Chapter 3

Physical Variables and Transducers

Physical variables and their types

Physical variables are the variables to be measured in an instrumentation system, which make its first contact with primary sensing elements. It is the information for the measurement system in the form of a physical phenomenon or an electrical signal.

Types of physical variables

i. Process variables
   Temperature, pressure and flow rate are the process variables which are widely employed in process and production plants.

ii. Electrical variables
   Current voltage resistance, inductance, capacitance, frequency, phase angle, power are the electrical variables.

iii. Mechanical variables
   Spring (Force to displacement), monometer (pressure to displacement), thermocouple (temperature to electric current), Hydrometer (specific gravity to displacement), Turbines (Linear to angular velocity) etc. are the mechanical variables.

iv. Bio-physical variables
   Here, the information is taken from living beings ECG (Electro Cardio Graph) for heart, EEG (Electro Encephilo Graph for Brain), EMG (Electro Mio Graph for muscular), excitation, restoring potential are the bio-physical variables.

Transducer

It is a device which converts non-electrical quantity into an electrical form in order to use electrical methods and techniques for measurement, manipulation or control.

Basic Requirement of Transducer

The main function of the transducer is to respond only for the measurement under specified limit for which it is designed.

a. Ruggedness
   It should be capable of withstanding overload and some safety arrangement should be provided for the overload protection.
b. Linearity
   It's input- output characteristics should be linear.
c. Repeatability

It should reproduce same output signals when the input signal is applied again and again under fixed environment condition.

d. High quality output signals
   The ratio of Signal to Noise (SNR) should be high and the amplitude of the output signal should be up to detectable level.
e. High Reliability and Stability (minimum error)
   We should get minimum error in measurement for temperature variations, vibrations and other various changes in surrounding.
f. Good Dynamic Response
   Its output should be faithful to input when taken as a function of time.

Selection of Transducers

a. Range
   The range of the transducer should be large enough to cover up all the expected magnitude of quantity under measurement.
b. Sensitivity
   Transducer should be given a sufficient output signal per unit of measured input in order to yield meaningful data.
c. Electrical output characteristics
   The electrical characteristics such as output impedance, frequency response should be compatible with the recording devices.
d. Physical Environment
   The transducer select should be able to withstand the environmental conditions to which it is likely to be exposed.
e. Error
   The error inherent in the operation of the transducer itself or those errors caused by environmental condition of the measurement should be small enough or controllable enough that they allow meaningful data to be taken.

Classification of transducers

1. On the basis of transduction form used
   The transducers can be classified on the basis of principle of transduction as resistive, inductive, capacitive etc. depending upon how they convert the input quantity into resistance, inductance or capacitance respectively. They can be classified as piezo-electric, thermoelectric, magneto restrictive, electrokinetic and optical.

2. As primary and secondary transducers
   There are two stages of transduction; firstly the pressure is converted into a displacement by Bourdon tube, then the displacement is converted into an analogous voltage by LVDT. The Bourdon tube is called primary transducer while LVDT is called a secondary transducer.

3. As passive and active transducers

Passive transducers derive the power required for transduction from an auxiliary power source. They are also known as externally powered transducers. Example: resistive (Potentiometer, Strain gauge), inductive (LVDT) and capacitive (variable capacitance pressure gauge, Dielectric gauge) transducers
Active transducers are those which don't require an auxiliary power source to produce their output, so called as self-generating type as they develop their own voltage or current output. Examples: thermocouple, photovoltaic cells and piezoelectric crystals.

4. As analog and digital transducers
   Analog transducers convert the input quantity into an analog output which is a continuous function of time. Example: strain gauge, LVDT, thermocouple, thermistor.

   Digital transducers convert the input quantity into an electrical output which is in the form of pulses (0 and 1).

5. As transducers and inverse transducers
   Transducer is a device which converts a non-electrical quantity into an electrical quantity.
   An inverse transducer is a device which converts an electrical quantity into a non-electrical quantity. Example: piezoelectric crystals (voltage to displacement)

Input characteristics of the transducers

1. Type of input and operating Range
   The type of input which can be any physical quantity is generally determined in advance. The useful operating range of the transducer may be decisive in selection of a transducer for a particular application. The upper limit is decided by the transducer capabilities while the lower limit of range is normally determined by the transducers error or by the unavoidable noise originating in the transducer

2. Loading Effect
   Ideally, transducers should be have no loading effect on the quantity being measured. In practice, it is impossible and hence steps may be taken to reduce the loading effects to negligible proportions.

Output Characteristics of Transducer

The three conditions in the output characteristics which should be considered are

i. Type of electrical output
   The output quantities (voltage, current, impedance or time function of these amplitudes) may or may not be acceptable to the latter stages of the instrumentation system. They may have to be manipulated so as to make them drive the subsequent stages of instrumentation system.

ii. Output impedance

The output impedance of a transducer determines to the extent the subsequent stages of instrumentation is loaded. Ideally, the value of output impedance should be zero if no loading effects are there on the subsequent stages. It also determines the amount of power that can be transferred to the succeeding stages of the instrumentation system for a given output signal of instrumentation system for a given output signal level.

iii. Useful Output Range

The output range of a transducer is limited at the lower end by noise signals and the upper limit is set by the maximum useful input level.

Error of transducers

The errors in transducer occur because they don't follow the input-output relationship in many situations. The error can be split into three components which are

i) Scale error
ii) Dynamic error
iii) Error on account of noise and drift

i) Scale error

It is of four different types

a. Zero error:

Here, the output deviates from the correct value by a constant factor over the entire range of the transducer.

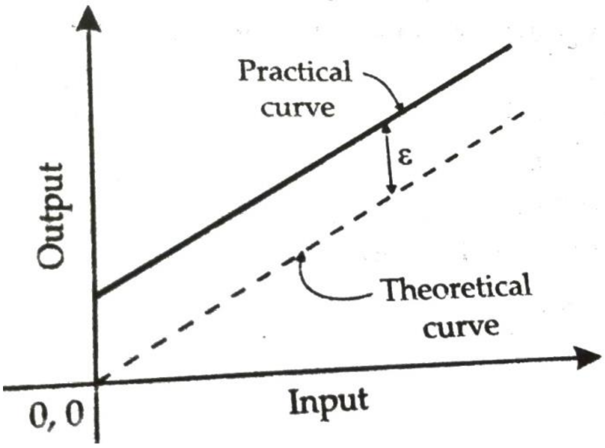


Fig. Transducer zero error

b. Sensitivity error

It occurs when the observed output deviates from the correct value by a constant value. Suppose, the correct output is q_O, the output would be kq_O over the entire range of the transducer, where k is constant.

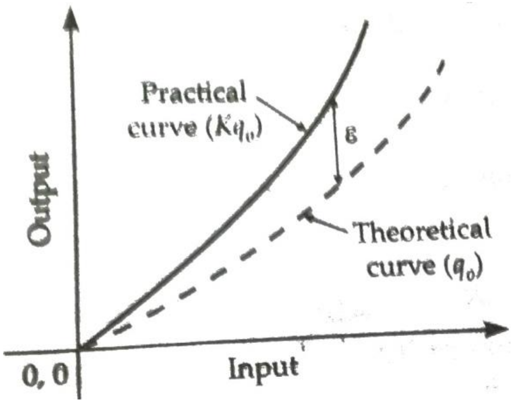


Fig. Transducer sensitivity error

c. Non-conformity
Here, the experimentally obtained transfer function deviates from the theoretical transfer function for almost every input .

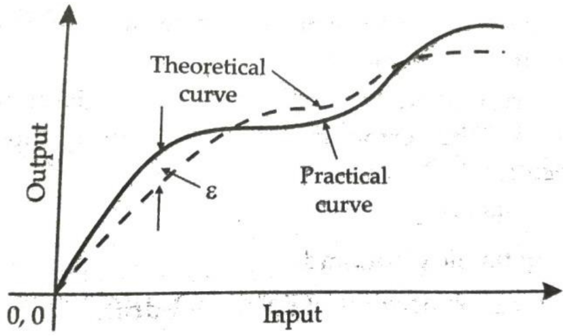


Fig. Transducer non-conformity

d. Hysteresis
The output of transducer not only depends upon the input quantity but also upon the input quantity previously applied to it. Therefore, a different output is obtained when same value of input quantity is applied depending upon whether it is increasing or decreasing. For decreasing values a greater output is obtained than with increasing value for the same value of the input quantity.

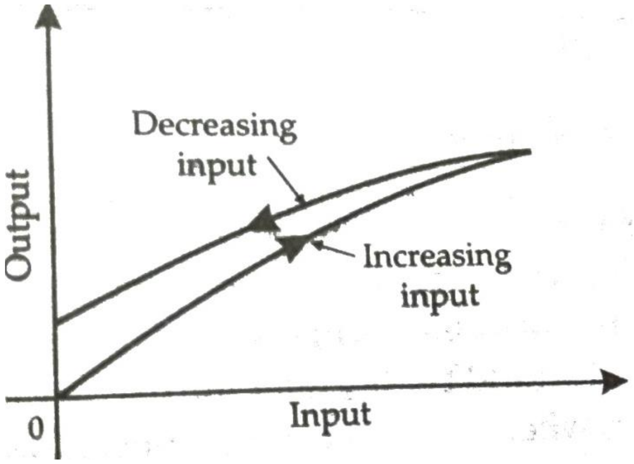


Fig. Transducer hysteresis

ii) Dynamic Error

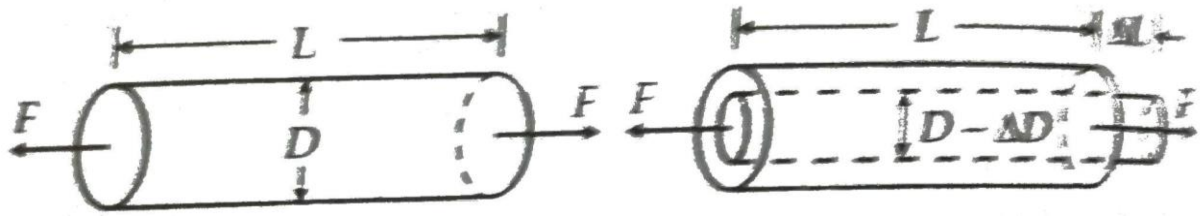
It occurs only when the input quantity is varying with time. It can be made small by having a small time constant. It reduces as the time after application of the input increases.

iii) Error due to noise and Drift

Noise is a signal of random amplitude and random frequency while drift is a slow change with time. The magnitude of the noise and drift is normally independent of the magnitude of the input signal.

