Using Simple Incentives to Improve Two-Sided Fairness in Ridesharing Systems (Supplement)

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This document serves as the appendix for the paper titled "Using Simple Incentives to Improve Two-Sided Fairness in Ridesharing Systems", published at ICAPS 2023. This document contains supplemental information in the form of theoretical properties. We also provide some additional discussion about our approach and the experiments.

Theoretical Properties

In this section, we discuss the full proofs of the theoretical properties discussed in the main body. Both proofs assume that vehicles can only accept a single request at any time step. With this assumption, each action a contains only one request r, allowing us to use them interchangeably.

Let $\mathcal{A}(w)$ be the matching solution produced by the ILP with fairness weight $w \in \{\beta, \delta\}$. Further, let $g_{\min} = \operatorname{argmin}_j z_j$ denote the group with the smallest metric value. We now have the following proofs.

Passenger-side Min-Fairness

We show that if vehicles can only service a single request at a time, and if there is at least one request from the group with the worst current service rate that is not served when $\beta = 0$, our approaches improve the worst service rate for a sufficiently high β (see Theorem 1). Note that the fairness score for a single request can written as

$$f(r) = \bar{z} - z(r), \tag{1}$$

Formally, suppose that each action a contains at most one request r, allowing us to use both a and r interchangeably. Under this assumption, the assignment $\mathcal{A}$ will contain at most one request per vehicle. For actions which contain only one request, $F_p(i, a) = f(r), a = \{r\}$.

Improving service rate: Recall that z_j are *historical* service rates for groups j, where groups are passenger origin-destination neighborhood pairs. We assume that $z_j \neq z_k$ whenever $j \neq k$ (an assumption that is nearly always true in practice). Let T_j^h be the total number of requests involving group j prior to the current matching round, and let T_j be the *current* number of requests involving j. All requests that are still outstanding (i.e., requests that have neither been canceled nor serviced yet) are counted as current rather than historical. Let $\mathcal{A}(\beta)$ be the matching solution produced by the ILP with fairness weight β, and let $S_j(\beta)$ be the number of requests from group j that are serviced in the matching $\mathcal{A}(\beta)$. We define $z'_j(\beta)$ as the service rate for group j *after* the matching computed by the ILP with fairness weight β. Since $z'_j(\beta) = \frac{z_j \cdot T_j^h + S_j(\beta)}{T_j^h + T_j}$, it is evident that $z'_j(\beta)$ is increasing in $S_j(\beta)$ for each group j i.e., serving more requests from a group at any time leads to higher improvement in service rate for that group.

We now introduce some additional notation. Let $g_{\min}$ be the group with the lowest service rate, i.e., $g_{\min} = \operatorname{argmin}_j z_j$, and let $\mathcal{R}_f = \{r\}_{g(r)=g_{\min}}$ be the set of current requests corresponding to group $g_{\min}$.

Any request r with $g(r) = g_{\min}$ will have the highest score (Eq. 1). Further, since $F_p(\cdot, r)$ only depends on the group service rate $z(r)$, every request from the same group has the same $F_p(\cdot, r)$. More formally:

Property 1. *All requests $r_f \in \mathcal{R}_f$ have the same fairness score $F_p(\cdot, r_f)$, which is larger than the fairness score $F_p(\cdot, r)$ of all other requests $r \notin \mathcal{R}_f$.*

In what follows, we show that as long as it is possible to improve the service rate of $z_{g_{\min}}$ relative to $\mathcal{A}(0)$ – a condition which we formalize as *passenger-min-unfairness* in the following definition – we can do so for a sufficiently high β. For ease of notation, let $Q(i, r) = V(i, r) + R(i, r)$.

Definition 1 (passenger-min-unfair)**.** *A matching $\mathcal{A}$ is* passenger-min-unfair *if there exists a request $r_f \in \mathcal{R}_f$ not served in $\mathcal{A}$, there is a vehicle $i \in \mathcal{V}$ such that $r_f \in A_i$, but the request assigned to i is $r_i \notin \mathcal{R}_f$.*

Theorem 1. *If $\mathcal{A}(0)$ is passenger-min-unfair, then there exists $\beta > 0$ such that $z'_{g_{\min}}(\beta) > z'_{g_{\min}}(0)$.*

We begin by first proving several lemmas that serve as useful building blocks.

