Question 2 Methodology (Principle of Economics)

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Chapter 1

Introduction

We are constructing a theoretical model based on the Theory of Consumer Choice from Microeconomics that will be used to study the substitution and income effect of the dairy market in the United States of America (U.S.) when the government imposes a subsidy to reduce the price of dairy milk.

We will first have a brief review to the concepts of Microeconomics (Chapter 2). A theoretical model with its details will be listed out in the next section. Last, an analysis to the graph will be included in this document.

The contents are taken from [2] and [1].

Chapter 2

Theoretical Review

2.1 Budget Constraint

Recall that from the 10 principles of economics, the consumers always faces trade off due to the scarcity in resources. When the context is focused to how consumers make decision to purchase goods in a market, the resources that limits the consumer's buying power is the budget they have. The scarcity of budget is described as the **budget constraint**. To describe it mathematically, for a market that has n products, the set of products $\{x_n\}$ which contains all the quantity of the product x_n, the set of price for each product is given $\{P_n\}$, given the total budget for the consumer to purchase any quantity of goods in $\{x_n\}$ as I, the budget constrain formulates a linear equation.

$$\sum_{i=1}^{n} P_n x_n = I. \tag{2.1}$$

$$P_1x_1 + P_2x_2 + P_3x_3 + \cdots + P_nx_n = I. \tag{2.2}$$

For simplicity to the model we want to consider, a two product consumption bundle is considered.

$$P_x x + P_y y = I. \tag{2.3}$$

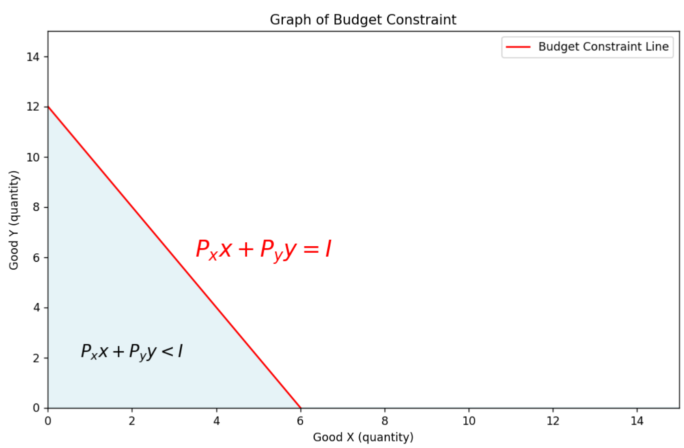


Figure 2.1: Example of Budget Constraint for Good X ($P_x = 12$) and Good Y ($P_x = 12$) under budget of \$ 72

Consumer are bounded by the budget line $P_x x + P_y y = I$, he can only by the quantity of goods x and y in a consumption bundle, (x, y) bounded to the area $P_x x + P_y y \leqslant I$. The area is called the budget set for budget I, which consists of all bundle that are affordable at the given price and income.

The slope of the budget line $\dfrac{\Delta y}{\Delta x} = -\dfrac{P_x}{P_y}$ demonstrates the ratio of the price of the good x to good, which measures the opportunity cost of giving up a number of goods y to consume an extra good x.

2.1.1 Shift of Budget Line

There are two types of shift for the budget lines as the parameters varies.

Parallel Shift of Budget Line The budget line has a parallel shift when the value of the budget I varies.

