Mathematical model of cycling performance

T. S. OLDS, K. I. NORTON, AND N. P. CRAIG
Human Bioenergetics Laboratory, School of Sport and Leisure Studies, University of New South Wales, Oatley, New South Wales 2223; and South Australian Sports Institute, Adelaide, South Australia, Australia

OLDS, T. S., K. I. NORTON, AND N. P. CRAIG. *Mathematical model of cycling performance.* J. Appl. Physiol. 75(2): 730–737, 1993.—A model of cycling performance is presented. The model is based on equating two expressions for the total amount of work performed. One expression is deduced from biomechanical principles deriving energy requirements from total resistance. The other models the energy available from aerobic and anaerobic energy systems, including the effect of oxygen uptake kinetics at the onset of exercise. The equation can then be solved for any of the variables. Empirically derived field and laboratory data were used to assess the accuracy of the model. Model estimates of 4,000-m individual pursuit performance times showed a correlation of 0.803 ($P \le 0.0001$) with times measured in 18 high-performance track cyclists, with a mean difference (predicted − measured) of 4.6 s (1.3% of mean performance time). The model enables estimates of the performance impact of alterations in physiological, biomechanical, anthropometric, and environmental parameters.

bioenergetics; biomechanics; sports performance

THE APPLICATION OF mathematical modeling to sports performance can shed light on the interaction of different physiological systems and can be used as a valuable adjunct in training. Such models can be used to *1*) predict performance, *2*) identify individuals who have the physiological requirements to excel in a given sport, *3*) quantify the effects on performance of changes in a given physiological parameter, and *4*) quantify other unknown or unmeasurable characteristics of the performance, such as the relative contribution of energy from aerobic and anaerobic sources. There are two general classes of quantitative models, goodness-of-fit models and analogue models. Goodness-of-fit models use mainly statistical procedures to predict a response variable from a group of regressors. On the other hand, analogue models start "from the ground up," with a qualitative model built from known or axiomatic logical, physical, and physiological laws. Quantitative expressions are then added, usually drawn from a number of empirical studies. Although analogue models are sensitive to the characteristics of the samples used for parameter estimates, and of necessity employ simplifying assumptions, they allow us to elucidate the ways in which various parameters influence the response variable and interact with each other. They also permit the elaboration of "what if" scenarios. The present study is an attempt to develop an analogue model of cycling performance.

Previous authors (6, 8, 11) have used biomechanical data to model cycling performance. This paper has adopted the approach of developing two independent expressions for the energetics of cycling. One is an expression for the energy requirement of cycling derived from biomechanical principles with the use of data presented by di Prampero et al. (6). The other is an expression for the maximal amount of energy that can be supplied by the cyclist. These two expressions are then equated, and the equation is solved iteratively. The model permits the prediction of the effect on cycling performance of changes in physiological, biomechanical, anthropometric, and environmental factors. To assess the accuracy of the model, we applied it to the test results of 18 high-performance track cyclists specializing in the 4,000-m individual pursuit.

METHODS

Calculation of cycling energy requirements. In steady-state work, the cyclist must do work to overcome rolling resistance (Rr, in N) and air resistance (Ra, in N). The total work performed is modified when riding up or down hills. Finally, extra work is required to accelerate the bicycle at the beginning of exercise.

Rr is proportional to the combined mass of the bicycle (M_b, in kg) and the rider (M, in kg) and to the coefficient of Rr (C_{Rr}, related to tire pressure and tread, wheel radius, and road surface; Ref. 8). Rr may be represented by the equation

$$\mathrm{Rr} = C_{\mathrm{Rr}} \cos\,[\arctan\,(\mathrm{S})](M + M_b) \qquad (1)$$

where S is the gradient of the course (rise/run). The expression cos [arctan(S)]($M + M_b$) is the normal force exerted by the combined mass of the bicycle and the rider. When riding on the flat, this is equal to $M + M_b$. In this expression, the friction of the gears, chain, and other moving parts is taken to be negligible. Di Prampero et al. (6) found Rr to be equivalent to 3.2 N. In that study, the mean combined mass of bike and rider was 70 kg and the terrain was level. Substituting these values into *Eq. 1* yields $3.2 = C_{\mathrm{Rr}} \cdot 70$ or $C_{\mathrm{Rr}} = 0.0457$. This is retained as the default value. The energy required to overcome Rr (E_{Rr}, in J) is given by

$$E_{\mathrm{Rr}} = C_{\mathrm{Rr}} \cos\,[\arctan\,(\mathrm{S})](M + M_b)d \qquad (2)$$

where d is the event distance in meters.

