CONTENTS

CHAPTER 3

PHYSICAL VARIABLES AND TRANSDUCERS

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3.1 PHYSICAL VARIABLES AND THEIR TYPES (ELECTRICAL, MECHANICAL, PROCESS, BIO-PHYSICAL VARIABLE)

Physical variables are the variables to be measured in an instrumentation system, which make its first contact with primary sensing elements. It is the information for the measurement system in the form of a physical phenomenon or an electrical signal.

Types of physical variables

a) Electrical variables

Current, voltage, resistance, inductance, capacitance, frequency, phase angle, power are the electrical variables.

b) Mechanical variables

Spring (force to displacement), monometer (pressure to displacement), thermocouple (temperature to electric current), Hydrometer (specific gravity to displacement), Turbines (linear to angular velocity) etc are mechanical variables.

c) Process variables

Temperature, pressure and flow are the process variables which are widely employed in process and production plants.

d) Bio-physical variables.

Here the information is taken from living beings ECG (electro-cardio graph) for heart, EEG (electroencephilo graph) for brain, EMG (electro-mio graph) for muscles, excitation, restoring potential are the bio-physical variables.

3.2 TRANSDUCER PRINCIPLE OF OPERATIONS

A device that converts non-electrical energy into electrical energy into electrical energy is known as transducers.

3.2.1 Characteristics of a Transducer

A known values of the measured (input) are applied to a sensor (measurement system) for the purpose of observing the sensor (system) output. The main characteristics of transducer are given below:

i) **Sensitivity**

It can be defined as the ratio of the incremental output and the incremental input. While defining the sensitivity, we assume that the input-output characteristic of the instrument is approximately linear in that range.

ii) **Range**

The range of the sensor is the maximum and minimum values of applied parameter that can be measured.

iii) **Precision**

The concept of precision refers to the degree of reproducibility of a measurement. In other words, if exactly the same value were measured a number of times, an ideal sensor would output exactly the same value every time. But real sensors output a range of values distributed in some manner relative to the actual correct value.

iv) **Resolution**

The smallest difference between measured values that can be discriminated. It can be expressed either as a proportion of the reading (or the full scale reading) or in absolute terms.

v) **Accuracy**

The accuracy of the sensor is the maximum difference that will exist between the actual value and the indicated value at the output of the sensor. The accuracy can be expressed either as a percentage of full scale or in absolute terms.

vi) **Linearity**

The linearity of the transducer is an expression of the extent to which the actual measured curve of a sensor departs from the ideal curve. It is often specified in terms of percentage of non-linearity, which is defined as,

$$\text{Non-linearity (\%)} = \frac{\text{Maximum input deviation}}{\text{Maximum full scale input}} \times 100$$

vii) **Hysteresis**

Hysteresis exists not only in magnetic circuits, but in instruments also. A transducer should be capable of following the changes of the input parameter regardless of which direction the change is made, hysteresis is the measure of this property.

3.2.2 Basic Requirement of Transducer

i) **Ruggedness**

It should be capable of withstanding overload and some safety arrangement should be provided for the over load protection.

ii) **Linearity**

Its input-output characteristics should be linear.

ii) **Repeatability**

It should reproduce same output signals when the input is applied again and again under fixed environment condition.

iv) **High quality output signals**

The ratio of signal to noise (SNR) should be high and the amplitude of the output signal should be up to detectable level.

v) **High reliability and stability**

We should get minimum error in measurement for temperature variations, vibrations and other various changes in surrounding.

vi) **Good dynamic response**

Its output should be faithful to input when taken as a function of time.

3.2.3 Selection of Transducers

Following are the factors which need to be considered while selecting a transducer.

a) **Range**

The range of the transducer should be large enough to cover up all the expected magnitude of quantity under measurement.

b) **Sensitivity**

Transducer should be given a sufficient output signal per unit of measured input in order to yield the meaningful data.

c) High input impedance and low output impedance to avoid loading effect

d) Good resolution over entire selected range

e) Prefarably small in size

f) High degree of accuracy and repectability

g) Selected transducer must be free from errors.

3.2.4 Classification of Electrical Transducers

There are many basis on which electrical transducers may be classified. They can be classified according to application, method of energy conversion, nature of output signals and so on. All these classifications mainly overlap each other and a sharp classifications mainly overlap each other and a sharp distinction between different types is rather difficult.

a) **Based on physical phenomenon.**

i) **Primary transducer**

When the input signal is directly sense by the transducer, the non-electrical energy is converted into electrical energy, then this type of transducer is known as primary transducer. *For example*; thermistor senses the temperature directly and causes the change in resistance with the change in temperature.

ii) **Secondary transducer**

When the input signal is first sensed by some sensor or detector, then its output signal is fed to the other instrument as an input. The output of this instrument is given as the input of transducer for converting into electrical energy, then this type of transducer is called secondary transducer.

For example; in the case of pressure measurement, we use bourdon tube to convert pressure into displacement, then the pressure is converted into output voltage with the help of LVDT. Here the LVDT act as secondary transducer.

b) **Based on the power types**

i) **Active transducer**

Active transducer is also known as self generating type transducer. It develops their own voltage or current from the physical phenomenon being measured. Active transducers generate electric current or voltage directly in response to environmental simulation. Active transducer does not require any auxiliary power source to produce their output. Examples of active transducer are thermocouples and piezoelectric accelerometers.

ii) **Passive transducer**

Passive transducer is also known as externally powered transducer. It derives the power required for energy conversion from an external power source. Passive transducer produce a change in some passive electrical quantity such as capacitance, resistance or inductance as a result of stimulation. These usually required additional electrical energy for excitation. *For example*; resistive (potentiometer, strain gauge), inductive (LVDT) and capacitive variable capacitance pressure gauge, dielectric gauge) transducers.

c) **Based on type of output**

i) **Analog transducer**

These transducer convert the input physical phenomenon into on analog output which is a continuous function of time. *For example*; strain gauge, a LVDT, a thermocouple or a thermistor may be called as analog transducer.

ii) **Digital transducer**

These transducers convert the input physical phenomenon into an electrical output which is in the form of pulses. *For example*; a digital linear displacement transducer that uses digital code marks to identify the position of a movable piece by a binary system of notation. The position is given out as a train of digital pulses.

d) **Based on the transduction phenomenon**

i) **Transducer**

Transducer is a device which converts a non electrical into an electrical quantity.

ii) **Inverse transducer**

An inverse transducer is a device which converts an electrical quantity into a non-electrical quantity. *For example*; piezo electric crystals (voltage to displacement).

e) **Based on the non-electrical phenomenon**

i) Linear displacement transducer

ii) Rotary displacement transducer

f) **Based on the electrical phenomenon**

i) Resistive transducer

ii) Capacitive transducer

iii) Inductive transducer

iv) Photo electric transducer

v) Photo voltaic transducer

3.3 INPUT AND OUTPUT CHARACTERISTICS OF TRANSDUCERS

3.3.1 Input Characteristics of Transducer

There are three factors which influence the input characteristics of the transducer.

a) Type of input

A transducer is based on input. There are different types of transducers available which measures the different quantities. The type of input which can be any physically quantity is generally determined in advance.

b) Operating range

The useful operating range of the transducer may be decisive in selection of a transducer for a particular application. The upper limit is decided by the transducer capabilities while the lower limit of range is normally determined by the transducer error or by the 'unavoidable noise originating in the transducer.

c) Loading effect

Ideally, transducers should be have no loading effect on the quantity being measured. But in practice, it is impossible and hence steps may be taken to reduce the loading effects to negligible proportions.

3.3.2 Output Characteristics of Transducer

The three conditions in the output characteristics which should be considered are

a) Type of electrical output

The output quantities (voltage, current, impedance or time function of these amplitudes) may or may not be acceptable to the latter stages of the instrumentation system. They may have to be manipulated so as to make them drive the subsequent stages of instrumentation system.

b) Output range

The output range of a transducer is limited at the lower end by noise signals and the upper limit is set by the maximum useful input level.

c) Output impedance

The output impedance of a transducer determines to the extent the subsequent stages of instrumentation is loaded. Ideally, the value of output impedance should be zero if no loading effects are there on the subsequent stages. It also determines the amount of power that can be transferred to the succeeding stages of the instrumentation system for a given output signal level.

3.4 CALIBRATION AND ERRORS IN TRANSDUCERS

Calibration consists of comparing the output of the instrument or sensor under test against the output of an instrument of known accuracy when the same input (the measured quantity) is applied to both instruments. This procedure is carried out for a range of inputs covering the whole

measurement range of the instrument or sensor. Calibration ensures that the measuring accuracy of all instruments and sensors used in a measurement system is known over the whole measurement range, provided that the calibrated instruments and sensors are used in environmental conditions that are the same as those under which they were calibrated.

Calibration refers to the adjustment of an instrument so its output accurately corresponds to its input throughout a specified range. This definition specifies the outcome of a calibration process. Instruments used as a standard in calibration procedures are usually chosen to be of greater inherent accuracy than the process instruments that they are used to calibrate. Because such instruments are only used for calibration purposes, greater accuracy can often be achieved by specifying a type of instrument that would be unsuitable for normal process measurements. For instance, ruggedness is not a requirement and freedom from this constraints opens up a much wider range of possible instruments.

Instrument calibration has to be repeated at prescribed intervals because the characteristics of any characteristics are brought about by such factors as mechanical wear and the effects of dirt, dust, fumes, chemicals and temperature changes in the operating point. Determination of the frequency at which instruments should be calibrated is dependent upon several factors that require specialist knowledge. As the environmental and usage conditions of an instrument may change beneficially as well as adversely, there is the possibility that the recommended calibration interval may decrease as well as increase. Any instrument that is used as a standard in calibration procedures must be kept solely for calibration duties and must never be used for other purposes.

3.4.1 Errors in Transducers

The errors in transducer occur because they don't follow the input-output relationship in many situations. The error can be split into four: Components which are;

a) Scale error
b) Dynamic error
c) Error on account of drift and noise
d) Error due to change of frequency

a) Dynamic error

It occurs only when the input quantity is varying with time. It can be made small by having a small time constant. It reduces as the time after application of the input increases.

b) Error on account of drift and noise

Noise is a signal of random amplitude and random frequency while drift if a slow change with time. The magnitude of the noise and drift is normally independent of the magnitude of the input signal. Noise and drift signals originating from the transducers vary with time and are superimposed on the output signal.

c) Error due to change of frequency

Frequency response and the high frequency cut off are the two specifications that describes the response of a transducer to a variable frequency sine wave input applied to it.

For a reasonably linear transducer, a sine wave input yields a sine wave output. As the frequency of the sine wave input is increased, the transducer is required to respond more and more quickly. Ultimately, beyond a particular frequency, the transducer can no larger respond as rapidly as its sinusoidal input is changing. So the output of the transducer becomes smaller and also the phase shift between the input and output increases. Thus as the frequency increases, the output of the transducer falls. This fall off of amplitude of output with an increase in input frequency is the frequency response.

d) Scale error

The scale error comprise of four different types of error in transducers.

i) Zero error

In this case, the output deviates from the correct value by a constant factor over the entire range of the transducer.

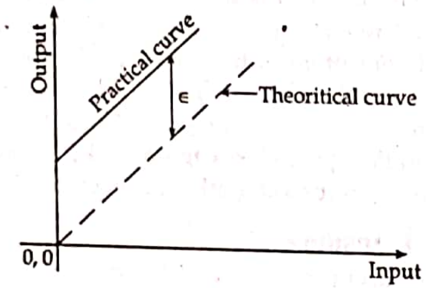


Figure: Transducer zero error

ii) Sensitivity error

It occurs when the observed output deviated from the correct value by a constant value. Suppose the correct output is q_0, the output would be kq_0 over the entire range of the transducer, where k is a constant.

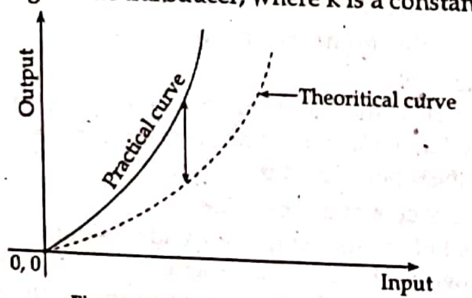


Figure: Transducer sensitivity error

iii) Non-conformity error

Here the experimentally obtained transfer function deviated from the theoretical transfer function for almost every input.

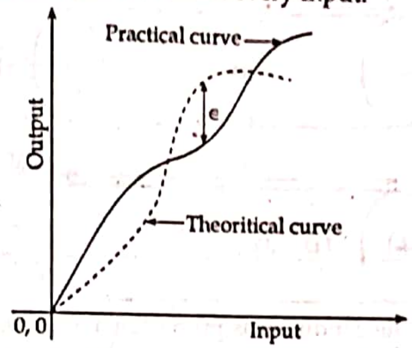


Figure: Transducer non-conformity error

iv) Hysteresis

The output of transducer not only depends upon the input quantity but also upon the input quantity previously applied to it. Therefore, a different output is obtained when same value of input quantity is applied depending upon whether it is increasing or decreasing. For decreasing values, a greater output is obtained than with increasing value for the same value of the input quantity.