Strain Gauges

The strain gauges are used for measurement of strain and associated stress in experimental stress analysis. If a metal conductor is stretched or compressed, its resistance changes on account of the fact that both length and diameter of conductor change. Also, there is a change in the value of resistivity of the conductor when it is strained and this property is called piezoresistive effect. Therefore, resistance strain gauges are also known as piezoresistive gauges.



Let us consider a strain gauge made of circular wire. The wire has the dimensions: length = L, area =A, diameter = D before being strained. The material of the wire has a resistivity ρ.

∴ , Resistance of unstrained gauge is given by $R = \rho \frac{L}{A}$

Let a tensile stress S be applied to the wire. This produces a positive strain causing the length to increase and area to decrease. Thus, when the wire is strained, there are changes in its dimensions. Let ΔL = change in length, ΔA = change in area, ΔD = change in diameter and ΔR = change in resistance. In order to find how ΔR depends upon the material physical quantities, the expression for R is differentiated w.r.to stress S i.e

$$\frac{dR}{dS} = \frac{\rho}{A}\frac{\partial L}{\partial S} - \frac{\rho L}{A^2}\frac{\partial A}{\partial S} + \frac{L}{A}\frac{\partial \rho}{\partial s} \ldots\ldots\ldots\ldots\ldots(1)$$

Dividing equation (1) by resistance $R = \rho \frac{L}{A}$, we get

$$\frac{1}{R}\frac{dR}{dS} = \frac{1}{L}\frac{\partial L}{\partial S} - \frac{1}{A}\frac{\partial A}{\partial S} + \frac{1}{\delta}\frac{\partial \rho}{\partial s} \ldots\ldots\ldots\ldots\ldots(2)$$

It is evident from equation (2) that the per unit change in resistance is due to (i) per unit change in length = ΔL/L, (ii) per unit change in area = ΔA/A and (iii) per unit change in resistivity = Δρ/ρ

Now, Area (A) = $\frac{\pi}{4}D^2$ $\therefore \frac{\partial A}{\partial S} = \frac{2\pi}{4}D\,\frac{\partial D}{\partial S}$

So, $\frac{1}{A}\frac{\partial A}{\partial S} = \frac{1}{\left(\frac{\pi}{4}D^2\right)} * \frac{2\pi}{4}D\,\frac{\partial D}{\partial S} = \frac{2}{D}\frac{\partial D}{\partial S}$

And equation (2) becomes

$$\frac{1}{R}\frac{dR}{dS} = \frac{1}{L}\frac{\partial L}{\partial S} - \frac{2}{D}\frac{\partial D}{\partial S} + \frac{1}{\rho}\frac{\partial \rho}{\partial s} \ldots\ldots\ldots\ldots(3)$$

Also, Poission's ratio (γ) = $\frac{Lateral\ Strain}{Longitudinal\ Strain}$

$= \frac{-\partial D/D}{\partial L/L}$

$-\frac{\partial D}{D} = \gamma * \frac{\partial L}{L}$ So equation (3) becomes

$$\frac{1}{R}\frac{dR}{dS} = \frac{1}{L}\frac{\partial L}{\partial s} + \gamma\frac{2}{L}\frac{\partial L}{\partial S} + \frac{1}{\rho}\frac{\partial \rho}{\partial S} \ldots\ldots\ldots(4)$$

For small variations, the equation (4) can be written as

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + 2\gamma\frac{\Delta L}{L} + \frac{\Delta \rho}{\rho} \ldots\ldots\ldots\ldots(5) ..$$

The Gauge Factor is defined as the ratio of per unit change in resistance to per unit change in length i.e.

Gauge factor (G_f) = $\frac{\Delta R/R}{\Delta L/L}$

or, $\frac{\Delta R}{R} = G_f \frac{\Delta L}{L} = G_f * \in$...........(6)

Solving equation (5) and (6) we get

$G_f * \in = \frac{\Delta R}{R} = \in + 2\gamma\epsilon + \frac{\Delta\rho}{\rho}$...................(7)

If the change in the value of resistivity of a material when strained is neglected i.e. piezoresistive effect is negligible, the gauge factor is

$G_f = 1+2\gamma$ ……………………………(8)

Poison's ratio for most metals lies in the range of 0.25 to 0.35 and the gauge factor would be on the order of 1.5 to 1.7 .

Hooke's law gives the relationship between stress and strain for a linear stress- strain curve in terms of the modulus of elasticity of the material under tension. Hooke's law is written as

Strain, $\epsilon = \frac{S}{E}$

Where, $\epsilon = \frac{\Delta L}{L}$ = strain (no units)

S= stress (kg/cm^2) and

E = Young's modulus/modulus of elasticity of material(kg/cm^2)

Applications of strain Gauges

i. Experimental stress analysis of machines and structure.

ii. Construction of force, torque, pressure, flow and acceleration transducers.

Example 1

A resistance strain gauge with a gauge factor(sensitivity of a strain gauge) of 2 is fastened to a steel member subjected to a stress of 1050 kg/cm^3. The modulus of elasticity of a steel is approximately 2.1*10^6 kg/cm^2. Calculate the change in resistance, ΔR of the strain gauge element due to the applied stress. Comment upon the result.

Solution:

Hooke's law yields,

$\epsilon = \frac{S}{E} = \frac{\Delta L}{L} = \frac{1050}{2.1*10^6} = 5*10^{-4}$

Again, the sensitivity of the strain gauge(gauge factor) is Gf= $\frac{\Delta R/R}{\Delta L/L}$

$\frac{\Delta R}{R}$=G_f *ϵ = 2*5*10^{-4} = 10^{-3} = 0.1%

∴ the change in resistance is only 0.1%.
The result shows that the relatively high stress results in a resistance change of only 0.1% , a very small change indeed. Lower stress produces still lower changes in resistance which may not be perceptible at all. To overcome this difficulty, the gauge factor of strain gauge should be high which produce large changes in resistance when strained.

Example 2

A compressive force is applied to a structural member. The strain is 5 micro-strain. Two separate strain gauge are attached to the structural member, one is nickel wire strain gauge having a gauge factor of -12.1 and other is nichrome wire strain gauge having factor of 2. Calculate the value of resistance of the gauges after they are strained. The resistance of strain gauges before being strained is 120 **Ω**.

Solution:

According to our convention, the tensile strain is taken as positive while the compressive strain is taken as negative.

∴ strain, ϵ = -5*10^{-6} (1 micro strain = 1μm/m)

Now, G_f = $\frac{\Delta R/R}{\epsilon}$ = $\frac{\Delta R}{R}$ = G_f *ϵ

∴ Change in value of resistance of nickel wire strain gauge is $\Delta R = G_f \in * R$

= -12.1 × (-5×10^{-6}) ×120

= 7.26×10^{-3} Ω

=7.26 mΩ

i.e. there is an increase of 7.26 mΩ in the value of resistance.

For nichrome, the change in the value of resistance is $\Delta R = G_f \in * R$

$= 2\times(-5\times10^{-6})\times120$

$= -1.2\times10^{-3} = -1.2$ mΩ

Thus, with compressive strain, the value of resistance gauge shows a decrease of 1.2 mΩ in the value of resistance.

Example 3

A Strain gauge is bonded to a beam 0.1m long and has across-sectional area 4 cm^2. Young's modulus for steel is 207 GN/m^2. The strain gauge has an unstrained resistance of 240 Ω and a gauge factor of 2.2. When a load is applied, the resistance of gauge changed by 0.013 Ω. Calculate the change in length of the steel beam and the amount of force applied to the beam.

Solution:

We have, the gauge factor, $G_f = \frac{\Delta R/R}{\Delta L/L}$

$\therefore \Delta L = \frac{\Delta R/R}{G_f} \times L = \frac{0.013/240}{2.2}\times 0.1 = 2.46\times10^{-6}$m

Change in length of the steel beam is 2.46×10^{-6} m

Now, Hooke's law yields

$$\in = \frac{S}{E}$$

$$S = \in * E = \frac{\Delta L}{L} * E = \frac{2.46\times10^{-6}}{0.1}\times 2.7\times10^{9} = 5.092\times10^{6} \text{ N/m}^2$$

So, the amount of force applied to the beam is,

F= SA as S = F/A

$= (5.092\times10^{6})\times4\times10^{-4}$

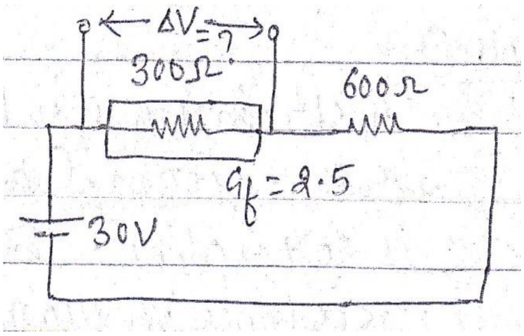
$= 2.037\times10^{3}$ N

Example 4

A strain gauge having a resistance of 300 Ω and gauge factor 2.5 is connected in series with a blast resistance of 600 Ω across 30V. Determine the change in output voltage when a stress of 140 MN/m^2 is applied. The modulus of elasticity is 200 GN/m^2.

