

$$y''(t) + 2y'(t) + y(t) = t u(t) \quad y(0) = -1$$

$$y'(0) = 0$$

$$s^2 Y(s) - s y(0) - y'(0) + 2(s Y(s) - y(0)) + Y(s) = \frac{1}{s^2}$$

$$\left(\frac{s^2 + 2s + 1}{(s+1)^2} \right) Y(s) = -s - 2 + \frac{1}{s^2} = -\frac{s^3 - 2s^2 + 1}{s^2}$$

$$Y(s) = \frac{-s^3 - 2s^2 + 1}{s^2 (s+1)^2} = \frac{(s+1)(-s^2 - s + 1)}{s^2 (s+1)^2}$$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} = -\frac{(s^2 + s - 1)}{s^2 (s+1)}$$

$$C = -\left(\frac{s^2 + s - 1}{s^2} \right) \Big|_{s=-1} = -\frac{(-1 - 1 - 1)}{1} = 1$$

$$B = -\left(\frac{s^2 + s - 1}{s+1} \right) \Big|_{s=0} = 1$$

Then in eq

$$A + C = -1$$

$$A = -1 - C = -2$$

$$= -\frac{2}{s} + \frac{1}{s^2} + \frac{1}{s+1}$$

$$\boxed{y(t) = (-2 + t + e^{-t}) u(t)}$$