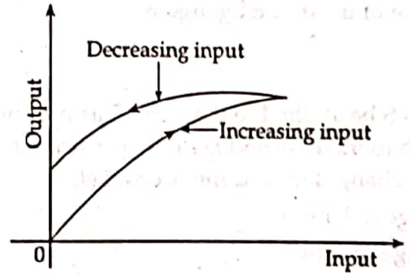


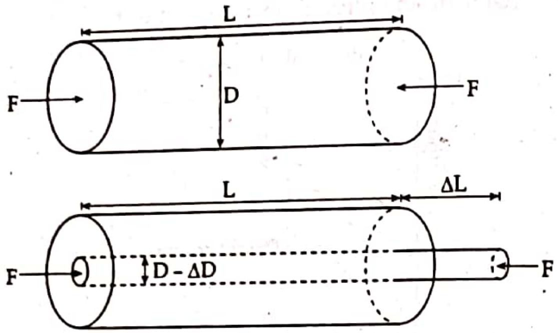
Figure: Transducer hysteresis

3.5 TRANSDUCER AND ITS APPLICATIONS

3.5.1 Strain Gauge

A strain gauge is a device used to measure the strain of an object. If a metal conductor is stretched or compressed, its resistance changes on account of the fact that both length and diameter of conductor change. Also there is a change in the value of resistivity of the conductor when it is strained and this property is called piezo resistive effect. Hence resistance strain gauge are also known as piezo resistive gauges.

The change in the value of resistance by straining the gauge may be partly explained by elastic material. Figure below shows a strip of elastic material, if the tension is applied it's longitudinal dimension will increase while there will be a reduction in the lateral dimension. When it is positive strain, its length increase while its area of cross-section decreases.



The resistance of the conductor is proportional to its length and inversely proportional to its area of cross section. The resistance of the gauge resistance increase with positive strain.

Let us consider a strain gauge made of circular wire. The wire has the dimensions:

L = Length
A = Area
D = Diameter before being strained
δ = Resistivity of the material

Then the resistance of unstrained gauge is

$$R = \delta\frac{L}{A}$$

Let a tensile stress S be applied to the wire. This produces a positive strain causing the length to increase and area to decrease. Thus when the wire is strained there are changes in its dimensions. Let,

ΔL = Change in length
ΔA = Change in area
ΔD = Change in diameter
ΔR = Change in resistance

Lets differentiate R with respect to S in order to find how ΔR depends upon the material physical quantities.

$$\frac{dR}{ds} = \frac{\rho}{A}\frac{\partial L}{\partial S} - \frac{\rho L}{A^2}\frac{\partial A}{\partial S} + \frac{L}{A}\frac{\partial \rho}{\partial S} \quad (1)$$

Dividing equation (1) by $R = \rho\frac{L}{A}$, we have

$$\text{or,} \quad \frac{1}{R}\frac{dR}{ds} = \frac{1}{L}\frac{\partial L}{\partial S} - \frac{\partial A}{A\partial S} + \frac{\partial P}{\rho \partial S} \quad (2)$$

It is evident that the per unit change in resistance is due to

i) Per unit change in length $= \frac{\Delta L}{L}$

ii) Per unit change in are $= \frac{\Delta A}{A}$

Area, $A = \frac{\pi}{4}D^2$

$$\text{or,} \quad \frac{\partial A}{\partial S} = 2\frac{\pi}{4}\cdot D\frac{\partial D}{\partial S}$$

$$\text{or,} \quad \frac{1}{A}\cdot\frac{dA}{dA} = \frac{\left(\frac{2\pi}{4}\right)D}{\left(\frac{\pi}{4}\right)D^2}\cdot\frac{\partial D}{\partial S}$$

$$\text{or,} \quad \frac{1}{A}\frac{dA}{ds} = \frac{2}{D}\frac{\partial D}{\partial S}$$

Hence equation (2) can be written as,

$$\frac{1}{R}\frac{dR}{ds} = \frac{1}{L}\frac{\partial L}{\partial S} - \frac{2}{D}\frac{\partial D}{\partial S} + \frac{1}{\rho}\frac{\partial \rho}{\partial S} \quad (3)$$

Now, Poisson's ratio

$$V = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = -\frac{\left(\frac{\partial D}{D}\right)}{\left(\frac{\partial L}{L}\right)}$$

$$\text{or,} \quad -\frac{\partial D}{D} = -V\cdot\frac{\partial L}{L}$$

Putting the value of $\frac{\partial D}{D}$ in equation (3);

For small variations,

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + 2V\frac{\Delta L}{L} + \frac{\Delta \rho}{\pi}$$

The gauge factor is defined as the ratio of per unit change in resistance to per unit change in length.

$$\text{Gauge factor, } G_f = \frac{\left(\frac{\Delta R}{R}\right)}{\left(\frac{\Delta L}{L}\right)}$$

$$\text{or,} \quad \frac{\Delta R}{R} = G_f\cdot\frac{\Delta L}{L} = G_f \times \epsilon$$

where, $\epsilon = \frac{\Delta L}{L}$

Thus the gauge factor can be expressed as,

$$G_f = \frac{\left(\frac{\Delta R}{R}\right)}{\left(\frac{\Delta L}{L}\right)} = 1 + 2V + \frac{\left(\frac{\Delta \rho}{\rho}\right)}{\left(\frac{\Delta L}{L}\right)} = 1 + 2V + \frac{\partial\rho/\rho}{\epsilon}$$

If the change in the value of resistivity of a material when strained is neglected, the gauge factor is, $G_f = 1 + 2V$

This is valid only when piezoresistive effect *i.e.*, change in resistivity due to strain is almost negligible. The Poisson's ratio for all metals lies between 0 and 0.5.

Types of strain gauge are

i) Foil type strain gauges
ii) Semi conductor strain gauges
iii) Wire wound strain gauges

Example 1

A resistance wire strain gauge with a gauge factor of 2 is bonded to a steel structural member subjected to a stress of 100 MN/m². The modulus of elasticity of steel is 200 GN/m². Calculate the percentage in value of the gauge resistance due to the applied stress.

Solution:

Gauge factor, $G_f = 2$

$S = 100\ \text{MN/m}^2$

$E = 200\ \text{GN/m}^2$

We know that,

$$\epsilon = \frac{S}{E} = \frac{100 \times 10^6}{200 \times 10^9} = 500 \times 10^{-3}$$

∴ $\epsilon = 500$ micro strain

Also,

$$\frac{\Delta R}{R} = G_f \cdot \epsilon$$

or, $$\frac{\Delta R}{R} = 2 \times (500 \times 10^{-6}) = 0.001$$

∴ $$\% \frac{\Delta R}{R} = 0.1\%$$

The result shows that the relatively high stress results in a resistance change of only 0.1%, a very small change indeed. Lower stress produces still lower changes in resistance which may not be perceptible at all. To overcome this difficulty, the gauge factor of strain gauge should be high which produces large change in resistance when strained.

Example 2

A compressive force is applied to a structural member. The strain is 5 micro-strain. Two separate strain gauges are attached to the structural member, one is nickel wire strain gauge having a gauge factor of −12.1 and the other is nichrome wire strain gauge having a gauge factor 2. Calculate the value of resistance of the gauge after they are strained. The resistance of strain gauges before being strained is 120 Ω.

Solution:

According to convention, the tensile strain is taken as positive while the compressive strain is taken as negative. Hence strain, $\epsilon = -5 \times 10^{-6}$ (1 micro strain = 1 μm/m).

Now,

$$\frac{\Delta R}{R} = G_f \cdot \epsilon$$

∴ Change in value of resistance of nickel wire strain gauge,

$$\Delta R = G_f \cdot \epsilon \times R = (-12.1) \times (-5 \times 10^{-6}) \times 120 = 7.26 \times 10^{-8}\ \Omega$$
$$= 7.26\ \text{m}\Omega$$

Thus there is an increase of 7.26 mΩ in the value of resistance.

For nichrome, the change in the value of resistance is,

$$\Delta R = (2) \times (-5 \times 10^{-6}) \times 120 = -1.2 \times 10^{-3}\ \Omega = -1.2\ \text{m}\Omega$$

Thus with compressive strain, the value of resistance gauge shows a decrease of 1.2 mΩ in the value of resistance.

Example 3

A single gauge having resistance of 120Ω is mounted on a steel cantilever beam at a distance of 0.15 m from the free end. An unknown force F applied at the free end produces a deflection of 12.7 mm of the free end. The change in gauge resistance is found to be 0.152 Ω. The beam is 0.25 m long width of 20 mm and a depth of 3 mm. The young's modulus for steel is 200 GN/m². Calculate the gauge factor.

Solution:

We know,

Moment of inertia of beam,

$$I = \frac{1}{12} \times b \times d^3 = \frac{1}{12} \times 0.02 \times (0.003)^3 = 45 \times 10^{-12}\ \text{m}^4$$

Deflection, $$x = \frac{FL^3}{3EI}$$

or, Force, $$F = \frac{3EI_x}{L^3} = \frac{3 \times 200 \times 10^9 \times 45 \times 10^{-12} \times 12.7 \times 10^{-3}}{(0.25)^3} = 21.945\ \text{N}$$

Now, bending moment at 0.15 m from free and M

$$= F \times x = 21.945 \times 0.15 = 3.291\ \text{Nm}$$

Stress at 0.15 m from free end,

$$S = \frac{m}{I} \cdot \frac{t}{2} = \frac{3.291}{45 \times 10^{-12}} \times \frac{0.003}{2} = 1.097 \times 10^2\ \text{MN/m}^2 = 109.70\ \text{MN/m}^2$$

Since, strain,

$$\epsilon = \frac{\Delta L}{L} = \frac{S}{E} = \frac{109.70 \times 10^6}{200 \times 10^9} = 5.485 \times 10^{-4}$$

Thus,

$$\text{Gauge factor} = \frac{\left(\frac{\Delta R}{R}\right)}{\left(\frac{\Delta L}{L}\right)} = \frac{\frac{0.152}{120}}{5.485 \times 10^{-4}} = 23.$$

Example 4

A strain gauge is bonded to a beam 0.1 m long and has cross sectional area 4 cm². Young's modulus for steel is 207 GN/m². The strain gauge has an unstrained resistance of 240 Ω and a gauge factor of 2.2. When a load is applied, the resistance of gauge changed by 0.013 Ω. Calculate the change in length of the steel beam and the amount of force applied to the beam.

Solution:

We know,

$$\text{Gauge factor, } G_f = \frac{\left(\frac{\Delta R}{R}\right)}{\left(\frac{\Delta L}{L}\right)}$$

or, $$\Delta L = \frac{\left(\frac{\Delta R}{R}\right)}{G_f} \times L = \frac{\left(\frac{0.013}{240}\right)}{2.2} \times 0.1$$

∴ $$\Delta L = 2.46 \times 10^{-6}\ \text{m}$$

Now,

$$\text{Hooke's law, } \epsilon = \frac{S}{E}$$

or, $$S = \epsilon \times E = \frac{\Delta L}{L} \times E = \frac{2.46 \times 10^{-6}}{0.1} \times 2.7 \times 10^{9}$$

∴ $$S = 5.092 \times 10^{-6}\ \text{N/m}^2$$

So, the amount of force applied to the beam is,

$$F = SA$$

or, $$F = (5.092 \times 10^{6}) \times 4 \times 10^{-4}$$

∴ $$F = 2.037 \times 10^{3}\ \text{N}$$

Example 5

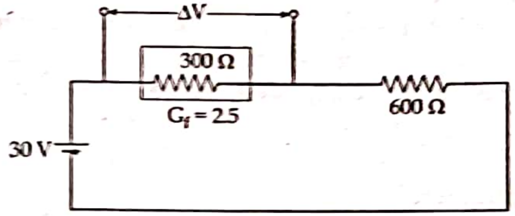
A strain gauge having a resistance of 300 Ω and gauge factor 2.5 is connected in series with a blast resistance of 600 Ω across 30 V. Determine the change in output voltage when a stress of 140 mN/m² is applied. The modulus of elasticity is 200 GN/m².

Solution:

$$R = 300\ \Omega$$

Gauge faction, $G_f = 2.5$

Modulus of elasticity, $E = 200\ \text{GN/m}^2 = 200 \times 10^9\ \text{N/m}^2$



$$\text{Hooke's law yields, } \epsilon = \frac{S}{E} = \frac{\Delta L}{L} = \frac{140 \times 10^{6}}{200 \times 10^{9}} = 7 \times 10^{-4}$$

$$\text{Again, gauge factor, } G_f = \frac{\left(\frac{\Delta R}{R}\right)}{\left(\frac{\Delta L}{L}\right)} = \frac{\left(\frac{\Delta R}{R}\right)}{\epsilon}$$

or, $\Delta R = G_f \cdot \epsilon \times R$

or, $\Delta R = 2.5 \times 7 \times 10^{-4} \times 300$

∴ $\Delta R = 0.525\ \Omega$

i.e., there is an increase of 0.525 Ω in the value of resistance. When unstrained, the voltage across strain gauge is,

Now,

$$V_1 = \left(\frac{300}{300 + 600}\right) \cdot 30 = 10\ \text{V}$$

When strained, the voltage across strain gauge is,

$$V_2 = \left[\frac{300 + \Delta R}{(300 + \Delta R) + 600}\right] \times 30 = \left[\frac{300 + 0.525}{(300 + 0.525) + 600}\right] \times 30$$

∴ $V_2 = 10.012$ volts

Thus the change in output voltage is,

$$\Delta = V_2 - V_1 = (10.012 - 10)\ \text{volts} = 0.0120\ \text{volts}$$

3.5.2 Piezo-Electric Transducers

A Piezo-electric material is one in which an electric potential appears across certain surfaces of a crystal if the dimensions of the crystal are changed by the application of a mechanical force. This potential is produced by the displacement of charges. This effect is reversible *i.e.,* conversely, if a varying potential is applied to the proper axis of the crystal, it will change the dimensions of the crystal there by deforming it. This effect is known as piezo-electric effect.

