

CM146, Fall 2019  
Problem Set 4: Learning Theory, Boosting, Multi-class  
Classification  
Due December 8, 2019, 11:59pm

**Submission instructions**

- Submit your solutions electronically on the course Gradescope site as PDF files.
- If you plan to typeset your solutions, please use the LaTeX solution template. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app.

**1 PAC Learning [25 pts]**

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$i$	Label	Hypothesis 1 (1st iteration)				Hypothesis 2 (2nd iteration)			
		$D_0$	$f_1 \equiv$ [ $x > \_$ ]	$f_2 \equiv$ [ $y > \_$ ]	$h_1 \equiv$ [ $\_\_\_\_\_\_$ ]	$D_1$	$f_1 \equiv$ [ $x > \_$ ]	$f_2 \equiv$ [ $y > \_$ ]	$h_2 \equiv$ [ $\_\_\_\_\_\_$ ]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	−								
2	−								
3	+								
4	−								
5	−								
6	−								
7	+								
8	−								
9	+								
10	+								

Table 1: Table for Boosting results

## 2 VC Dimension [15 pts]

## 3 Boosting [40 pts]

$i$	$x$	$y$	Label
1	0	8	−
2	1	4	−
3	3	7	+
4	-2	1	−
5	-1	13	−
6	9	11	−
7	12	7	+
8	-7	-1	−
9	-3	12	+
10	5	9	+

## 4 Multi-class classification [60 pts]

Consider a multi-class classification problem with  $k$  class labels  $\{1, 2, \dots, k\}$ . Assume that we are given  $m$  examples, labeled with one of the  $k$  class labels. Assume, for simplicity, that we have  $m/k$  examples of each type.

Assume that you have a learning algorithm  $L$  that can be used to learn Boolean functions. (E.g., think about  $L$  as the Perceptron algorithm). We would like to explore several ways to develop learning algorithms for the multi-class classification problem.

There are two schemes to use the algorithm  $L$  on the given data set, and produce a multi-class classification:

- **One vs. All:** For every label  $i \in [1, k]$ , a classifier is learned over the following data set: the examples labeled with the label  $i$  are considered “positive”, and examples labeled with any other class  $j \in [1, k], j \neq i$  are considered “negative”.
- **All vs. All:** For every pair of labels  $\langle i, j \rangle$ , a classifier is learned over the following data set: the examples labeled with one class  $i \in [1, k]$  are considered “positive”, and those labeled with the other class  $j \in [1, k], j \neq i$  are considered “negative”.

