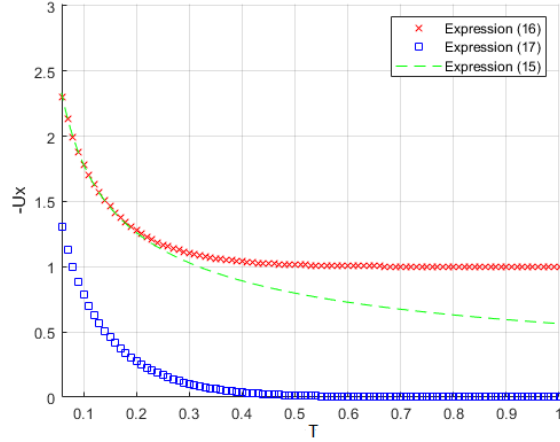


Figure 4: $-U_X(0, T)$ against T for (15), (16), and (17).

From the graph we see that $-U_X(0, T)$ tends to 1, 0, and 0 for (16), (17), and (15) with a descending rate of convergence. This is consistent with the discussion we have for Figure 1, 2, and 3.

Question 3:

Let X -derivative be approximated by central difference quotients²

$$U_X(X, T) = \frac{U(X + \delta X, T) - U(X - \delta X, T)}{2\delta X} = \frac{U_{n+1}^m - U_{n-1}^m}{2\delta X} \quad (18)$$

Substitute $n = N$ in (18). We see that $U_X(1, T) = \frac{U_{N+1}^m - U_{N-1}^m}{2\delta X} = 0$ by (6), which implies $U_{N+1}^m = U_{N-1}^m$. Similarly we approximate U_0^0 by

$$U_0^0 = \frac{1}{2} \left(\lim_{X \rightarrow 0} U(X, 0) + \lim_{T \rightarrow 0} U(0, T) \right) = 0.5 \quad (19)$$

Now we use the recurrence relation to find the numerical solution:

For the case $N = 8, C = 0.5$ the result for different values of T is tabulated below.

Table 2: The Numerical and Analytic Solutions at $T = 0.125, 0.25$

	$T = 0.125$			$T = 0.25$		
	Numerical	Analytic	Error	Numerical	Analytic	Error
$X = 0$	1.00000	1.00000	0.00000	1.00000	1.00000	0.00000
$X = 0.125$	0.80365	0.80274	0.00091	0.86495	0.86504	0.00009
$X = 0.25$	0.61832	0.61753	0.00079	0.73522	0.73554	0.00032
$X = 0.375$	0.45502	0.45441	0.00061	0.61614	0.61666	0.00052
$X = 0.5$	0.31870	0.32001	0.00131	0.51200	0.51299	0.00099
$X = 0.625$	0.21429	0.21726	0.00297	0.42707	0.42838	0.00131
$X = 0.75$	0.14102	0.14603	0.00501	0.36402	0.36584	0.00182
$X = 0.875$	0.09808	0.10457	0.00649	0.32549	0.32748	0.00199
$X = 1$	0.08420	0.09100	0.00680	0.31235	0.31455	0.00220

Table 3: The Numerical and Analytic Solutions at $T = 0.5, 1$

	$T = 0.5$			$T = 1$		
	Numerical	Analytic	Error	Numerical	Analytic	Error
$X = 0$	1.00000	1.00000	0.00000	1.00000	1.00000	0.00000
$X = 0.125$	0.92778	0.92766	0.00012	0.97914	0.97893	0.00021
$X = 0.25$	0.85830	0.85810	0.00020	0.95907	0.95868	0.00039
$X = 0.375$	0.79432	0.79400	0.00032	0.94058	0.94001	0.00057
$X = 0.5$	0.73818	0.73781	0.00037	0.92436	0.92365	0.00071
$X = 0.625$	0.69219	0.69170	0.00049	0.91108	0.91022	0.00086
$X = 0.75$	0.65792	0.65744	0.00048	0.90118	0.90024	0.00094
$X = 0.875$	0.63692	0.63635	0.00057	0.89511	0.89410	0.00101
$X = 1$	0.62973	0.62922	0.00051	0.89303	0.89202	0.00101