Because Ra increases with the square of speed, the general expression for the Ra that has to be overcome is

$$\mathrm{Ra} = k(v_{ss} + v_w)^2 \qquad (3)$$

where Ra is in Newtons, v_w is head (positive) or tail (negative) wind speed (in m/s), v_{ss} is the steady-state bike

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speed (in m/s), and k is a constant. This can be expanded (19) as

$$\mathrm{Ra} = 0.5C_D A\rho(v_{ss} + v_w)^2 \tag{4}$$

where C_D represents the coefficient of drag, A is the projected frontal area (in m^2) of the rider, and ρ is air density (in kg/m^3). C_D is related to factors such as the aerodynamic shape of the bicycle and rider. Some of the factors affecting it are clothing, frame design, accessories, wheel radius, helmets, and disk wheels (5, 11).

A is related to the body surface area (BSA) of the rider and bicycle shape. A varies with riding position, but in the racing position it is ~25% of BSA (22). In the study of di Prampero et al., the BSA of the rider was 1.77 m^2. Assuming A is proportional to BSA, we can correct for this by multiplying by a correction factor (CF_A). The BSA of a rider is a function of mass (M) and height (Ht, in cm) and is given by the Dubois formula (7)

$$\mathrm{BSA} = M^{0.425}\mathrm{Ht}^{0.725}0.007184 \tag{5}$$

CF_A will therefore be equivalent to BSA/1.77.

ρ is proportional to barometric pressure (PB, in mmHg) and inversely proportional to temperature (T, in °K). In the study of di Prampero et al., PB was 755 Torr and T was 288°K. A second correction factor for air density (CF_ρ) can therefore be used

$$CF_\rho = (\mathrm{P_B}288)/(755T) \tag{6}$$

Di Prampero et al. estimated the value of k in *Eq. 3* to be 0.19. This constant can be rewritten as $0.19CF_\rho CF_A$. Because speed is constant only after acceleration is completed, the energy required to overcome Ra (E_{Ra}, in J) during the steady-state phase is

$$E_{Ra} = 0.19CF_\rho CF_A(v_{ss} + v_w)^2(d - d_{acc}) \tag{7}$$

where d_{acc} is the distance (in m) required to achieve the final v_{ss}.

If the course is not flat, allowance must be made for work performed against or with the grade. In riding uphill, the work done is equivalent to the weight of the rider and the bike in newtons multiplied by the vertical distance. The vertical distance is given by d multiplied by sin [arctan(S)]. That is, the energy required to ride on a grade (E_{grade}, in J) is

$$E_{grade} = (M + M_b)g \sin[\arctan(S)]d \tag{8}$$

where g is the acceleration due to gravity (9.807 m/s^2 at sea level).

A different algorithm must be used to calculate the energy required for acceleration from the start. Applying Newton's second law, force = mass × acceleration, the mass accelerated is the mass of the rider plus the mass of the bicycle ($M + M_b$). Speed changes from 0 to v_{ss} in some (unknown) time (t_{acc}, in s). We can therefore express the force required to increase the kinetic energy (F_k, in N) as

$$F_k = (M + M_b)v_{ss}/t_{acc} \tag{9}$$

The work performed in acceleration (E_k) is the product of F_k and d_{acc} during the acceleration phase. In the simulations used here, it was assumed the d_{acc} was equal to 100 m. d_{acc} is the product of the mean speed during this phase ($0.5v_{ss}$) and t_{acc}. In other words

$$E_k = 0.5(M + M_b)v_{ss}^2 \tag{10}$$

which is, as expected, the change in kinetic energy of the system during the acceleration phase.

During this phase, however, extra energy is required to overcome Ra, which is proportional to $(v + v_w)^2$, where v is the instantaneous speed (in m/s). Because speed is constantly changing during this phase, the work required depends on the pattern of power development and hence of Ra. If we assume that the total work required to overcome Ra during this phase is equivalent to the mean Ra multiplied by the total d, then the potential for error will be small. At the start, Ra will be $0.19CF_\rho CF_A v_w^2$. At the end of the acceleration phase, Ra will be $0.19CF_\rho CF_A \times (v_{ss} + v_w)^2$. The mean resistance, assuming linear acceleration, will be $0.19CF_\rho CF_A[v_w^2 + (v_{ss} + v_w)^2]/2$. The expression for the energy required for acceleration (E_{acc}, in J) must therefore be reformulated (combining the "kinetic" element E_k and the Ra component)

$$E_{acc} = 0.5(M + M_b)v_{ss}^2 + 0.19CF_\rho CF_A \times [v_w^2 + (v_{ss} + v_w)^2]d_{acc}/2 \tag{11}$$

Rr, which has to be overcome during acceleration, has been accounted for by *Eq. 2*.

The total energy requirement of cycling ($E_{tot} = E_{Rr} + E_{Ra} + E_{grade} + E_{acc}$, in J) can therefore be represented as

$$\begin{aligned} E_{tot} = {} & d\{C_{Rr}\cos[\arctan(S)](M + M_b) \\ & + (M + M_b)g\sin[\arctan(S)]\} \\ & + (d - d_{acc})0.19CF_\rho CF_A \\ & \times (v_{ss} + v_w)^2 + 0.5(M + M_b)v_{ss}^2 \\ & + 0.5d_{acc}0.19CF_\rho CF_A[v_w^2 + (v_{ss} + v_w)^2] \end{aligned} \tag{12}$$

The following assumptions have been made in deriving E_{tot}: *1)* C_{Rr} and C_D are similar to those found by di Prampero et al.; *2)* A is a constant fraction of BSA; *3)* acceleration up to v_{ss} is constant; *4)* v_{ss} is maintained once it has been achieved; *5)* "drafting" does not occur; *6)* in this application of the model, d_{acc} of 100 m is assumed; and *7)* friction from moving bicycle parts is considered to be negligible.

