

$$6y[n-2] + y[n-1] - y[n] = 0 \quad y[-1] = -1/6$$

$$y[-2] = 1/36$$

$$6(z^{-2}Y(z) + z^{-1}Y(z) + Y(z)) + z^{-1}Y(z) + Y(z) - Y(z) = 0$$

$$(6z^{-2} + z^{-1} - 1)Y(z) = -\frac{6z^{-1}Y[-1]}{z^{-1}} - \frac{6Y[-2]}{z^{-1}} - Y[-1]$$

$$(3z^{-1} - 1)(2z^{-1} + 1)Y(z) = \frac{z^{-1}}{(3z^{-1} - 1)(2z^{-1} + 1)}$$

$$Y(z) = \frac{-z^{-1}}{(1 - 3z^{-1})(1 + 2z^{-1})}$$

$$= \frac{A}{(1 - 3z^{-1})} + \frac{B}{(1 + 2z^{-1})}$$

$$A = \frac{-z^{-1}}{(1 + 2z^{-1})} \bigg|_{z^{-1} = 1/3} = \frac{-1/3}{1 + 2/3} = -\frac{1}{5}$$

$$B = \frac{-z^{-1}}{1 - 3z^{-1}} \bigg|_{z^{-1} = -1/2} = \frac{1/2}{1 + 3/2} = 1/5$$

$$Y(z) = -1/5 \frac{1}{1 - 3z^{-1}} + 1/5 \frac{1}{1 + 2z^{-1}}$$

$$y[n] = 1/5 \left((-2)^n - (3)^n \right) u[n]$$

$$y[n] = 6y[n-2] + y[n-1]$$

$$n=0 \quad y[0] = 6y[-2] + y[-1] \quad \text{OK}$$

$$n=1 \quad y[1] = 6y[-1] + y[0] \quad \text{OK}$$