The graphs of the analytic and numerical solutions for $T = 0.0625, 0.125, 0.25, 0.5, 1$, and 2 are plotted:

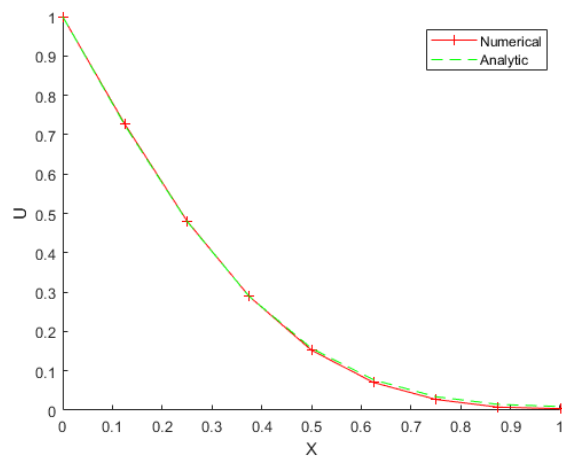


Figure 5: $T = 0.0625$.

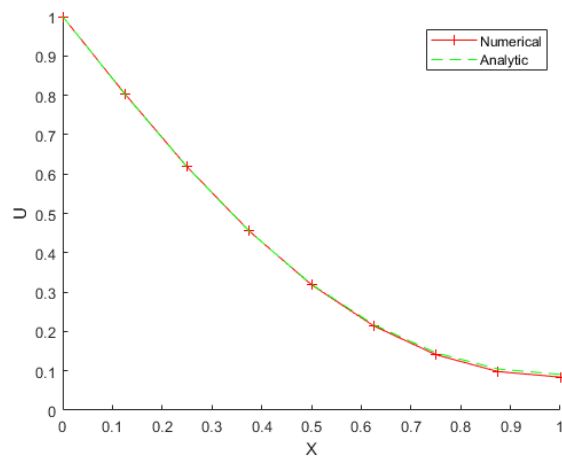


Figure 6: $T = 0.125$.

²MSc Course in Mathematics and Finance, Mark Davis http://wwwf.imperial.ac.uk/~mdavis/FDM11/LECTURE_SLIDES2.PDF

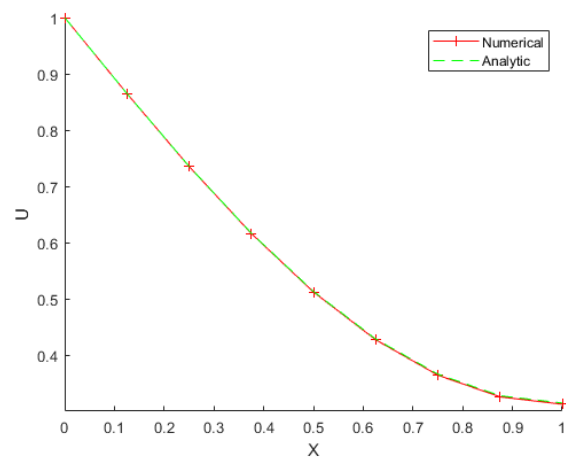


Figure 7: $T = 0.25$.

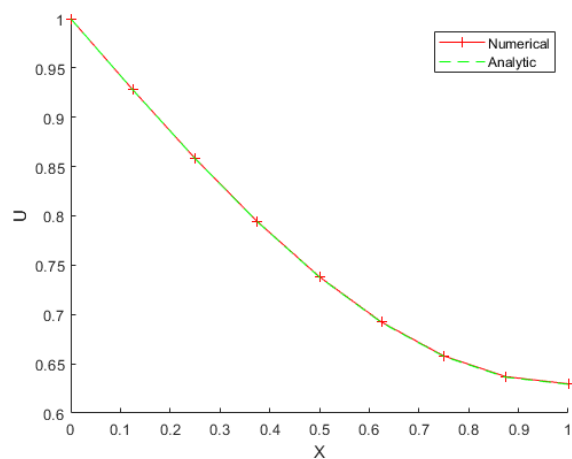


Figure85: $T = 0.5$.