Calculation of available energy. The energy available to perform this work must be derived from aerobic and anaerobic sources and converted into external work. For exercise lasting <2–3 min, and less perhaps in specifically trained individuals (3), the maximal contribution from "anaerobic" energy sources (including stored oxygen) may be quantified by the maximal accumulated oxygen deficit (O_2def, in liters; Refs. 15, 16). This may amount to 3–6 liters and will become less important as the performance distance increases. The amount of energy that can be produced aerobically is related to maximal aerobic power ($\dot{V}O_{2\,max}$, l/min) and to the maximal fraction of $\dot{V}O_{2\,max}$ that can be sustained, once steady-state is achieved, over the performance period (f). For short periods (<10 min), it is possible for highly trained athletes to sustain a steady state of close to 100% $\dot{V}O_{2\,max}$. Rowers, for example, sustain a $\dot{V}O_2$ close to $\dot{V}O_{2\,max}$ throughout the 6–7 min of the race (see Fig. 12 in Ref. 21). For prolonged periods this is not possible. For work periods of between 30 min and 2–3 h, anaerobic threshold

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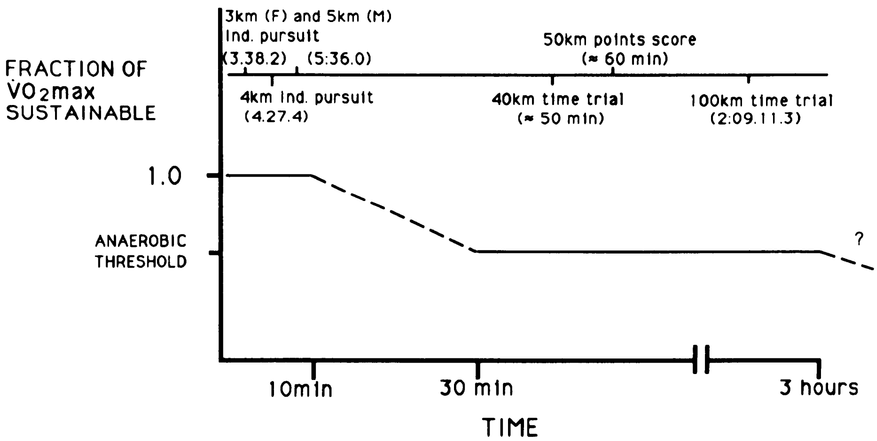

FIG. 1. Theoretical relationship describing fraction of maximal O_2 uptake ($\dot{V}O_{2\,max}$) sustainable during maximal cycling performances of varying duration. Current world record times for various track and road race distances are included for reference purposes only. F, female; M, male.

has been shown to be a significant predictor of cycling performance (3, 17). For intermediate periods (between 10 and 30 min) f decreases in an unknown fashion. Until this function can be quantified, f is considered in this model to be a continuous nonlinear function of race time, taking a value of 1 for race times of ≤10 min and a value equivalent to the individual anaerobic threshold for race times of ≥30 min. For race times between 10 and 30 min, f is determined by linear interpolation (Fig. 1). The pattern of f described in Fig. 1 is speculative but will serve as an interim model until further studies clarify the relationship. Almost all competitive cycling events last either <10 min or between 30 min and 2–3 h (Fig. 1). An estimate of the total available internal energy (E_{int}, liters O_2) is therefore

$$E_{int} = O_2def + t\dot{V}O_{2\,max}f \qquad (13)$$

where t is the performance time (in min) and O_2def is the maximal accumulated oxygen deficit as determined by the method of Medbø et al. (15).

At the onset of exercise, oxygen uptake ($\dot{V}O_2$) does not rise immediately to its final steady-state level. Although more complex models describe $\dot{V}O_2$ kinetics better at higher work rates (WR; Ref. 12), $\dot{V}O_2$ kinetics at the onset of exercise can be approximately described by

$$\dot{V}O_{2\,t} = (\dot{V}O_{2\,f} - \dot{V}O_{2\,init})(1 - e^{-t/\tau}) \qquad (14)$$

where $\dot{V}O_{2\,t}$ is the $\dot{V}O_2$ at *time t*, $\dot{V}O_{2\,f}$ is the final (steady-state) $\dot{V}O_2$, $\dot{V}O_{2\,init}$ is the initial (prerace) $\dot{V}O_2$, and τ is a time constant (12). The O_2def incurred before $\dot{V}O_2$ plateaus is $\tau(\dot{V}O_{2\,max}f\text{-}\dot{V}O_{2\,init})$ (24). The total available internal energy (as an O_2 equivalent) can therefore be expressed as

$$E_{int} = O_2def + tf\dot{V}O_{2\,max} - \tau(\dot{V}O_{2\,max}f - \dot{V}O_{2\,init}) \qquad (15)$$

