$$y'' + 4y' + 3y = 100e^{-2t}u(t) \qquad \begin{cases} y(0) = -20 \\ y'(0) = 0 \end{cases}$$

$$s^2Y(s) - sy(0) - y'(0) + 4\left[sY(s) - y(0)\right] + 3Y(s) = \frac{100}{s+2}$$

$$\underbrace{(s^2 + 4s + 3)}_{(s+1)(s+3)}Y(s) = (s+4)y(0) + \frac{100}{s+2}$$

$$Y(s) = \frac{s+4}{(s+1)(s+3)}y(0) + \frac{100}{(s+1)(s+3)(s+2)}$$

$$= \frac{-20(s+4)}{(s+1)(s+3)} + \frac{100}{(s+1)(s+3)(s+2)}$$

$$= \frac{A}{s+1} + \frac{B}{s+3} + \frac{\bar{A}}{s+1} + \frac{\bar{B}}{s+3} + \frac{\bar{C}}{s+2}$$

$$A = \left.\frac{-20(s+4)}{s+3}\right|_{s=-1} = \frac{-20(3)}{2} = -30$$

$$B = \left.\frac{-20(s+4)}{(s+1)}\right|_{s=-3} = \frac{-20(1)}{-2} = 10$$

$$\bar{A} = \left.\frac{100}{(s+3)(s+2)}\right|_{s=-1} = \frac{100}{2(1)} = 50$$

$$\bar{B} = \left.\frac{100}{(s+1)(s+2)}\right|_{s=-3} = \frac{100}{(-2)(-1)} = 50$$

$$\bar{C} = \left.\frac{100}{(s+1)(s+3)}\right|_{s=-2} = \frac{100}{(-1)(1)} = -100$$

$$Y(s) = \frac{20}{s+1} + \frac{60}{s+3} - \frac{100}{s+2} \qquad y(0) = -20 \ \checkmark$$

$$y(t) = \left[20e^{-t} + 60e^{-3t} - 100e^{-2t}\right]u(t)$$

$$\mathcal{L}\{y''(t)\} = s\,\mathcal{L}\{y'(t)\} - y'(0) \qquad \mathcal{L}\{y'(t)\} = sY(s) - y(0)$$

$$= s\left(sY(s) - y(0)\right) - y'(0)$$

$$= s^2Y(s) - sy(0) - y'(0)$$