

Chapter-3 - Numerical x Past problem
ex-1 capacitance transducer

Q19E

1.6 Three resistances have the following ratings:

$$R_1 = 20\Omega \pm 0.1$$

$$R_3 = 60\Omega \pm 0.25$$

Soln $\rightarrow R_2 = 20\Omega \pm 0.1$

Determine magnitude and limiting error in Ω if resistance are connected in series. Also obtain % relative error in resultant.

Soln:-

writing resistance in % form.

$$R_1 = 20 \pm 0.5\%$$

$$R_3 = 60\Omega \pm 0.42\%$$

$$R_2 = 20\Omega \pm 0.5\%$$

$$R_{eq} = R_1 + R_2 + R_3$$

$$\text{nominal value of } R_{eq} = 20 + 20 + 60 = 100\Omega$$

For limiting error in Ω , value of R_1, R_2, R_3 be highest.

$$R_{eq} = 20.1 + 20.1 + 60.25$$
$$= 100.45\Omega$$

$$\begin{aligned}\text{limiting error in } \Omega &= \text{nominal value} - \text{highest value} \\ &= 100 - 100.45 \\ &= -0.45\end{aligned}$$

$$\therefore \text{limiting error in } \Omega = \pm 0.45\Omega$$

$$\% \text{ limiting error} = \frac{\text{limiting error}}{\text{nominal value}} \times 100$$

$$= \frac{\pm 0.45}{100} \times 100 = \pm 0.45\%$$

transducer → 2/3 (1)

2. (a) AB: $R_1 = 1000 \Omega \parallel C = 0.5 \mu F$

BC: $R = 1000 \Omega \parallel C = 0.5 \mu F$

CD: $L = 30 mH + R = 200 \Omega$

DA: $R \parallel C \text{ or } L$ $f = 1000 Hz$

$\omega = 2000 \pi$

Here,

$Z_1 = R_1 \parallel C_1$

$Y_1 = \frac{1}{R_1} + j\omega C_1$

$Z_2 = R_2 \parallel C_2$

$Y_2 = \frac{1}{R_2} + j\omega C_2$

$Z_3 = R_3 + j\omega L_3$

$Z_4 = ?$

$Z_1 Z_3 = Z_2 Z_4$

$Y_2 Z_3 = Y_1 Z_4 \Rightarrow Z_4 = \frac{Y_2 Z_3}{Y_1} = \frac{(\frac{1}{R_2} + j\omega C_2)(R_3 + j\omega L_3)}{(\frac{1}{R_1} + j\omega C_1)}$

$= \frac{R_3 - \omega^2 C_2 L_3 + j\omega \frac{L_3}{R_2} + j\omega R_3 C_2}{\frac{R_2}{R_1}}$

$= \left(\frac{1000}{200} - (2000\pi)^2 \cdot 0.5 \times 10^{-6} \times 30 \times 10^{-3} \right) + j \left(\frac{2000\pi \cdot 30 \times 10^{-3}}{1000} + \frac{2000\pi \cdot 0.5 \times 10^{-6}}{200} \right)$

$= \frac{4.408 + 0.8169j}{0.001 + 3.142 \times 10^{-3}j} = 641.52 - 1198.75j$

$R = 641.52 \Omega$ and, $\frac{1}{\omega C} = 1198.75$

$C = \frac{1}{2000\pi \times 1198.75} = 1.32 \times 10^{-7} = 0.132 \mu F$

3. (a) A capacitive transducer uses two quartz diaphragms of area 750 mm^2 separated by a distance of 3.5 mm . A pressure of 900 kN/m^2 when applied to the top diaphragm produces a deflection of 0.6 mm . The capacitance is 370 pF when no pressure is applied to the diaphragms. Find capacitance after applⁿ of pressure.