Lemma 1. *If $s_\beta(i, r) > s_\beta(i, r')$ for all $r' \in A_i \setminus \{r\}$ of a vehicle i with available actions A_i, then request r will always be assigned to some vehicle.*

Proof. If another vehicle services request r, the result clearly holds. So, suppose that no other vehicle does. Assume that there is an optimal matching in which vehicle i also does not take request r, with the resulting total objective value $\sum_k s_\beta(k, a_k^*)$. However, since $s_\beta(i,r) > s_\beta(i,r_k^*)$, this solution cannot be optimal. □

Let $Q_{\max} = \max_{i,r} Q(i,r)$; $Q_{\min} = \min_{i,r} Q(i,r)$; and $\epsilon_{\min} = \min_{j\neq k} |z_j - z_k|$. Then, we define a *threshold* $\bar{\beta}$:

$$\bar{\beta} = \frac{Q_{\max} - Q_{\min}}{\epsilon_{\min}}. \tag{2}$$

Lemma 2. *If $r_f \in \mathcal{R}_f$ is optimal for vehicle i for some β, then it is optimal for all $\beta' > \beta$ values.*

Proof. We prove this lemma by contradiction. Assume there exists a $\beta' > \beta$ such that, for some r':

$$\begin{aligned} s_{\beta'}(i,r_f) &\leq s_{\beta'}(i,r') \\ Q(i,r_f) + \beta' F_p(i,r_f) &\leq Q(i,r') + \beta' F_p(i,r') \\ \beta'(F_p(i,r_f) - F_p(i,r')) &\leq Q(i,r') - Q(i,r_f) \end{aligned} \tag{3}$$

Since r_f is optimal for β, we similarly have:

$$\begin{aligned} Q(i,r_f) + \beta F_p(i,r_f) &\geq Q(i,r') + \beta F_p(i,r') \\ Q(i,r') - Q(i,r_f) &\leq \beta(F_p(i,r_f) - F_p(i,r')) \end{aligned} \tag{4}$$

Combining Eqs. (3) and (4), we get:

$$\beta'(F_p(i,r_f) - F_p(i,r')) \leq \beta(F_p(i,r_f) - F_p(i,r')) \tag{5}$$

If $F_p(i,r_f) - F_p(i,r') = 0$, then they are tied for both β and β'. As $F_p(i,r_f) - F_p(i,r') \geq 0$ (Property 1), Eq. (5) implies that $\beta' \leq \beta$, which contradicts our assumption. □

Lemma 3. *If $\beta > \bar{\beta}$, then $s_\beta(i,r) > s_\beta(i,r')$ if $F_p(i,r) > F_p(i,r')$, for all vehicles $i \in \mathcal{V}$ and requests $r \in \mathcal{R}$.*

Proof. Assume, for a proof by contradiction, that there exists a pair of requests r and r' for vehicle i with $F_p(i,r) > F_p(i,r')$ such that $s_\beta(i,r) \leq s_\beta(i,r')$. Then:

$$\begin{aligned} Q(i,r) + \beta F_p(i,r) &\leq Q(i,r') + \beta F_p(i,r') \\ \bar{\beta} < \beta &\leq \frac{Q(i,r') - Q(i,r)}{F_p(i,r) - F_p(i,r')} \\ \frac{Q_{\max} - Q_{\min}}{\epsilon_{\min}} &< \frac{Q(i,r') - Q(i,r)}{F_p(i,r) - F_p(i,r')} \\ \frac{Q_{\max} - Q_{\min}}{Q(i,r') - Q(i,r)} &< \frac{\epsilon_{\min}}{F_p(i,r) - F_p(i,r')} \end{aligned}$$

However, this last inequality cannot hold because the left side ≥ 1 and the right side ≤ 1. □

Lemma 4. *Every request $r_f \in \mathcal{R}_f$ served in $\mathcal{A}(0)$ is also served in $\mathcal{A}(\beta)$ when $\beta > \bar{\beta} \geq 0$.*