The maximal rate at which this energy can be supplied [internal power (P_{int}; in l/min)] over the course of an event lasting t min is

$$P_{int} = [O_2def + t\dot{V}O_{2\,max}f - \tau(\dot{V}O_{2\,max}f - \dot{V}O_{2\,init})]/t \qquad (16)$$

The total time (in min) for the event is the sum of the time required for acceleration [taken to be $d_{acc}/(0.5 \times 60v_{ss})$] and the time required for the steady-state phase $[(d\text{-}d_{acc})/60v_{ss}]$, that is, $(d + d_{acc})/60v_{ss}$. Therefore, *Eq. 16* may be written as

$$P_{int} = [O_2def + (d + d_{acc})/60v_{ss}\dot{V}O_{2\,max}f - \tau(\dot{V}O_{2\,max}f - \dot{V}O_{2\,init})]/[(d + d_{acc})/60v_{ss}] \qquad (17)$$

E_{int} represents the available internal energy supply. However, the amount of energy that must be produced internally to perform the required external work will be considerably greater, because of the inefficiency of the human-machine system. Gross mechanical efficiency on the bicycle can be measured on an ergocycle, which simulates the riding position, pedal frequency, and angles of the racing bicycle. From a series of submaximal trials, a linear regression of $\dot{V}O_2$ (l/min) on WR (in W) can be established of the form

$$\dot{V}O_2 = \text{gradient} \cdot \text{WR} + \text{intercept}$$

or

$$\text{WR} = (\dot{V}O_2 - \text{intercept})/\text{gradient} \qquad (18)$$

where gradient is the slope of the regression line and intercept is the y-intercept of the regression line. To convert the maximal P_{int} to a maximal external power, we insert the expression for P_{int} (*Eq. 17*) into *Eq. 18*

$$\begin{aligned} \text{WR} &= (P_{int} - \text{intercept})/\text{gradient} \\ &= \{[O_2def + (d + d_{acc})/60v_{ss}\dot{V}O_{2\,max}f - \tau(\dot{V}O_{2\,max}f - \dot{V}O_{2\,init})]/ \\ &\quad [(d + d_{acc})/60v_{ss}] - \text{intercept}\}/\text{gradient} \end{aligned} \qquad (19)$$

To convert this to the total amount of energy available for external work (E_{supply}, in J) in t min, we multiply by $(d + d_{acc})/v_{ss}$

$$\begin{aligned} E_{supply} = {} & [(d + d_{acc})/v_{ss}]\{[O_2def + (d + d_{acc})/ \\ & 60v_{ss}\dot{V}O_{2\,max}f - \tau(\dot{V}O_{2\,max}f - \dot{V}O_{2\,init})]/ \\ & [(d + d_{acc})60v_{ss}] - \text{intercept}\}/\text{gradient} \end{aligned} \qquad (20)$$

The following assumptions have been made in deriving the equation for E_{supply}: *1*) f follows the pattern outlined in Fig. 1, *2*) mechanical efficiency is similar at supramaximal and submaximal levels, *3*) mechanical efficiency as measured on a modified ergocycle is similar to mechanical efficiency measured on the track or road, and *4*) me-

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TABLE 1. *Effect of changing physical and physiological parameters on performance outcome in 4,000-m individual cycling pursuit*

Variable	4,000-m Time, s	Change, s	Change, %
$\dot{V}O_{2\,max}$ (l/min)			
5.914	328.5	−15.5	−4.5
5.143	344.0		
4.586	357.3	+13.3	+3.9
O_2 deficit (liters)			
5.917	337.0	−7.0	−2.0
4.414	344.0		
2.872	350.0	+6.0	+1.7
d_{acc} (m)			
50	341.9	−2.1	−0.6
100	344.0		
150	346.2	+2.2	+0.6
$\dot{V}O_{2\,init}$ (l/min)			
1.105	342.9	−1.1	−0.4
0.681	344.0		
0.439	344.6	+0.6	+0.2
M (kg)			
65.25	335.2	−8.8	−2.6
75.29	344.0		
86.00	352.6	+8.6	+2.5
Ht (cm)			
173.0	341.2	−2.8	−0.8
179.3	344.0		
184.6	346.2	+2.2	+0.7
τ (min)			
0.460	341.4	−2.6	−0.7
0.594	344.0		
0.710	346.2	+2.2	+0.7
Mechanical efficiency			
0.0121; 0.125	330.8	−13.2	−3.8
0.0129; 0.318	344.0		
0.0142; 0.742	367.6	+23.6	+6.9
M_b (kg)			
6.0	343.8	−0.2	−0.1
7.0	344.0		
8.0	344.2	+0.2	+0.1

Criterion values are mean levels for the subjects used to determine predictive accuracy of the model. $\dot{V}O_{2\,max}$, maximal O_2 uptake; d_{acc}, acceleration distance; $\dot{V}O_{2\,init}$, initial O_2 uptake; M, mass of rider; Ht, height of rider; τ, time constant; M_b, bike mass. Mechanical efficiency is comprised of gradient (in $l \cdot min^{-1} \cdot W^{-1}$) and intercept (in l/min).

chanical efficiency during the acceleration phase is similar to that during the steady-state phase.