Here,

$$A = 750 \text{ mm}^2 = 750 \times 10^{-6} \text{ m}^2$$

originally Distance betⁿ plates (d_1) = 3.5 mm

Pressure $P = 900 \text{ kN/m}^2$

deflection $d = 0.6 \text{ mm}$

now, distance betⁿ two plates after pressure applied is,

$$d_2 = d_1 - d = 3.5 - 0.6 = 2.9 \text{ mm}$$

Original capacitance $C = 370 \text{ pF}$

Capacitance after pressure $C' = ?$

$$C = \frac{\epsilon A}{d} \quad \text{--- (1)}$$

For no pressure applied,

$$C = \frac{\epsilon A}{d_1} \quad \text{--- (2)}$$

After pressure applied,

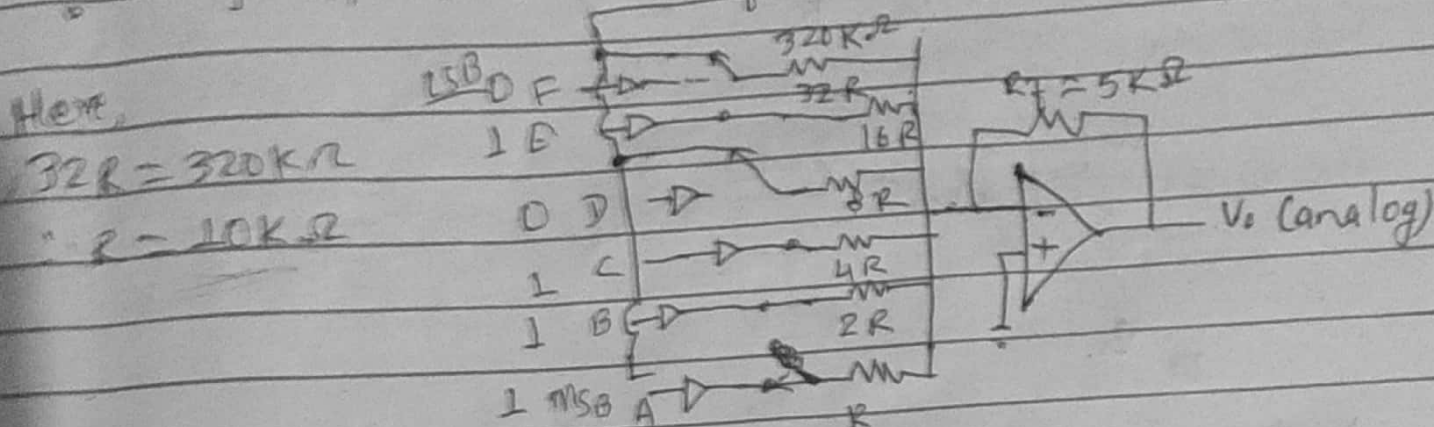
$$C' = \frac{\epsilon A}{d_2} \quad \text{--- (3)}$$

$$\frac{C'}{C} = \frac{d_1}{d_2}$$

$$C' = \frac{3.5}{2.9} \times 370 \times 10^{-12} \text{ F}$$

=

5. @ 6-bit DAC Resistance of $320\text{ k}\Omega$ in LSB position.
 weighted register network: $V_{ref} = 10\text{V}$. o/p of resistive network
 is connected to an op-Amp with a feedback resistor of $5\text{ k}\Omega$.
 ? Analog o/p of 111010?



we do this by super position theorem.
 For only MSB is 1, and keeping all other switches open,
 we get,

$$V_A = -\frac{R_f}{R} V_{in} \quad (\because \text{inverting amplifier}).$$

$$V_A = -\frac{R_f}{R} V_{ref}$$

$$= -\frac{5\text{ k}\Omega}{10\text{ k}\Omega} \times 10 = -5\text{V}$$

$$\text{i.e. } V_A = -\frac{V_{ref}}{2}$$

$$\text{Similarly, } V_B = -\frac{R_f}{2R} V_{ref} = -\frac{V_{ref}}{4}$$

$$V_C = -\frac{V_{ref}}{8} \quad V_D = -\frac{V_{ref}}{16} \quad V_E = -\frac{V_{ref}}{32} \quad V_F = -\frac{V_{ref}}{64}$$