Proof. Let i denote the vehicle that served request r_f in $\mathcal{A}(0)$; $\hat{r}$ denote an arbitrary request with $Q(i,\hat{r}) > Q(i,r_f)$, which was assigned to another vehicle j in $\mathcal{A}(0)$; and $\check{r}$ denote an arbitrary request with $Q(i,\check{r}) < Q(i,r_f)$. There are the following two cases: **(Case 1)** Request r_f is optimal for vehicle i when $\beta = 0$. In this case, the request is still optimal for vehicle i when $\beta > \bar{\beta} \geq 0$ (Lemma 2) and it will thus be served in $\mathcal{A}(\beta)$ (Lemma 1). **(Case 2)** Request r_f is not optimal for vehicle i when $\beta = 0$. We first show why request r_f remains preferred over request $\check{r}$ when $\beta > \bar{\beta}$. $s_\beta(i,r_f) = Q(i,r_f) + \beta F_p(i,r_f) > Q(i,\check{r}) + \beta F_p(i,\check{r}) = s_\beta(i,\check{r})$. The inequality is because $Q(i,\check{r}) < Q(i,r_f)$ and r_f has the largest fairness score $F_p(\cdot, r_f)$ because it is in $\mathcal{R}_f$ (Property 1).

We now show that all requests $\hat{r}$ will not be served by vehicle i through the following two cases:

- Request $\hat{r} \notin \mathcal{R}_f$. As $r_f \in \mathcal{R}_f$, when $\beta > \bar{\beta}$, $F_p(i,r_f) > F_p(i,\hat{r})$ (Property 1) and $s_\beta(i,r_f) > s_\beta(i,\hat{r})$ (Lemma 3). Therefore, vehicle i prefers r_f over $\hat{r}$.
- Request $\hat{r} \in \mathcal{R}_f$. There are the following two subcases:
  - Request $\hat{r}$ is assigned to a vehicle j in $\mathcal{A}(0)$ for which the request is optimal. The request remains optimal when $\beta > \bar{\beta} \geq 0$ (Lemma 2) and it will thus be served in $\mathcal{A}(\beta)$ (Lemma 1). Further, since $\hat{r}$ was assigned to vehicle j instead of vehicle i in $\mathcal{A}(0)$, we know that $Q(j,\hat{r}) = s_0(j,\hat{r}) \geq s_0(i,\hat{r}) = Q(i,\hat{r})$. Combining this inequality and the fact that $\hat{r}$ has the largest fairness score $F_p(\cdot,\hat{r})$ when $\beta > \bar{\beta} \geq 0$ because it is in $\mathcal{R}_f$ (Property 1), we get $s_\beta(j,\hat{r}) = Q(j,\hat{r}) + \beta F_p(j,\hat{r}) \geq Q(i,\hat{r}) + \beta F_p(i,\hat{r}) = s_\beta(i,\hat{r})$. Therefore, it is still better to assign the request to vehicle j instead of vehicle i when $\beta > \bar{\beta} \geq 0$.
  - Request $\hat{r}$ is assigned to a vehicle j in $\mathcal{A}(0)$ for which the request is not optimal; there exists another request r' with $Q(j,r') = s_0(j,r') > s_0(j,\hat{r}) = Q(j,\hat{r})$. If r' is assigned to a vehicle k in $\mathcal{A}(0)$ for which the request is optimal, then the arguments from the previous subcase holds, and $\hat{r}$ is assigned to vehicle j because the more preferred r' is assigned to a different vehicle. If r' is assigned to a vehicle k in $\mathcal{A}(0)$ for which the request is not optimal, then one can recursively apply the same reasoning until the more preferred request is assigned to a vehicle for which the request is optimal.

  Consequently, all requests $\hat{r}$ with higher scores than r_f will be served by other vehicles, and r_f has the highest score among the remaining available requests for vehicle i. Therefore, r_f will be assigned to vehicle i or some other vehicle k for which $s_\beta(k,r_f) \geq s_\beta(i,r_f)$.

Thus, requests in $\mathcal{R}_f$ served in $\mathcal{A}(0)$ are served in $\mathcal{A}(\beta)$. □

We are now ready to prove Theorem 1.

