Trouver la transformée de Laplace inverse de

$$X(s) = \frac{2s+1}{\underbrace{(s^2+5s+6)}_{(s+2)(s+3)}(s^2+4s+8)}$$

$$= \frac{A}{(s+2)} + \frac{B}{(s+3)} + \frac{Cs+D}{s^2+4s+8}$$

$$A = \left.\frac{(2s+1)}{(s+3)(s^2+4s+8)}\right|_{s=-2} \qquad \frac{-3}{(1)(4)} = -3/4$$

$$B = \left.\frac{2s+1}{(s+2)(s^2+4s+8)}\right|_{s=-3} \qquad \frac{-5}{(-1)5} = 1$$

$C = ?$ termes en s^3.

$$As^3 + Bs^3 + Cs^3 = 0 \Rightarrow C = -1/4$$

$D = ?$ termes indépendant

$$24A + 16B + 6D = 1$$
$$-18 + 16 + 6D = 1 \rightsquigarrow D = 1/2$$

$$= \frac{-3/4}{(s+2)} + \frac{1}{s+3} - 1/4 \; \frac{s-2}{\underbrace{s^2+4s+8}_{(s+2)^2+4}}$$

$$= \frac{-3/4}{s+2} + \frac{1}{s+3} - 1/4 \left(\frac{s+2}{(s+2)^2+4} - \frac{4}{(s+2)^2+4} \right)$$

$$x(t) = \left(-3/4\, e^{-2t} + e^{-3t} - 1/4\left(e^{-2t}\cos 2t - 2e^{-2t}\sin 2t\right)\right)u(t)$$

$$= \left(-3/4\, e^{-2t} + e^{-3t} - 1/4\, e^{-2t}\cos 2t + 1/2\, e^{-2t}\sin 2t\right)u(t)$$

Résoudre $y'''(t) + y''(t) = (e^t + t + 1)\,u(t)$

$y(0) = y'(0) = y''(0) = 0$

$$s^3 Y(s) + s^2 Y(s) = \frac{1}{s-1} + \frac{1}{s^2} + \frac{1}{s}$$
$$= \frac{s^2 + (s-1) + s(s-1)}{s^2(s-1)}$$
$$= \frac{2s^2 - 1}{s^2(s-1)}$$

$$Y(s) = \frac{2s^2 - 1}{s^4(s-1)(s+1)}$$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s^4} + \frac{E}{(s+1)} + \frac{F}{s-1}$$

$$A s^3(s-1)(s+1) + B s^2(s-1)(s+1) + C s(s-1)(s+1)$$
$$+ D(s-1)(s+1) + E s^4(s-1) + F s^4(s+1)$$
$$= 2s^2 - 1$$

$s = 0 \quad -D = -1 \Rightarrow D = 1$

$s = 1 \quad 2F = 1 \Rightarrow F = 1/2$

$s = -1 \quad -2E = 1 \Rightarrow E = -1/2$

Termes en s^5 $\quad A + E + F = 0 \Rightarrow A = 0$

en s^4 $\quad B \underbrace{- E + F}_{1} = 0 \Rightarrow B = -1$

$s^3 \quad C = 0$

$$Y(s) = \frac{-1}{s^2} + \frac{1}{s^4} - 1/2\,\frac{1}{s+1} + 1/2\,\frac{1}{s-1}$$

$$\Rightarrow \boxed{y(t) = \left(-t + \frac{t^3}{3!} - 1/2\,e^{-t} + 1/2\,e^{t}\right)u(t)}$$