$$\therefore V_o = -(V_A + V_B + V_C + V_D + V_E + V_F)$$

$$= -\left(\frac{1}{2} V_{ref} + \frac{1}{4} V_{ref} + \frac{1}{8} V_{ref} + 0 + \frac{1}{32} V_{ref} + 0\right)$$

$$= -\left(\frac{1}{2} \times 10 + \frac{1}{4} \times 10 + \frac{1}{8} \times 10 + \frac{1}{32} \times 10\right) = -5 - 2.5 - 1.25 - 0.3125$$

$$= -9.0625$$

(ubit) (d3 d2 d1 d0)

above question can be done as,

we have,

$$I_{out} = \frac{V_{ref}}{R} D_{n-1} + \frac{V_{ref}}{2R} D_{n-2} + \frac{V_{ref}}{4R} D_{n-3} + \dots + \frac{V_{ref}}{2^{n-2}R} D_1 + \frac{V_{ref}}{2^{n-1}R} D_0$$

$$= \frac{V_{ref}}{R} \left(D_{n-1} + \frac{D_{n-2}}{2} + \frac{D_{n-3}}{4} + \dots + \frac{D_{n-(n-1)}}{2^{n-2}} + \frac{D_{n-(n-0)}}{2^{n-0-1}} \right)$$

Q. 1.

$$V_{out} = I_{out} \times R_f \quad \text{--- (2)}$$

$V_{ref} = 10V$,

$$\text{For } 111010, I_{out} = \frac{10}{10K} \left(1 + \frac{1}{2} + \frac{1}{4} + 0 + \frac{1}{16} + 0 \right)$$

$$\therefore V_{out} = \frac{10}{10K} \times 5K (1 + 0.5 + 0.25 + 0.0625)$$

$$= 9.0625 \text{ V}$$

Q18 F

1. (b) A 0-10A ammeter has a guaranteed accuracy of $\pm 1.5\%$ full scale reading. The current measured by this instrument is 2.5A calculate limiting value of current and \pm limiting error.

Solⁿ:-

Here,

$$\text{Magnitude of limiting error} = 1.5\% \text{ of } 10A = 0.15A$$

\therefore \pm error limiting errors at indication of 2.5A is,

$$\pm \text{error} = \frac{0.15}{2.5} \times 100 = \pm 6\% \text{ \#}$$

2. (b) AB: $R_1 = 1000 \Omega$ // $C_1 = 0.159 \mu F$ Z_1

BC: $R_2 = 1000 \Omega$ Z_2

CD: $R_3 = 500 \Omega$ Z_3

DA: $C_4 = 0.636 \mu F + R_4$ Z_4

$f = ?$

$R = ?$

$$Z_1 Z_3 = Z_2 Z_4$$

$$\frac{Z_3}{Z_2} = Z_4 Y_1$$

$$\frac{500}{1000} = \left(R_4 - \frac{j}{\omega C_4} \right) \left(\frac{1}{R_1} + j\omega C_1 \right)$$

$$\omega R_4 C_1 = \frac{1}{\omega R_1 C_4}$$

$$\omega^2 = \frac{1}{R_1 R_4 C_1 C_4} = 39555397.33$$

$$f = \frac{\sqrt{\omega}}{2\pi} = \frac{\sqrt{39555397.33}}{2\pi}$$

$$f = 1000.97$$

$$f = 1000 \text{ Hz}$$

$$0.5 = \frac{R_4}{R_1} + \frac{C_1}{C_4} + j\omega C_1 R_4 - \frac{j}{\omega R_1 C_4}$$

Equating,

$$\frac{R_4}{R_1} + \frac{C_1}{C_4} = 0.5 \Rightarrow R_4 = \left(0.5 - \frac{C_1}{C_4} \right) R_1 = 250 \Omega \text{ \#}$$

3. (a) Capacitor of same problem $C = \frac{\epsilon A}{d_1}$ $C' = \frac{\epsilon A}{d_2}$

Ans $C' = \frac{d_1}{d_2} \times C = \frac{3.5}{2.9} \times C$ #

5. (a) Compressive force is applied to a structural member. Strain is 5 micro strain. Two separate strain gauge are attached to structural member, one is nickel wire strain gauge having $K_1 = -12.1$ and other is nichrome wire strain gauge having $K_2 = 2$. Calculate value of Resistance of gauges after they are strained.