Proof of Theorem 1. We show that $z'_{g_{\min}}(\beta) > z'_{g_{\min}}(0)$ because every request in $\mathcal{R}_f$ served in $\mathcal{A}(0)$ is also served in $\mathcal{A}(\beta)$ (Lemma 4) and there is one other request in $\mathcal{R}_f$ not served in $\mathcal{A}(0)$ but is served in $\mathcal{A}(\beta)$. Since $\mathcal{A}(0)$ is passenger-min-unfair, there exists a vehicle k that could serve some request $r_f \in \mathcal{R}_f$ that was dropped, instead serving $r' \notin \mathcal{R}_f$. However, when $\beta > \bar{\beta}$, requests $\mathcal{R}_f \cap A_k$ have the largest fairness score (Property 1) and will thus have larger s_β scores compared to other requests (Lemma 3). As $Q(k,r_f) > Q(k,r_{f'})$ for all $r_{f'} \in \mathcal{R}_f \cap A_k \setminus \{r_f\}$ (Definition 1), r_f is now optimal for vehicle k and is served in $\mathcal{A}(\beta)$ (Lemma 1). □

Table 1: Notation reference for proofs. Note that here we have used r and a interchangeably, as per the assumption for the proofs

Symbol	Meaning
A_i	Set of available actions for vehicle i
$R(i,r)$	Immediate reward if vehicle i takes action $a=\{r\}$
$V(i,r)$	VFA prediction of future value if vehicle i takes action $a=\{r\}$
$Q(i,r)$	Score for vehicle i taking action $\{r\}$, as used in NeurADP ($= R(i,r)+V(i,r)$)
$F_p(i,r)$	Passenger-side fairness score for assigning request r to vehicle i
$F_d(i,r)$	Driver-side fairness score for assigning request r to vehicle i
$s_\beta(i,a)$	Modified score function that takes passenger fairness into account ($= Q(i,r)+\beta F_p(i,r)$)
β	Passenger fairness hyperparameter. Defines the relative weight of fairness
$\mathcal{A}(w)$	Assignment; a matching of all vehicles to valid actions using fairness weight $w \in \{\beta,\delta\}$.
$s_\delta(i,a)$	Modified score function that takes driver fairness into account ($= Q(i,r)+\delta F_d(i,r)$)
δ	Driver fairness hyperparameter. Defines the relative weight of fairness
z_j	The historical metric for group j
$z'_j(\beta)$	The updated metric for the group j after assignment $\mathcal{A}(\beta)$
$z'_j(\delta)$	The updated metric for the group j after assignment $\mathcal{A}(\delta)$
$g(r)$	The group that the request r belongs to
$g_{\min}$	The group with the worst metric value
$\mathcal{R}_f$	The set of requests from group $g_{\min}$

The above proof holds for both SIP and SIP(+) methods. For SIP(+), Eq. 1 is written as:

$$f(r) = \max(\bar{z} - z(r), 0) \tag{6}$$

The rest of the proof remains the same.

While we focus on service rate as a way to measure group-specific performance z_j, our results apply for any performance measure that increases linearly with the number of requests served for a group j. Specifically, these groups do not need to be defined based on geographic areas.

However, our results are only for a particular matching round and do not consider service rates over a series of many time steps. We discuss this further in the next section, but our empirical results demonstrate that the proposed approaches do, in fact, significantly improve fairness over time on multiple metrics of inequality and can do so without significantly reducing overall efficiency.

Driver-side Min-Fairness

We show that when using SID(+), worse-off drivers are guaranteed to get the request that benefits them the most, as long as no fellow worse-off driver competes for that request. As a further generalization, we also show that each disadvantaged driver gets their highest-ranked request that does not benefit another driver more, for a high enough delta.

We show that if any driver with worse-off income has a higher-preferred request they are not assigned when $\delta = 0$, such that no other worse-off driver can serve that request, our approaches improve the income of that driver by assigning their most-preferred request for a sufficiently high δ (see Theorem 2).

Note that the fairness score for a driver j can be written as:

$$f(j) = (\bar{z} - z_j), \tag{7}$$

Formally, suppose that each action a contains at most one request r, allowing us to use both a and r interchangeably. Under this assumption, the assignment $\mathcal{A}$ will contain at most one request per vehicle. For actions which contain only one request, $F_d(j,a) = f(j)R(j,r), a = \{r\}$. For actions with zero requests, $F_d(j,a) = 0$. For the following proofs, we add the assumption that each request provides the same income to all drivers (i.e., $R(j,r) = R(r)$ for all drivers j). This assumption is reasonable as customers are not expected to pay different amounts when matched to different drivers.

Improving driver income: Recall that z_j is *historical* income for driver j, and each driver can be thought of as their own group. We assume that $z_j \neq z_k$ whenever $j \neq k$ (an assumption that is nearly always true in practice).