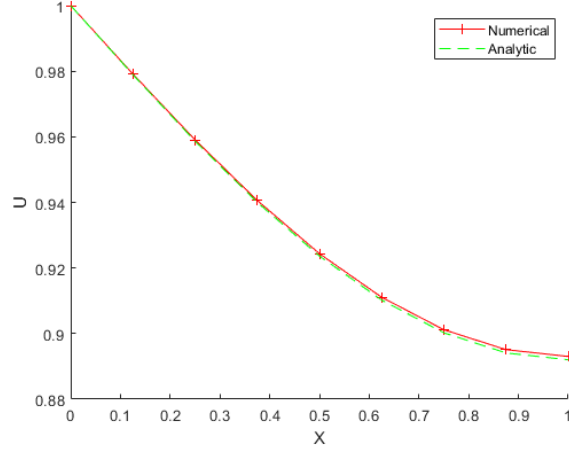


Figure 9: $T = 1$.

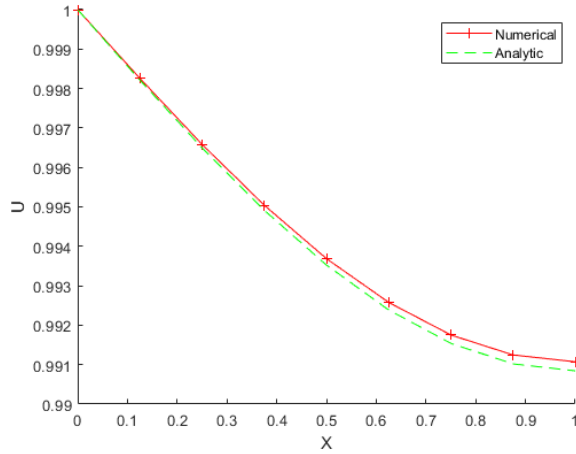


Figure 10: $T = 2$.

Stability of the Numerical Scheme:

Note that the error due to numerical approximation is accumulated at $X = 1$. We use the absolute difference between the analytic and numerical solution at $X = 1$ to give an estimation of error at a particular time.

We run our program for different values of N and C and observe that when $C = \frac{2}{3}$, the numerical scheme is unstable for all values of N (The iterated value of $U(1, 1)$ diverges in each case).

However, when the Finite Element Method does work, the result converges to the analytic solution.

The above phenomena is expected: The theoretic order of accuracy of the

method is $O(\delta T^2)$.³ When C becomes larger while δX is small, δT will become large, which leads to a significant $O(\delta^2)$ term.

Source Code

```
1 function y = FS1(n,x,t) %fourier series solution 1
2 y = 1-x;
3 s = 0;
4 for i = 1:n
5 s = s - (2/(i*pi))*exp(-pi^2*i^2*t)*sin(i*pi*x);
6 end
7 y = y+s;
8 end
```

```
1 function y = FS2(n,x,t) %fourier series solution 2
2 y = 1;
3 s = 0;
4 for i = 1:n
5 s = s - (4/((2*i-1)*pi))*exp(-pi^2*(i-1/2)^2*t)*sin((i-1/2)*pi*x);
6 end
7 y = y+s;
8 end
```

```
1 S1 = zeros(1,8);
2 S2 = zeros(1,8);
3 S3 = zeros(1,8);
4 for j = 1:8
5 format long
6 S1(j) = FS1(3,0.125*j,0.25);
7 S2(j) = FS2(3,0.125*j,0.25);
8 S3(j) = AS(0.125*j,0.25);
9 end
```

```
1 function y = AS(x,t)
2 p = x/sqrt(t);
3 y = erfc(p/2);
4 end
```

```
1 function y = DAS(t)
```