Resistance of strain gauges before strained is 120Ω .

Sol: We have,

We shall consider compressive force as -ve & Tensile as positive convention.

$$\text{Strain (s)} = -5 \times 10^{-6}$$

$$K_1 = -12.1$$

$$K_2 = 2$$

$$R = 120$$

$$\text{We have, } K = \frac{\Delta R/R}{\text{strain}}$$

$$\therefore \Delta R_1 = R \times K \text{ strain}$$

$$= 120 \times -12.1 \times -5 \times 10^{-6}$$

$$= 7.26 \text{ m}\Omega$$

$$\Delta R_2 = 120 \times 2 \times -5 \times 10^{-6} = -1.2 \text{ m}\Omega$$

Q185

1. (b) A resistor is measured by the voltmeter-ammeter method. The voltmeter reading is 123.4V on the 250V scale & ammeter reading is 283.5mA on 500mA scale. Both meters are guaranteed to be accurate within $\pm 1\%$ of full scale.

(i) measured value of resistance.

(ii) limits within which you can guarantee results.

Solⁿ: Here,

$$V_R = 123.4V \quad V_S = 250V$$

$$I_R = 283.5mA \quad I_S = 500mA$$

(i) measured value of resistance

$$R = \frac{V_R}{I_R} = \frac{123.4}{283.5m} = 0.435k\Omega$$

(ii) magnitude of limiting error

For current,

$$= \pm 1\% \text{ of } 500mA$$

$$= \frac{1}{100} \times 500 = \pm 5mA$$

For voltage,

$$= \pm 1\% \text{ of } 250$$

$$= \pm 2.5V$$

2(b) A variable potential divider has a total resistance of $2k\Omega$ and is fed from $10V$ DC supply $V = 10V$. The o/p is connected to load resistance $R_L = 5k\Omega$. Determine loading errors for wiper positions corresponding to $K = 0, 0.25, 0.5, 0.75$, and $K = 1$. Use your result to plot graph (1) error vs K .

Solⁿ:

$$\text{loading error } (E_r) = \frac{K(1-K)}{\alpha + K(1-K)} \quad \text{--- (1) error vs } K$$

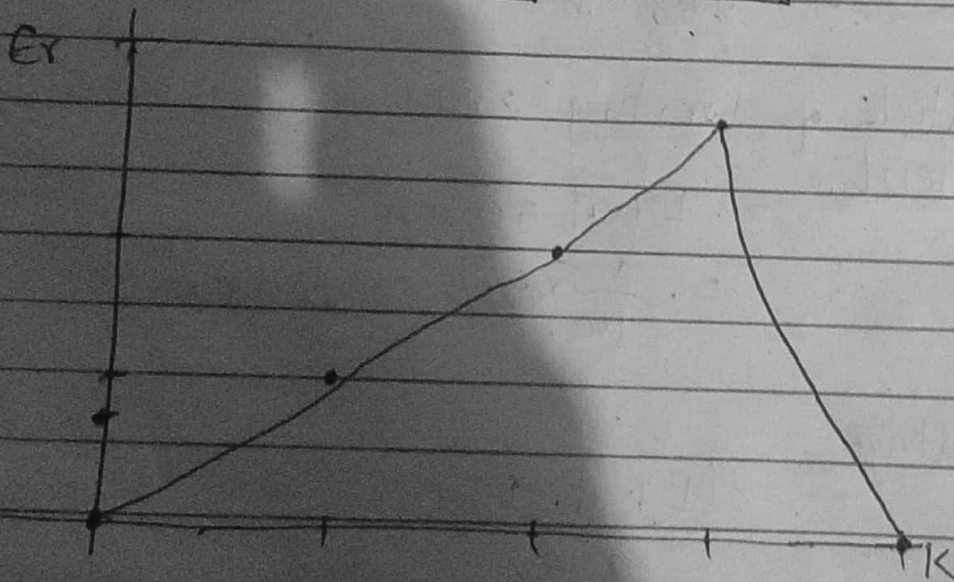
$$\alpha = \frac{R_m}{R_p}; \quad R_m = 5k\Omega$$