Solution :

Hooke's law yields



$\epsilon = \frac{S}{E} = \frac{\Delta L}{L} = \frac{140\times10^{6}}{200\times10^{9}} = 7\times10^{-4}$

Again, gauge factor is,

$G_f = \frac{\Delta R/R}{\Delta L/L} = \frac{\Delta R/R}{\epsilon}$ or $\Delta R = G_f\, \epsilon * R$

$\therefore \Delta R = 2.5\times7\times10^{-4}\times300 = 0.525\Omega$

i.e there is an increase of 0.525 **Ω** in the value of resistance. When unstrained, the voltage across strain gauge is

$V_1 = \frac{30}{300+600} * 30 = 10$ Volts

When strained , the voltage across strain gauge is

$V_2 = [\frac{300+\Delta R}{(300+\Delta R)+600}]*30 \;= [\frac{300.525}{300.525+600}]*30$

$= 10.012$volts

Piezoelectric Transducers

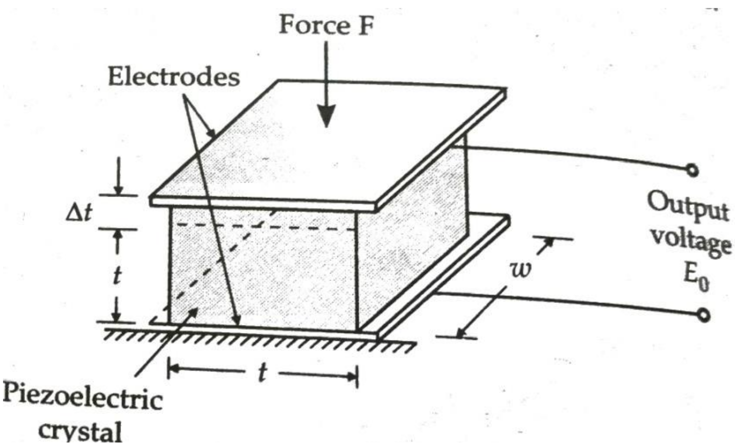


Fig. Piezoelectric Transducer

A piezoelectric material is one in which an electric potential appears across certain surfaces of crystal if the dimensions of the crystal are changed by the application of mechanical force. The potential is produced by the displacement of charges and the effect is reversible.

Common piezo-electric materials include Rochelle salts, quartz, Lithium sulphate. A piezo-electric element used for converting mechanical motion to electrical signals may be thought as charge generator and a capacitor. Mechanical deformation generates a charge and this charge appears as voltage (Q/C) across the electrodes. The magnitude and polarity of the induced surface charges are proportional to the magnitude and direction of the applied force F given by the expression,

Charge, Q = d*F Coulomb ……..(1)

where, d = charge sensitivity of the crystal, it is constant for a given crystal ; C/N

and F = applied force ; N

The force F causes a change in thickness of the crystal. So

$F=\frac{AE}{t} * \Delta t$ Newton ……………………..(2)

where, A = area of crystal ; m^2

t = thickness of the crystal ; m and

E= Young's modulus; N/m^2

$\epsilon=\frac{stress}{strain}=\frac{\frac{F}{A}}{\frac{\Delta t}{t}}=\frac{Ft}{\Delta tA}$ N/m^2

where A=wl

w= width of crystal; m

and l = length of crystal ; m

From equation (1) and (2) , we have , charge

$Q = dAE\left(\frac{\Delta t}{t}\right)$ ………………….(3)

The charge at the electrodes give rise to an output voltage E_O as

$E_O = Q/C_P$ ………………..(4)

where, $C_P = \epsilon_r\epsilon_O A/t$ ……………(5)

C_P = capacitance between electrodes; F

A = area of crystal; m^2

$\epsilon=\epsilon_r\epsilon_O$ = permittivity of medium ; F/m

ϵ_r = relative permittivity

ϵ_O = permittivity of free space; $8.85*10^{-12}$ F/m

Solving equation (1), (4) and (5), we get

$E_O = \frac{Q}{Cp} = \frac{dF}{\epsilon r \epsilon o \frac{A}{t}} = \frac{dt}{\epsilon r \epsilon o} (\frac{F}{A})$

But , F/A= P =pressure or stress in N/m^2

$\therefore E_O = \frac{d}{\epsilon r \epsilon o} tp = gtp$

where,

g = d/ϵrϵo is the voltage sensitivity of crystal which is constant for a given crystal; Vm/N

Now, $g = \frac{Eo}{tp} = \frac{Eo/t}{P}$

But, Eo/t = electric field strength; V/m. hence, crystal voltage sensitivity g is defined as the ratio of electric field intensity to pressure or stress.

Uses of piezo-electric Transducer

The use of piezo-electric transducer elements is confined primarily to dynamic measurements. The voltage developed by application of strain is not held under static conditions. Hence, the elements are primarily used in the measurement of such quantities as surface roughness and in accelerometers and vibration pickups.

Example

A Barium Titanate pickup (piezo-electric crystal) has the dimensions of 5mm*5mm*1.25mm. The force acting on it is 5N. The charge sensitivity of Barium Titanate is 150 pC/N and its permittivity is $12.5*10^{-9}$ F/m. If the modulus of elasticity of Barium Titanate is $12*10^6$ N/m^2, Calculate the strain, the charge and the capacitance.

Solution:

Area of plates(A) = $5*10^{-3}*5*10^{-3} = 25*10^{-6}$ m^2

Pressure(P) = F/A = $\frac{5}{25*10^6} = 0.2*10^6 N/m^2$

Now, Voltage sensitivity (g) = $\frac{d}{\epsilon r \epsilon o} = \frac{150*10^{-12}}{(12.5*10^{-9})} = 0.012\ Vm/N$

Voltage generated(E_o) = $\frac{Stress(s)}{Young's\ modulus(E)} = \frac{\frac{F}{A}}{E} = \frac{\Delta t}{t}$

$= \frac{0.2*10^6}{12*10^6} = 0.0167$

Charge(Q) = dF = $150*10^{-12}*5 = 750*10^{-12}$ C

and Capacitance(C_p) = $\frac{Q}{Eo} = \frac{750*10^{-12}}{3} = 250*10^{-12}F$

Merits and demerits of Piezo-electric transducers

Merits:

- They are generally small in size, light weight and very rugged in construction.
- They are active transducers as they don't need external supply.
- They have very good frequency response and output is quite large.
- They can be operated over a wide range of temperature without appreciable temperature induced.

Demerits:

- The output of piezo-electric transducer is affected by the temperature variation of the crystal.
- There is a problem of leakage of charges through the surface of crystal.

Capacitance Transducers

The principle of operation of capacitive transducer is based upon the familiar equation for capacitance of a parallel plate capacitor i.e

$C = \epsilon A/d = \epsilon_r \epsilon_o A/d$

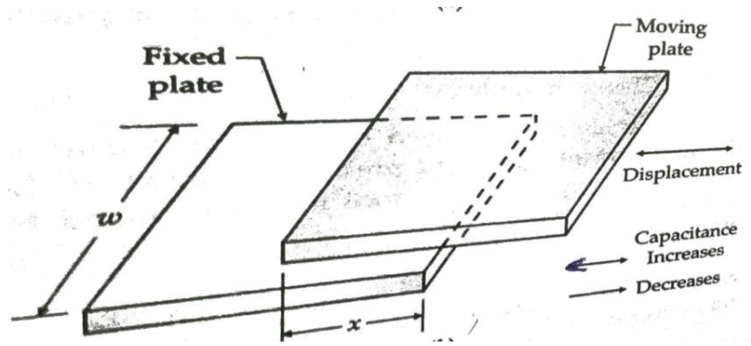
where,

C= capacitance of a parallel plate capacitor; F
A= overlapping area of plates ; m^2
D= distance between two plates; m
$\epsilon = \epsilon_r \epsilon_o$ = permittivity of medium; F/m
ϵ_r= relative permittivity
ϵ_o = permittivity of free space; $8.85*10^{-12}$F/m
A capacitance transducer works on the principle of change of capacitance which may be used by

i) Change in overlapping area 'A'
ii) Change in distance d between the plates and
iii) Change in dielectric constant

These changes are caused by physical variables like displacement, force and pressure. The capacitive transducers are commonly used for measurement of linear displacement whose output impedance is high ($Xc = \frac{1}{2\pi fc}$)

i) Measurement of displacement by varying the overlapping area of the plates:
For a parallel plate capacitor,



the capacitance is C $=\frac{\epsilon A}{d}=\frac{\epsilon lw}{d}$

where,

l =length of overlapping part of plates; m

w = width of overlapping part of plates; m

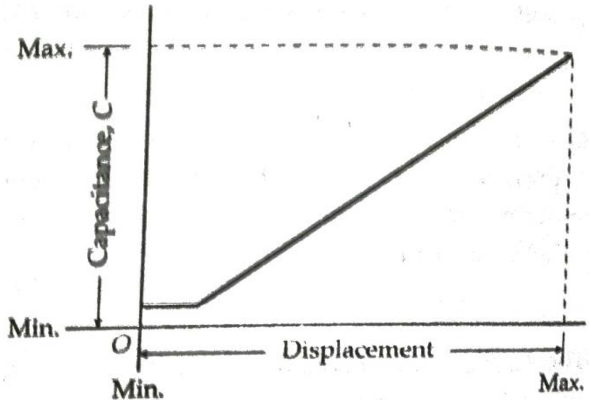
$\epsilon = \epsilon_r\epsilon_o$ = permittivity of medium; F/m

d = distance between two plates; m

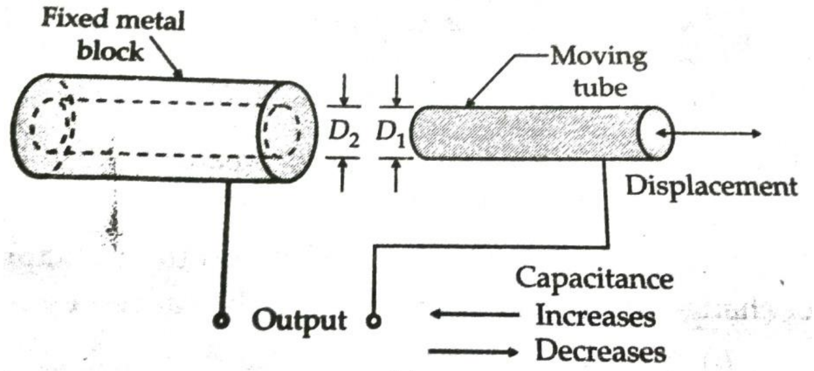
and sensitivity (S) = $\frac{\partial c}{\partial l}=\frac{\epsilon w}{d}\ F/m$

$\therefore$S= $\epsilon_r\epsilon_o\frac{w}{d}$= constant

i.e. there is linear relationship between capacitance and displacement. But , near l=0 due to edge effect, there is certain capacitance developed.