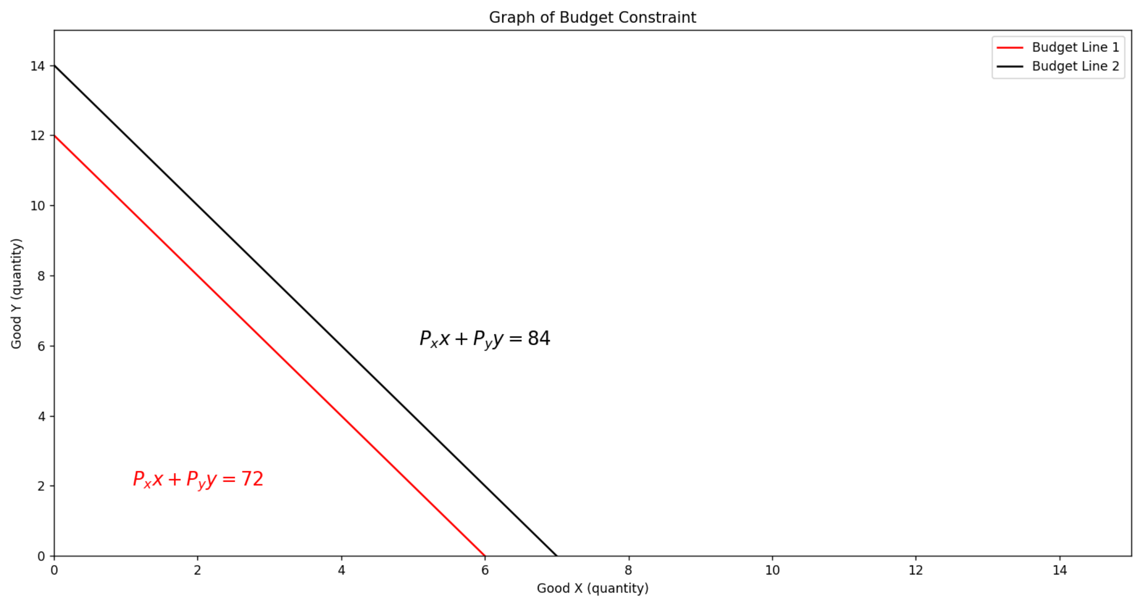


Figure 2.2: Parallel shift of budget line by increase of budget

Shift under different gradient As the ratio of the prices of the two goods P_x, P_y, $\dfrac{\Delta y}{\Delta x} = -\dfrac{P_x}{P_y}$ varies, the graph becomes steeper or less steep.

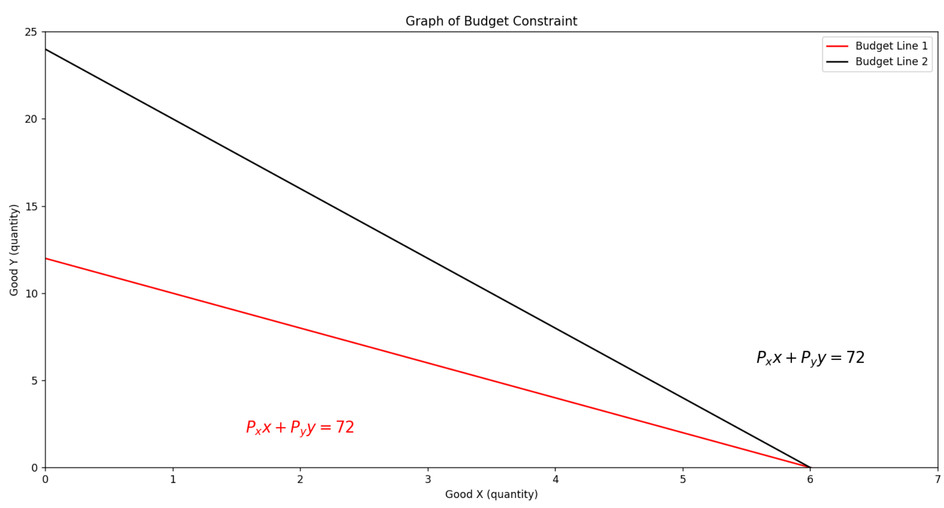


Figure 2.3: Shift of Budget Line when price ratio changes ($P_y = 6 \rightarrow P_y = 3$, P_x unchanged)

2.2 Preference

In analyzing the decision made by consumers, an essential factor that is taken into consideration is the preference of the consumer, which is which goods does the consumer prefer over a good. The concept of preference exists over the comparison between different goods.