We have therefore derived an expression for the total energy demand of the work (*Eq. 12*) and another for the energy supply (*Eq. 20*). By equating these two expressions, we can solve directly or iteratively for any of the variables given values for all the other variables.

Model accuracy. To assess the predictive accuracy of the model, empirically derived data were substituted into *Eqs. 12* and *20* and solved for v_{ss} and hence t. The data were collected on a group of 18 elite male track cyclists (Table 1) as part of another study (3). All subjects were cycling scholarship holders at either the Australian or South Australian Institutes of Sport. Measurements included both laboratory and track testing. The ergometer used for laboratory testing was a wind-braked cycle ergometer (4). Continuous measurements of instantaneous power (in W) and total work done (in kJ) and other work indexes were calculated and stored using specifically designed software. The ergometer was calibrated dynamically throughout the physiological range of measurements by use of an electronic torque meter.

Briefly, the laboratory measurements involved three tests. *1*) Standard anthropometric measurements were made, followed by an incremental test using 5-min durations to determine steady-state $\dot{V}O_2$ at various submaximal WRs. Subjects began at 100 W and progressively increased WRs in 50-W increments until they could no longer maintain the required pedal cadence. The required pedal cadences were between 85 and 120. The $\dot{V}O_2$ measured during the last 2 min of each work period was used as the steady-state value and was regressed against power output. The slope and intercept of this relationship were used as indicators of cycling efficiency. The correlation coefficients for the regression lines were ≥0.995 (standard error of estimate = 128 ml O_2). *2*) An incremental test was made using 1-min durations to determine maximal aerobic power. Subjects began at 200 W and increased by 25 W/min. $\dot{V}O_{2\,max}$ was identified when the $\dot{V}O_2$ for successive WRs differed by <0.15 l/min. *3*) On a separate day, within 1 wk of the maximal test, a supramaximal test at an intensity estimated to require 115% $\dot{V}O_{2\,max}$ was performed. This test to exhaustion was used to determine maximal accumulated O_2 deficit according to the procedures of Medbø et al. (15) and the rate of $\dot{V}O_2$ kinetics. The actual WR required was estimated from the $\dot{V}O_2$ vs. power output relationship determined for each subject. The mean duration of these tests was 174 ± 47 (SD) s. During each of the above testing procedures, $\dot{V}O_2$ was assessed on a breath-by-breath basis (Ametek OCM-2 metabolic assessment system) and then averaged over longer periods according to the requirements of the test: 10 s for O_2 deficit and $\dot{V}O_2$ kinetics tests, 30 s for the $\dot{V}O_{2\,max}$ test, and 60 s for the cycling efficiency test.

Track testing consisted of a timed standing-start 4,000-m individual pursuit performance on a 400-m outdoor concrete velodrome. Subjects used their road bikes and clothing. The mean time was 339.7 ± 14.1 (SD) s (range 314.4–362.2 s). On the day of the track cycling, environmental conditions averaged P_B = 768.9 Torr, T = 296.4°K, and maximal v_w < 2.5 m/s (mean = 0.9 m/s).

This series of tests enabled derivation of the variables outlined in the model above. Prerace $\dot{V}O_2$ ($\dot{V}O_{2\,init}$) was the measured $\dot{V}O_2$ immediately preceding the start of the supramaximal test to exhaustion. The combined mass of bike and rider (in kg) was measured. τ was obtained by iterative fitting of a curve of the form

$$\Delta\dot{V}O_2 = (\dot{V}O_{2\,max} - \dot{V}O_{2\,init})(1 - e^{-t/\tau})$$

to the 10-s values recorded during the supramaximal test. The fits were all excellent, with correlation coefficients ranging from 0.94 to 0.98.