³Taylor expand U_n^{m+1} we see that $U_n^{m+1} = U_n^m + \delta T \frac{\partial U}{\partial T} + O(\delta T^2) = U_n^m + \delta T \left(\frac{U_n^{m+1} - U_n^m}{\delta T} + \delta T \right) = U_n^m + \delta T \left(\frac{U_n^{m+1} - U_n^m}{\delta T} \right) + O(\delta T^2)$. Use the numerical scheme, $U_n^{m+1} = U_n^m + C(U_{n+1}^m - 2U_n^m + U_{n-1}^m) + O(\delta T^2)$

```

2 y = 1/sqrt(pi*t)
3 end

```

```

1 function y = DFS1(n,t)
2 y = 1;
3 s = 0;
4 for j=1:n
5 s = s + 2*exp(-pi^2*j^2*t);
6 end
7 y = y + s;
8 end

```

```

1 function y = DFS2(n,t)
2 y = 0;
3 s = 0;
4 for j=1:n
5 s = s + 2*exp(-pi^2*j^2*t);
6 end
7 y = y + s;
8 end

```

```

1 function [e] = Err(N,C,T)
2 [s,A] = FE(N,C,T);
3 X = listofAS(N,T);
4 e = A(s+1,N+1) - X(N+1);
5 end

```

```

1 function [s,A] = FE(N,C,T)
2 dx = 1/N
3 dt = C*(dx)^2
4 s = round(T/dt)
5 A = zeros(s+1,N+2);
6 A(1,1) = 0.5;
7 for i = 2:s+1
8 A(i,1) = 1;
9 for j = 2:N+1
10 A(i,j) = A(i-1,j)+C*(A(i-1,j+1) - 2*A(i-1,j) + A(i-1,j-1));
11 end
12 A(i,N+2) = A(i,N);
13 end
14 end

```

```

1 function [X] = listofAS(N,T)
2 X = zeros(1,N);
3 dx = 1/N;
4 for i = 1:N+1
5 X(i) = FS2(5, (i-1)*dx,T);
6 end

```

```
7 end
```

```
1 A = zeros(1,100);
2 B = zeros(1,100);
3 C = zeros(1,100);
4 D = zeros(1,100);
5 E = zeros(1,100);
6 F = zeros(1,100);
7 G = zeros(1,100);
8 for i = 1:100
9 A(i) = 0.01*i;
10 end
11 for j = 1:100
12 B(j) = FS1(4,A(j),0.0625);
13 C(j) = FS1(4,A(j),0.125);
14 D(j) = FS1(4,A(j),0.25);
15 E(j) = FS1(4,A(j),0.5);
16 F(j) = FS1(4,A(j),1);
17 G(j) = FS1(4,A(j),2);
18 end
19 hold on
20 xlabel('X');
21 ylabel('U')
22
23 hold on
24 plot(A,B,'rx')
25 plot(A,C,'bx')
26 plot(A,D,'gx')
27 plot(A,E,'r*')
28 plot(A,F,'b*')
29 plot(A,G,'g*')
30 hold off
31
32 grid on
33
34 %second sol
35 A = zeros(1,100);
36 B = zeros(1,100);
37 C = zeros(1,100);
38 D = zeros(1,100);
39 E = zeros(1,100);
40 F = zeros(1,100);
41 G = zeros(1,100);
42 for i = 1:100
43 A(i) = 0.01*i;
44 end
45 for j = 1:100
46 B(j) = FS2(4,A(j),0.0625);
47 C(j) = FS2(4,A(j),0.125);
48 D(j) = FS2(4,A(j),0.25);
49 E(j) = FS2(4,A(j),0.5);
50 F(j) = FS2(4,A(j),1);
51 G(j) = FS2(4,A(j),2);
52 end
53 hold on
54 xlabel('X');
```

```

55 ylabel('U')
56
57 hold on
58 plot(A,B,'rx')
59 plot(A,C,'bx')
60 plot(A,D,'gx')
61 plot(A,E,'r*')
62 plot(A,F,'b*')
63 plot(A,G,'g*')
64 hold off
65
66 grid on
67 %3rd sol
68 A = zeros(1,100);
69 B = zeros(1,100);
70 C = zeros(1,100);
71 D = zeros(1,100);
72 E = zeros(1,100);
73 F = zeros(1,100);
74 G = zeros(1,100);
75 for i = 1:100
76 A(i) = 0.01*i;
77 end
78 for j = 1:100
79 B(j) = AS(A(j),0.0625);
80 C(j) = AS(A(j),0.125);
81 D(j) = AS(A(j),0.25);
82 E(j) = AS(A(j),0.5);
83 F(j) = AS(A(j),1);
84 G(j) = AS(A(j),2);
85 end
86 hold on
87 xlabel('X');
88 ylabel('U')
89
90 hold on
91 plot(A,B,'rx')
92 plot(A,C,'bx')
93 plot(A,D,'gx')
94 plot(A,E,'r*')
95 plot(A,F,'b*')
96 plot(A,G,'g*')
97 hold off
98
99 grid on

