$$y[n+2] - 4y[n+1] + 4y[n] = 0 \qquad y[0] = 1,\ y[1] = 3$$

$$z^2 Y(z) - z^2 y[0] - z y[1] - 4\left[z Y(z) - z\, y[0]\right] + 4 Y(z) = 0$$

$$Y(z)\left(\frac{z^2 - 4z + 4}{(z-2)^2}\right) = z^2 \underbrace{y[0]}_{1} + z\left(\underbrace{y[1]}_{3} - 4\underbrace{y[0]}_{1}\right)$$

$$Y(z) = \frac{z^2 - z}{(z-2)^2} \qquad \frac{Y(z)}{z} = \frac{z-1}{(z-2)^2}$$

$$= \frac{A}{z-2} + \frac{B}{(z-2)^2}$$

$$B = z-1\big|_{z=2} = 1$$

$$A = [z-1]'\big|_{z=2} = 1$$

$$Y(z) = \frac{z}{z-2} + \frac{z}{(z-2)^2}$$

$$\boxed{y[n] = 2^n\left(1 + \frac{n}{2}\right) u[n]}$$

$y[0] = 2^0 = 1$
$y[1] = 2\left(1 + \frac{1}{2}\right) = 3$
ok

$$\mathcal{Z}[2^n n] = -z\frac{d}{dz}\left(\frac{z}{z-2}\right)$$

$$= -z\,\frac{z-2-z}{(z-2)^2} = \frac{2z}{(z-2)^2}$$

$y[n+2] = 4y[n+1] - 4y[n]$

$n = 0 \rightsquigarrow y[2] = 12 - 4 = 8$ $\qquad 2^2\left(1 + \frac{2}{2}\right) = 8$ ok