Common piezo-electric materials include Rochelle salts, ammonium dihydrogen phosphate, lithium sulphate, dipotassium tartarate, potassium dihydrogen phosphate, quartz and ceramics A and B. The piezo-electric effect can be made to response mechanical deformations of the material in many different modes. The modes can be thickness expansion, transverse expansion, thickness shear and face shear. The mode of motion effected depends on the shape of the body relative to the crystal axis and location of the electrodes. The mechanical deformation generates a charge and this charge depends upon the direction of applied force. The charge is given by,

$$Q = d \times F \qquad (1)$$

where, F = Applied force, N

d = Charge sensitivity of crystal, C/N

Q = Charge, C

The force F causes in thickness of the crystal

$$\therefore \quad F = \frac{AE}{t} \cdot \Delta t \text{ Newton} \qquad (2)$$

where, A = Area of crystal, m^2

t = Thickness of crystal, m

E = Young's modulus, N/m^2

$$\text{Young's modulus, } E = \frac{\text{Stress}}{\text{strain}} = \frac{Ft}{A\Delta T} N/m^2 \qquad (3)$$

Area, $A = W \cdot L$

where, W = Width of crystal, m

L = Length of crystal, m

From equation (1) and (2);

$$Q = d\frac{AE}{t} \cdot \Delta t \text{ coulomb} \qquad (4)$$

The charge at the electrodes give rise to an output voltage E_0,

$$E_0 = \frac{Q}{C_p} \text{ volt} \qquad (5)$$

where, C_p = Capacitance between electrodes, F

Capacitance between electrodes,

$$C_p = \frac{\epsilon_r \epsilon_0 A}{t}$$

From equation (1) and (5),

$$E_0 = \frac{dF}{\frac{\epsilon_r \epsilon_0 A}{t}} = \frac{dt}{\epsilon_r \epsilon_0} \cdot \frac{F}{A}$$

But $\frac{F}{A}$ = P = Pressure or stress in N/m^2

$$\text{or,} \quad E_0 = \frac{d}{\epsilon_r \epsilon_0} t \cdot p = g\,t\,p$$

where, $g = \frac{d}{\epsilon_r \epsilon_0}$ = Sensitivity of the crystal. This is constant for a crystal cut. Its unit are Vm/N.

Now, $g = \frac{E_0}{t\,p} = \frac{E_0/t}{p}$

But, $\frac{E_0}{t}$ = Electric field strength, V/m

Let, $\epsilon = \frac{E_0}{t}$ = Electric field

$$\therefore \quad g = \frac{\text{Electric field}}{\text{Stress}} = \frac{E}{P}$$

Thus, Charge sensitivity, $d = \epsilon_r \epsilon_0 g$ F/N.

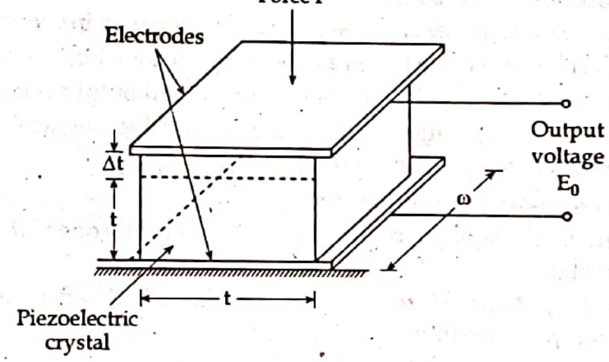


Figure: Piezo electric transducer

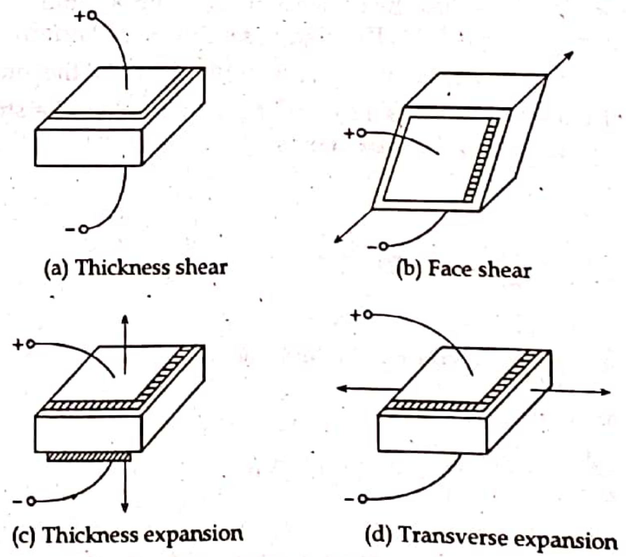


Figure: modes of operation of piezo-electric crystals

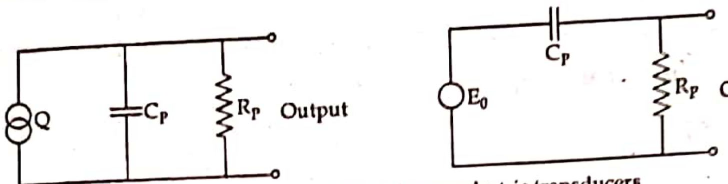


Figure: Equivalent circuit of piezo-electric transducers

Uses of piezo-electric transducer

The use of piezo-electric transducer elements is confined primarily to dynamic measurements. The voltage developed by applications of strain is not held under static conditions. Hence the elements are primarily used in the measurements of such quantities as surface roughness and in accelerometers and vibration pickups.

Merits of piezo- electric transducers

i) They are small in size, light weight and very rugged in construction.
ii) They are active transducers as they don't need external supply.
iii) They have very good frequency response and output is quite large.
iv) They can be operated over a wide range of temperature without appreciable temperature induced.

Demerits of piezo-electric transducers

i) There is a problem of leakage of charges through the surface of crystal.
ii) The output of piezo-electrical transducer is affected by the temperature variation of the crystal.

Example 6

A Piezo-electric crystal has the dimensions of 5 mm × 5 mm × 1.25 mm. The force acting on it is 5 N. The charge sensitivity of barium titanate is 150 PC/N and its permittivity is 12.5×10^{-9} F/m. If the modulus of elasticity of bariumtitan ate is 12×10^{-6} N/m². Calculate the strain. Also calculate the charge and the capacitance.

Solution:

Area of plates,

$$A = \omega L = 5 \times 5\ \text{mm}^2 = 25 \times 10^{-6}\ \text{m}^2$$

Pressure,

$$P = \frac{F}{A} = \frac{5}{25 \times 10^{-6}}\ \text{N/m}^2 = 0.2\ \text{MN/m}^2$$

Voltage sensitivity,

$$g = \frac{d}{\epsilon_0 \epsilon_r} = \frac{150 \times 10^{-12}}{12.5 \times 10^{-9}} = 12 \times 10^{-3}\ \text{Vm/N}$$

Voltage generated,

$$E_0 = g\,tp = 12 \times 10^{-3} \times 1.25 \times 10^{-3} \times 0.2 \times 10^6 = 3\ \text{volts}$$

Now,

$$\text{Strain, } \epsilon = \frac{\Delta t}{t} = \frac{\left(\frac{F}{A}\right)}{E} = \frac{\text{Stress}}{\text{Young's modulus}} = \frac{0.2 \times 10^6}{12 \times 10^6} = 0.0167$$

$$\text{Charge, } Q = dF = 150 \times 10^{-12} \times 5\ C = 750\ \text{PC}$$

and, $$\text{Capacitance, } C_p = \frac{Q}{E_0} = \frac{750 \times 10^{-12}}{3} = 250\ \text{pF.}$$

3.5.3 Capacitive Transducers

The capacitive transducer is the capacitor with variable capacitance. The capacitive transducer comprises of two parallel metal plates that are separated by the material such as air, which is called as the dielectric material. The principle of operation of capacitive transducer is based upon the familiar equation for capacitance of a parallel plate capacitor.

i.e., $$C = \epsilon_0 \frac{A}{d} = \epsilon_r \cdot \epsilon_0 \frac{A}{d}$$

where, C = Capacitance of a parallel plate capacitor, F
A = Overlapping area of plates, m²
D = Distance between two plates, m
$\epsilon = \epsilon_r \cdot \epsilon_0$ = Permittivity of medium, F/m
ϵ_r = Relative permittivity
ϵ_0 = Permittivity of free space = 8.85×10^{-12} F/m

A capacitance transducer works on the principle of change of capacitance which may be caused by

i) Change in overlapping area 'A'
ii) Change in dielectric constant
iii) Change in distance 'd' between the plates

These changes are caused by physical variables like displacement, force and pressure. The capacitive transducers are commonly used for measurement of linear displacement whose output impedance is high. Depending on the parameter that changes for the capacitive transducers, there are three types of capacitive transducers. They are;

a) Changing area of the plates of capacitive transducers

The capacitance of the variable capacitance transducer also changes with the area of the two plates. The capacitance is directly proportional to the area of plates, thus the capacitance changes linearly with change in area of plates. This type of transducer is used for measurement of displacement. Figure below shows the parallel plate capacitor in which one plate of the capacitor is fixed while other plate moves according to the displacement applied.

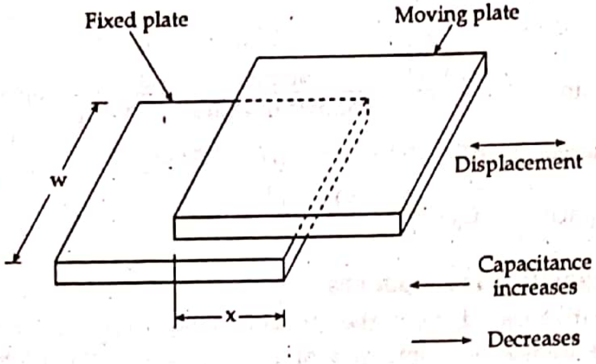


Figure: Parallel plate capacitor

The capacitance is given by

$$C = \epsilon \frac{A}{d} = \epsilon \frac{xw}{d}$$

where, x = Length of overlapping part of plates
w = Width of overlapping part of plates

Sensitivity of the transducer is given by,

$$s = \frac{\partial C}{\partial x} = \epsilon \frac{w}{d}$$

Hence sensitivity is constant and therefore there is linear relationship between capacitive and displacement. The sensitivity for a fractional change in capacitance is,

$$S' = \frac{\partial C}{C \partial x} = \frac{1}{x}$$

This type of capacitance transducer is suitable for measurement of linear displacements.

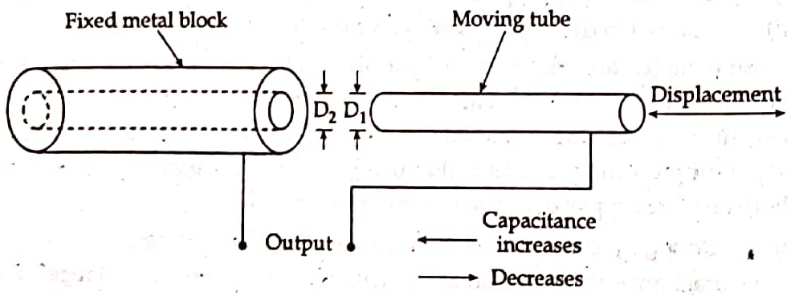


Figure: Cylindrical capacitance

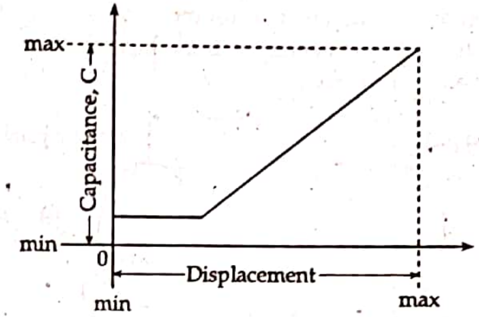
D_2 = Inner diameter of outer cylindrical electrode
D_1 = Outer diameter of inner cylindrical electrode

Hence, $$C = \frac{2\pi \epsilon x}{\log_e \left(\frac{D_2}{D_1}\right)}$$

and, The sensitivity,

$$s = \frac{\partial c}{dx} = \frac{2\pi \epsilon}{\log_e \left(\frac{D_2}{D_1}\right)}$$

The sensitivity is constant and the relationship between capacitance and displacement is linear as shown below,



The principle of change of capacitance with change in area can be employed for measurement of angular displacement. Figure shows a two plate capacitor. One plate is fixed and the other is movable. The angular displacement changes the effective area between the plates and thus the capacitance changes. The capacitance is maximum when two plate overlap with each other at angular displacement $\theta = 180°$.

