Signals and systems

Signals, systems and tools
Signals, systems and telecommunications

Exercises 1: signals and systems

Franky De Bruyne

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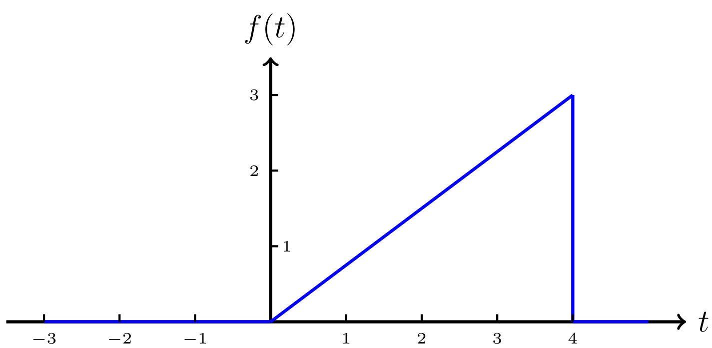
1. Sketch the following functions:

    (a) $f(t-2)$

    (b) $f(2t)$

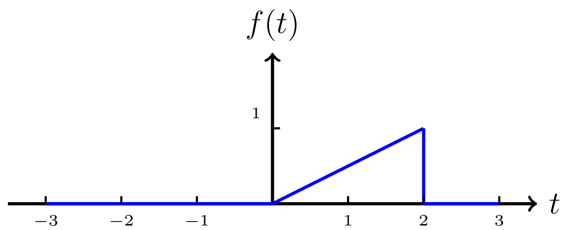
    (c) $f(\frac{t}{2})$

    (d) $f(-\frac{t}{2}+2)$

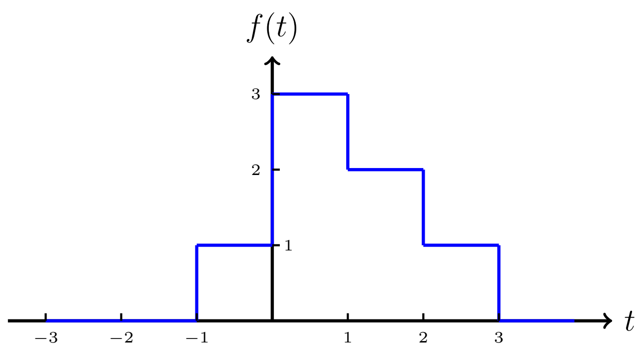


2. Express the following functions using elementary basic functions. Compute and sketch the derivatives.

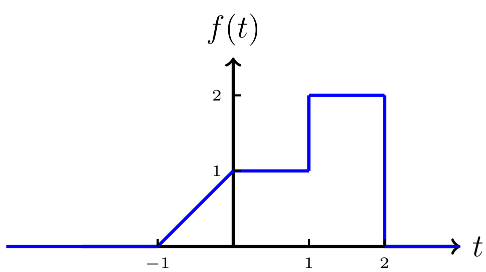
   (a)



   (b)



3. Consider the following functions.



   Sketch

   (a) $f(t)\,u(1-t)$
   (b) $f(t)\,(u(t)-u(t-1))$
   (c) $f(t)\,\delta(t-\frac{3}{2})$

4. Sketch the following functions:

   (a) $\Pi(2t+6)$

   (b) $r(-0.5t+2)$

   (c) $2\,\Lambda(0.5t-4)$

5. Determine if the following functions are periodic. If yes, what is the underlying period ?

   (a) $\cos(\pi t)+\cos(t)$

   (b) $\cos(\pi t)+\cos(\frac{3\pi}{2}t)$

   (c) $\cos(t)+\cos(\frac{5}{2}t)$

6. Evaluate the following expressions:

   (a) $\int_{-\infty}^{\infty} e^{-\alpha t^2}\delta(t-10)\,dt$

   (b) $\int_{0}^{\infty} e^{-\alpha t^2}\delta(t-10)\,dt$

   (c) $\int_{0}^{\infty} e^{-\alpha t^2}\delta(t+10)\,dt$

   (d) $t\,\delta(t-1)$

   (e) $\sin(t)\,\delta(t-\frac{\pi}{2})$

   (f) $\cos(t)\,\delta(t-\pi)$

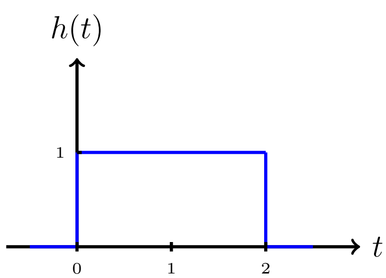
7. Determine if the following signals have finite energie or power:

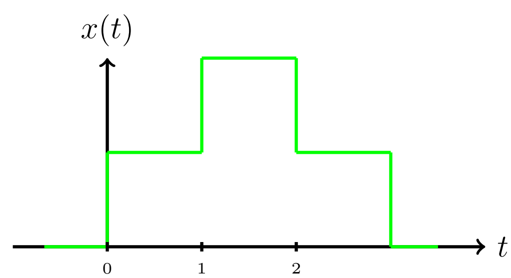
   (a) $A\,e^{-at}\,u(t)$

   (b) $A\cos(\omega_0 t+\theta)$

   (c) $t\,u(t)$

8. Evaluate the convolution of $h(t)$ and $x(t)$ graphically:





9. A system has the step response

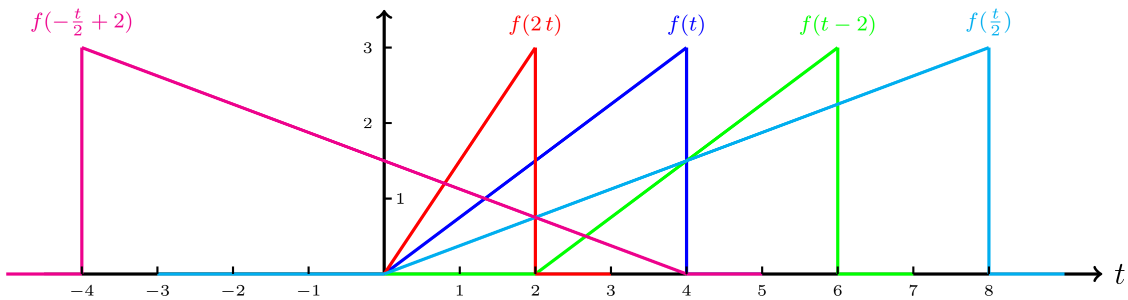
$$y(t) = e^{-t}\,u(t).$$

Determine and sketch its response to the input signal

$$x(t) = u(t-1) - u(t-3).$$

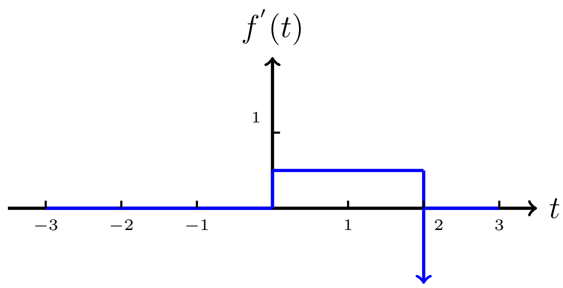
Solutions:

1.



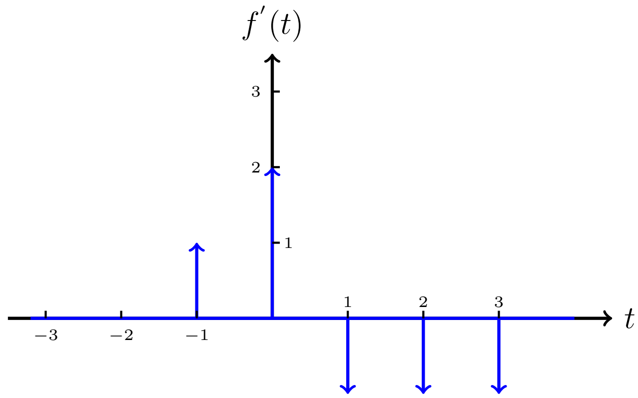
2. (a) $f(t) = \frac{t}{2}(u(t) - u(t-2))$

$$\frac{df(t)}{dt} = -\delta(t-2) + \frac{1}{2}\left(u(t) - u(t-2)\right)$$

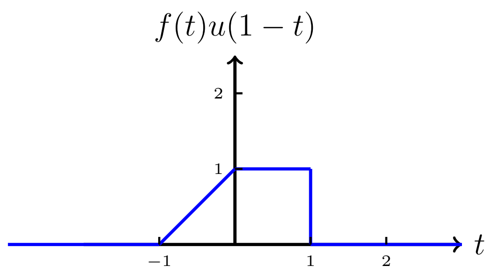


(b) $f(t) = u(t+1) + 2u(t) - u(t-1) - u(t-2) - u(t-3))$

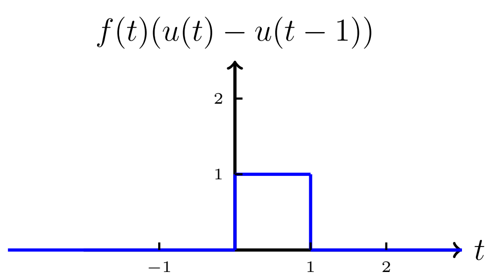
$$\frac{df(t)}{dt} = \delta(t+1) + 2\delta(t) - \delta(t-1) - \delta(t-2) - \delta(t-3)$$



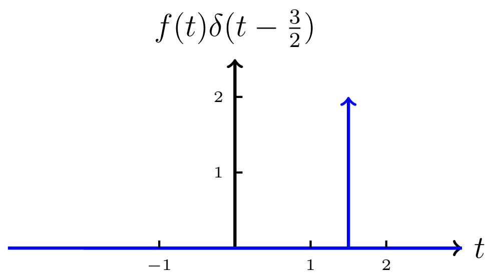
3. (a)