$$R_p = 2k\Omega$$

$$\alpha = \frac{5k\Omega}{2k\Omega} = 2.5$$

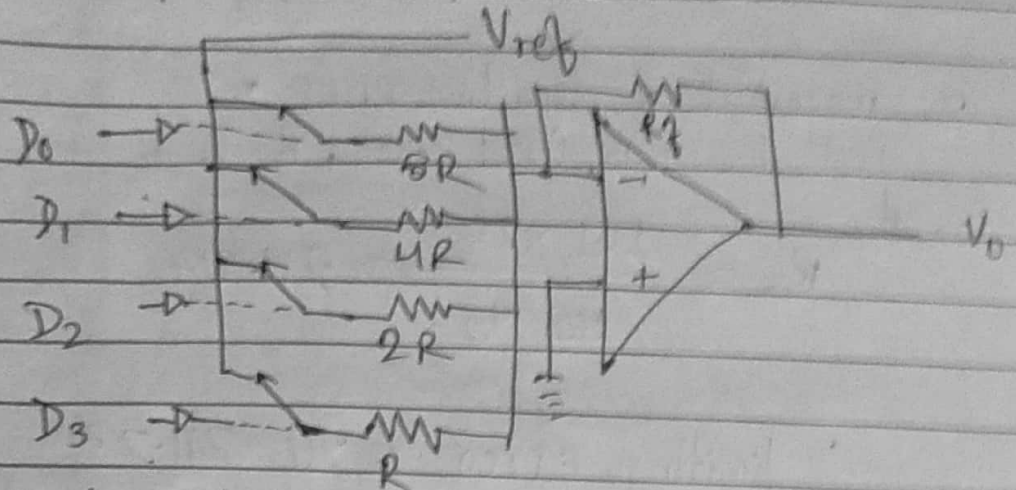
| K | 0 | 0.25 | 0.5 | 0.75 | 1 |
|-------|---|-------|-------|-------|---|
| E_r | 0 | 0.069 | 0.090 | 0.699 | 0 |

$\times 100$ 6.9 9 69.8



$$\text{Resolution of DAC} = \frac{\text{full-scale voltage}}{2^N - 1}$$

46) Design 4-bit weighted-resistor DAC, whose full scale o/p voltage is -10V . Logic levels are $1 = +5\text{V}$, $0 = 0\text{V}$. What is o/p when input is 1010?



we have full-scale voltage o/p is -10V , i.e. for 1111, o/p analog is 10V . (we don't care -ve okay.)
we know,

$$I_{\text{out}} = \frac{V_{\text{ref}}}{R} \left(D_3 + \frac{D_2}{2} + \frac{D_1}{4} + \frac{D_0}{8} \right) \quad \text{--- (1)}$$

For 1111, $V_{\text{out}} = 10\text{V}$.

$$V_{\text{out}} = I_{\text{out}} R_f = \frac{V_{\text{ref}}}{R} \cdot R_f \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \right)$$

$$10 = V_{\text{ref}} (1.875) \quad \boxed{\text{Let } R_f = R}$$

$$\therefore V_{\text{ref}} = 5.33\text{V}$$

Assume, $R_f = 1\text{K}\Omega$

o/p for 1010.

$$V_0 = 533 \left(1 + 0 + \frac{1}{4} + 0 \right)$$

$$= 533 \times 1.25$$

$$= 6.667\text{V} \#$$

0.1%

$$\frac{0.5\% \times 200}{100}$$

$$100 \pm 1$$

$$200 \pm 5$$

$$100 \pm 2$$

Q1

$$R_x = \frac{R_1 R_2}{R_3}$$

$$R_1 = 100 \pm 1\%$$

$$R_2 = 200 \pm 2.5\%$$

$$R_3 = 100 \pm 2\%$$

① Nominal value $R_x = \frac{100 \times 200}{100} = 200$

② magnitude of limiting error

For R_x max (best case), R_1 & R_2 be highest & R_3 be lowest.