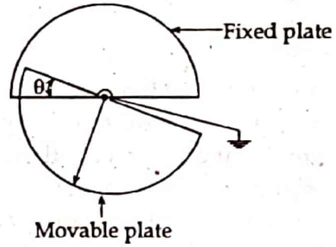


Figure: Two plate capacitor

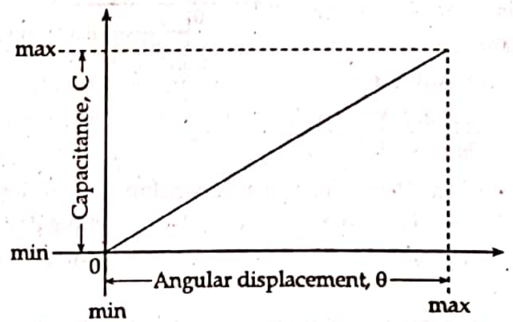


Figure: Relationship between capacitance and angular displacement

Maximum value of capacitance is given by,

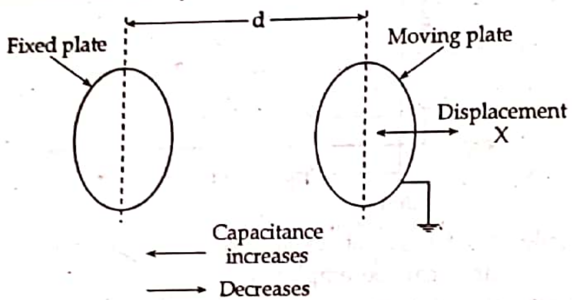
$$C = \epsilon \frac{A}{d} = \pi \epsilon \frac{r^2}{2d}$$

Capacitance at angle θ is, $C = \frac{\pi \theta r^2}{2d}$

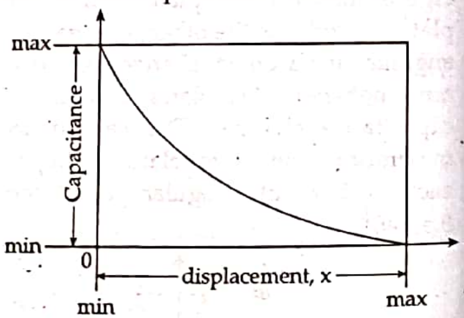
and, Sensitivity, $s = \frac{\partial c}{\partial \theta} = \frac{\pi r^2}{2d}$

b) Changing distance between the plates of capacitive transducers

In these capacitive transducers, the distance between the plate is variable, while the area of the plates and the dielectric constant remain constant. This is the most commonly used type of variable capacitance transducer. Figure below shows that the measurement of the displacement of the object, one plate of the capacitance transducer is kept fixed while the other is connected to the object.



When the object moves, the plate of the capacitance transducer also moves, this results in change in distance between the two plates and the change in the capacitance. The capacitance varies inversely proportional to the distance between the plates. The response of the transducer is not linear as shown in figure.



Sensitivity of the transducer,

$$s = \frac{\partial c}{\partial x} = \frac{\partial c}{\partial x}\left(\frac{\in A}{x}\right) = -\frac{\in A}{x^2}$$

The sensitivity of this transducer is not constant but varies over the range of the transducer. The relationship between capacitance, C and the distance between plates x is hyperbolic. This is linear over the small range of displacement.

c) Changing dielectric constant type of capacitive transducer

In this capacitive transducer, the dielectric material between the two plates changes, due to which the capacitance of the transducer also changes. When the input quantity to be measured changes, the value of the dielectric constant also changes so the capacitance of the instrument changes. This capacitance, calibrated against the input quantity, directly gives the value of the quantity to be measured.

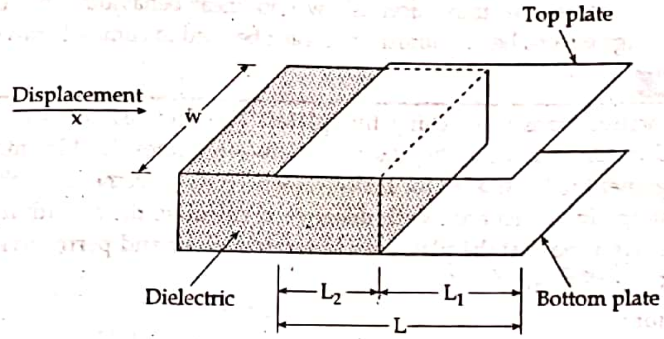


Figure: Capacitive transducer for measurement of linear displacement.

If has dielectric of relative permittivity $\in_r$. Initial capacitance of transducer,

$$C = \in_0 \frac{wL_1}{d} + \in_0 \in_r \frac{wL_2}{d} = \in_0 \frac{w}{d}(L_1 + \in_r \cdot L_2)$$

Let the dielectric is moved through the distance x in the direction indicated,

$$C + \Delta C = \frac{w}{d}(L_1 - x) + \in_0 \in_r \frac{w}{d}(L_2 + x)$$

or, $$C + \Delta C = \in_0 \frac{w}{d}[(L_1 - x) + \in_r (L_2 + x)]$$

or, $$C + \Delta C = \in_0 \frac{w}{d}(L_1 + \in_r L_2) + \in_0 \frac{wx}{d}(\in_r - 1)$$

or, $$C + \Delta C = C + \in_0 \frac{wx}{d}(\in_r - 1)$$

Now, change in capacitance,

$$\Delta C = \in_0 \frac{wx}{d}(\in_r - 1)$$

This transducer is used for the measurement of the level in the hydrogen container where the change in level of hydrogen between the two plates results in change of the dielectric constant of the capacitance transducer. Apart from level, this principle can also be used for measurement of humidity and moisture content of the air.

Advantages of capacitive transducer

i) They are very sensitive
ii) Good frequency response
iii) The force requirement of capacitive transducer is very small and therefore it requires small power to operate.

Disadvantages of capacitive transducer

i) The capacitive transducer are temperature sensitive.
ii) The instrumentation circuit is very complex.
iii) The output impedance of capacitive transducer tends to be on high on account of their small capacitance value. This leads to loading effects.

iv) The capacitive transducers show non-linear behaviour on account of edge effects, hence gurard rings must be used to eliminate this effect.

Example 7

A capacitive transducer using five plates. The dimensions of each plate are 25 × 25 mm and the distance between plates is 0.25 mm. This arrangement is to be used for measurement of displacement by observing the change in capacitance with distance x. calculate the sensitivity of the device. Assume that the plates are separated by air and permittivity of air is 8.85×10^{-12} F/m.

Solution:

Given that;

$$A = 25 \times 25 \text{ mm}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$d = 0.25 \text{ mm} = 0.25 \times 10^{-3} \text{ m}$$

We know for five plates transducers forms four capacitors connected in parallel,

$$C = 4C' = 4\frac{\epsilon_0 (L - x)w}{d}$$

Sensitivity of transducer,

$$\frac{\partial c}{\partial x} = -\frac{4\epsilon_0 w}{d} = -\frac{4 \times 8.85 \times 10^{-12} \times 25 \times 10^{-3}}{0.25 \times 10^{-3}}$$

$$\therefore \quad \frac{\partial c}{\partial x} = -3{,}540 \text{ pF/m}$$

Example 8

A capacitive transducer uses two quartz diaphragms of area 750 mm² separated by a distance of 3.5 mm. A pressure of 900 kN/m² when applied to the top diaphragm produces a deflection of 0.6 mm. The capacitance is 370 pF when no pressure is applied to the diaphragms. Find the value of capacitance after the application of a pressure of 900 kN/m².

Solution:

Let C_1 and C_2 are respectively the values of capacitance before and after application of pressure. Let d_1 and d_2 be the values of distance between the diaphragms for the corresponding pressure conditions.

$$C_1 = \frac{\epsilon A}{d_1}$$

and, $$C_2 = \frac{\epsilon A}{d_2}$$

or, $$\frac{C_2}{C_1} = \frac{d_1}{d_2}$$

or, $$C_2 = C_1 \times \frac{d_1}{d_2}$$

or, $$C_2 = 370 \times \frac{3.5 \text{ mm}}{(3.5 - 0.6)\text{mm}}$$

$$\therefore \quad C_2 = 446.56 \text{ pF}$$

Hence value of capacitance after application of pressure is 446.56 pF.

Example 9

A capacitive transducer is made up of two concentric cylindrical electrodes. The outer diameter of the inner cylindrical electrode is 3 mm and the dielectric medium is air. The inner diameter of the outer electrode is 3.1 mm. Calculate the dielectric stress when a voltage of 100 V is applied across the electrodes. Is it within safe limits? The length of electrodes is 20 mm. calculate the change in capacitance if the inner electrode is moved through a distance of 2 mm. The breakdown strength of air is 3 kV/mm.

Solution:

Length of the air gap between the two electrodes is,

$$= \left(\frac{3.1 - 3}{2}\right) \text{mm} = 0.05 \text{ mm}$$

$$\therefore \quad \text{Dielectric stress} = \frac{V}{\text{Lengt of air gap}} = \frac{100}{0.05} = 2000 \text{ kV/mm}$$

The break down strength of air is 3 kV/mm, hence the dielectric is safe.

$$C = \frac{2\pi\epsilon L}{\log_e\left(\frac{D_2}{D_1}\right)} = \frac{2\pi \times 8.85 \times 10^{-12} \times 20 \times 10^{-3}}{\log_e\left(\frac{3.1}{3}\right)} = 33.9 \text{ pF}$$

Thus the moving in electrode is shifted through a distance of 2 mm.

$$\therefore \quad l = 20 \text{ mm} - 2 \text{ mm} = 18 \text{ mm} = 18 \times 10^{-3} \text{ m}$$

Then the new value of capacitance,

$$C' = \frac{2\pi \times 8.85 \times 10^{-12} \times 18 \times 10^{-3}}{\log_e\left(\frac{3.1}{3}\right)} = 30.5 \text{ pF}$$

∴ Change in value of capacitance,

$$\Delta C = (33.9 - 30.5) \text{ pF} = 3.40 \text{ pF}.$$

3.5.4 Linear Variable Differential Transformer (LVDT)

Linear variable differential transformer (LVDT) is an inductive type position sensor which works on the same principle as the AC transformer that is used to measure movement. It is a very accurate device for measuring linear displacement and whose output is proportional to the position of its movable core.

LVDT consists of a single primary winding (P) and two secondary winding (S_1 and S_2) wound on a cylindrical former. The secondary windings have equal number of turns and are identically placed on either side of the primary winding which is connected to on a.c source. A movable soft iron core is placed inside the former. The displacement to be measured is applied to the arm attached to the soft iron core. Since the primary winding is excited by an a.c source, it produces an alternating magnetic field which in turn induces alternating current voltages in the two secondary windings.

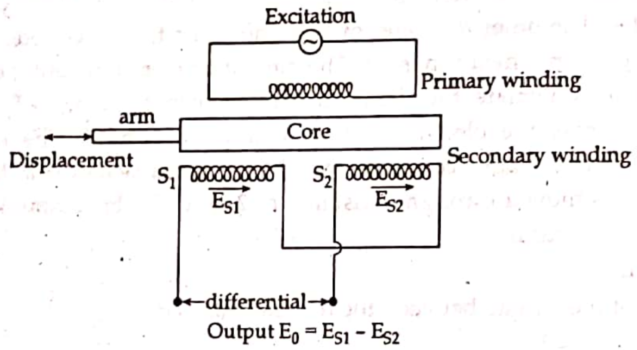


Figure: Circuit of a LVDT

E_{s1} = output voltage of secondary S_1

E_{s2} = output voltage of secondary S_2

The two secondary S_1 and S_2 are connected in series opposition order to convert the output from S_1 and S_2 into single voltage signal. Thus the output voltage of the transducer is the difference of two voltages.

∴ Differential output voltage, $E_0 = E_{s1} - E_{s2}$ (1)

When the core is at its null position, the flux linking with both the secondary winding is equal and hence equal emfs are induced in them. Thus at null position,

$E_{s1} = E_{s2}$ and $E_0 = 0$

Now if the core is moved to the left of the null position, more flux links with winding S_1 than with winding S_2 *i.e.*, $E_{s2} > E_{s1}$, so equation (1) becomes $E_0 > 0$ and positive. This implies that the output voltage is in phase with primary voltage. Again if the core is moved to the right of the null position, the flux linking with winding S_2 becomes large than that linking with winding S_1. Thus, $E_{s2} > E_{s1}$ and equation (1) becomes E_0 and negative. This implies that the output voltage is 180° out of phase with the primary voltage.

By comparing the magnitude and phase of the differential output voltage with that of the source, the amount and direction of the core (displacement) may be determined. The output voltage of the LVDT is a linear function of core displacement within a limited range of motion.

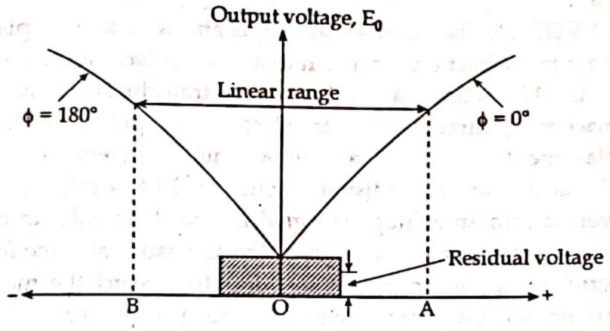


Figure: Variation of output voltage with linear displacement for an LVDT.