The preference of the consumer over two goods can only fall into three determined answers.

1. The consumer **prefers good 1 over good 2**.
2. The consumer **prefers good 2 over good 1**.
3. The consumer **feels indifferent** for both good 1 and good 2.

For a consumption bundle, the symbol $\succ$ is defined as strict preference. Given $(x_1, y_1) \succ (x_2, y_2)$ means the consumer is **strictly prefer** bundle 1, (x_1, y_1) over bundle 2, (x_2, y_2). If the consumer feels indifferent between bundle 1 and bundle 2, its mathematical notations is $(x_1, y_1) \doteq (x_2, y_2)$.

2.2.1 Assumption of Preference

The consumer theory adapts three axioms of consumer preference:

Complete For any comparable bundles, the consumer preference must be either $(x_1, y_1) \succeq (x_2, y_2)$ or $(x_2, y_2) \succeq (x_1, y_1)$, or both, in which case the consumer is indifferent between two bundles $(x_1, y_1) \doteq (x_2, y_2)$. The consumer is clear of his/her preferences, either prefer a good over another good or indifferent for both goods.

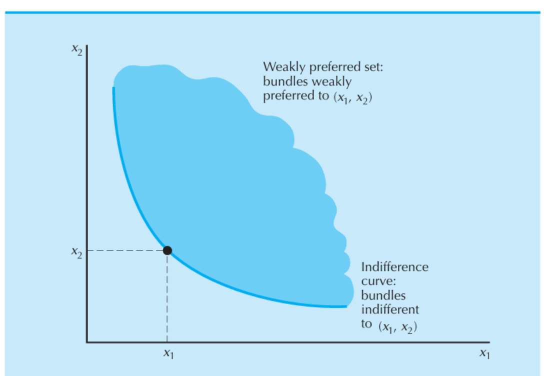
Reflexive Any bundle is at leas as good as itself: $(x_1, y_1) \succeq (x_1, y_1)$.

Transitive For three bundles, if $(x_1, y_1) \succeq (x_2, y_2)$ and $(x_2, y_2) \succeq (x_3, y_3)$, then $(x_1, y_1) \succeq (x_3, y_3)$.

2.2.2 Indifference Curve

Definition 2.2.1: Indifference curve

The ***indifference curve*** is the mathematical representation of the preference of the consumer over the goods, which the consumer bundles on the curve are indifferent mutually.



Weakly preferred set. The shaded area consists of all bundles that are at least as good as the bundle (x_1, x_2).

Figure 2.4: Weakly preferred set and the indifference curve.

Recall in figure 2.4, the shaded area is the consumption bundles allowed by the relation $(y_1, y_2) \succeq (x_1, x_2)$, is called the **Weakly Preferred Set**, which is the consumption bundles that better off or indifferent to bundle (x_1, x_2). The indifference curve exactly sits on the boundary of the weakly preferred set.

We don't want to consider too general indifference curve, since it is not practical in the context afterwards. We choose some features of indifference curve and label the indifference curve that adopt these features as the **well-behaved indifferent curves**.

Below are the features of a "well-behaved" indifference curve.

Monotonicity More is better. If (x_1, x_2) is a consumer bundle and (y_1, y_2) is a bundle of good with at least the number of goods in the first bundle, $(y_1, y_2) \succ (x_1, x_2)$,

Convexity Averages are preferred to extremes. Implies that indifference curves have negative slope.

Strict Convexity The weighted average of any two points on the indifference curve is strictly preferred to the extreme bundles. Which, the indifference cannot have flat spots. (This feature is very useful in the upcoming chapter when we want to find the optimal bundles.)