For cylindrical capacitor,



The capacitance is

$$C = \frac{2\pi\epsilon l}{log_e\left(\frac{D2}{D1}\right)} \text{ F}$$

where , l = length of overlapping part of cylinders; m

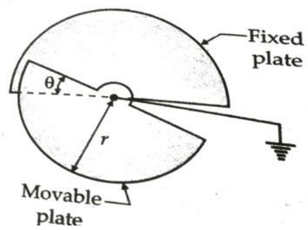
D_2 = inner diameter of outer cylindrical electrode; m

D1 = outer diameter of inner cylindrical electrode; m

Now,

Sensitivity(S) = $\frac{\partial C}{\partial l} = \frac{2\pi\epsilon}{log_e\left(\frac{D2}{D1}\right))} \; F/m$ is constant and the relationship between capacitance and displacement is linear.

The principle of change of capacitance with the change in overlapping area can be employed for measurement of angular displacement.



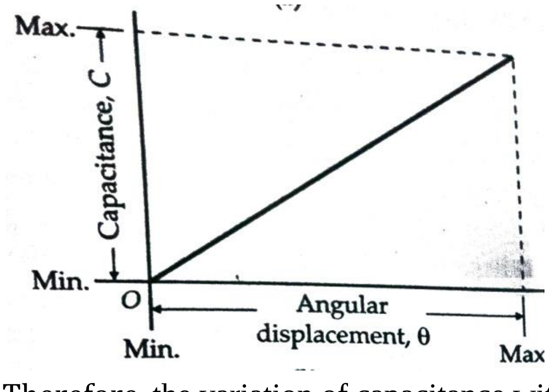
The capacitance is maximum when two plates completely overlap each other i.e. when θ=180° .

$\therefore C_{max} = \frac{\epsilon A}{d} = \frac{\pi\epsilon r^2}{2d}$

At any angle θ, $C_\theta = \frac{\epsilon\theta r^2}{2d}$

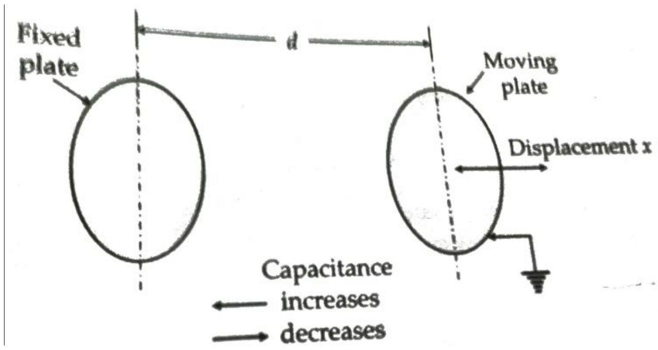
where, θ = angular displacement in radian

and Sensitivity (S) = $\frac{\partial C_\theta}{\partial\theta} = \frac{\epsilon r^2}{2d}$, so the sensitivity of the device is constant



Therefore, the variation of capacitance with angular displacement is linear.

ii) Measurement of displacement by varying the distance between two plates

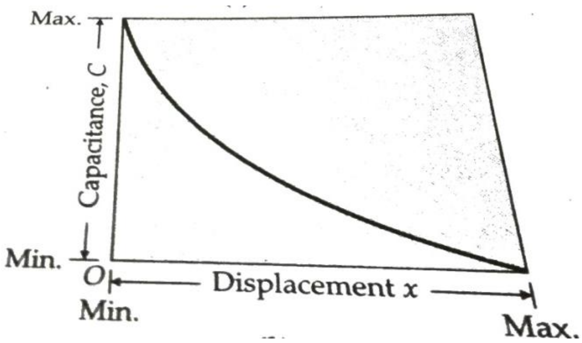


Since , the capacitance(C) varies inversely as the distance (d) between the plates, the response of this transducer is not linear. Thus, it is useful only for measurement of extremely small displacement.

We have,

$C = \frac{\in A}{d}$ and

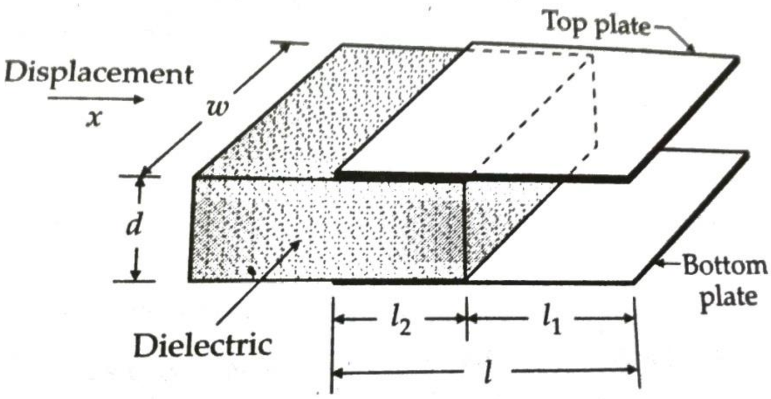
$sensitivity(S) = \frac{\partial C}{\partial d} = -\in \frac{A}{d^2}$



To increase the sensitivity or linearity

1) Area can be increased
2) Distance between the plates(d) can be decreased, but there is practical limit for decreasing (d) i.e. when the electric field strength in the gap exceeds the breakdown voltage of 3KV/mm

iii) Measurement of displacement by varying the dielectric constant



Initial capacitance of transducer is

$$C = C1 + C2 = \frac{\epsilon_o\ wl_1}{d} + \frac{\epsilon_o \epsilon_r\ wl_2}{d}$$
$$= \frac{\epsilon_o\ w}{d}(l_1 + \epsilon_r\ l_2) \ldots\ldots\ldots\ldots..(1)$$

Let the dielectric be moved through a distance of 'x' in the right direction. The capacitance changes from C to C+ΔC.

$$C+\Delta C = \frac{\epsilon ow}{d}[(l_1 - x) + \epsilon_r\ (l_2 + x)]$$
$$= \frac{\epsilon ow}{d}(l_1 + \epsilon_r\ l_2) + \frac{\epsilon owx}{d}(\epsilon_r - 1)$$
$$= C + \frac{\epsilon owx}{d}(\epsilon_r - 1)\ [\,from\ equation\ (1)]$$

Hence, $\Delta C = \frac{\epsilon_o wx}{d}(\epsilon_r - 1)$ i.e. the change in Capacitance (Δc) ∝ displacement(x) and

Sensitivity(S) = $\frac{\partial C}{\partial x} = \frac{\epsilon_o w(\epsilon r - 1)}{d}$

Advantages of Capacitive Transducers:

i. They require extremely small force to operate them all and hence are very useful for use in small system.
ii. They are extremely sensitive and have a good frequency response.
iii. They have a high input impedance and therefore the loading effects are minimum.
iv. They require small power to operate as the force requirements is very small
v.

Disadvantages of Capacitive Transducers:

i. The metallic parts of the capacitive transducers must be insulated from each other. In order to reduce the effects of stray capacitances, the frames must be earthed.
ii. The capacitive transducers show non-linear behaviour on account of edge effects. Therefore, guard rings must be used to eliminate this effect.
iii. The output impedance of capacitive transducer tends to be on high on account of their small capacitance value. This leads to loading effects.
iv. The cable connecting the transducer to the measuring point is also a source of error as the cable may be source of loading resulting loss of the sensitivity.

Applications of capacitive transducers

i. It can be used for measurement of both linear and angular displacement, for the measurement of force and pressure and for measurement of humidity in gases, since the dielectric constant of gases changes with change in humidity thereby producing a change in capacitance.
ii. It can be used directly as pressure transducers in all those cases where dielectric constant of a medium changes with pressure.
iii. They are commonly used in conjunction with mechanical modifiers for measurement of volume, density, liquid level, weight etc.

Inductive transducers

Inductive transducers measure the displacement

i. by variation of self-inductance
ii. by variation of mutual inductance and
iii. by production of eddy current.

i) By variation of self-inductance

The self inductance of a coil is given by

$L=\frac{N^2}{R} \ldots \ldots \ldots \ldots \ldots (1)$

Where, N= number of turns and

R = reluctance of the magnetic circuit = $\frac{l}{\mu A}$

Thus, equation (1) becomes

$L = \frac{N^2}{\frac{1}{\mu A}} = N^2\mu\left(\frac{A}{l}\right) = N^2\mu G \ldots \ldots \ldots \ldots \ldots .. (2)$

Where, μ = effective permeability of the medium in and around the coil; H/m ($\mu=\mu_O\mu_r$)

G =Δ l / l = geometric form factor

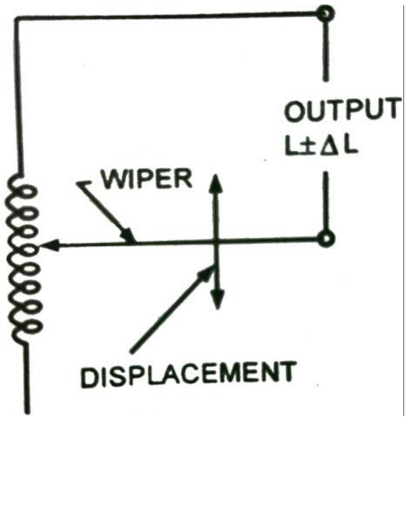
A= area of cross-section; m^2

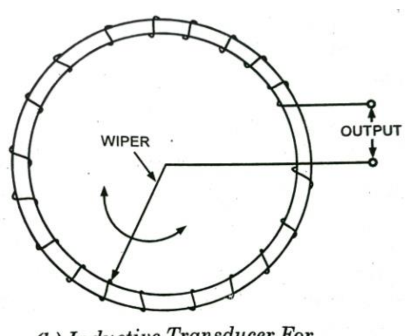
L =length of coil; m

It is cleared from equation (2) that the variation in induction may be caused by :

a. Change in number of turns(N)
b. Change in geometric configurations (G) and
c. Change in permeability (μ)

a. Change in number of turns

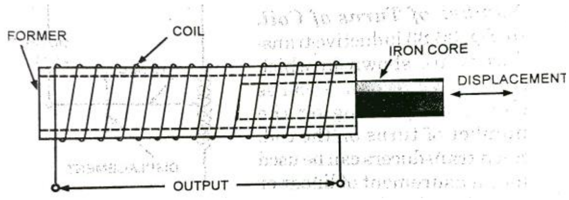




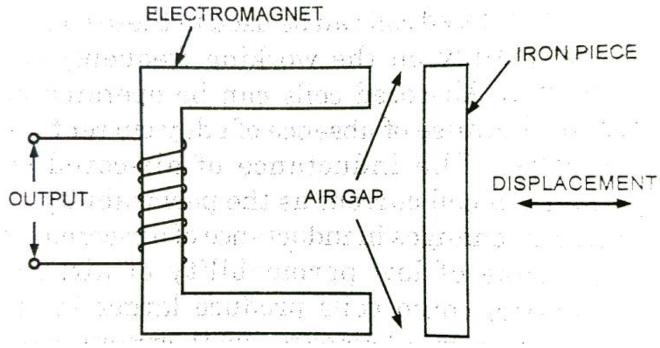
(b) *Inductive Transducer For Measurement of Angular Displacement*