The curve is practically linear for small displacement. Beyond this range of displacement, the curve starts to deviate from a straight line.

Advantages of LVDTs

i) Low power consumption
ii) Low hysteresis
iii) Immunity from external effects
iv) No friction and electrical isolation gives truly infinite resolution.
v) It gives high output and therefore many times there is no need for intermediate amplication devices.
vi) It possesses high sensitivity as high as 40 V/mm.
vii) These transducers can usually tolerate a high degree of shock and vibration without any adverse effects. They are simple and by virtue of being small and light in weight, they are stable and easy to align and maintain.
viii) The output voltage of this transducer is practically linear for displacements up to 5 mm.
ix. The change in output voltage is step less as there are no mechanical elements to change the output voltage in discrete steps. The effective resolution depends more on test equipment that on the transducer.

Disadvantages of LVDTs

i) Relatively large displacements are required for appreciable differential output.
ii) They are sensitive to stray magnetic fields but shielding is possible. This is done by providing magnetic shields with the longitudinal slots.
iii) Temperature affects the performance of the transducer.
iv) The dynamic response is limited mechanically by the mass of the core and electrically by the frequency of applied voltage. The frequency of the carrier should be at least ten times the highest frequency component to be measured.
v) The receiving instrument must be selected to operate on ac signals or demodulator network must be used if a dc output is required.

Uses of LVDTs

i) The LVDT can be used in all applications where displacement ranging from fractions of a mm to a few cm have to be measured. The LVDT acting as a primary transducer converts the displacement directly into an electrical output proportional to displacement. This is a fundamental conversion, *i.e.*, the mechanical variable (displacement in this case) is directly converted into an analogous signal in one stage only. In contrast, the electrical strain gauge requires the assistance of some form of a material to act as primary transducer to convert the mechanical displacement into strain which in turn is converted into an electrical signal by the strain gauge acting as a transducer. Hence two stages of signal conversion are involved in strain gauge while there is only one in case of LVDT when displacement is being measured.

ii) Acting as a secondary transducer it can be used as a device to measure force, weight and pressure etc. The force measurement can be done by using a load cells as the primary transducer while fluid pressure can be measured by using bourdon tube which acts as primary transducer. The force or the pressure is converted into displacement which when applied to an LVDT is converted into a voltage. In these applications, the high sensitivity of LVDIs is a major attraction.

Example 10

The output of an LVDT is connected to a 5 V voltmeter through an amplifier whose amplification factor is 250. An output of 2 mV appears across the terminals of LVDT when the core moves through a distance of 0.5 mm. Calculate the sensitivity of the LVDT and that of the whole set-up. The milli-voltmeter scale has 100 divisions. The scale can be read to 1/5 of a division. Calculate the resolution of the instrument in mm.

Solution:

$$\text{Sensitivity of LVDT} = \frac{\text{Output voltage}}{\text{Displacement}} = \frac{2 \times 10^{-3}}{0.5} = 4 \times 10^{-3}\ \text{V/mm}$$

$$= 4\ \text{mV/mm}$$

Now,

$$\text{Sensitivity of instrument} = \text{Amplification factor} \times \text{Sensitivity of LVDT}$$

$$= 4 \times 10^{-3} \times 250 = 1\ \text{V/mm} = 1{,}000\ \text{mV/mm}$$

$$1\ \text{scale division} = 5/100\ \text{V} = 5\ \text{mV}$$

Minimum voltage that can be read on the voltmeter

$$= \frac{1}{5} \times 50 = 1\ \text{mV}$$

$$\therefore \quad \text{Resolution of instrument} = 1 \times \frac{1}{1{,}000} = 1 \times 10^{-3}\ \text{mm}$$

Example 11

An LVDT produces an rms output voltage of 2.6 V for displacement of 0.4 μm. Calculate the sensitivity of LVDT.

Solution:

$$V_{rms} = 2.6\ \text{V}$$

Displacement = 0.4 μm

We know,

$$\text{Sensitivity} = \frac{V_{rms}}{\text{Displacement}} = \frac{2.6}{0.4} = 6.5\ \text{V/μm}$$

Example 12

An LVDT has a secondary voltage of 5 V for a displacement of ± 12.5 mm. determine the output voltage for a core displacement of 8 mm from its central position.

Solution:

Given that;

$$V_s = 5\text{V}$$

Displacement = ± 12.6 mm

Displacement from its central position = 8 mm

We know,

$$\text{Sensitivity, S} = \frac{V_s}{\text{Displacement}} = \frac{5}{12.5} = 0.4\ \text{V/mm}$$

and, Output voltage for core displacement of 8 mm

$$= S \times 8\ \text{mm} = 0.4\ \text{V/mm} \times 8\ \text{mm} = 3.2\ \text{V}.$$

3.5.5 Hall Effect Transducer

This transducer works on the principle of hall effect. Figure below shows the hall effect element.

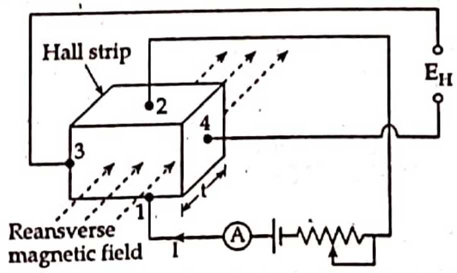


Figure: Hall effect element

When a current conducting material is placed in the transverse magnetic field then the difference of potential is produced between the opposite edges of the conductor. This effect is known as hall effect. The magnitude of the voltage depends upon the current, the strength of magnetic field and the property of the conductor.

Let the current pass through edge 1 and 2 of the conductor and the output leads is connected to edge 3 and 4. The edge 3 and 4 are at same potential where there is no transverse magnetic field passing through the conductor. When a transverse magnetic field pass through a conductor, output voltage appears across the output leads. This output voltage is proportional to the current and the field strength. The output voltage is given by,

$$E_H = \frac{K_H IB}{t}$$

where, K_H = Hall effect constant

t = Thickness of the conductor

I = Current in the circuit

B = Flux density

Thus the voltage produced may be used for measurement of either current (I) or the magnetic field strength (B). Hall effect emf is very small in conductors and is difficult to measure. So Hall effect is mostly pronounced in semiconductor than in conductor.

Applications

i) Measurement of displacement, current and power

ii) Can be used as magnetic to electric transducer

Example 13

A Hall effect transducer is used for the measurement of magnetic field or 0.5 wb/m². The 2 mm thick slab is made of Bismuith for which the hall effect coefficient is -1×10^{-6} Vm and the current is 3 A.

Solution:

Given that;

$K_H = -1 \times 10^{-6}$ Vm

$I = 3$ A

$B = 0.5$ wb/m²

$t = 2$ mm $= 2 \times 10^{-3}$ m

We know that,

$$E_H = \frac{K_H \cdot B I}{t} = \frac{(-1 \times 10^{-6}) \times 3 \times 0.5}{2 \times 10^{-3}} = 0.75 \times 10^{-3}$$

$\therefore \quad E_H = 0.75$ mV

3.5.6 Resistive Transducer or Potentiometers (POT)

A resistive potentiometer consists of resistance element provided with a sliding contact. This sliding contact is called a wiper. The motion of sliding contact may be translator or rotational. Some pots use the combination of the two motions *i.e.*, translation as well as rotational. The POT is a passive transducer since it requires an external power source for its operation.

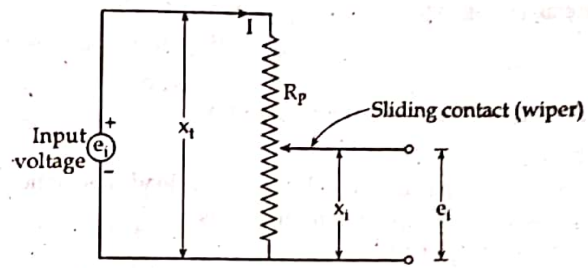


Figure: Translation potentiometer

Let, e_i = Input voltage, V

R_p = Total resistance of potentiometer, Ω

x_t = Displacement of a translational pot, m

x_i = Displacement of the slider from its 0 position, m

e_0 = Output voltage, V

If the distribution of the resistance with respect to translational movement is linear, the resistance per unit length is R_p/x_t. The output voltage under ideal condition is,

$$e_0 = \left(\frac{\text{Resistance at the output terminals}}{\text{Resistance at the input terminals}}\right) \times \text{Input voltage}$$

$$= \left[\frac{(R_p/x_t \times x_i)}{R_p}\right] \times e_i$$

$$= \frac{x_i}{x_t} \times e_i$$

Thus under ideal conditions, the output voltage varies linearly with displacement as shown in figure below.

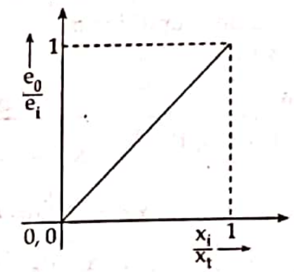


Figure: Unloaded POT characteristics

$$\text{Sensitivity, } s = \frac{\text{Output}}{\text{Input}} = \frac{e_0}{x_i} = \frac{e_i}{x_t}$$

Thus under ideal conditions the sensitivity is constant and the output is faithfully reproduced and has a linear relationship with input. The same is true for rotational motion.

Let, θ_I = Input angular displacement in degrees

θ_t = Total travel of the wiper in degrees

Then the output voltage is,

$$e_0 = \left(\frac{\theta_i}{\theta_t}\right) \times e_i$$

Which is true of single turn potentiometer only.

Loading Effect

The resistance of the parallel combination of load resistance and the portion of the resistance of the potentiometer is,

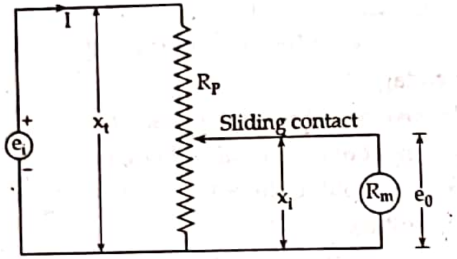


Figure: Loaded potentiometer

$$\frac{\frac{x_i}{x_t} \times R_p R_m}{\frac{x_i}{x_t} R_p + R_m} = \frac{K R_p R_m}{KR_p + R_m}$$

The total resistance seen by the source is,

$$R = R_p (1 - K) + \frac{KR_pR_m}{KR_p + R_m} = \frac{KR_p^2 (1 - K) + R_p R_m}{KR_p + R_m}$$

so, $\quad$ Current, $i = \dfrac{e_i}{R} = \dfrac{e_i (KR_p + R_m)}{KR_p^2 (1 - K) + R_pR_m}$

The output voltage under loaded condition is,

$$e_0 = e_t - i R_m (1 - K)$$

$$= \frac{e_i \cdot K}{1 + (R_p/R_m) K - (R_p/R_m) K^2} = \frac{e_i K}{K(1 - K)\left(\frac{R_p}{R_m}\right) + 1}$$

This equation shows that there exists a non-linear relationship between output voltage e_0 and input voltage e_i. In case $R_m = \infty$, $\dfrac{e_0}{e_i} = K$

Another Method of Derivation

Here R_m is the internal resistance of measuring device. Now,

$$R_i = \left(\frac{x_i}{x_t}\right) R_p = K \cdot R_p$$

where, $\dfrac{x_i}{x_t} = K$

$$R_2 = R_p - R_i = R_p - KR_p = (1 - K) R_p$$

The resistance of the parallel combination of load resistance and the portion of the resistance of the POT is,

$$R_{eq} = R_m \parallel R_i = R_m \parallel KR_p$$

The total resistance seen by the source is,

$$R_t = R_{eq} + R_2$$

$$= R_{eq} + (1 - K) R_p$$

$$= \frac{R_m R_p . K}{R_m + KR_p} + (1 - K) R_p$$

$$= \frac{R_m KR_p + (1 - K) (R_m + KR_P) R_p}{R_m + KR_p}$$

$$= \frac{R_m KR_p + (R_m R_p - KR_m R_p - K^2R_p^2)}{R_m + KR_p}$$

$$= \frac{KR_p^2 (1 - K) + R_pR_m}{R_m} + KR_p$$

Now,

$$\text{Current, } i = \frac{e_i}{R_T} = \frac{e_i (R_m + KR_p)}{KR_p^2 (1 - K) + R_p R_m}$$

The output voltage under load condition is,

$$E_0 = i \times R_{eq}$$

$$= \frac{[e_i (R_m + KR_p)]}{[KR_p^2 (1 - K) R_mR_P]} \times \frac{(R_m . KR_p)}{R_m + KR_p}$$

$$= \frac{e_iK R_p R_m}{KR_p^2 (1 - K) + R_m R_p}$$

Dividing by R_p^2 on numerator and denominator,

$$= \frac{e_i K \left(\frac{R_m}{R_p}\right)}{K (1 - K) + \left(\frac{R_m}{R_p}\right)}$$

Now the ratio of output voltage to input voltage under load condition is,

$$\frac{e_0}{e_i} = \frac{K\left(\frac{R_m}{R_p}\right)}{K (1 - K) + \left(\frac{R_m}{R_p}\right)}$$

or, $\quad \dfrac{e_0}{e_i} = \dfrac{K\alpha}{K (1 - K) + \alpha}$

where, $\alpha = \dfrac{R_m}{R_p}$

Thus,

$$\frac{e_0}{e_i} = \frac{K}{K(1-K)\left(\frac{R_p}{R_m}\right)+1}$$

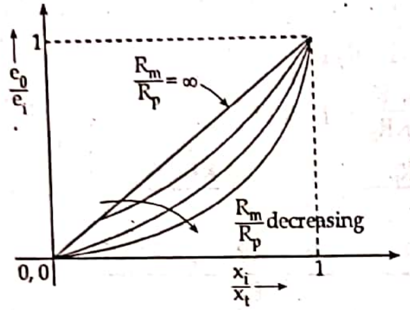


Figure: Loaded POT characteristics of potentiometers

As the ratio of $\frac{R_m}{R_p}$ decreases, the non-linearity goes on increasing. Thus, in order to keep linearity, the value of $\frac{R_m}{R_p}$ should be as large as possible. However when we have to measure the output voltage with a given meter, the resistance of the potentiometer, R_p should be as small as possible.