Let $\mathcal{A}(\delta)$ be the matching solution produced by the ILP with fairness weight δ. We define $z'_j(\delta)$ as the updated income estimate for driver j *after* the matching computed by the ILP with fairness weight δ. The action which maximizes z'_j is the one with the largest $R(j,r)$, leading to the highest increase in income.

Consider the *ranking* of requests elicited by the reward function $R(j,a)$ for each driver. Let $r^*_j = \operatorname{argmax}_r R(j,r)$ denote the request that maximizes the reward for driver j. Note that this order is not affected by δ. We call r^*_j the preferred request for driver j. Being assigned the preferred request maximizes z'_j for driver j. Finally, we also assume that r^*_j is unique (i.e., $R(r^*_j) > R(r')$ for all $r' \in A_j \setminus \{r^*_j\}$).

We say a driver j is *worse-off* or disadvantaged when $z_j < \bar{z}$, or when $f(j) > 0$. Similarly, we say a driver j is *better-off* or advantaged when $z_j \geq \bar{z}$, or when $f(j) \leq 0$.

In what follows, we show that for a large enough δ, each worse-off driver j is assigned their highest preference r^*_j as long as no other worse-off driver can serve r^*_j. This is especially impactful if, in the initial matching $\mathcal{A}(0)$, j was not matched to r^*_j, a condition we formalize as *driver-min-*

unfairness in the following definition. Just like the previous proof, let $Q(i,r) = V(i,r) + R(i,r)$.

Definition 2 (driver-min-unfair)**.** *A matching $\mathcal{A}$ is* driver-min-unfair *if any worse-off driver j is assigned a request $r \neq r_j^*$ in $\mathcal{A}$ and there exists no other worse-off driver k that can serve r_j^*.*

Theorem 2. *If $\mathcal{A}(0)$ is driver-min-unfair for any driver j, then there exists $\delta > 0$ such that $z'_j(\delta) > z'_j(0)$, when using SID(+).*

We build some helpful intuition before proving this theorem. Note that Lemma 1 still holds for this case, as it is a property of the ILP. Let j be the worse-off driver with preferred request r_j^*. Since $\mathcal{A}(0)$ is min-unfair, we know r_j^* was not assigned to j. First, we show that for some large value of δ, $s_\delta(j, r_j^*) \geq s_\delta(j, r')$ for all $r' \in A_j \setminus \{r_j^*\}$.

Lemma 5. *For any worse-off driver j with $f(j) > 0$, there exists a threshold weight δ' such that for any $\delta > \delta'$, $s_\delta(j, r_j^*) \geq s_\delta(j, r')$ for all $r' \in A_j \setminus \{r_j^*\}$.*

Proof. The proof for this is constructive. Take any $r' \in A_j \setminus \{r_j^*\}$. Since $R(r_j^*) > R(r')$, we have, for some δ:

$$\begin{aligned} s_\delta(j, r_j^*) &\geq s_\delta(j, r') \\ Q(j, r_j^*) + \delta f(j) R(r_j^*) &\geq Q(j, r') + \delta f(j) R(r') \\ \delta f(j) \left(R(r_j^*) - R(r')\right) &\geq Q(j, r') - Q(j, r_j^*) \\ \delta &\geq \frac{Q(j, r') - Q(j, r_j^*)}{f(j)\left(R(r_j^*) - R(r')\right)} \\ \delta \geq \delta' &= \frac{Q_{\max} - Q_{\min}}{f(j)\left(R(r_j^*) - R(r')\right)} \end{aligned}$$

Since j is a worse-off driver, $f(j)$ is positive, and thus, the denominator is positive. If $Q(j, r_j^*) > Q(j, r')$, the numerator is negative, and the property holds even at $\delta = 0$. Otherwise, there exists some positive threshold δ' above which r_j^* is guaranteed to have the highest score for vehicle j. □

By Lemma 1, we also know that for such a δ, r_j^* is guaranteed to be assigned to some driver. Next, we prove that under the conditions of driver-min-unfairness, this driver is guaranteed to be driver j. To show this, consider another driver k that is competing for r_j^*. Let r_j and r_k be the corresponding requests that j and k will be assigned respectively if they fail to get r_j^*. Then, to favor the scenario where j gets r_j^*, we need the score for that assignment to be larger than the score for when k gets r_j^*:

$$\begin{aligned} s_\delta(j, r_j^*) + s_\delta(k, r_k) &\geq s_\delta(j, r_j) + s_\delta(k, r_j^*) \\ s_\delta(j, r_j^*) - s_\delta(j, r_j) &\geq s_\delta(k, r_j^*) - s_\delta(k, r_k) \end{aligned} \tag{8}$$