$$R_x = \frac{101 \times 205}{98} = 211.27$$

$$\therefore \text{limiting error} = 211.27 - 200$$

$$= 11.27 \Rightarrow \pm 11.27$$

$$\therefore \% = \frac{11.27}{200} \times 100\%$$

$$= \pm 5.63\%$$

③ Here,

Q175

3 (b) A strain gauge is bonded to a beam of 0.1 m long and has $x\text{-sect}^{\text{nal}}$ Area 4 cm^2 . $Y = 207.6 \text{ N/mm}^2$. Strain gauge has unstrained resistance of 240Ω and $g.f. (K) = 2.2$. When load applied Resistance changes by 0.013Ω . Calculate ΔL of steel beam. Amount of Force applied to beam.

Soln:-

$$L = 0.1 \text{ m}$$

$$A = 4 \text{ cm}^2 = 4 \times 10^{-4} \text{ m}^2$$

$$Y = 207 \times 10^9 \text{ N/m}^2$$

$$R = 240 \Omega$$

$$K = 2.2$$

$$\Delta R = 0.013 \Omega$$

We have,

$$K = \frac{\Delta R/R}{\text{Strain} \left(\frac{\Delta L}{L} \right)}$$

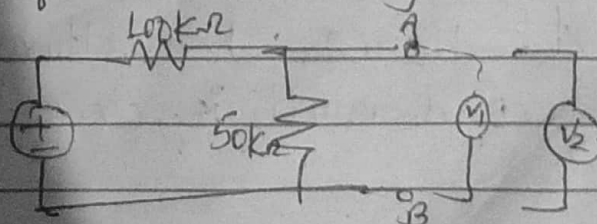
$$\frac{\Delta L}{L} = \frac{\Delta R/R}{K}$$

$$\Delta L = \frac{\Delta R \times L}{R \times K} = \frac{0.013 \times 0.1}{240 \times 2.2}$$

$$Y = \frac{\text{Stress}}{\text{Strain}} \Rightarrow \frac{F}{A \times \text{Strain}}$$

$$F = Y A \text{ Strain}$$

1 (b) For given network ($R_1 = 100 \text{ k}\Omega$, $R_2 = 50 \text{ k}\Omega$) find the voltage reading on voltmeter (V_1), if voltmeter sensitivity is $1 \text{ k}\Omega/\text{volt}$. If the voltmeter is replaced by another voltmeter (V_2) having sensitivity $25 \text{ k}\Omega/\text{volt}$. find new reading. Comment on answer.



$$R_1 = 100\text{K}\Omega \quad R_2 = 50\text{K}\Omega$$

$$S_1 = 1\text{K}\Omega/\text{volt} \quad S_2 = 25\text{K}\Omega/\text{volt}$$

Resistance of voltmeter.

$$R_{V1} = 5\text{V}$$

$$\text{The true value of voltage} = \frac{R_2}{R_1 + R_2} \times V_{in} = \frac{50}{150} \times 150 = 50\text{V}$$

So, In worst case, the value of V_1 & V_2 reading never exceed 50V. So, full-scale range of meter is 50V.

n.c.v.

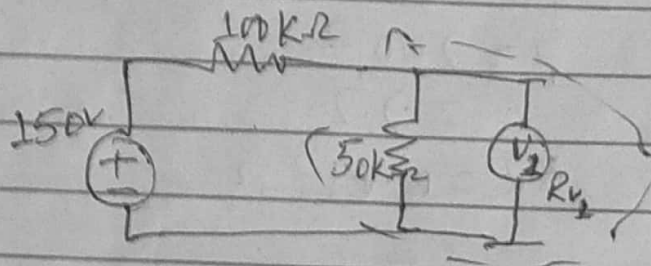
$$R_{V1} = S_1 V = 1\text{K}\Omega/\text{volt} \times 50\text{volt} = 50\text{K}\Omega$$

$$R_{V2} = S_2 V = 25 \times 50 = 1250\text{K}\Omega$$

For V_1 reading.