The iterative procedure used in simulations was a simple brute-force method. A plausible value for the predicted variable (v_{ss}) was guessed and iteratively adjusted according to the sign of the difference between the predicted values of E_{tot} and E_{supply}. Increments or decre-

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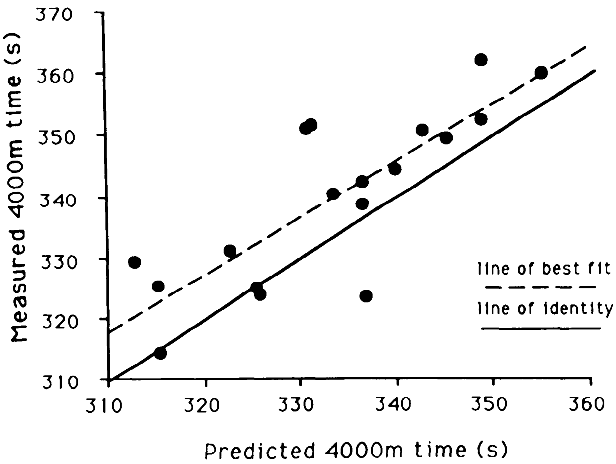

FIG. 2. Relationship between measured and predicted 4,000-m time.

ments of 0.001 m/s were used until the sign of the difference changed. The residual difference between E_{tot} and E_{supply} never exceeded 0.2% of the E_{supply} value.

RESULTS

Velocities and hence times predicted from the model were compared with measured times for the 4,000-m time trial. The correlation between predicted and measured times was 0.803 ($P \leq 0.0001$; 95% confidence limits $0.54 \leq \rho \leq 0.92$; Fig. 2). The mean difference between measured and predicted times (measured − predicted) was 4.6 s (1.3% of the mean 4,000-m time), the mean absolute difference was 7.7 s (2.3%), and the root mean square error was 9.5 s (2.8%).

DISCUSSION

A mathematical model was developed using biomechanical, environmental, and physiological considerations to estimate energy requirement (and subsequently time) for cycling. The basis for this model was the study of di Prampero et al. (6). Because of the similarities in bicycles used (modern road bicycles) and riding surface (concrete tracks), the drag and Rr characteristics encountered in the present study were assumed to be very similar to those of the results of di Prampero et al. Assessment of the predictive accuracy of this model was performed using data collected on 18 elite cyclists. A strong correlation (r = 0.803) was found between predicted and measured times with small error measurements.

The accuracy of the model to predict performance in this study is encouraging, since the subjects represented a relatively homogeneous group. It is likely that an even stronger correlation would have been found in a more heterogeneous group. If, for example, the sample times represented only the top half of a distribution, the putative population correlation would be r = 0.91 (10). The small mean difference, root mean square error, and mean absolute error between predicted and measured times suggest that model-predicted times are good estimates of actual times.

The small remaining errors are perhaps ascribable in part to factors such as motivation and fatigue, which cannot easily be modeled. We have also not modeled changes in C_D that may be due to differing cycling clothes and cycle design (11). Furthermore, a model component for possible changes in mechanical efficiency during supramaximal exercise has not been included (20).

A simplifying assumption in this model is that acceleration at the start of the race is constant. It is in fact unlikely that this is the case. The main energy requirement during this phase is to impart E_k to the bicycle-rider system. In accelerating to a typical 4,000-m individual pursuit pace (14 m/s) in 14 s, a 75-kg rider on a 7-kg machine would have to produce 8,036 J for E_k. The E_{tot} during the accelerative phase will depend on the function relating velocity to time. When a number of different functions were modeled (linear, exponential, quadratic, logarithmic, hyperbolic, various polynomials, two linear segments) the estimated E_{tot} (E_k plus E_{Ra} and E_{Rr}) to accelerate to 14 m/s in 14 s ranged between 8,263 and 11,370 J. The estimated energy requirement under the assumption of linear acceleration was 10,223 J. The potential error using this assumption is therefore small (±2,000 J or ~2% of E_{tot} even for short events such as the 4,000-m pursuit). For the mean values in the current study, the maximal possible error amounts to ±1.4 s. In general, there is a trade-off between optimizing total energy cost and minimizing peak power. If velocity is an exponential function of time, for example, the total energy required during the accelerative phase is relatively small (8,263 J) but the peak 1-s power output required is 7,196 J. Such a pattern of power generation is clearly impossible. The actual pattern of acceleration may be chosen so as to even out power requirements throughout the accelerative phase. When velocity is a logarithmic function of time, power production is quite even and energy cost is moderate. Further empirical studies are required to determine the actual pattern of acceleration at the start of races.

Despite these assumptions and limitations, the percentage of variance explained by the model (64%) was high and greater than that explained by any of the model parameters taken separately or when used in combination. Stepwise regression retained two variables as significant predictors of 4,000-m time ($\dot{V}O_{2\,max}$ and mass), but the combined coefficient (R = 0.76) was less than that obtained using the model.

As well as showing good fit with the data, the model is based on physical and physiological analogues, with the left-hand side representing the work required to overcome external resistance and the right-hand side the energy generated by the aerobic and anaerobic systems. Each of the components of the model is rigidly operationalized in standard tests, which show good reliability (13).