Solving the left-hand side (LHS) first, we have the following:

$$\text{LHS} = Q(j, r_j^*) - Q(j, r_j) + \delta f(j) \left(R(r_j^*) - R(r_j)\right)$$

Similarly, the right-hand side (RHS) becomes:

$$\text{RHS} = Q(k, r_j^*) - Q(k, r_k) + \delta f(k) \left(R(r_j^*) - R(r_k)\right)$$

We use the following shorthand notations for ease of comprehension:

$$\begin{aligned} \Delta Q_j &= Q(j, r_j^*) - Q(j, r_j) \\ \Delta Q_k &= Q(k, r_j^*) - Q(k, r_k) \\ \Delta R_j &= R(r_j^*) - R(r_j) \\ \Delta R_k &= R(r_j^*) - R(r_k) \end{aligned}$$

Substituting these into Eq. 8, we get:

$$\begin{aligned} \Delta Q_j + \delta f(j) \Delta R_j &\geq \Delta Q_k + \delta f(k) \Delta R_k \\ \delta f(j) \Delta R_j &\geq \delta f(k) \Delta R_k + \Delta Q_k - \Delta Q_j \\ f(j) \Delta R_j x &\geq f(k) \Delta R_k + \frac{1}{\delta}\left(\Delta Q_k - \Delta Q_j\right) \end{aligned} \tag{9}$$

Note that the effect of the ΔQ terms goes to zero as $\delta \to \infty$. We are now ready to prove Theorem 2.

Proof. Let j be the worse-off driver with preferred request r_j^*, such that $\mathcal{A}(0)$ is driver-min-unfair for j. We know r_j^* was not assigned to j in $\mathcal{A}(0)$, and j was matched to some other request r_0. By Lemma 5, we know that for any δ larger than a threshold value δ', r_j^* has the largest score $s_\delta(j, r_j^*)$ among all actions available to j. By Lemma 1, we can say that r_j^* is assigned to some driver in $\mathcal{A}(\delta)$.

Further, since $\mathcal{A}(0)$ is driver-min-unfair, no other worse-off driver can serve r_j^*. Thus, any drivers competing for r_j^* are better-off drivers. For SID(+), recall that we clip the fairness score to be positive. So, $f(k) = 0$ for all better-off drivers. Plugging this into Eq. 9, we see j is guaranteed to be a preferred match for r_j^* over k if the following condition is satisfied:

$$f(j) \Delta R_j \geq \frac{1}{\delta}\left(\Delta Q_k - \Delta Q_j\right)$$

ΔR_j is guaranteed to be positive since r_j^* is the highest preference. $f(j)$ is also positive since j is a worse-off driver. Thus, we can always find a large enough $\delta > \delta'$ such that this inequality holds, and thus guarantee that j gets their highest preferred request. Therefore, since $R(r_j^*) > R(r_0)$, it must be the case that $z'_j(\delta) > z'_j(0)$ for all drivers j for which $\mathcal{A}(0)$ is driver-min-unfair. □

If we interpret ΔR_x to be the *benefit* that driver x gets from being assigned r_j^*, Eq. 9 tells us that any worse-off driver j will get r_j^* as long as they benefit from it more than any other driver, weighted by their fairness score. This also leads to an interesting behavior when $f(k)$ is negative. In this case, the matching may prefer to give r_j^* to k iff $R(r_j^*) < R(r_k)$, i.e., this assignment harms k. So, for SID, it prefers pulling down better-off drivers as much as pulling up worse-off drivers, and may forego improving disadvantaged drivers as long as it can worsen the better off groups more, pulling them towards the mean.