S_1 & R_{V1} are in // so,

$$R_{eq} = R_1 \parallel R_{V1} = 25\text{K}\Omega$$



$$\therefore V_1 \text{ reading} = \frac{25\text{K}}{25\text{K} + 100\text{K}} \times 150 = 30\text{V}$$

For V_2

$$R_q = R_1 \parallel R_{V2} = 50\text{K} \parallel 1250\text{K} = 48.07\text{K}\Omega$$

$$\therefore V_2 = \frac{48.07}{100 + 48.07} \times 150 = 48.70\text{V}$$

comment: Hence, when voltmeter of higher sensitivity is used, result is close to true value.

017E 50 OR

The o/p of an LVDT is connected to a 5V voltmeter through an amplifier whose amplification factor is 100. o/p of 1mV appears across terminals of LVDT, when the core moves through distance of 0.4mm. Calculate sensitivity of LVDT and that of whole set up. The mv scale has 100 divisions. The scale can be read to $\frac{1}{5}$ of a division. Calculate resulting resolution of instrument in mm.

Solⁿ:

$$\text{Sensitivity of LVDT} = \frac{\text{o/p voltage}}{\text{displacement}} = \frac{1\text{mV}}{0.4\text{mm}} = 2.5\text{mV/mm}$$

$$\begin{aligned}\text{Sensitivity of set up} &= \text{Amplification factor} \times \text{Sensitivity of LVDT} \\ &= 100 \times 2.5\text{mV/mm} \\ &= 250\text{mV/mm}\end{aligned}$$

we have,

$$100\text{ div.} = 5\text{V}$$

$$1\text{ div.} = \frac{5}{100}\text{V} = 50\text{mV}$$

$$\therefore \text{Smallest value that can be measure } \left(\frac{1}{5}\text{ div}\right) = \frac{50\text{mV}}{5} = 10\text{mV.}$$

$$\therefore \text{Resolution} = \frac{\text{Smallest value that can be read}}{\text{sensitivity of set up}}$$

$$= \frac{10\text{mV}}{250\text{mV/mm}} = 0.04\text{mm}$$

||

016 Fall

$$V_0 = g \cdot t \cdot P \rightarrow \text{voltage sensitivity.}$$

- 3 (b) A quartz piezoelectric pickup has dimension of $5\text{mm} \times 5\text{mm} \times 1.5\text{mm}$ has voltage sensitivity of 0.012 Vm/N . The relative permittivity of quartz is 1600 and modulus of elasticity of quartz is 12 N/Nm^2 . Force applied is 10 N . determine,
- o/p voltage,
 - charge sensitivity
 - strain
 - charge generated & capacitance of pickup.

$$(1) V_0 = g \cdot t \cdot P = 0.012 \times 1.5 \times 10^{-3} \times \frac{10}{25 \times 10^{-4}} =$$

$$(2) \text{ charge sensitivity } (d) = \frac{Q}{F} = \frac{8.85 \times 10^{-12}}{1600 \times 0.012}$$

$$(3) \text{ strain} = \frac{F}{AY}$$

$$(4) \text{ charge generated } (Q) = d \cdot F$$

$$(5) C = \frac{\epsilon_0 \epsilon_r A}{t}$$

- 4 (a) ^{resistance} thermister at 27°C is 1050Ω with constant $\beta = 3140$. Calculate value of temp. when the thermister resistance becomes 2330Ω in Kelvin scale & find sensitivity of transducer at given operating point.

Resistance R of a thermister at temp. T can be expressed as,

$$R_T = R_0 e^{\beta \left(\frac{1}{T} - \frac{1}{T_0} \right)}$$

resistance at T resistance at T_0

Here,

$$T_0 = 27^\circ\text{C} + 273 = 300\text{K}$$

$$\beta = 3140 \quad R_0 = 1050$$

$$T = ? \quad R_T = 2330$$

$$R_T = R_0 e^{\beta \left(\frac{1}{T} - \frac{1}{T_0} \right)}$$

$$2330 = 1050 e^{3140 \left(\frac{1}{T} - \frac{1}{300} \right)}$$

$$\Rightarrow T = 5.77^\circ\text{C} \quad \#$$