Kyle (11) has developed a model of the energy demand of cycling based on biomechanical principles. However, this model is restricted in that it does not incorporate physiological aspects (i.e., energy supply). By calculating changes in resistance due to grade, wind, added mass, and drag, Kyle has been able to make some specific predictions about the effect of these changes on performance time. The present model produces results that are

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quite similar to those of Kyle. For example, Kyle predicts that an extra bicycle mass of 0.454 kg would add 0.05 s to the time of a 4,000 m pursuiter completing the race in 290 s. The present model would predict an increase of 0.08 s. Kyle estimates that if drag could be reduced by 120 g, 40-km time-trial performance would be reduced by 39 s from a baseline of 50 min. The present model predicts a 30 s reduction. The effects of simulated grade and wind are also quite similar between the two models. Riding with a power output that would maintain a speed of 32 km/h on the flat in still conditions would, according to Kyle, result in a speed of 23.4 km/h on a 2% uphill slope and 40.9 km/h on a 2% downhill slope. The corresponding predictions from the present model are 21.3 and 42.5 km/h. Kyle's model suggests that riding at the same power output into a head wind of 8 km/h would result in a speed of 28 km/h (as opposed to 27.4 km/h for the present model). Riding with a tail wind of 8 km/h would produce a speed of 36.2 km/h (36.7 km/h for the present model). Péronnet et al. (18) have outlined a physiological model of the energy cost of cycling quite similar to the one employed here. The model is used to predict the optimal altitude for a 1-h cycling performance. With the help of supplementary algorithms estimating the effect of altitude on $\dot{V}O_{2\,max}$, $\dot{V}O_2$ kinetics, ρ, weight force, and other model parameters, the present model can also be used to predict optimal altitude. The suggested altitude (3,850 m) is similar to the altitude predicted by the present model using similar assumptions (~3,400 m).

The general modeling strategy employed here (the equating of expressions for energy demand and energy supply) may be generalizable to other sporting events of a similar closed-skill nature (for example, speed and roller skating, rowing, running, and race walking). The energy demand of the task may be established from biomechanical first principles or from empirical determinations of $\dot{V}O_2$ (19). A similar model of energy demand to the one used here may be applied to speed skating (23). Quite different physicobiomechanical models (incorporating, for example, hydrodynamics) are available for other sports. The limitations of the current model include events of <2 min in duration where there are questions of alterations in mechanical efficiency and the ability to fully express maximal anaerobic capacity. The question of a possible dependence of $\dot{V}O_2$ kinetics on WR at supramaximal WRs should also be addressed. For prolonged events (>2–3 h), the potential for substrate availability to limit performance may make the use of this model inappropriate. In real-life situations, changes in slope and environmental conditions make modeling events, such as triathlons and stage races, very difficult. One strategy would be to subdivide the race into time or distance slices and model each slice individually. Wind speed and temperature changes could be modeled in a similar fashion post hoc, although the calculations would be extremely complex and of theoretical interest only. General questions such as the effect of wind or grade on out-and-back courses can be addressed (11).

The cycling event modeled here is in some ways an "ideal" event in that it assumes that a constant racing speed is maintained once it has been achieved. This has

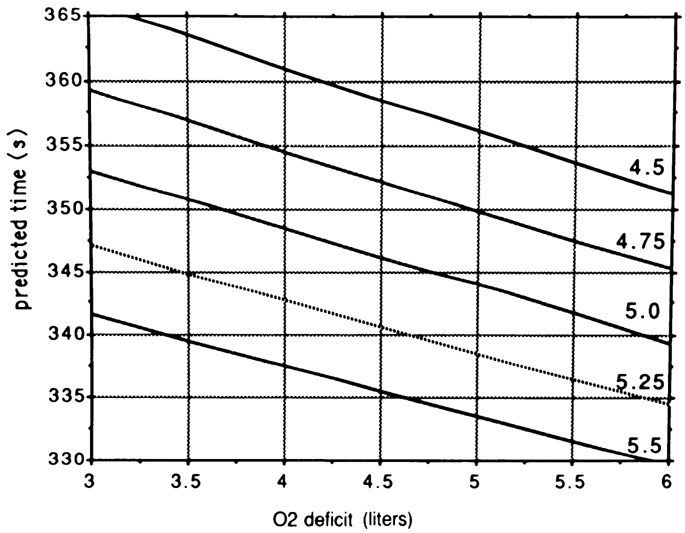

FIG. 3. Effect of changing O_2 deficit and $\dot{V}O_{2\,max}$ on performance time. Curves represent $\dot{V}O_{2\,max}$ values of 4.5, 4.75, 5, 5.25, and 5.5 l/min.

been shown to be the case with elite track cyclists (in whom lap speeds vary by ± <1 s). Riders of lesser ability show slightly greater variability (25). Furthermore, no allowance is made for "drafting," which has been shown to reduce the oxygen cost of cycling at 32-40 km/h by 20-62% (14). However, drafting is illegal in the cycle leg of triathlons, individual time trials, and time-trial legs of stage races and is not relevant in most individual track endurance events.