$\therefore$ Error = Output voltage under load - Output voltage under no load

$$= \frac{e_i K}{K(1-K)\left(\frac{R_p}{R_m}\right)+1} - e_i K$$

$$= e_i\left[\frac{K^2(K-1)}{K(1-K)+\left(\frac{R_m}{R_p}\right)}\right]$$

so, Absolute error, $|\varepsilon| = \frac{e_0}{e_i}\Big|\text{loaded} - \frac{e_0}{e_i}\Big|\text{unloaded}$

$$= \frac{K\alpha}{K(1-K)+\alpha} - K$$

$$= \frac{K\alpha - K^2(1-K) - K\alpha}{K(1-K)+\alpha}$$

$$= \frac{-K^2(1-K)}{K(1-K)+\alpha}$$

where, $\alpha = \frac{R_m}{R_p}$

Except for the two ends where K = 0 *i.e.*, $x_i = 0$ and K = 1 *i.e.*, $x_i = x_t$, the error is always negative.

The maximum error is about 12% of full scale if $\frac{R_m}{R_p} = 1$ and drops down to

about down to about 1.5% when $\frac{R_m}{R_p} = 10$. For the value of $\frac{R_m}{R_p} > 10$, the position of maximum error occurs in the vicinity of $\frac{x_i}{x_t} = 0.67$.

Thus, maximum percentage error is % $E_{max} = 15 \times \left(\frac{R_p}{R_m}\right)$%

Advantages

i) They are inexpensive
ii) They are simple to operate and are very useful for applications where the requirements are not particularly sever.
iii) They are useful for measurement of large amplitudes of displacement.
iv) Their electrical efficiency is very high and they provide sufficient output to permit control operations without further amplification.

Disadvantages

i) They require a large force to move their sliding contacts (wipers).
ii) The wipers can be contaminated, can wear out, become misaligned and generate noise. So the life of the transducer is limited.

Example 14

A variable potential divider has a total resistance of 2 kΩ and is fed from a 10 V supply. The output is connected across a load resistance of 5 kΩ. Determine the loading error for the slider positions corresponding to $\frac{x_i}{x_t}$ = 0, 0.25, 0.5, 0.75 and 1.0. Use the results to plot a rough graph of loading error against the ratio $\frac{x_i}{x_t}$

Solution:

We know,

$$\text{Percentage loading error} = \left[\frac{K^2(K-1)}{K(1-K)+\left(\frac{R_m}{R_p}\right)}\right] \times 100$$

In terms of input voltage,

$$\text{The error} = \left[\frac{K^2(K-1)}{K(1-K)+\left(\frac{R_m}{R_p}\right)}\right] e_i$$

i) When $\frac{x_i}{x_t} = K = 0$,

$$\text{Error} = \left[\frac{0(0-1)}{0(1-0)+\frac{5{,}000}{2{,}000}}\right] \times 10 = 0$$

ii) When K = 0.25

$$\text{Error} = \left[\frac{(0.25)^2(0.25-1)}{0.25(1-0.25)+\frac{5{,}000}{2{,}000}}\right] \times 10 = -0.174 \text{ V}$$

iii) When $K = 0.50$

$$\text{Error} = \left[\frac{(0.5)^2\,(0.5-1)}{0.5\,(1-0.5)+\frac{5{,}000}{2{,}000}}\right] \times 10 = -0.454 \text{ V}$$

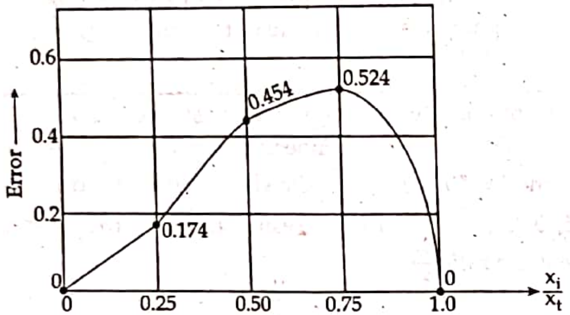
iv) When $K = 0.75$

$$\text{Error} = \left[\frac{(0.75)^2\,(0.75-1)}{0.75\,(1-0.75)+\frac{5{,}000}{2{,}000}}\right] \times 10 = -0.524 \text{ V}$$

v) When $K = 1$,

$$\text{Error} = \left[\frac{(1)^2\,(1-1)}{1\,(1-1)+\frac{5{,}000}{2{,}000}}\right] \times 10 = 0 \text{ V}$$

A rough graph of error versus $\frac{x_i}{x_t}$ is shown below. (Errors are negative)



Example 15

A linear potentiometer is 50 mm long and is uniformly wound with a wire having a resistance 10,000 Ω. Under normal conditions, the slider is at the centre of the potentiometer. Find the linear displacement when the resistance of the potentiometer as measured by a Wheatstone bridge for two case is; (1) 3,850 Ω and (2) 7,560 Ω Are two displacements are in same direction?

If it is possible to measure a minimum value of 10 Ω resistance with the above arrangement, find the resolutions of the potentiometer in mm.

Solution:

Resistance of potentiometer at the normal position $= \frac{10{,}000}{2} = 5{,}000\ \Omega$

i) Resistance of potentiometer wire per unit length

$$= \frac{10{,}000}{50} = 200\ \Omega/\text{mm}$$

ii) Change in resistance of potentiometer from its normal position

$$= 5{,}000 - 3{,}850 = 1{,}150\ \Omega$$

$$\therefore \quad \text{Displacement} = \frac{1{,}150}{200} = 5.75 \text{ mm}$$

iii) Change in resistance of potentiometer from its normal position

$$= 7{,}560 - 5{,}000 = 2{,}560\ \Omega$$

$$\text{so} \quad \text{Displacement} = \frac{2{,}560}{200} = 12.55 \text{ mm}$$

Since one of the displacements represents a decrease and the other represents an increase in resistance of the potentiometer as compared with the resistance of the potentiometer at its normal position, the two displacements are in the opposite direction.

Thus,

$$\text{Resolution} = \text{Minimum measurable value of resistance} \times \frac{\text{mm}}{\Omega}$$

$$= 10 \times \frac{1}{200}$$

$$= 0.05 \text{ mm}$$

3.5.7 Measurement of Temperature

Temperature is a physical property of any matter that quantitatively expresses the common notions of hot and cold. Heat spontaneowly flows from bodies of a higher temperature to bodies of lower temperature at a rate that increases with the temperature difference and the thermal conductivity. No heat will be exchanged between bodies of the same temperature; such bodies are said to be in thermal equilibrium.

3.5.7.1 Thermistor

Thermistor are the semiconductor type resistance thermometers. They have high sensitivity but highly non-linear characteristics. It also have positive temperature coefficient but generally the resistor having negative temperature coefficient are called thermistor. Thermistors are widely used in such applications especially in the temperature range of -60°C to + 15°C. The resistance of thermistor ranges from 0.5 Ω to 75 m Ω.

Construction

Thermistors are composed of sintered mixture of metallic oxide such as manganese, nickel, cobalt, iron, copper and uranium. They are available in variety of sizes and shapes. The thermistors may be in the form of beads, rods or discs. A thermistor in the form of a bead is smallest in size and the bead may have a diameter of 0.015 mm to 12.5 mm. Beads may be sealed in the tips of solid glass rods to form probes which may be easier to mount than the beads. Glass probes have a diameter of about 2.5 mm and a length which varies from 6 mm to 5 mm. Discs are made by pressing material under high pressure into cylindrical flat shapes with diameters ranging from 2.5 mm to 25 mm.

The mathematical expression for the relationship between the resistance of a thermistor and absolute temperature of thermistor is,

$$R_T = R_0\, e^{\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)}$$

where, R_T = resistance at temperature T (K)

R_0 = resistance at temperature T_0

T_0 = reference temperature at 25°C

β = constant

Voltage - Current characteristics

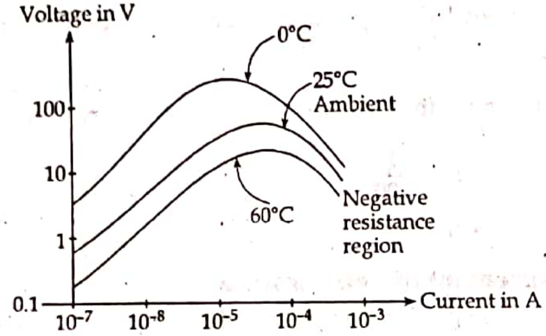


Figure: Voltage-current characteristics of thermistor

As the current increases, the voltage across the thermistor increases. After reaching the peak value voltage starts decreasing and the negative resistance region starts.

Current-time characteristics

At very low voltages, the thermistor takes long time to reach peak current As the voltage level increases, the time to reach peak current decreases.

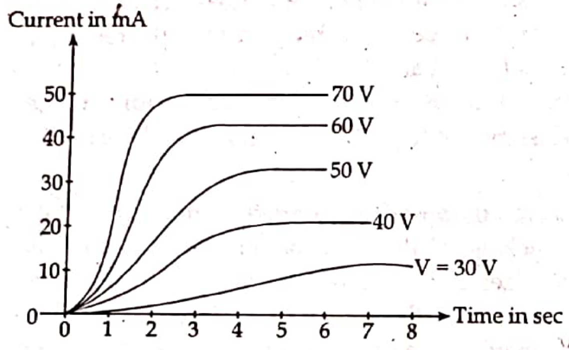


Figure: Current-time characteristics of thermistors.

Advantages

i) Low cost

ii) Sensitivity is high

iii) Small in size

iv) Good stability

v) High output signal

Disadvantages

i) Requires external power supply

ii) Not suitable for high temperature measurement

Applications

i) Used for providing time delay

ii) Used for measuring thermal conductivity of a medium

iii) Used as temperature compensation in electronic equipments

iv) Used for measurement and control of temperature.

Example 16

At R_0 = 1,050 Ω at 27°C, the corresponding β = 3140. What is the temperature when the thermistor resistance is 2,330 Ω.

Solution:

$R_0 = 1{,}050\ \Omega$

$\beta = 3{,}140$

$T = ?$

$R_T = 2330\ \Omega$

We know that,

$$R_T = R_0^e\, e^{\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)}$$

or, $2{,}330 = 1{,}050\, e^{3{,}140\left(\frac{1}{T} - \frac{1}{300}\right)}$

or, $3{,}140\left(\frac{1}{T} - \frac{1}{300}\right) = \log_e (2{,}219)$

or, $3{,}140\left(\frac{1}{T} - \frac{1}{300}\right) = 0.797$

or, $\left(\frac{1}{T} - \frac{1}{300}\right) = \frac{0.797}{3{,}140}$

or, $\frac{1}{T} = 2.538 \times 10^{-4} + 3.34 \times 10^{-3}$

or, $\frac{1}{T} = 3.587 \times 10^{-3}$

or, $T = \frac{1}{3.587 \times 10^{-3}}$

∴ $T = 278.77$ K

Hence, temperature at 2,330 Ω is 278.77 kelvin

and, Sensitivity, $\frac{\delta R}{\delta T} = R\left(\frac{-\beta}{T_2}\right) = (-2{,}330) \times \frac{(3{,}140)}{(278.77)^2} = -94.14\ \Omega/K.$

3.5.7.2 Resistance Temperature Detector (RTD)

Resistance temperature detectors (RTD) are used to measure temperature by correlating the resistance of the RTD element with temperature. The resistance of a conductor changes when its temperature changes. The variation of resistance R with temperature T can be expres :l by following relationship,

$$R = R_0 (1 + \alpha_1 T + \alpha_2 T^2 + \dots\dots\dots + \alpha_n T^n) \quad (1)$$

where, R_0 = resistance at T = 0 K

$\alpha_1, \alpha_2 + \dots\dots\dots + \alpha_n$ = constants.

By increasing the temperature, the electrical resistance of different metals increases in the direct relation to the rise of temperature. The basic element of RTD is very simple. Mostly used material is platinum, so it is also called platinum RTD (PRTD). Platinum is used because if has got linear characteristics and good chemical inertness. For platinum RTD working in linear range,

$$R_T = R_0 (1 + \alpha T) \quad (2)$$

At temperature 0°C with resistance R_0 and 100°C with resistance R_{100},

$$R_{100} = R_0 (1 + \alpha \times 100)$$

$$\therefore \quad \alpha = \frac{(R_{100} - R_0)}{100\, R_0} \quad (3)$$

Thus the value of α is determined. The change in resistance with the change in temperature can be observed by using wheatstone bridge circuit.