Here, the variation in induction is due to the change in the number of turns of the coil. Such transducers are used for the measurement of linear as well as angular displacement.

b. Change in permeability of the medium
Here, for x=0 i.e when there is no displacement
$\mu = l$ /RA. But when there is the displacement (x) there is a change in permeability of the medium and hence inductance varies i.e
$\mu_x = \frac{(l-x)}{RA}$ and $L_x = N^2\mu_x G$



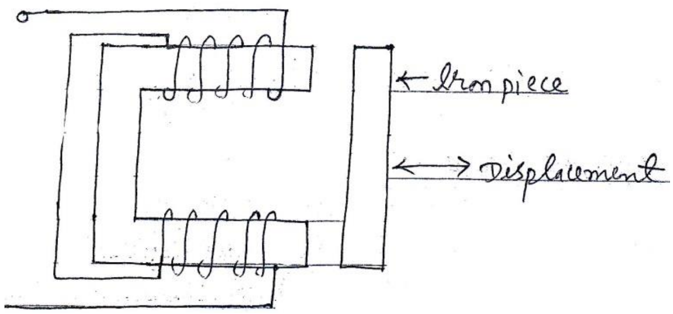
c. Change in geometric configurations or reluctance of magnetic circuit.



Here, the displacement of iron piece changes the value of reluctances R_2 and R_4 so that the net reluctances of magnetic circuit gets change. Thus, the inductance varies with respect to the change of reluctance as $L = N^2/R$

ii) By variation of mutual inductance

An inductive working on the principle of variation of mutual inductance uses multiple coils. The mutual inductance between two coils is given as $m = K\sqrt{L1L2}$



Where, L_1 and L_2 = self-inductances of two coils
K = Co-efficient of coupling

Thus, the mutual inductance between the coils can be varied by variation of self-inductances or the co-efficient of coupling. However, the mutual inductance can be converted into a self-inductance by connecting the coils in series.

Linear Variable Differential Transformer (LVDT)

It is most widely used inductive transducer to translate the linear motion into electrical signals.

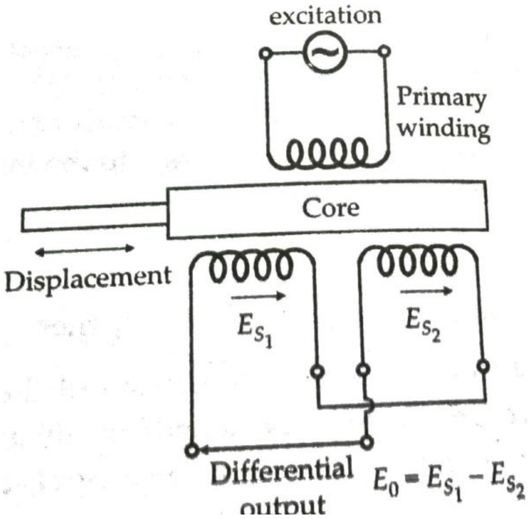


Fig. Circuit of an LVDT

LVDT consists of a single primary winding (P) and two secondary winding (S_1 and S_2) wound on a cylindrical former. The secondary windings have equal number of turns and are identically placed on either side of the primary winding which is connected to an a.c source. A movable soft iron core is placed inside the former. The displacement to be measured is applied to the arm attached to the soft iron core.

Since the primary winding is excited by an a.c source, it produces an alternating magnetic field which in turn induces alternating current voltages in the two secondary windings.

The output voltage of secondary S_1 is Es_1 and that of secondary S_2 and is Es_2. In order to convert the outputs from S_1 and `S_2 into single voltage signal, the two secondaries S_1 and S_2 are connected in series opposition as shown and thus, the output voltage of the transducer is the difference of the two voltages.

∴ Differential output voltage (E_O)= $Es_1 – Es_2$ …………(1)

When the core is at its null position, the flux linking with both the secondary windings is equal and hence equal emfs are induced in them.

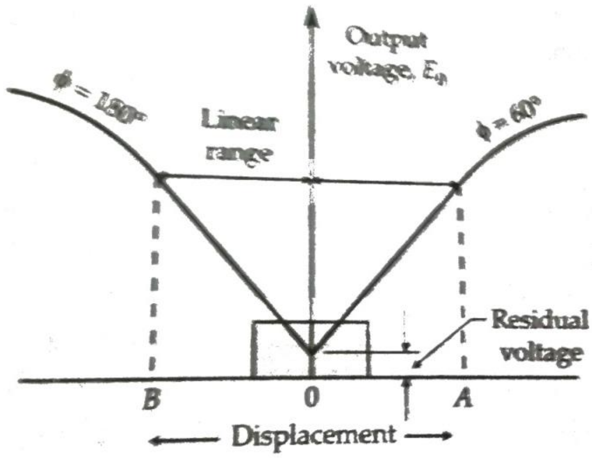
Thus at null position,

$Es_1=Es_2$ and equation (1) becomes $E_O = 0$

Now, if the core is moved to the left of the null position, more flux links with winding S_1 than with winding S_2i.e $Es_1> Es_2$ So equation (1) becomes $E_O>0$ and positive. This implies that the output voltage is in phase with primary voltage. Again, if the core is moved to the right of the null position, the flux linking with winding S_2 becomes larger than that linking with winding S_1. Thus, $Es_2>Es_1$ and equation(1) becomes $E_O<0$ and negative. This implies that the output voltage is 180° out of phase with the primary voltage.

By comparing the magnitude and phase of the differential output voltage with that of the source, the amount and direction of the movement of the core (displacement) may be determined.

The output voltage of the LVDT is a linear function of core displacement within a limited range of motion.



Advantages of LVDT

i. Have high range for measurement of displacement
ii. No friction and electrical isolation gives truly infinite resolution
iii. Immunity from external effects
iv. High input and high sensitivity
v. can tolerate high degree of shock and vibrations (Ruggedness)
vi. Low Hysteresis
vii. Low power consumption.

Disadvantages of LVDT

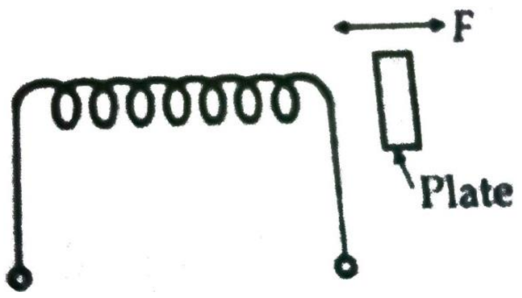
i. are sensitive to stray magnetic fields but shielding is possible
ii. The dynamic response is limited mechanically by the mass of the core and electrically by the frequency of applied voltage.
iii. Temperature affects the performance of the transducer
iv. The receiving instrument must be selected to operate on a.c. signals or a demodulator network must be used if a d.c output is required.

Applications of LVDT

Some of the major applications of LVDT are:

i. can be used in all applications where displacement ranging from fraction of a mm to a few cm have to be measured.
ii. Acting as a secondary transducer, it can be used as a device to measure force, weight and pressure etc.

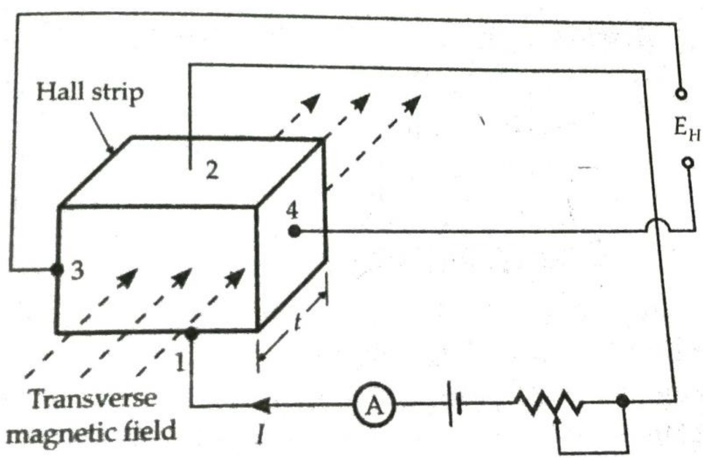
iii) By production of eddy current



When a conducting plate is placed near a coil carrying alternating current, the eddy current are induced in the conducting plate producing its own magnetic field which acts against the magnetic field produced by the coil. This results in reduction of flux and hence the inductance of the coil decreases. The nearer is the plate to the coil, the higher are the eddy currents and thus higher is the reduction in the induction of the coil. Thus, the inductance of the coil alters with variation of distance between the plate and the coil.

Voltage and current Transducer (Hall-effect Transducer):

The working principle of a Hall Effect Transducer is that if a strip of conducting material carries a current in the presence of a transverse magnetic field, a difference of potential is produced between the opposite edges of the conductor.



The magnitude of the voltage depends upon the current, the strength of magnetic field and the property of the conductor called hall Effect. The output voltage (E_H) is

$E_H = K_H IB/t$

where K_H = Hall Effect Coefficient ;$\frac{V-m}{A-wbm^{-2}}$

I = current; A

B= flux density ;wb/m^2

t=thickness of strip; m

Thus the voltage produced may be used for measurement of either current (I) or the magnetic field strength (B). Hall Effect emf is very small in conductors and is difficult to measure. So, Hall Effect is mostly pronounced in semiconductor than in conductor.

