$y[-1] = 3,\ y[-2] = 2$

$$y[n] - 5y[n-1] + 6y[n-2] = u[n]$$

$$Y(z) - 5\left(z^{-1}Y(z) + y[-1]\right) + 6\left(z^{-2}Y(z) + z^{-1}y[-1] + y[-2]\right) = \frac{1}{1-z^{-1}}$$

$$\underbrace{\left(1 - 5z^{-1} + 6z^{-2}\right)}_{(1-2z^{-1})(1-3z^{-1})}Y(z) = \underbrace{5y[-1]}_{15} - \underbrace{6y[-2]}_{12} - 6y[-1]z^{-1} + \frac{1}{1-z^{-1}}$$

$$= 3 - 18z^{-1} + \frac{1}{1-z^{-1}}$$

$$= \frac{3 - 18z^{-1} - 3z^{-1} + 18z^{-2} + 1}{1-z^{-1}}$$

$$= \frac{4 - 21z^{-1} + 18z^{-2}}{(1-z^{-1})}$$

$$Y(z) = \frac{4 - 21z^{-1} + 18z^{-2}}{(1-2z^{-1})(1-3z^{-1})(1-z^{-1})}$$

$$= \frac{A}{1-2z^{-1}} + \frac{B}{1-3z^{-1}} + \frac{C}{1-z^{-1}}$$

$$A = \left.\frac{4 - 21z^{-1} + 18z^{-2}}{(1-3z^{-1})(1-z^{-1})}\right|_{z^{-1}=1/2} = \frac{4 - \frac{21}{2} + \frac{18}{4}}{(-1/2)(1/2)}$$

$$= \frac{\frac{8-21+9}{2}}{-1/4}$$

$$= 8$$

$$B = \left.\frac{4 - 21z^{-1} + 18z^{-2}}{(1-2z^{-1})(1-z^{-1})}\right|_{z^{-1}=1/3} = \frac{4 - 7 + 2}{\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)}$$

$$= -9/2$$

$$C = \left.\frac{4 - 21z^{-1} + 18z^{-2}}{(1-2z^{-1})(1-3z^{-1})}\right|_{z^{-1}=1} = \frac{4 - 21 + 18}{(-1)(-2)}$$

$$= +1/2$$

$$Y(z) = \frac{8}{1-2z^{-1}} + \frac{-9/2}{1-3z^{-1}} + \frac{1/2}{1-z^{-1}}$$

$$\boxed{y[n] = \left[8\,2^n - 9/2\,3^n + 1/2\right]u[n]}$$

Considerez un système linéaire invariant dans le temps

$$H(z) = \frac{z}{z-1/2}$$

a) Réponse à $x[n] = u[n]$

b) Réponse à $x[n] = n\,a^n u[n]$ $(a \neq 1/2)$

a) $X(z) = \frac{z}{z-1} \Rightarrow Y(z) = \frac{z}{z-1/2}\,\frac{z}{z-1}$

$$\frac{Y(z)}{z} = \frac{A}{z-1/2} + \frac{B}{z-1} = \frac{z}{(z-1/2)(z-1)}$$

$$A = \left.\frac{z}{z-1}\right|_{z=1/2} = \frac{1/2}{-1/2} = -1$$

$$B = \left.\frac{z}{z-1/2}\right|_{z=1} = \frac{1}{1/2} = 2$$

$$Y(z) = \frac{2z}{(z-1)} - \frac{z}{z-1/2}$$

$$\boxed{y[n] = \left[2 - (1/2)^n\right] u[n]}$$

b) $X(z) = \frac{az}{(z-a)^2} \Rightarrow Y(z) = \frac{z}{z-1/2}\,\frac{az}{(z-a)^2}$

$$\frac{Y(z)}{z} = \frac{A}{(z-1/2)} + \frac{B}{(z-a)} + \frac{C}{(z-a)^2} = \frac{az}{(z-1/2)(z-a)^2}$$

$$A = \left.\frac{az}{(z-a)^2}\right|_{z=1/2} = \frac{a/2}{(1/2-a)^2} = \frac{2a}{(1-2a)^2}$$

$$C = \left.\frac{az}{z-1/2}\right|_{z=a} = \frac{a^2}{a-1/2} = \frac{2a^2}{2a-1}$$

$$B = \left.\left[\frac{az}{z-1/2}\right]'\right|_{z=a} = a\left.\frac{(z-1/2)-z}{(z-1/2)^2}\right|_{z=a} = \frac{-1/2\,a}{(a-1/2)^2} = \frac{-2a}{(2a-1)^2}$$

$$Y(z) = \frac{2a}{(1-2a)^2}\,\frac{z}{z-1/2} - \frac{2a}{(2a-1)^2}\,\frac{z}{z-a} + \frac{2a}{2a-1}\,\frac{az}{(z-a)^2}$$

$$\boxed{y[n] = \left[\frac{2a}{(1-2a)^2}\left(\frac{1}{2}\right)^n - \frac{2a}{(2a-1)^2}(a)^n + \frac{2a}{(2a-1)}\,n\,a^n\right] u[n]}$$