The requirements of a conductor material to be used in RTD are given below;

i) The change in resistance of material per unit change in temperature should be as large as possible.

ii) The material should have a high value of sensitivity so that minimum volume of material is used for the construction of RTD.

iii) The resistance of materials should have a continuous and stable relationship with temperature.

3.6 BOARD EXAM QUESTIONS SOLUTION

1. **What characteristics do you consider while selecting a transducer? Explain them in brief. [2011/F, 2013/F]**

Solution: See the definition of 3.2.1.

2. **Write short notes on transducer and its classification. [2011/F, 2014/F, 2015/S, 2018/F, 2019/F]**

Solution: See the definition of 3.2.4.

3. **Write short notes on Hall effects transducer. [2011/S]**

Solution: See the definition of 3.5.5.

4. **What do you mean by capacitive transducer? How can you measure the value of angular displacement and linear displacement by the use of this capacitive transducer? [2011/S, 2013/F]**

Solution: See the definition of 3.5.3 and 3.5.3(a).

5. **What do you mean by piezoelectric effect? Describe how such effect can be used to measure displacements. [2011/S]**

Solution: See the definition of 3.5.2.

6. **A piezo-electric crystal has the dimensions of 5 mm × 5 mm × 1.25 mm. The force acting on it is 5 N. The charge sensitivity of the crystal is 150 PC/N and its permittivity is 12.5×10^{-9} F/m. If the modulus of elasticity is 12×10^6 N/m². If the modulus of elasticity is 12×10^6 N/m². Calculate the strain. Also calculate the charge and the capacitance. [2011/F, 2011/S, 2013/F]**

Solution: See the solution of example number 6.

7. **Define piezo-resistive effect? Derive the relationship between gauge factor and Poisson's ratio. [12/F, 14/F, 16/S]**

OR

Define gauge factor. Derive an expression for gauge factor of a strain gauge. [2011/F, 2013/F, 2013/S, 2015/F, 2015/S, 2019/F]

Solution: See the definition of 3.5.1.

8. **Write short notes on LVDT. [2012/F, 2012/S]**

Solution: See the definition of 3.5.4.

9. **Explain the appropriate transducer for the measurement of weight. [2012/F]**

Solution: See the definition of 3.5.1 *i.e.*, Strain gauge.

10. **Justify that the linear relationship between input and output in a potentiometer is disturbed due to loading of a linear potentiometer. [2012/F]**

OR,

A potentiometer displacement transducer having total resistance $R_p\ \Omega$ and a dc excitation voltage V_{in} is to be used with a measurement system having an input resistance $R_L\ \Omega$. Show that the measured output voltage V_{out} is related to the fractional displacement of the wiper as

$$V_{out} = V_{in} \times \frac{\alpha K}{K(1-K)+\alpha} \text{ where } \alpha = \frac{R_L}{R_p}$$

Also, show that maximum relative error occurs at K = 0.5. [2016/F]

Solution: See the definition of 2.5.6.

We know relative error, $\epsilon = \left|\frac{V_m - V_0}{V_0}\right|$

or, $\epsilon = \frac{\alpha(1-\alpha)}{K+\alpha(1-\alpha)}$

or, $\frac{d\epsilon}{d\alpha} = \frac{d}{d\alpha}\left(\frac{\alpha(1-\alpha)}{K+\alpha(1-\alpha)}\right)$

$$= \frac{[K+\alpha(1-\alpha)](1-2\alpha) - [\alpha(1-\alpha)](1-2\alpha)}{[K+\alpha(1-\alpha)]^2} = \frac{K(1-2\alpha)}{[K+\alpha(1-\alpha)]^2}$$

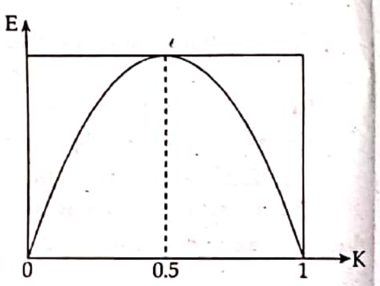
Maximum error is reached when

$1 - 2\alpha = 0$

i.e., $\alpha = \frac{1}{2} = 0.5$

so, Maximum error,

$\epsilon(0.5) = \frac{0.25}{K+0.25}$



The percent error is maximum when the wiper is in the middle of the potentiometer *i.e.*, K = 0.5.

11. **Write short notes on errors in transducer. [2014/S, 2016/F, 2017/F]**

Solution: See the definition of 3.4.1.

12. **The output of an LVDT is connected to a 5 V voltmeter though and amplifier with a gain of 250. The voltmeter scale has 100 divisions and the scale can be read up to 1/5th of a division. An output of 2 mV appears across 0.5 mm. Calculate and determine:**
    i) **Sensitivity of LVDT and entire setup**
    ii) **The resolution of the instrument [2012/S 2013/S, 2014/F, 2015/F]**

Solution

Solution: See the solution of example number 1.

13. ***Illustrate* the working of Hall effect transducer with its necessary diagram. [2013/S, 2014/S, 2017/S, 2018/F]**

Solution: See the definition of 3.5.5.

14. **Write short notes on input characteristics of transducer. [2015/F]**

Solution: See the definition of 3.3.1.

15. **What are physical variables? Explain primary and secondary transducers in brief. [2015/F]**

Solution: See the definition of 3.1 and 3.2.4(a).

16. **The resistance of thermistor at 27°C is 1,050 Ω with constant β = 3140. Calculate the value of temperature when the thermistor resistance becomes 2,330 Ω in °C and Kelvin scale. [2015/S, 2016/F]**

Solution: See the solution of example number 16.

Temperature at 2,330 Ω = 278.77 Kelvin.

Thus,

Temperature at 2,330 Ω in °Celsius

= Kelvin − 273.15 = 278.77 − 273.15 = 5.62° Celsius

17. **Explain the principle operation of LVDT. List out its area of applications and merits and demerits. [2016/S, 2018/S]**

Solution: See the definition of 3.5.4.

18. **Write short notes on basic requirement of transducer. [2016/S]**

Solution: See the definition of 3.2.2.

19. **Define piezoelectric effect. Derive an expression for output voltage when a stress is applied to a piezoelectric transducer. [2018/S]**

Solution: See the definition of 3.5.2.

Output voltage, E_0 = gt p

20. **A variable potential divider has a total resistance of 2 kΩ and is fed from 10 V DC supply. The output is connected to load resistance of 5 kΩ. Determine the loading errors for the wiper positions corresponding to K = 0, 0.25, 0.5, 0.75 and 1.0. Use your results to plot the graph of error versus K. [2018/S]**

Solution: See the solution of example number 14.

21. **A capacitance transducers uses two quartz diaphragms of area 750 mm² separated by a distance of 3.5 mm. A pressure of 900 kN/m² when applied to the top diaphragm produces a deflection of 0.6 mm. The capacitor has capacitance 370 pF when no pressure is applied to the diaphragm. Find the value of capacitance after the application of pressure of 900 kN/m². [2018/F, 2019/F]**

Solution: See the solution of example number 8.

22. **A compressive force is applied to a structural memfrane. The strain is 5 micro-strain. Two separate strain gavges are attached to the structural member, one is a nikel wire strain gavge having a gavge factor of − 12.1 and the outer is nichrome wire strain**

gavge having a gavge factor of 2. Calculate the value of resistance of the gavge after they are strained. The resistance of strain gavges before staired is 120 Ω. **[2017/F]**

Solution: See the solution of example number 2.

23. **A potentiometer displacement transducer having total resistance R_p Ω and a dc excitation voltage V_{in} is to be used with a measurement system having an input resistance R_L Ω. Show that the maximum % error is approximately 14.81 $\frac{R_p}{R_L}$.** **[2017/S]**

Solution: See the definition of 3.5.6.

Maximum percentage error = $14.81 \times \left(\frac{R_p}{R_l}\right) \approx 15 \times \left(\frac{R_p}{R_L}\right)$ $(\because R_L = R_m)$

24. **A strain gauge is bonded to a beam of 0.1 m long and has a cross sectional area of 4 cm². Young's modulus for steel is 207 GN/m². The strain gauge has an unstrained resistance of 240 Ω and a gauge factor of 2.2. When a load is applied, the resistance of gauge changes by 0.013 Ω. Calculate the change in length of steel beam and the amount of force applied to the beam.** **[2012/F, 2017/S]**

Solution: See the solution of example number 4.

25. **"Error of a potentio metric device is always negative except at two extreme points". Justify if with necessary derivations.** **[2019/F]**

Solution: See the definition of 3.5.6.

26. **A strain gauge having the resistance of 400 Ω and gauge factor of 1.5 is connected in series with a ballast resistance of 500 Ω. Determine the change in output when a stress of 200 mN/m² is applied and take the value of modulus of elasticity as 250 GN/m².** **[2014/S]**

Solution:

Resistance, R = 400 Ω

Gauge factor, $G_f = 1.5$

Change in output, ΔV =?

Stress, P = 200 mN/m² = 200×10^6 N/m²

Modulus of elasticity, E = 250 GN/m² = 250×10^9 N/m²

We know,

$$\text{Hook's law, } \in = \frac{S}{E} = \frac{200 \times 10^6}{250 \times 10^9} = 0.00080$$

Now,

$$\text{Gauge factor, } G_f = \frac{\left(\frac{\Delta R}{R}\right)}{\left(\frac{\Delta L}{L}\right)} = \frac{\left(\frac{\Delta R}{R}\right)}{\in}$$

or, $\Delta R = G_f \times \in \times R$

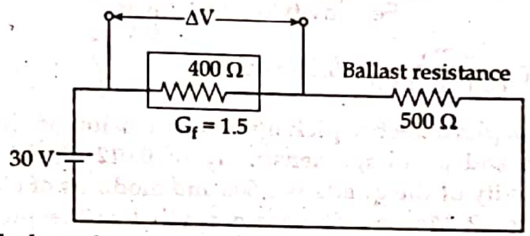
or $\Delta R = 1.5 \times 0.00080 \times 400$

∴ $\Delta R = 0.480\ \Omega$

i.e., there is an increase of 0.480 Ω in the value of resistance. When unstrained, voltage across strain gauge is,

$$V_1 = \left(\frac{400}{400 + 500}\right) \times 30 = 13.3333\ \text{V}$$

(∵ Voltage not given so let V = 30 V)



When strained, the voltage across strain gauge is,

$$V_2 = \left(\frac{400 + \Delta R}{(400 + \Delta R) + 500}\right) \times 300 = \left(\frac{400 + 0.480}{400 + 0.480 + 500}\right) \times 30$$

$$= \left(\frac{400.480}{900.480}\right) \times 30$$

∴ $V_2 = 13.3422$ V

Thus, change in output = $V_2 - V_1$ = (13.3422 − 13.3333) V = 0.0088 V

27. **The output of an LVDT is connected to a 5V voltmeter through an amplifier whose amplification factor is 150. An output of 5 mv appears across the terminals of LVDT, when the core moves through a distance of 0.5 mm. Calculate the sensitivity of LVDT and that of the whole setup. The milli-voltmeter scale has 100 divisions. The scale can be read to $\frac{1}{5}$ of a division. Calculate the resolution of the instrument in mm.** **[2014/S, 2015/S, 2017/F]**

Solution:

Given that;

Amplification factor = 150

Output voltage = 5 mV = 0.005 V

Sensitivity = ?

Resolution of instrument =?

Displacement = 0.5 mm

We know,

i) Sensitivity of LVDT = $\frac{\text{Output voltage}}{\text{Displacement}} = \frac{5\text{ mV}}{0.5\text{ mm}} = 10$ mV/mm

ii) Sensitivity of instrument

= Amplification factor × Sensitivity of LVDT

= 150 × 10 mV/mm = 1,500 mV/mm

iii) 1 scale division $= \frac{5}{100}$ V = 50 mV

Minimum voltage that can be read on voltmeter

$= \frac{1}{5} \times 1$ scale division $= \frac{1}{5} \times 50$ mV = 10 mV

Then,

Resolution of instrument

$= \frac{\text{Minimum voltage that can be read on voltmeter}}{\text{Sensitivity of instrument}}$

$= \frac{10 \text{ mV}}{1500 \text{ mV/mm}} = 0.00666$ mm.

28. **A quartz piezoelectric pick up has dimension of 5 mm × 5 mm × 1.5 mm and a voltage sensitivity of 0.012 Vm/N. The relative permittivity of the quartz is 1,600 and modulus of elasticity of the quartz is 12 MN/m². The force applied to the pickup is 10 N. Determine,** **[2016/F]**
   i) **Output voltage**
   ii) **Charge sensitivity**
   iii) **Strain**
   iv) **Charge generated and the capacitance of the pick-up**

Solution:

Thickness, $t = 1.5$ mm $= 1.5 \times 10^{-3}$ m

Area of plate, $A = 5$ mm × 5 mm $= 25 \times 10^{-6}$ m²

Voltage sensitivity, g = 0.012 Vm/N.