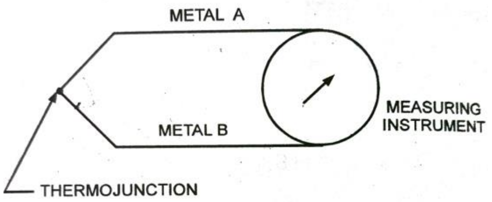
Application of Hall-Effect Transducer

i. Measurement of displacement, current and power
ii. Can be used as magnetic to Electric transducer

Measurement of Temperature

1. **Thermocouple**

A thermocouple is formed when two dissimilar metallic conductor are joined together to form two junctions. The operation of thermocouple is based on Seebeck effect i.e if a closed circuit is formed of two metal junctions keeping them at different temperature, an electric current flows in the circuit . The magnitude of the electric current is caused by an emf called thermo emf setup in the circuit and is proportional to the temperature difference between two junctions (hot junction whose temperature to be measured and cold or reference junction which is maintained at constant temperature $0°C$ normally).
For example, if two metals copper and iron forms thermocouple, then the current flows from copper to iron at hot junction and iron to copper at cold junction.



Now, Induced emf is given by,

$E=a(T_2-T_1)+b(T_2-T_1)^2$

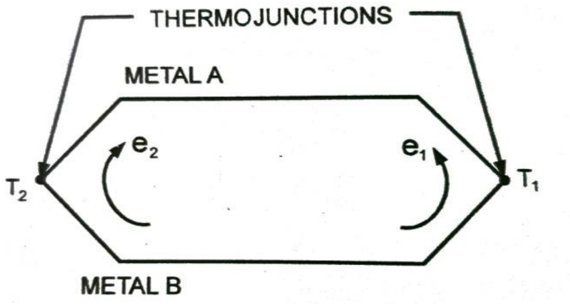
$= a(\Delta T)+b(\Delta T)^2$ ………………..(1)

where
T_1 = temperature of cold thermocouple junction; $°C$
T_2 = temperature of hot thermocouple junction; $°C$
and a and b are constants.
Since a>>>b, b can be neglected . So equation(1) becomes

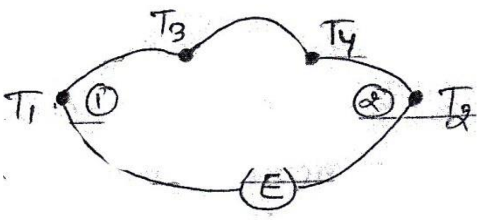
$$E = a(T_2 - T_1) = a(\Delta T)$$
$$\text{or, } \Delta T = E/a \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Laws of thermocouple

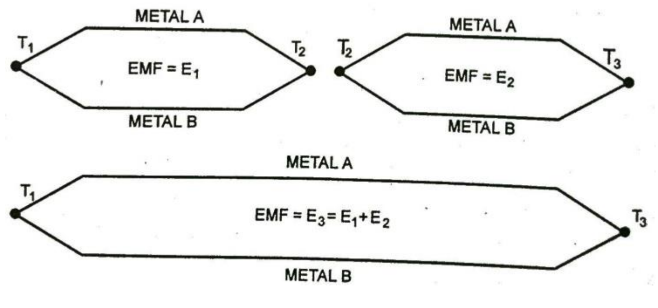
The emf generated depends on the temperature difference between junction (1)and junction (2) and independent of temperature of conductor A and conductor B.



It the third element is joined in series with the conductor having a junction temperature T_3 and T_4 then thermal emf will be constant if T_3=T_4.

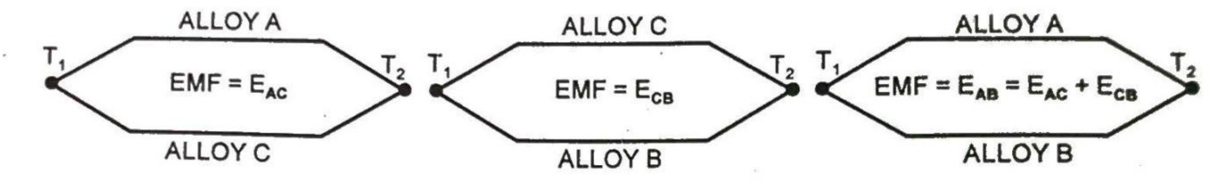


Law of intermediate temperature



If two dissimilar metals produce thermoelectric emf E_1 when junctions are at T_1 and T_2 and emf $E_2$when junctions are at T_2 and T_3, then emf generated with junction at temperature T_1 and T_3 will be E= E_1 + E_2

Laws of intermediate metals



A third element can be added to a circuit with no effect on the net emf as long as the extremities are at same temperature.

Types of thermocouple

Types	Material used	Temp(℃) range	Sensitivity μV/℃
E	Chromel / Constantan	0 -500	68
J	Iron/Constantan	-40-750	52
T	Copper/Constantan	-200-350	43
K	Chromel/Alumel	-200-1200	41
S	Platinum+10%Rhudium/Platinum	30-1600	10

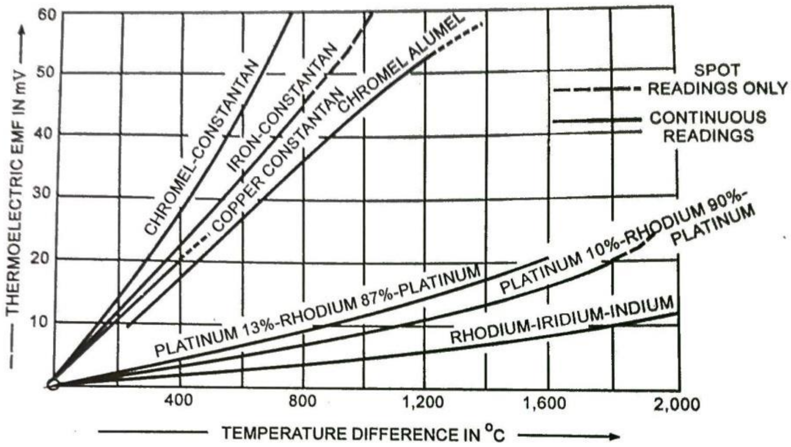


Fig. Temperature emf curves for thermocouple with reference junction at 0 °C

Thermopile

When the number of thermocouples are connected in series the sensitivity of the instrument gets increased which is known as thermopile.

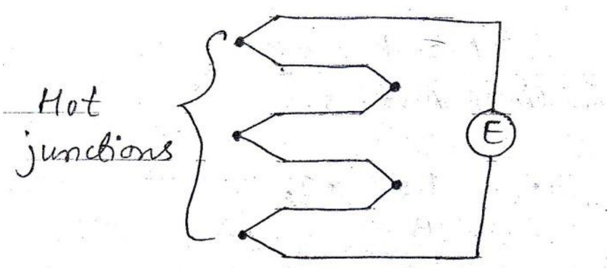


Fig. Thermopile.

Advantages of thermocouple

- Thermocouple are cheaper than the resistance thermometers
- Thermocouples follow the temperature changes with a small time lag and such are suitable for recording comparatively rapid changes in temperature.

Disadvantages of Thermocouple

- They have lower accuracy and hence cannot be used for precision work
- To ensure long life of thermocouples in their operating environments they should be protected in an open or closed end metal protecting tube or well
- The thermocoples are placed remote from measuring devices and hence the circuitry is very complex and compensating leads are needed.
- Reference junction compensation is required in thermocouple.

Application of thermocouple

Thermocouple's particular application is the measurement of surface temperature.

Note: thermocouple is an active transducer whereas thermistors and RTD are passive transducer.

2. Thermistors (Thermally Sensitive Resistors)

Thermistors (thermal resistors) area semiconductor device that behaves as resistors with high, usually negative temperature co-efficient of resistance. Its usual range is -100℃ to 300$℃$. Thermistors are composed of mixtures of metallic oxide such as manganese, nickel, Cobalt , Copper etc. Thermistors are commercially available in wide varieties of shape and size. The following are the important characteristics of thermistors.

a. Resistance Temperature Characteristics

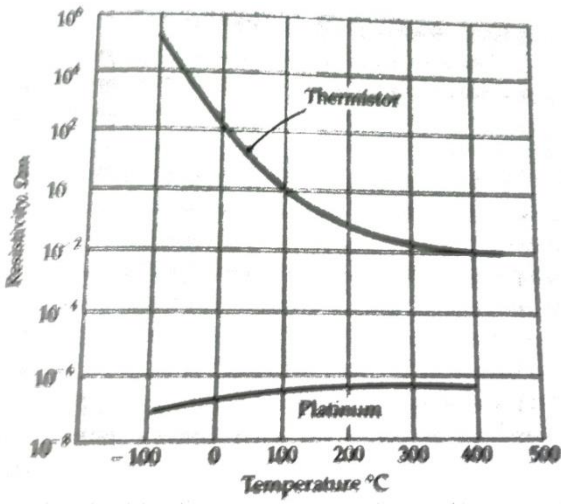
The mathematical expression for the relationship between the resistance of the thermistor and absolute temperature of the thermistor is

$$R_T = R_O e^{\beta(1/T - 1/To)}$$

where R_T = resistance of the thermistor at absolute temperature T°K

R_o = resistance of the thermistor at absolute temperature T_0°K which is the reference absolute temperature 298°K and

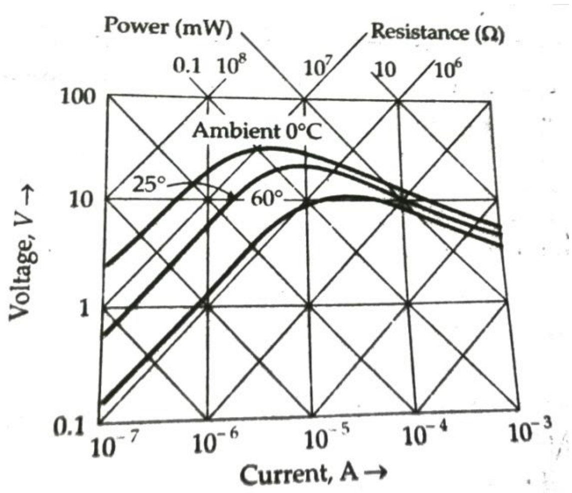
β = a constant depending upon the material of thermistor (typically 3500 to 4500°K)



In the range of -100 to 400℃ , the thermistor changes its resistance by factor of 10^7 i.e. from 10^5 to 10^{-2}. In the same range, Platinum changes by the factor of 10 i.e. from 10^{-7} and 10^{-6}. Hence, it concludes that thermistor is highly sensitive device.