2.2.3 Marginal Rate of Substitution

Definition 2.2.2: Marginal Rate of Substitution

The ***marginal rate of substitution (MRS)*** is the slope of the indifference curve, which measures the rate which the consumer is wiling to substitute one good for another (ratio of substituting good y to good x).

$$MRS = -\frac{\Delta y}{\Delta x}. \tag{2.4}$$

The MRS is sometimes addressed as the marginal willingness to pay.

2.3 Utility

The concept of utility is used to compare the preference of the consumer over the consumption bundles.

2.3.1 Utility Function

The Utility function assigns a number to each indifference curve. The magnitude of the utility orders the preferences of the consumer bundle and the indifference curve. Its importance of the ordering emphasis its ordinal properties.

Since the order is the essentiality of assigning numbers to the indifference curve, any transformation that preserves the 'ranking' of the utility is accepted in using it to describe the same order of preferences. We call these type of transformation as the monotonic transformation.

Definition 2.3.1: Monotonic Transformation

The ***monotonic transformation*** is a transformation to the utility function such that it preserves the order of the numbers assigned to different indifference curve.

Theorem 2.3.1

A monotonic transformation of a utility function is a utility function that represents the same preferences as the original utility function.

Some utility equations are typical, and is intuitively obvious by its geometric view.

Perfect Substitutes In general, utility of perfect substitutes could be represented by linear functions.

$$U(x_1, x_2) = ax_1 + bx_2.$$

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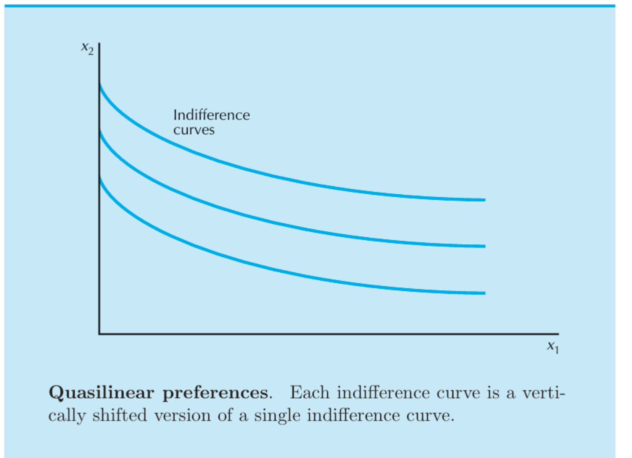
Perfect Complements The utility of the perfect compliments takes the minimum of both the quantity of the goods.

$$U(x_1, x_2) = a_0 min\{x_1, x_2\}.$$

Quasilinear Preference The indifference function are different functions that are just merely vertical translate of each other.

$$U(x_1, x_2) = v(x_1) + x_2 = k.$$



Quasilinear preferences. Each indifference curve is a vertically shifted version of a single indifference curve.

Figure 2.5: Example of Quasilinear Indifference curves

Cobb-Douglass Preferences The Cobb-Douglas utility function demonstrates a well being function that assigns numbers to well-behaved indifference curves.

$$U(x_1, x_2) = x_1^c x_2^d.$$

c, d are positive numbers which measures the preference of the consumer to each product.

By taking a monotonic transformation $T : U \to U^{\frac{1}{c+d}}$, the Cobb-Douglas utility function to the same preference is transformed into a new functional form.

$$U(x_1, x_2) = x_1^{\frac{c}{c+d}} x_2^{\frac{d}{c+d}}$$

Since,

$$\frac{c}{c+d} + \frac{d}{c+d} = 1.$$

We let $a = \dfrac{c}{c+d}$, $b = \dfrac{d}{c+d} = 1 - a$, a normalized form of the Cobb-Douglas utility function is generated.

$$U(x_1, x_2) = x_1^a x_2^{1-a}.$$

Where a is a positive number which $0 \leqslant a \leqslant 1$.