This observation also lets us make an additional, looser corollary statement about SID(+):

Corollary 1. *All worse-off drivers j that do not receive their highest priority request r_j^* in $\mathcal{A}(0)$ are assigned that request using SID(+) for some large enough δ, as long as no other worse-off driver k can benefit more from receiving it, i.e. $f(j)\Delta R_j > f(k)\Delta R_k$.*

Addendum for Theoretical Properties

We note that Theorems 1 and 2 guarantee that the *current* $z_{g_{\min}}$ sees the most improvement, but it is possible that $\operatorname{argmin}_i(z'_i) \neq g_{\min}$. This is due to the fact that the fairness scores are calculated using previous values of Z, but due to batch allocation, the order might change. However, at the next time step, the new minimum would be improved, and so on, and thus we expect $\min(Z)$ to rise over time. This is backed by our experimental results.

For the theoretical analysis in the previous section, we assume that an action contains only one request at a time. We provide an example here to illustrate why this assumption is necessary for Theorem 1.

Example 1. *Consider a vehicle with two available actions a_1 and a_2. Let a_1 contain one action from the worst off group $g_{\min}$, and a_2 contain 2 requests from some other group g such that $z_g < (z_{g_{\min}} + \bar{z})/2$. We can see that $F(\cdot, a_2) > F(\cdot, a_1)$, thus it is possible that for some choice of value function, a_2 will be selected over a_1, thus voiding the property proved in Theorem 1.*

However, in our experiments and algorithm setup, we relax this assumption to allow a vehicle to pick up multiple requests at one time. While this does violate the conditions for Theorem 1 to hold, we note that this decision will still improve fairness for groups other than $z_{g_{\min}}$. Since the other group in the above example needs to lie between $z_{g_{\min}}$ and $\bar{z}$, it is one of the under-served groups, and thus even this alternate decision still helps improve some disadvantaged group, even if it doesn't improve $g_{\min}$.

Non-Myopic Example

As discussed above, Theorem 1 holds for the myopic setting, where we want to maximize the current worst-off zone. While we do observe that using our approach improves the minimum service rate over multiple time steps, we cannot guarantee it because it is possible to create a contrived example as follows.

Example 2. *Consider a vehicle i, capable of serving one passenger belonging to $g_{\min}$ at time t, with the trip taking N time steps to complete. This request will be necessarily served, for $\beta > \bar{\beta}$. We can now imagine a scenario where, at every time step starting from $t+1$, multiple requests from $g_{\min}$ are available, such that they take only one time step to complete. It is clear to see how foregoing the original request would have been a better long-term decision for improving $z_{g_{\min}}$.*

In the example above, N can be arbitrarily large, and thus we cannot bound the loss in fairness.

A similar example can be constructed for Theorem 2.

Extended Discussion

Selection of β and δ

We discussed one way of selecting $\beta \geq \bar{\beta}$ to ensure the fairness term dominates the value function, using $\epsilon_{\min}$, the minimum gap between two groups' current service rates. While $\epsilon_{\min}$ may change across time, it might instead be of interest to specify a minimum difference between the fairness scores of two groups such that any difference greater than that is considered for fairness, akin to the slack parameter for statistical parity. This is useful in scenarios where we have a desirable threshold of fairness ϵ, and we can select a threshold β_ϵ as:

$$\beta_\epsilon = \frac{Q_{\max} - Q_{\min}}{\epsilon} \tag{10}$$

Increase in Service Rate with Fairness

We noted in our experiments that in some cases, our approaches are able to improve service rate of the base algorithm, for small values of β and δ, especially in the SI(+) variants of our approach.

While this effect is small, we observed it consistently. This leads us to believe that adding SI-based fairness can have a regularizing effect on the value function, especially when the value function approximation is not perfect. For large values of δ and β, we start to see a drop in service rate, as the tradeoffs in cumulative value become larger. However, for small hyperparameter values, the algorithms can be made much fairer at almost no cost to the efficiency objective. This is also shown by our ablation experiment, suggesting that small amounts of fairness spread the system's resources (e.g., preventing taxis from accumulating in the busier areas) and allow for a better overall solution.

Discounting and the Selection of Metrics

For both passengers and drivers, we use historical metrics to ensure we are being fair over a longer time window, instead of only consider the current decisions. While this allows us to use information from the past and adds temporal flexibility to our fairness criterion, it also comes with the issue of under-weighting the present. Having overly long histories can hide recent inequalities, and maybe even bolster them.