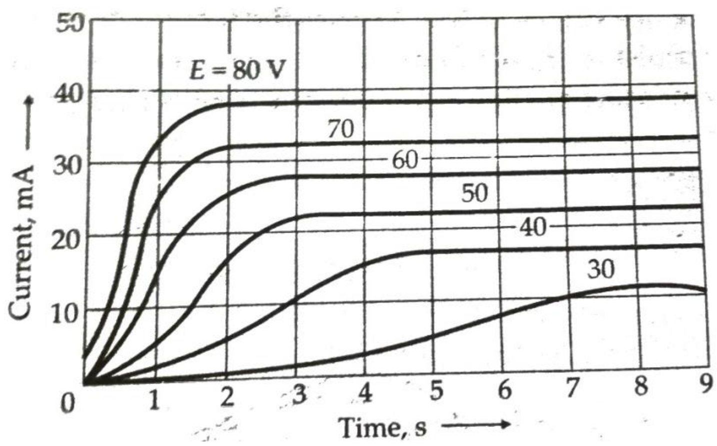
b. Voltage current Characteristics

The voltage drop across the thermistor increases with increase current until it reaches the peak value beyond which the voltage drop decreases with increase in current. This portion of curve exhibits the negative resistance characteristics of thermistors.



c. Current Time characteristics

This indicates that there is a time delay to reach the maximum current as a function of applied voltage.



Application of Thermistors

1. Measurement of temperature
2. Control of temperature
3. Temperature compensation
4. Measurement of power at high frequencies and so on

3. Resistance Temperature Detector (RTD)

RTD operates upon the fact that almost all pure metals have the property of varying their resistance with temperature and the change in resistance is almost directly proportional to the temperature. The range of temperature over which this transducer valid is decided by the co-efficient of resistance, chemical composition and its crystal structure which should not undergo permanent changes within this range.

Electrical resistance with temperature for most metallic material can be represented by an expression

$R_T = R_o(1+\alpha_1 T+ \alpha_2 T^2 + \ldots\ldots\ldots + \alpha_n T^n)$ …………….(2)

where, R_o = Resistance at reference temperature (usually 0° C)

$\alpha_1, \alpha_2, \ldots\ldots, \alpha_n$ are the constants called co-efficients of material being used.

T = Temperature; ℃ and

R_T = Resistance at temperature T℃

Mostly used material is platinum, so it is also called PRTD(Platinum RTD). Platinum is used because it has got linear characteristics and good chemical inertness.

For platinum RTD working in linear range , equation(1) becomes

$R_T = R_o(1+ \alpha T)$ ………………………(2)

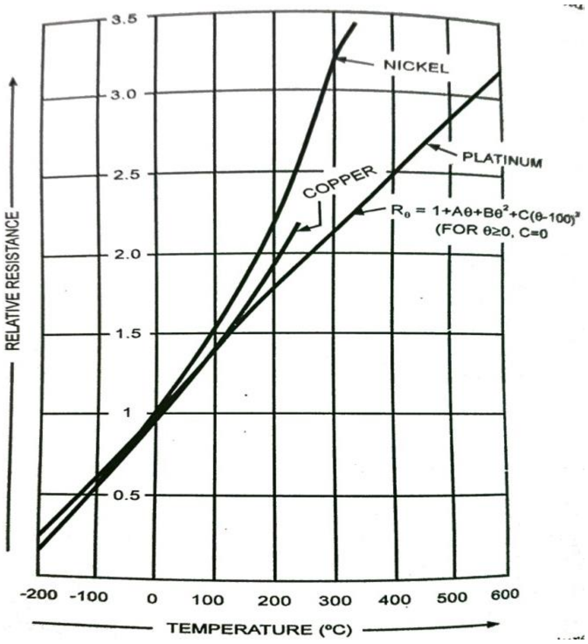
At temperature 0℃ with resistance R_o and 100℃ with resistance R_{100} , equation (2) becomes

$R_{100} = R_o(1+ \alpha *100)$

$\therefore\ \alpha = \frac{(R_{100}-R_0)}{100R_0}$ ……………..(3) thus the value of α is determined.

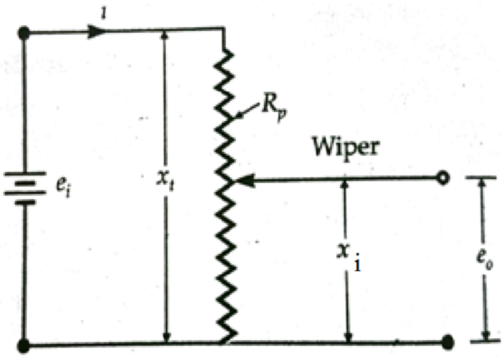
The change in resistance with the change in temperature can be observed by using Wheat Stone Bridge circuit.

The performance of RTD for various metal is shown below:



Resistive Transducer (Potentiometers(POT))

It is the simplified form of passive transducer widely used. It caonverts linear or agnular displacement into output voltage.



Let, e_i and e_o = input and output voltages respectively; V

x_t = total length of POT; m

x_i = displacement of wiper from its zero position; m and

R_p = total resistance of POT; **Ω**

We know,

$R_p = \rho\left(\frac{x_t}{A}\right)$ R=ρ($\frac{l}{A}$) …………(1) and

R_i = ρ($\frac{x_i}{A}$) ………………….(2)

where, R_i = resistance due to wiper displacement (x_i); **Ω**

From equation (1) and (2) , we get

$\frac{R_p}{x_t} = \frac{R_i}{x_i}$

$R_i = R_p\left(\frac{x_i}{x_t}\right)$ ……………(3)

The output voltage under ideal condition is,

$e_o = \left(\frac{Resistance\ at\ the\ output\ terminal}{Resistance\ at\ the\ input\ terminal}\right) *$ input voltage

$= \frac{Rp\left(\frac{x_i}{x_t}\right)}{Rp} * e_i$

$= \left(\frac{x_i}{x_t}\right) e_i$

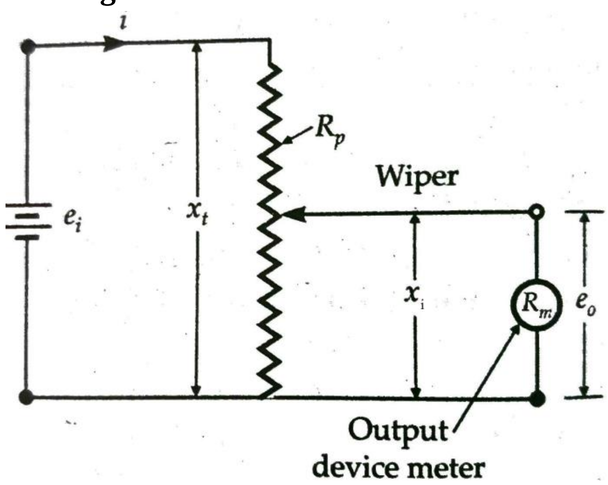
Thus, under ideal circumstances, the output voltage varies linearly with displacement as

and Sensitivity (S) = $\frac{output}{input} = \frac{e_o}{x_i} = \frac{e_i}{x_t}$

Thus , under ideal conditions, the sensitivity is constant and the output is faithfully reproduced and has a linear relationship with input. The same is true of rotational motion.

Let θ_i = input angular displacement in degrees and θ_t = total travel of the wiper in degrees. Then, the ouput voltage is $e_o = \left(\frac{\theta i}{\theta t}\right) * e_i$ which is true of single turn potentiometer only.

Loading effect



Here, R_m is the internal resistance of measuring device/output device.

Now, $R_i = R_p\left(\frac{x_i}{x_t}\right)$ from equation (3)

$= KR_p$

$\frac{x_i}{x_t} = K$ and $R2 = R_p - R_i = R_p - KR_p = (1-K)R_p$

The resitance of the parallel combination of load resistance and the portion of the resistance of the POT is

$R_{eq} = R_m || R_i = R_m || KR_p$

The total resistance seen by the source is

$R_T = R_{eq} + R_2 = R_{eq} + (1\text{-}K)R_p = \frac{R_m.KR_p}{R_m+KR_p} + (1-K)R_p$

$$= \frac{R_m.KR_p+(1-K)(R_m+KR_p)R_p}{R_m+KR_p}$$

$$= \frac{R_m.KR_p+(R_mR_p+-KR_mR_p-K^2R_p^2)}{R_m+KR_p}$$

$$= \frac{KR_p^2(1-K)+R_pR_m}{R_m} + KR_p$$

The equivalent current, i is

$$i = \frac{e_i}{R_T} = \frac{e_i(R_m+KR_p)}{KR_p^2(1-K)+R_pR_m}$$

The output voltage under load condition is

$$E_o = i*R_{eq} = \frac{\left(e_i(R_m+KR_p)\right)}{[KR_p^2(1-K)+R_mR_p]} * \left(\frac{R_m.KR_p}{R_m+KR_p}\right)$$

$$= \frac{eiKRpRm}{KR_p^2(1-K)+RmRp}$$

$$= \left(\frac{e_iK\left(\frac{Rm}{Rp}\right)}{K(1-K)+\left(\frac{Rm}{Rp}\right)}\right)$$ dividing by R_p^2 on numerator and denominator

Now, the ratio of output voltage to input voltage under load condition is

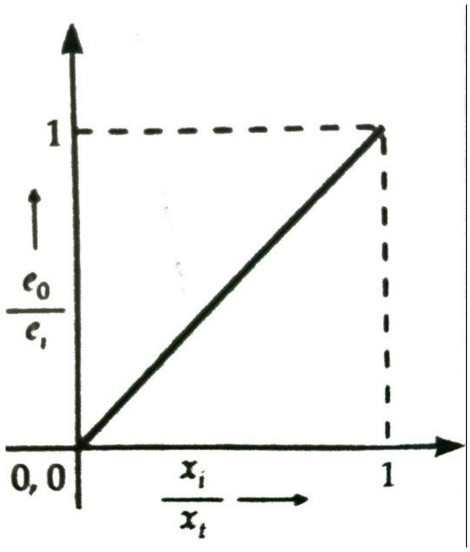
$$\frac{e_0}{e_i} = \frac{K\left(\frac{Rm}{Rp}\right)}{K(1-K)+\left(\frac{Rm}{Rp}\right)} \text{ ....(4)}$$