2.3.2 Marginal Utility

Given a consumption bundle of two goods, the marginal utility of the bundle with respect to good 1 is denoted as below.

$$MU_1 = \left.\frac{\partial U}{\partial x_1}\right|_{x_2}$$

Similar for the marginal utility with respect to goods two.

$$MU_2 = \left.\frac{\partial U}{\partial x_2}\right|_{x_1}.$$

By writing into their differential form,

$$\Delta U \approx MU_1 \Delta x_1 + MU_2 \Delta x_2.$$

Question: How is MU related to MRS from the previous chapter?

Recall that in the previous sections, the MRS of the consumer bundle is given by

$$MRS = \frac{\Delta x_2}{\Delta x_1}.$$

(Assume x_1 is x, x_2 is y.)

Consider a movement on the same indifferent curve which the utility is constant, then

$$MU_1 \Delta x_1 + MU_2 \Delta x_2 = 0.$$
$$\frac{\Delta x_2}{\Delta x_1} = -\frac{MU_1}{MU_2}$$

Hence,

$$MRS = -\frac{MU_1}{MU_2}$$

Note that for under monotonic transformations, the MRS are still preserved (However, its prove for generality is left as an exercise for the readers.)

2.4 Optimal Choice

Given a budget constraint $p_x x + p_y y = I$ and an indifference curve in the form of Cobb-Douglas utility function $U(x, y) = x^a y^{1-a}$. The optimal bundle falls at the tangency point of the two curves. Given the features of the Cobb-Douglass preferences with strict convexity, the indifference curve tangents the budget line on only a unique point. (The mathematical proof is left as an exercise for the readers.)

At the optimal point, the slope of the indifference curve and the budget line is equal, which implies that

$$MRS = Price\ Ratio$$
$$-\frac{MU_1}{MU_2} = \frac{\Delta y}{\Delta x}.$$

Chapter 3

Contructing Model

We want to construct a model to study the substitution and income effect when the government reduces the price of dairy milk - Class I Fluid Beverage Milk (abbreviated as Fluid Milk) by subsidy. We have selected a non-dairy product - Almond Milk to form a consumption bundle.

In this model, we included some assumptions to fit it with the economics theory.

- The Class I Fluid Beverage Milk and Almond milk are both assumed to be considered as normal goods, which obeys the demand law.
- The Class I Fluid Beverage Milk and Almond Milk are assumed to be non-perfect substitutes.
- The Class I Fluid Beverage Milk and Almond Milk are non-satiation, as their utility function are strictly monotonic non-linear function, assumed to be described by Cobb-Douglass preferences.

We inserted real-life data into the model. The market selected for data is the U.S. dairy industry, specified to the Class I Fluid Beverage Milk and the non-dairy market, the almond milk. The model adapts specifically the price of both products in 2023 as a parameter, with its total consumption. The market total consumption of the two milks are selected to be the optimal bundle of the consumption bundle, as believed to be selected by the U.S. consumers.

Product	Consumption Per Capita (cwt)	Price per cwt ($,USD)
Class I Fluid Beverage Milk	1.2800	19.20
Almond Milk	0.0385	75.95

Table 3.1: Data collected by USDA Published Documents

The data of the price and consumption are taken average over 12 months in the 2023. In order to analyze the substitution and income effect, we designed a python program to plot the graph of the consumption bundle. The theory applied is shown as below.

The preference of the consumption bundle is assumed to be the Cobb-Douglas Preference.

$$U(x, y) = x^a y^{1-a}.$$

The budget line is given by

$$p_x x + p_y y = I$$

The tangent point of the budget line and the indifference curve is

$$\begin{cases} x^* &= \dfrac{aI}{p_x} \\ y^* &= \dfrac{(1-a)I}{p_y} \end{cases}$$

Chapter 4

Discussion

The graph plotted is displayed as below.