Force applied, F = 10 N

Relative permittivity, $\epsilon_r = 1{,}600$

Modulus of elasticity, $E = 12 \times 10^6$ N/m².

We know,

i) Output voltage,

$$V_{out} = g\,tp = 0.012 \times 1.5 \times 10^{-3} \times \frac{F}{A}$$

$$= 0.012 \times 1.5 \times 10^{-3} \times \frac{10}{25 \times 10^{-6}}$$

$\therefore$ $V_{out} = 7.2$ volts

ii) Charge sensitivity, $d = \epsilon_0 \epsilon_r g$

$= 8.85 \times 10^{-12} \times 1{,}600 \times 0.012 = 1.70 \times 10^{-10}$ C/N = 170 PC/N

iii) Strain $= \frac{P}{E} = \frac{F/A}{E} = \frac{10}{12 \times 10^6 \times 25 \times 10^{-6}} = 0.0334$

iv) Charge generated on pick up,

$Q = d.F = 170 \times 10^{-12} \times 10$

Q = 1,700 PC

and, Capacitance of pick-up,

$$C_c = \frac{Q}{V_{out}} = \frac{1{,}700 \times 10^{-12}}{72} = 2.3611 \times 10^{-11} \text{ F} = 23.611 \text{ pF.}$$

29. **A resistance strain gauge with gauge factor of 2 is fastened to a steel member subjected to a stress of 1250 kg/cm². The modulus of elasticity of steel is approximately 2.1 × 10⁶ kg/cm².** **[2012/S]**
Calculate:
   i) **Percentage change in resistance**
   ii) **The Poisson's ratio**
   iii) **The strain value**

Solution:

Gauge factor, $G_f = 2$

Stress, P = 1,250 kg/cm²

Modulus of elasticity, $E = 2.1 \times 10^6$ kg/cm²

We know,

$$E = \frac{\text{stress}}{\text{strain}}$$

or, strain $= \frac{\text{stress}}{E} = \frac{1{,}250}{2.1 \times 10^6}$

$\therefore$ Strain $= 595.238 \times 10^{-6}$

i) Hence strain value is 595.238×10^{-6}

ii) Gauge factor,

$$G_f = \frac{\left(\frac{\Delta R}{R}\right)}{\left(\frac{\Delta L}{L}\right)} = \frac{\left(\frac{\Delta R}{R}\right)}{\text{Strain}}$$

or, $\frac{\Delta R}{R} = G \times \frac{\Delta L}{L}$

or, $\frac{\Delta R}{R} = 2 \times 595.238 \times 10^{-6}$

or, $\% \frac{\Delta R}{R} = 1.190 \times 10^{-3} \times 100\%$

$\therefore$ Percentage change in strain is 0.1190%

iii) Also,

$G = 1 + 2\mu$

or, $2\mu = G - 1$

or, $\mu = \frac{G-1}{2} = \frac{2-1}{2}$

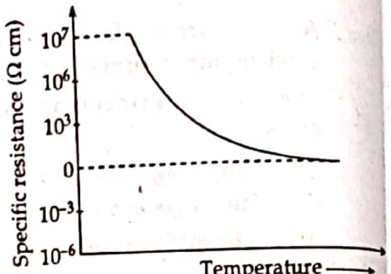
$\therefore$ $\mu = 0.5$

Hence Poisson's ratio $= \mu = 0.5$.

30. What are thermisters? How are they constructed? Discuss their resistance temperature characteristics. **[2012/S]**

Solution: See the definition of 3.5.7.1.

Thermistors has a very high negative temperature coefficient of resistance making it an ideal temperature transducer.



The characteristics of thermistors are no doubt non-linear approximation of the resistance temperature curve can be obtained over a small range of temperature.

Thus for the limited range of temperature, the resistance of a thermistor varies as,

$$R_\theta = R_{\theta 0}(1 + \alpha_{\theta 0}\Delta\theta)$$

A thermistor exhibits a negative resistance temperature coefficient which is typically 0.05 Ω/Ω - °C. In place of linear approximation, an appropriate logarithmic relationship may be used for resistance temperature relationship for a thermistor. *i.e.*, $R_\theta = aR_0 e^{b/T}$

where, R_θ = resistance at ice point

R_0 = resistance at temperature T

a, b = constants.

31. A Piezo- electrical crystal has the dimensions of 4 m × 4 m × 2 mm. The force acting on the crystal is 15 N. The charge sensitivity of the crystal is 150 PC/N and its permittivity is 12 × 10^{-9} F/m. If the modulus of elasticity is 10 × 10^6 N/m². Calculate the value of charge and capacitance. **[2016/S]**

Solution:

Area of crystal (A) = L × b = 4 m × 4 m = 16 m²

Thickness, t = 2 mm = 0.002 m

Charge sensitivity, d = 150 Pc/N = 150 × 10^{-12} C/N

Modulus of elasticity, E = 10 × 10^6 N/m²

Permittivity, $\epsilon_r \epsilon_0 = 12 \times 10^{-9}$ F/m.

Force, F = 15 N

so, Voltage sensitivity, $g = \dfrac{d}{\epsilon_0\epsilon_r} = \dfrac{150 \times 10^{-12}}{12 \times 10^{-9}} = 1.250 \times 10^{-2}$ Vm/N

Now, Voltage generated,

$$E_0 = g\,t\,p = 1.250 \times 10^{-2} \times 0.002 \times \frac{F}{A} = 1.250 \times 10^{-2} \times 0.002 \times \frac{15}{16}$$

$\therefore \quad E_0 = 2.3437 \times 10^{-5}$ volts

i) Charge,

$Q = d \times F = 150 \times 10^{-12} \times 15 = 2.25 \times 10^{-9}$

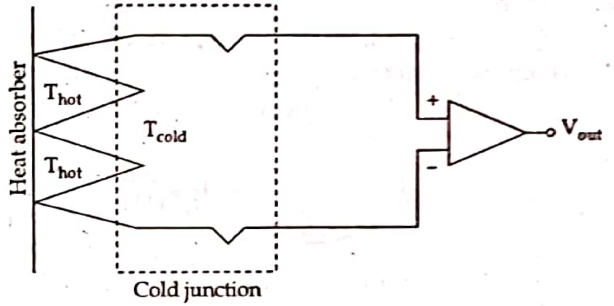
$\therefore \quad Q = 225$ PC

ii) Capacitance $= \dfrac{Q}{E_0} = \dfrac{225 \times 10^{-12}}{2.3437 \times 10^{-5}} = 9.60 \times 10^{-5}$ F = 96 μF.

32. What is thermopile? Explain the principle of thermistor to measure the temperature. **[2014/F]**

Solution: See the definition of 3.5.7.1.

A thermopile is an electronic device that converts thermal energy into electrical energy. It is composed of several thermocouples connected usually in series or less commonly in parallel.



$V_{out} = n \times G \times (T_{hot} - T_{cold})$ where, G = Seeback constant

Figure: thermopile

The thermistor acts as the temperature sensor and is placed on the body whose temperature is to be measured. It is also connected in the electric circuit. When the temperature of the body changes, the resistance of the thermistor also changes, which is indicated by the circuit directly as the temperature since resistance is calibrated against the temperature.

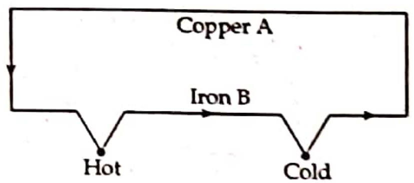
33. Explain how can the response of a capacitive transducer which works on the principle of variation of capacitance with displacement between plates can be made linear. Also give the sensitivity and resolution of such an arrangement. **[2013/S]**

Solution: See the definition of 3.5.3 (a).

34. Discuss seeback effect for temperature measurement mention the laws governing in it. List out the sources of errors occurred in it during temperature measurement. **[2017/S]**

Solution:

Two wires of different metals A and B are joined together to form two junctions and if the two junctions are at different temperatures, an electric current will flow around the circuit. This is the see beck effect.



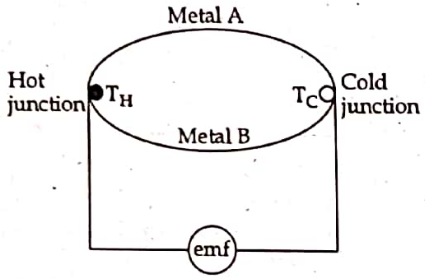
If metal A is of copper and metal B is of iron, then the current flows from copper to iron at the hot junction and from iron to copper at the cold junction. The voltage produced by seeback effect are small usually only a few micro volts per Kelvin of temperature difference at the junction. The seeback effect is responsible for the behavior or thermocouples which are used to approximately measure temperature difference or to actuate electronic switches that can turn large systems on and off.

A circuit formed from two dissimilar metals joined at both ends develop an voltage proportional to the difference in the two junction temperatures. So if the temperature of one junction is kept an a known value, the temperature of the other junction can be determined by the amount of voltage produced. Mathematically.

$$\text{Emf} = -S(T_h - T_c)$$

where, S = Seebeck coefficient, T_h = Temperature at hot end

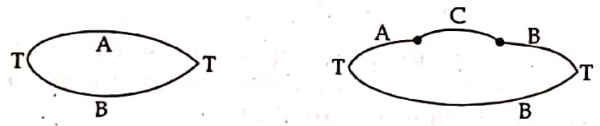
T_c = Temperature at cold end



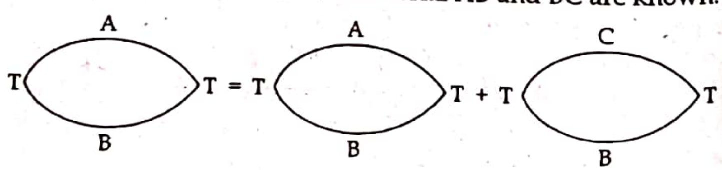
The laws of thermocouples:

i) The emf of a thermocouple depends only on the temperature of the junctions and is dependent of the wires connecting the junctions. This means that the leads connecting the instrument can be exposed to temperature fluctuations without affecting the measurement.

ii) The emf is not changed when a new conductor is introduced into one of the metals provided that both of the new junctions are at the same temperature. The practical bonus from this law is that a voltmeter can be added to the circuit without affecting emf.



iii) The law of intermediate metals states that the emf of material AC can be predicted if the emf of material AB and BC are known.



Six most common causes of thermocouple errors are;

i) Selecting the wrong type of thermocouple on the transmitter
ii) Problems related to the thermocouple extension wire
iii) Inherent variations in alloys
iv) Temperature variations around the reference junction connection
v) Thermocouple age
vi) Thermocouple grounded at more than one location

35. Explain the different principles of working of capacitive transducers. [2017/F]

Solution: See the definition of 3.5.3.

36. Explain the criteria for selection of transducers for a particular application. Describe the method for measurement of temperature with use of RTDs. [2017/F]

Solution: See the definition of 3.2.3 and 3.5.7.2.

37. Write short notes on sensor and its type. [2015/S]

Solution:

Sensors are sophisticated devices that are frequently used to detect and respond to electrical or optical signals. A sensor converts the physical parameter (*For example;* temperature, blood pressure, humidity, speed etc into a signal which can be measured electrically.

Different types of sensors are:

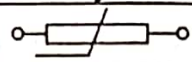
- Temperature sensor
- Proximity sensor
- Accelerometer
- Infrared sensor
- Pressure sensor
- Light sensor
- Ultrasonic
- Smoke, gas and alcohol sensor
- Touch sensor
- Color sensor
- Tilt sensor
- Flow and level sensor

Stimuli → Sensor → electrical signal

Sensors are omnipresent. They are embedded in our bodies, auto mobiles, cellular telephones, radio, chemical plants, industrial plants and countless other applications. Without the use of sensors, there would be no automation.

38. Differentiate RTD and thermistor for the measurement of temperature. Show the necessary equations for each. [2018/F]

Solution: See the definition of 3.5.7.1 and 3.5.7.2.

	RTD	Thermistor
a)	It is a device used for measuring the change in temperature.	It is a thermal resistor whose resistance changes with the temperature.
b)	Its response time is slow.	It's response time is fast.
c)	It has less accuracy.	It has high accuracy.
d)	Its symbol is 	Its symbol is 
e)	Its temperature range of working is from – 230°C to 660°C.	Its temperature range of working is from – 60°C to 15°C
f)	It has linear characteristic graph.	It has non-linear characteristic graph.
g)	It has low sensitivity.	It has high sensitivity.
h)	It is cheap.	It is expensive.
i)	It has high resistivity.	It has low resistivity.

39. Write short notes on RTD. [2012/S]

Solution: See the definition of 3.5.7.2.

40. Difference between active and passive transducers. [2012/S]

	Active transducers	Passive transducers
i)	These transducers are self generating transducers. They produce voltage or current proportional to the physical quantity to be measured.	These transducers produce variation in resistance, capacitance or inductance in response to the physical quantity being measured and are not self generating transducers.
ii)	They do not need external power supply for their operation.	They need external power supply for their operation.
iii)	*Example;* Thermocouple, piezo electrical crystal etc	*Example;* Potentiometer, thermistor etc
iv)	They have low resolution	They have high resolution
v)	Signal conversion is simpler	Signal conversion is more complicated
vi)	They are known as self-generating transducers.	They are known as externally energized transducers.