The predictive power of the model could be considerably augmented by introducing algorithms relating to the effect of environmental factors, such as altitude and temperature, on physiological characteristics. More discriminating models of the drag characteristics of different types of bicycles, better estimates of A based on anthropometric measures, and data on C_{Rr} for different surfaces (for example, concrete vs. wooden velodromes) could also be incorporated into the model. Data on the significantly improved drag characteristics of modern equipment could be incorporated into the model as it is presently structured. Tentative drag values for modern track bicycles have been suggested (18). An understanding of deficit kinetics, the rate at which the maximal accumulated O_2def can be expressed, would help to make the model more applicable to events of very short (<2 min) duration.

Notwithstanding these caveats, the general modeling strategy employed would allow training decisions to be based on the performance impact of physiological parameters and the trainability of those parameters. For example, the model can be used to quantify the contribution of various physiological parameters to a 4,000-m cycling performance. Table 1 shows the effect of changing a model parameter while holding others constant. The impact of these theoretical alterations on performance will clearly depend on the magnitude of the changes. We chose approximations of the highest and lowest values found in our study for each variable while holding the other variables constant and about equal to the mean value recorded. A further example of the application of

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the model is illustrated in Fig. 3. This graph represents the predicted performance outcomes when changing two variables, O_2 def and $\dot{V}O_{2\,max}$, while holding the other variables constant.

These analyses demonstrate that the major determinants of 4,000-m individual pursuit performance include maximal aerobic power and mechanical efficiency (reflected in gradient and intercept). This is not surprising, since we have calculated that the aerobic system contributes 84 ± 1% of the energy demand during a 5-min simulated pursuit (3). Moreover, track cyclists will sometimes ride in excess of 35,000 km/yr in training. The major value of such training may be in improvement in mechanical efficiency (in the sense of a reduced rate of $\dot{V}O_2$ per unit of external power output rather than a reduced rate of substrate utilization), as has been shown for other groups of athletes (9). Recently, however, some of the differences in mechanical efficiency in trained endurance cyclists have been ascribed to differences in fiber type (1). The coefficient of determination relating gross cycling efficiency to percentage of type I muscle fibers was found to be 0.56. The determinants of the remaining variability (44%) are unknown. It is of interest that a significant relationship was found between years of endurance training and mechanical efficiency, but no significant relationship was found between years of cycle training and mechanical efficiency. One possible interpretation is that the gain in mechanical efficiency has physiological rather than biomechanical origins. Anaerobic capacity and BSA (determined from mass and height) are also important contributors to 4,000-m cycling performance, whereas $\dot{V}O_2$ kinetics and initial $\dot{V}O_2$ levels make relatively small (but probably physiologically significant) contributions to performance.

Mass can affect the energy demand of cycling in a number of ways. First, additional mass will increase the energy required to overcome Rr, E_{acc} required for the bicycle-rider system, and E_{grade}. Second, increased body mass is accompanied by increased BSA and hence *A*. Using the mean values reported in the present study, our model suggests that a 5-kg change in body mass will result in a change of about ±0.07 min (4.2 s) in 4,000-m time. About 27–28% of the total change in performance time can be ascribed to the effects of increased mass on Rr and E_k. The remainder is due to increases in *A* and thereby Ra. The exact fractions will vary as speed, distance, and slope vary.

One interesting question is the effect of wind speed (that is, constant head or tail winds). By using the mean values found in the present study, a 5-km/h head/tail wind would change 4-km performance time by ±7–8% (~25 s) if the race were not conducted on a circular track. On a circular track, head and tail winds will tend to cancel each other out. A lighter wind of ±1 km/h would alter performance times by ±1.5% (5 s). Wind speed is therefore a critical factor in predicting performance time. It will be less important in comparing athletes who are all subject to the same or similar wind drag. It is not possible at the moment to model for the effect of crosswinds.

In summary, the model presented in this paper, when applied to the 4,000-m individual pursuit, shows a high degree of predictive accuracy. The model provides information regarding performance outcomes after alterations in physical and physiological characteristics of the athlete. The strategy employed may also be adaptable to sports of similar energy demands.

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An Apple MacIntosh computer program (requiring HyperCard 2.0) of the algorithms used in this paper is available from T. S. Olds, St. George Campus, Univ. of New South Wales, PO Box 88, Oatley, New South Wales 2223, Australia.

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Address for reprint requests: K. I. Norton, School of Sport and Leisure Studies, St. George Campus, Univ. of New South Wales, PO Box 88, Oatley, New South Wales 2223, Australia.

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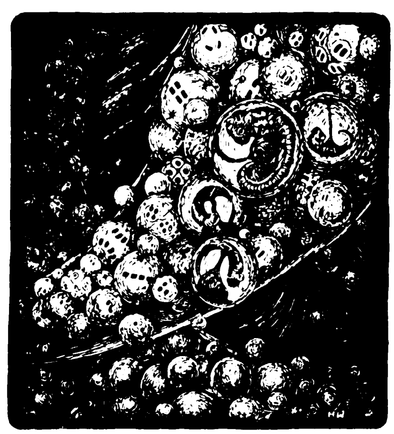

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