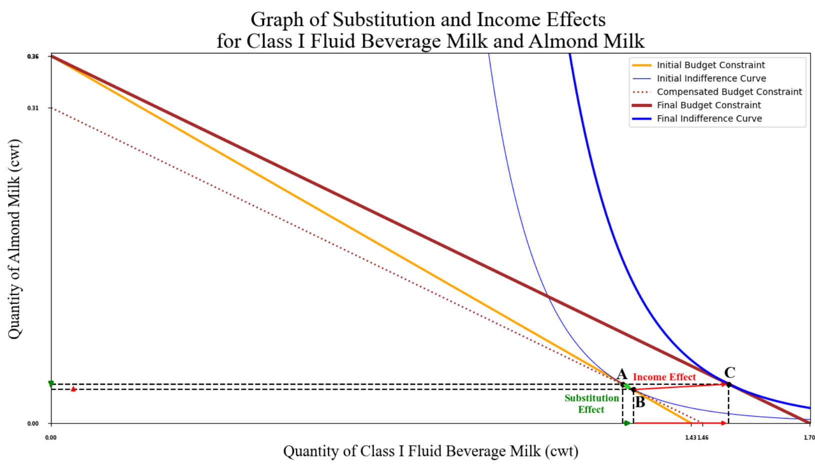


Figure 4.1: Original graph of Fluid Milk and Almond Milk

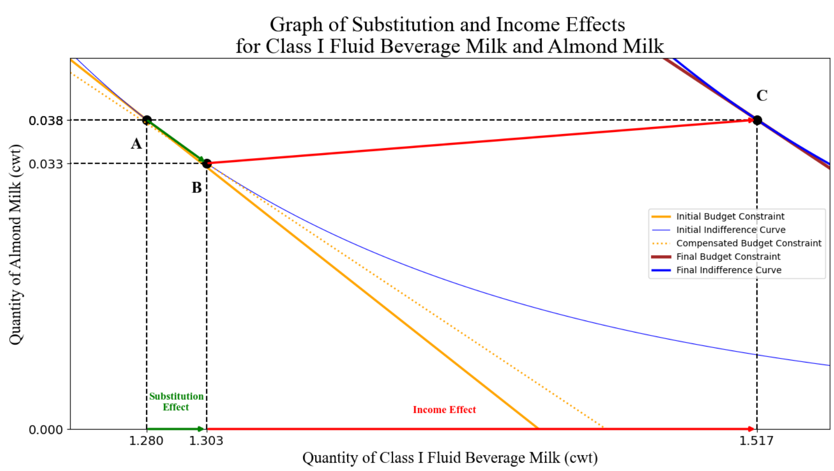


Figure 4.2: Enlarged Graph

The income effect is represented by the shift of optimal points (B to C), the parallel shift from the compensated budget line to the final budget line. This implies an increment in the purchasing power. Moreover, since the optimal indifference curve is higher, the consumer can achieve higher satisfaction through this shift. Besides that, although the consumer's total budget remains the same, the purchasing power of the consumer has improved due to the decrease in the price of fluid milk. This means that with the same budget, the consumer can buy more of both milks. As a result, the higher purchasing power results in consumers increasing their consumption of both milks. Particularly in this model, the increase in the consumption of almond milk in the income effect is the same as the decrease in the substitution effect, which results in the final consumption of almond milk being unchanged.

To conclude, the decrease of the fluid milk price leads to the substitution effect, where the consumer buys more fluid milk and less almond milk, and the income effect, where increased purchasing power leads to more purchase of both milks. Ultimately, through subsidizing, it would increase the quantity demanded of fluid milk, and stimulate the growth of the dairy market, as it encourages more consumption of fluid milk.

Chapter 5

Conclusion

In this paper, we applied the theories of economics to construct model for explaining possible incidents that will happen in the market. However, the models constructed may not always fit with the real-life market, additional conditions and assumptions are required to adjust the model for it to have a more sufficient description to the real-life market.

Bibliography

[1] N Gregory Mankiw. *Principles of microeconomics*. Vol. 1. Elsevier, 1998.

[2] Hal R Varian. *Intermediate microeconomics with calculus: a modern approach*. WW norton & company, 2014.