POTW #1 - Systems

Similar to Newton's laws, work and energy are done for systems. That is, before starting with anything, we draw an imaginary box around a certain set of objects and define it as our "system".

Then, all the theorems derived prior (Work-energy theorem, Newton's laws, and such) all apply to this particular system. This problem of the week will explore how different systems may make calculations easier or harder.

Note that today's POTW will not cover work - we'll look into how systems play a role in that next week!

Newton's Laws

(warmup) 1. Five boxes of masses m_1, m_2, $\ldots$ m_5 are lined up along a surface with coefficient of friction μ_k with all five boxes in contact with one another. A net force F acts on the leftmost box.

a If all boxes accelerate uniformly with nonzero acceleration a, what is a? (Hint: Let the system be all 5 of the boxes)

b Suppose the blocks are connected with strings instead, with F acting on the right most block. Find the tension force in the 2nd string from the left

c Solve part b. by considering each block individually.

Solution. Using the hint, we let the system be all five of the blocks.

a For this system the Free Body Diagram has only 2 forces in the horizontal direction: Friction and F. Writing the $F = ma$ equation for this, we arrive at:

$$F - \mu g \sum m_i = \sum m_i a \tag{1}$$

Solving, we find that

$$a = \frac{F - \mu g \sum m_i}{\sum m_i} \tag{2}$$

b Note that the acceleration is the same in this case - if we apply the same system above, the strings do not play a role in our $F = ma$ equation. Using this, we let the system be the first two blocks.

Drawing the Free body diagram, we see that the only two forces are Tension and Friction. Thus,

$$T - \mu(m_1 + m_2)g = (m_1 + m_2)a \tag{3}$$

Solving,

$$T = (m_1 + m_2)(a + \mu g) \tag{4}$$

c We write the f=ma equation for the first block, then the second block. Let the tension in the first rope be T_1 and the tension in the second rope be T_2. Our equations are then:

$$\begin{cases} T_1 - \mu m_1 g = m_1 a \\ T_2 - T_1 - \mu m_1 g = m_2 a \end{cases} \tag{5}$$

Adding the two, and solving for T_2, we arrive at the previous result.

2. A rod's left end is attached to the wall by some superglue. For any part of the rod, there are two forces on its left and right end in the perpendicular direction. These forces, known as shear forces, are used to maintain balance. It is an internal force, meaning that the shear force is due to some parts of the rod acting on other components (think: The tension force).

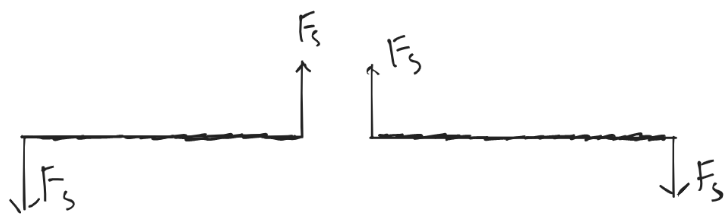

Figure 1: For the rod on the left, the shear force is defined to be negative. For the rod on the right, the shear force is defined to be positive. Note that the two shear forces acting on a rod are not necessarily equal.

a. Suppose a rod of length l and mass per length λ has its left end glued to the wall. Let S(x) be the shear force in the rod as a function of x, the distance from the left end. Express S(x) In terms of x , λ, and Earth's gravitational acceleration, g.

b. Suppose that the rod is now supported at its two ends by the same level support. Find S(x) now.

c. Solve part a. by considering a small piece of the rod with length Δx. Let this be your system. [1]

Solution. This is quite a tricky problem! Read through it carefully and let me know if you have any questions.

a. Consider a length $l - x$ of the rod from the right side. This is our system. (When we say consider XYZ, we mean that it is our system)

   The only forces on such a rod are the shear force from the left side and its weight. The two clearly must balance, so we have that
$$S(x) = \lambda(l - x)g \tag{6}$$

b. By symmetry, the two supports must provide the same upward force of $\lambda lg/2$. Consider a piece of rod extending from the left side with length x. We see that there are only 3 forces forces are the shear force on the right, gravity, and the force from the support. Balancing all three, we find that
$$S(x) = \lambda g(\frac{l}{2} - x) \tag{7}$$
   Where we used the sign conventions noted in the figure to write this equation.

c. Note that this part is similar to 1c). We're choosing a different system as a way to see that we can arrive at the same results through different methods!

   Consider a piece of rod in the middle of the rod: it will have two shear forces and the gravitational force. Balancing the forces:
$$S(x + \Delta x) - S(x) = \lambda g \Delta x \tag{8}$$
   This tells us that the function $S(x)$ increases at a constant rate (is a linear function in x), since when x increases by Δx, S(x) increases by an amount proportional to Δx! Using the two initial conditions that $S(0) = \lambda lg/2$ and that $S(l) = -\lambda lg/2$, we arrive at the desired result.

[1] This is more challenging and thus is optional (Some familiarity with calculus will help)