```

```

1 A = zeros(1,100);
2 B = zeros(1,100);
3 C = zeros(1,100);
4 D = zeros(1,100);
5 for i = 1:100
6 A(i) = 0.01*i;
7 end
8 for j = 1:100
9 B(j) = DFS1(4,A(j));
10 C(j) = DFS2(4,A(j));

```



```

11 D(j) = DAS(A(j));
12 end
13 hold on
14 xlabel('X');
15 ylabel('-Ux')
16
17 hold on
18 plot(A,B,'rx')
19 plot(A,C,'bs')
20 plot(A,D,'g--')
21 hold off
22
23 grid on

```

```

1 [s,A] = FE(8,0.5,0.0625); %t=0.0625
2 for j = 1:9
3 K(j) = (j-1)*0.125;
4 end
5 X = A(s+1,:);
6 for i = 1:9
7 Y(i) = X(i);
8 end
9 Z = listofAS(8,0.0625);
10 hold on
11 xlabel('X')
12 ylabel('U')
13 plot(K,Y,'r-+')
14 plot(K,Z,'g--')
15 hold off
16 %t = 0.125
17 [s,A] = FE(8,0.5,0.125);
18 for j = 1:9
19 K(j) = (j-1)*0.125;
20 end
21 X = A(s+1,:);
22 for i = 1:9
23 Y(i) = X(i);
24 end
25 Z = listofAS(8,0.125);
26 hold on
27 xlabel('X')
28 ylabel('U')
29 plot(K,Y,'r-+')
30 plot(K,Z,'g--')
31 hold off
32 %t = 0.25
33 [s,A] = FE(8,0.5,0.25);
34 for j = 1:9
35 K(j) = (j-1)*0.125;
36 end
37 X = A(s+1,:);
38 for i = 1:9
39 Y(i) = X(i);
40 end
41 Z = listofAS(8,0.25);
42 hold on

```

```

43 xlabel('X')
44 ylabel('U')
45 plot(K,Y,'r-+')
46 plot(K,Z,'g--')
47 hold off
48 %T=0.5
49 [s,A] = FE(8,0.5,0.5);
50 for j = 1:9
51 K(j) = (j-1)*0.125;
52 end
53 X = A(s+1,:);
54 for i = 1:9
55 Y(i) = X(i);
56 end
57 Z = listofAS(8,0.5);
58 hold on
59 xlabel('X')
60 ylabel('U')
61 plot(K,Y,'r-+')
62 plot(K,Z,'g--')
63 hold off
64 %t=1
65 [s,A] = FE(8,0.5,1);
66 for j = 1:9
67 K(j) = (j-1)*0.125;
68 end
69 X = A(s+1,:);
70 for i = 1:9
71 Y(i) = X(i);
72 end
73 Z = listofAS(8,1);
74 hold on
75 xlabel('X')
76 ylabel('U')
77 plot(K,Y,'r-+')
78 plot(K,Z,'g--')
79 hold off
80 %t = 2
81 [s,A] = FE(8,0.5,2);
82 for j = 1:9
83 K(j) = (j-1)*0.125;
84 end
85 X = A(s+1,:);
86 for i = 1:9
87 Y(i) = X(i);
88 end
89 Z = listofAS(8,2);
90 hold on
91 xlabel('X')
92 ylabel('U')
93 plot(K,Y,'r-+')
94 plot(K,Z,'g--')
95 hold off

```