As an example, consider that group i was highly served for the past 2 years, leading to a negative fairness incentive, but over the last two weeks, their service rate has been very low. If using an infinite historical horizon, SI would still continue to penalize group i, while we should be giving them bonuses. It would also take a long time for the historical average to catch onto this new behavior. Thus, to combat such issues, we *discount the past* for calculating the metrics used for assigning the fairness incentive. Older events are weighed less than more recent events. This discount can be tuned to change the weight of temporal distance.

We show how we compute the discounted passenger side metric of service rate below. Let T_i be the total requests for group i previously seen; S_i be the total requests for group i previously served; t_i be the number of new requests and s_i be the number of new requests served from group i. Then, without discounting, the update for z_i looks as follows:

$$S_i' = S_i + s_i$$
$$T_i' = T_i + t_i$$
$$z_i' = \frac{S_i'}{T_i'}$$

Now, assume we have a past-discount factor γ. Then, we write the update for z_i as:

$$S_i' = \gamma S_i + s_i$$
$$T_i' = \gamma T_i + t_i$$
$$z_i' = \frac{S_i'}{T_i'}$$

Similarly, for drivers, our metric is historical income scaled by the largest driver income. Let D_i be the historical income of driver i; d_i be the income at the current time step, and z_i be the metric value. Then, without discounting, we compute the update and z_i as:

$$D_i' = D_i + d_i$$
$$z_i' = \frac{D_i'}{\max_j D_j'}$$

If we consider an infinite past-horizon, all of these values would be very close to 1. With discounting, the update takes the following form:

$$D_i' = \gamma D_i + d_i$$
$$z_i' = \frac{D_i'}{\max_j D_j'}$$

Another alternate way to do this is the use of moving windows. However, computing metrics over moving windows is expensive, especially if the windows are meaningfully large. Discounting the past provides us with a smooth and easy to compute way of accounting for recent events.

Further, since history is always a factor in real life, we rarely start with zeros for the metric value for everyone. We can add an initial metric value to provide a warm start based on some history. For example, we can consider that drivers had some historical income (from previous days of working), thus preventing the first few jobs served in the simulation from appearing like a massive disparity.

Discounting ensures that the warm start values eventually stop contributing to the metric calculation. We note that we only used discounted metric values for calculating the fairness incentives. For comparing our results, we used the actual metric value computed over the simulation period without discounting. We found that discounting, combined with warm starts, improved the stability of the algorithms, especially for periods of low activity.

In general, we observe that metrics normalized to be within the $[0, 1]$ range will work well with our SI framework. Having a consistent range of metric values also ensures that the behavior of the hyperparameters stays consistent regardless of history size.

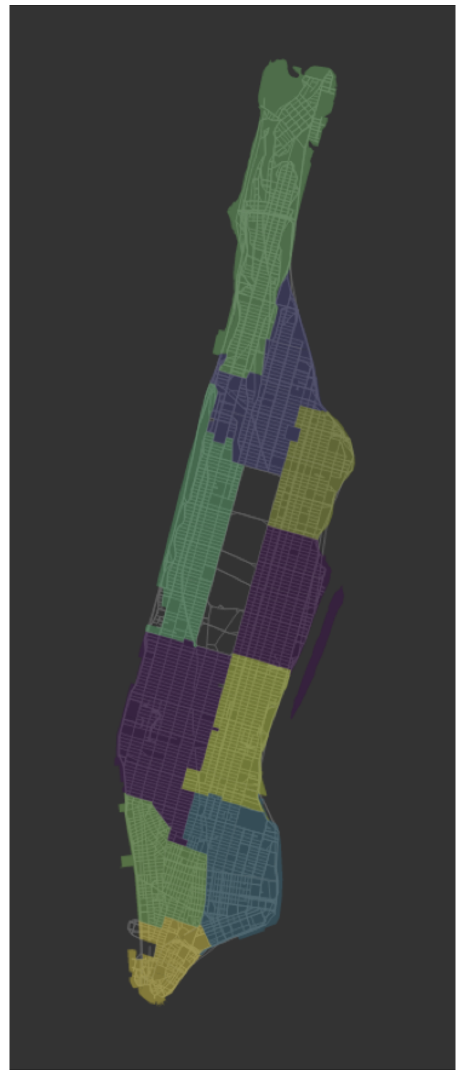

Fig. 1: A map of Manhattan island, with each independent colored region representing one area used for our passenger grouping. There are 10 areas, meaning there are 100 passenger groups.