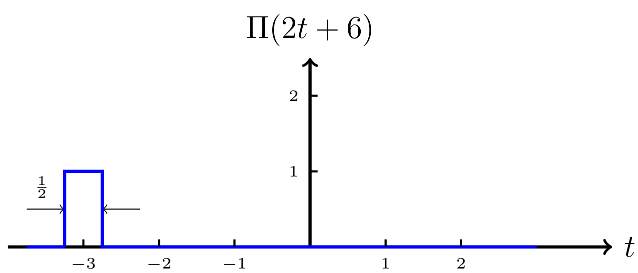
(b)



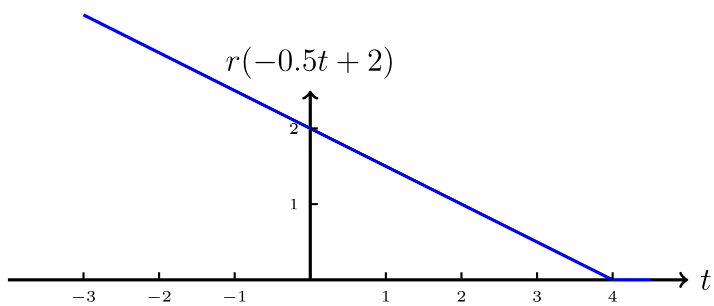
(c)


$f(t)\delta(t-\frac{3}{2})$
2
1
−1
1
2
t

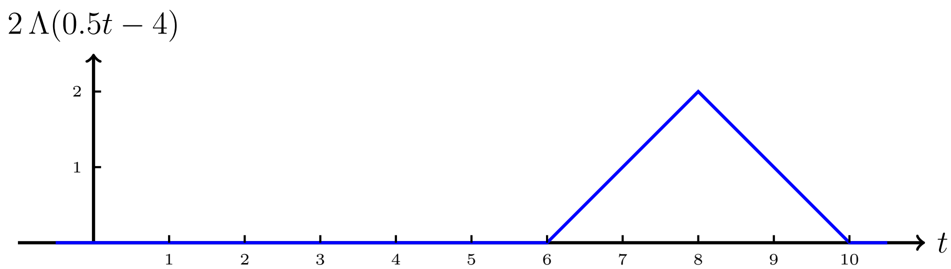
4. (a)


$\Pi(2t+6)$
2
1
$\frac{1}{2}$
−3
−2
−1
1
2
t

(b)


$r(-0.5t+2)$
2
1
−3
−2
−1
1
2
3
4
t

(c)



5. The sum of 2 periodic signals, $x_1(t)$ of period T_1 and $x_2(t)$ of period T_2 is periodic if

$$\frac{T_1}{T_2} = \frac{N}{M}$$

where N, M are integers. If N and M have no common dividers then the period of the sum $x_1(t) + x_2(t)$ is

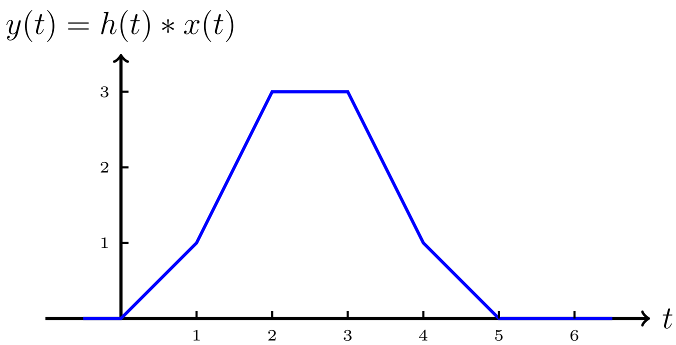
$$M\,T_1 = N\,T_2.$$

(a) $x_1(t) + x_2(t) = \cos(\pi\, t) + \cos(t)$ is aperiodic.

(b) $x_1(t) + x_2(t) = \cos(\pi\, t) + \cos(\frac{3\pi}{2}\, t)$ is periodic of period $T = 4$.

(c) $x_1(t) + x_2(t) = \cos(t) + \cos(\frac{5}{2}\, t)$ is periodic of period $T = 4\pi$.

6. (a) $\int_{-\infty}^{\infty} e^{-\alpha t^2} \delta(t-10)\, dt = e^{-100\alpha}$
   (b) $\int_{0}^{\infty} e^{-\alpha t^2} \delta(t-10)\, dt = e^{-100\alpha}$
   (c) $\int_{0}^{\infty} e^{-\alpha t^2} \delta(t+10)\, dt = 0$
   (d) $t\, \delta(t-1) = \delta(t-1)$
   (e) $\sin(t)\, \delta(t-\frac{\pi}{2}) = \delta(t-\frac{\pi}{2})$
   (f) $\cos(t)\, \delta(t-\pi) = -\delta(t-\pi)$
7. (a) $A\, e^{-at}\, u(t), \quad E = \frac{A^2}{2a}$
   (b) $A \cos(\omega_0 t + \theta), \quad E = \infty, \quad P = \frac{A^2}{2}$
   (c) $t\, u(t), \quad E = \infty, \quad P = \infty$
8.



9. The response $y_p(t)$ of the system to $x(t) = u(t-1) - u(t-3)$ is

$$y_p(t) = e^{-(t-1)}\, u(t-1) - e^{-(t-3)}\, u(t-3).$$