Signals and systems

Signals, systems and tools
Signals, systems and telecommunications

Exercises 2: Laplace transform*

Franky De Bruyne

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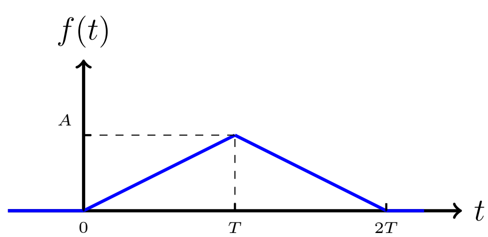
1. Compute the Laplace transforms and associated regions of convergence of the following functions:

   (a) $f(t) = (1 - 3\,t + 4t^5)\,u(t)$

   (b) $f(t) = e^{-2t}\sin(t)\,u(t)$

   (c) $f(t) = e^{-2t}t^2\,u(t)$

   (d)



   (e) $f(t) = A\,\sin(t), 0 < t < 2\pi; \qquad f(t) = 0, t \leq 0$ ou $t \geq 2\pi$

   (f*) $f(t) = e^{-t}\,u(t) + e^{2t}\,u(-t)$

*Exercises **marked with an * are there FYI only** as they concern **non causal signals**.

2. Determine the inverse Laplace transforms of the following functions:

   (a) $F(s) = \dfrac{3s+5}{s^2+3s+2}$

   (b) $F(s) = \dfrac{2s+3}{s^2+2s+4}$

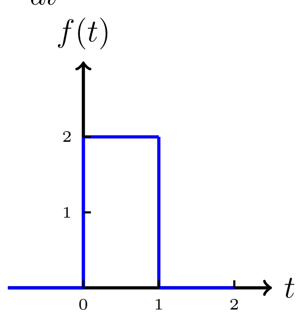
   (c) $F(s) = \dfrac{4}{s^3+4s^2+4s}$

3. Solve the following differential equations:

   (a) $\dfrac{d^2y(t)}{dt^2} + 3\,\dfrac{dy(t)}{dt} + 2\,y(t) = u(t), y(0) = 1, \dfrac{dy(0)}{dt} = 0$

   (b) $\dfrac{d^2y(t)}{dt^2} + 3\,\dfrac{dy(t)}{dt} + 2\,y(t) = e^t\,u(t), y(0) = 1, \dfrac{dy(0)}{dt} = 0$

   (c) $\dfrac{dy(t)}{dt} + 3\,y(t) = f(t), y(0) = 0$



4. Compute the convolution of the following functions:

   (a) $x(t) = \delta(t-1), h(t) = 3e^{-2t}\,u(t)$

   (b) $x(t) = u(t-1), h(t) = 3e^{-2t}\,u(t)$

   (c) $x(t) = r(t-1), h(t) = 3e^{-2t}\,u(t)$

5. Compute the impulse and step responses of the following systems described by their transfer function:

   (a) $F(s) = \dfrac{T_1\, s + 1}{T_2\, s + 1}$

   (b) $F(s) = \dfrac{1}{(T_1\, s + 1)(T_2\, s + 1)}$

Solutions:

1. (a) $F(s) = \dfrac{1}{s} - \dfrac{3}{s^2} + \dfrac{480}{s^6}$

   (b) $F(s) = \dfrac{1}{s^2 + 4s + 5}$

   (c) $F(s) = \dfrac{2}{(s+2)^3}$

   (d) $F(s) = \dfrac{A}{T\,s^2}\,(1 - e^{-Ts})^2$

   (e) $F(s) = \dfrac{A}{s^2+1}\,(1 - e^{-2\pi s})$

   (f) $F(s) = \dfrac{3}{(s+1)(2-s)}, \quad \mathrm{DC} = \{-1 < \mathcal{R}_e[s] < 2\}$

2. (a) $f(t) = (2\,e^{-t} + e^{-2t})\,u(t)$

   (b) $f(t) = \left(\frac{1}{\sqrt{3}} \sin(\sqrt{3}t) + 2\cos(\sqrt{3}t\right)\,e^{-t}\,u(t)$

   (c) $f(t) = (1 - e^{-2t} - 2te^{-2t})u(t)$

3. (a) $y(t) = (\frac{1}{2} + e^{-t} - \frac{1}{2}\,e^{-2t})\,u(t)$

   (b) $y(t) = (\frac{1}{6}\,e^{t} + \frac{3}{2}\,e^{-t} - \frac{2}{3}\,e^{-2t})\,u(t)$

   (c) $y(t) = \frac{2}{3}(1 - e^{-3t})u(t) - \frac{2}{3}(1 - e^{-3(t-1)})u(t-1)$

4. (a) $y(t) = 3\,e^{-2(t-1)}\,u(t-1)$

(b) $y(t) = 1.5\,(1 - e^{-2(t-1)})\,u(t-1)$

(c) $y(t) = \frac{3}{4}\,(e^{-2(t-1)} - 1 + 2(t-1))\,u(t-1)$

5. (a) $h(t) = \dfrac{T_1}{T_2}\,\delta(t) + \dfrac{T_2 - T_1}{T_2^2}\,e^{-\frac{t}{T_2}}\,u(t)$

$$s(t) = \left(1 + \frac{T_1 - T_2}{T_2}\,e^{-\frac{t}{T_2}}\right)\,u(t)$$

(b) $h(t) = \dfrac{1}{(T_2 - T_1)}\,\left(e^{-\frac{t}{T_2}} - e^{-\frac{t}{T_1}}\right)\,u(t)$

$$s(t) = \left(1 + \frac{1}{(T_2 - T_1)}\,\left(T_1\,e^{-\frac{t}{T_1}} - T_2\,e^{-\frac{t}{T_2}}\right)\right)\,u(t)$$