Kepler's Third Law Proof

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1 Intro

This proof uses basic methods that precede calculus. Other than the ellipse formula, everything else comes from physical principles - without any convoluted calculus.

This is a sharp contrast to the conventional proof of Kepler's Third Law shown Here

2 Proof

We have that from Kepler's second law: $\frac{dA}{dt} = \frac{L}{2m}$.

Thus, the total period is $T = \frac{A}{L/2m}$. Since the area of an ellipse is πab, we have:

$$T = \frac{\pi ab}{L/2m}$$

The total angular momentum of the system can be determined as follows: Consider the object when it is at a distance b from the center of the ellipse. Then, the total angular momentum is:

$$L = mbv_{\perp}$$

The velocity can be found from the energy equation (at this point, the focus is a distance a from the object):

$$\frac{1}{2}mv^2 - \frac{GMm}{a} = \frac{GMm}{2a} \implies v = \sqrt{\frac{GM}{a}}$$

Note that this velocity is perpendicular to b.

Plugging this in:

$$T = \frac{\pi ab}{(mb\sqrt{\frac{GM}{a}})/2m} = 2\pi\sqrt{\frac{a^3}{GM}}$$