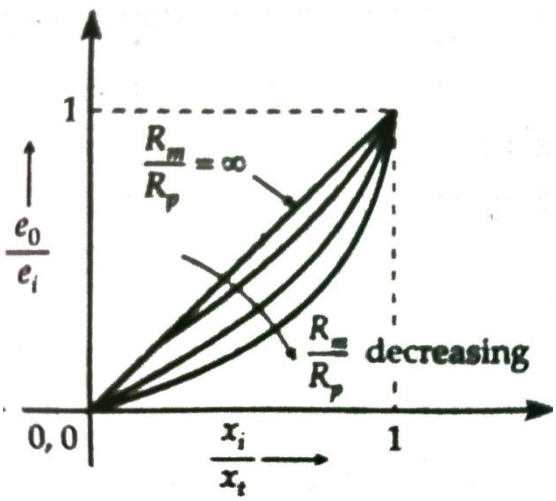
$$\therefore \frac{eo}{ei} = \frac{K\alpha}{K(1-K)+\alpha} \; where, \alpha = \frac{R_m}{R_p}$$

Equation (4) can be written as

$$\frac{e_0}{e_i} = \frac{K}{K(1-K)\left(\frac{Rp}{Rm}\right)+1} \text{ ....(5)}$$

Equation (5) shows that there exists a non-linear relationship between output voltage $e_0$and input displacement x_i since K = $\frac{x_i}{x_t}$. In case R$_m$ = ∞, $\frac{e_0}{e_i}$ = K





The ratio of $\frac{R_m}{R_p}$ decreasing means the non-linearity goes on increasing. Thus, in order to keep linearity, the value of $\frac{R_m}{R_p}$ should be as large as possible.

Again, Error = output voltage under load - output voltage under no load

$$= \frac{e_i K}{\left[K(1-K)\left(\frac{R_p}{R_m}\right)+1\right]} - e_i K$$

Absolute error $|\varepsilon| = \frac{e_0}{e_i}\left|loaded - \frac{e_0}{e_i}\right| unloaded$

$$= \frac{K\alpha}{(K(1-K)+\alpha)} - K$$

$$= \frac{K\alpha - K^2(1-K) - K\alpha}{K(1-K)+\alpha}$$

$$= \frac{-K^2(1-K)}{K(1-K)+\alpha} \; where \; \alpha = \left(\frac{R_m}{R_p}\right)$$

except for the two end points where K=0 i.e x_i=0 and K = 1 i.e $x_i = x_t$, the error is always negative.

The maximum error is about 12% of full scale if Rm/Rp = 1 and drops down to about 1.5% when $\frac{R_m}{R_p}$= 10. For the values of $\frac{R_m}{R_p}$>10, the position of aximum error occurs in the vicinity of $\frac{x_i}{x_t}$ = 0.67.

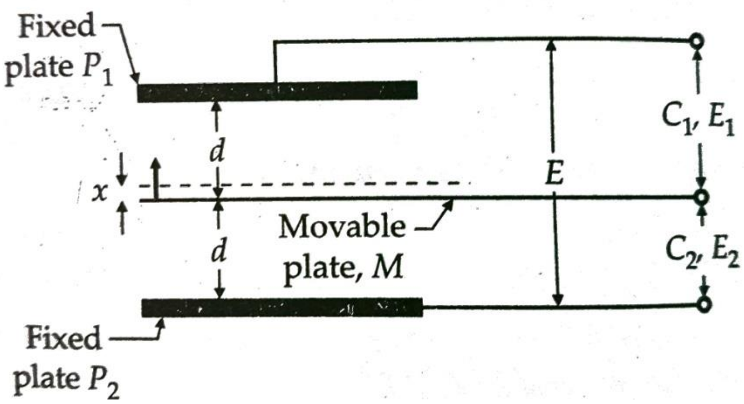
Maximum percentage error

% $E_{max} = 15\left(\frac{R_p}{R_m}\right)$ %

Differential arrangement of parallel plate Capacitor

"Differential arrangement of parallel plate capacitor eliminating the non-linearity between input and output". Prove this statement

Solution



Here, P_1 and P_2 are fixed plates and M is the movable plate which is midway between the two fixed plates having capacitance C_1 of plate P_1 and C_2 of fixed plate P_2 .Under this condition,

$d_1 = d_2 = d$ and

$C_1 = C_2 = \in \frac{A}{d} = \in_o \in_r \frac{A}{d}$

An alternating current voltage 'e' is applied across plates P_1 and P_2 and the difference of the voltages across the two capacitances is measured i.e

Voltage across C_1 is

$$e_1 = \left(\frac{C_2}{C_1+C_2} * e\right) = \frac{e}{2} \text{ and}$$

Voltage across C_2 is

$$e_2 = \frac{C_1}{C_1+C_2} * e = \frac{e}{2}$$

Therefore, Differential output when the movable plate is at center is $\Delta e_o = e_2 - e_1 = 0$

Let , the movable plate be moved up due to displacement x. Then d_1 = d-x and d_2=d+x. Therefore, the values C_1 and C_2 become different resulting in a differential voltage output.

Now,

$C_1 = \frac{\in A}{d-x}$ and $C_2 = \frac{\in A}{d+x}$

Therefore , $e_1 = \frac{C_2 e}{C_1+C_2} = \frac{\frac{\epsilon A}{d+x}}{\frac{\epsilon A}{d-x}+\frac{\epsilon A}{d+x}} * e$

$$= \left(\frac{\in Ae(d-x)(d+x)}{(\in A)(d+x+d-x)(d+x)} = \frac{d-x}{2d} * e\right.$$

And

$e_2 = \frac{C_1 e}{C_1+C_2} = \frac{d+x}{2d} * e$

Hence, differential output voltage is

$\Delta e_o = e_2 - e_1 = \frac{d+x}{2d} e - \left(\frac{d-x}{2d}\right) e$

or, $\Delta e_o = \left(\frac{x}{d}\right) e$

i.e the output voltage varies linearly with the displacement.

Also , Sensitivity(S) = $\frac{\Delta eo}{x} = \frac{e}{d}$

Example 1

A capacitance transducer uses two quartz diaphragms of area 750mm^2 seperated by a distance of 3.5 mm. A pressure of 900 KN/m^2 when applied to the top diaphragm produces a deflection of 0.6 mm. The capacitor has capacitance 370 pF when no pressure is applied to the diaphragms. Find the value of capacitance after the application of pressure of 900 KN/m^2

Solution:

Suppose C_1 and C_2 are respectively the values of capacitance before and after the applications of pressure. Let d_1 and d_2 be the value of distance between the diaphragms for the corresponding pressure conditions. Then,

$C_1 = \epsilon A/d_1$ and $C_2 = \epsilon A/d_2$

So, $\frac{C2}{C1} = \frac{d1}{d2}$ $\therefore C2 = \frac{d1}{d2} * C1$ ………………….(1)

But, d_1 = 3.5mm and d_2 = (3.5-0.6) = 2.9mm

Hence, value of capacitance after application of pressure is $C_2 = \frac{d1}{d2} * C1 = \frac{3.5}{2.9} * 370 = 446.6pF$

Example 2

The output of LVDT is connected to a 5V voltmeter through an amplifier whose amplification factor is 250. An output of 2mv appears across the terminals of LVDT when the core moves through a distance of 0.5mm. Calculate the sensitivity of LVDT and that of the whole set up. The mili-voltmeter scale has 100 divisions and the scale can be read to1/5 of the division. Calculate the resolution of the instrument in mm.

Solution:

Sensitivity of LVDT = $\frac{(output\ voltage)}{displacement} = \frac{2*10^{-3}}{0.5} = 4 * 10^{-3}\ V/mm$

Sensitivity of instrument = amplification factor * sensitivity of LVDT

$= 250*4*10^{-3}$

= 1000mv/mm

1 scale division = 5/100 V = 50mv [100scale division = 5V]

Minimum voltage that can be read on the voltmeter is

1/5 scale division = (1/5)*50mv = 10mv

Again,

1000mv = 1mm

1mv = 1/1000 mm and

10mv = (1/1000)*10 = 0.01mm is the resolution of the instrument i.e

Resolution of instrument= $\frac{minimum\ voltage\ that\ can\ be\ read\ on\ the\ voltmeter}{sensitivity\ of\ instrument}$

$$= \frac{10mv}{1000\ mv/mm} = 0.01mm$$

PMMC (Permanent magnet Moving coil) Mechanism

The basic PMMC movement is often called the d'Arsonval movement after its inventor. This design offers the largest magnet in a given space and is used when maximum flux in the air gap is required. It provides an instrument with very low power consumption and low current required for full scale deflection. PMMC instrument is a linear for full-scale deflection. PMMC instrument is a linear reading dc device as the torque(the pointer deflection) is directly proportional to the coil current. It is unsuitable for ac measurements unless the current is rectified before application to the coil.

DC ammeters:

1. **Shunt Resistor**

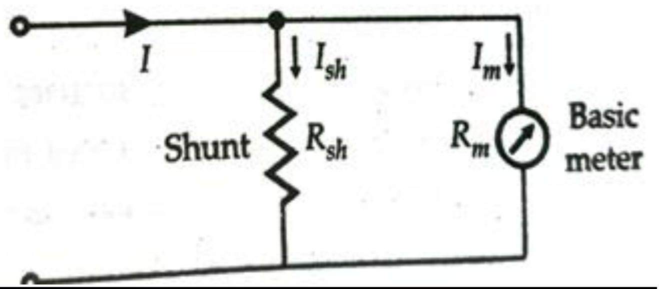
   The resistance of the shunt (R_s) =$\frac{\mathrm{ImRm}}{\mathrm{I_{sh}}} = \frac{ImRm}{I-Im}$

   where, I_m = full scale deflection current of the movement

   R_m = interval resistance of the movement (coil),

   I = full scale current of the ammeter including shunt and

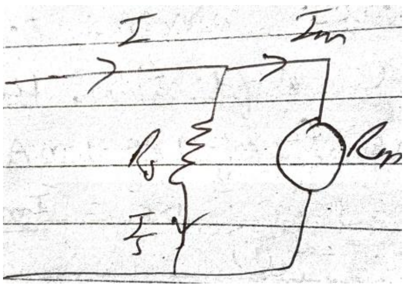
   I_{sh} = shunt current



Example

A 1-mA meter movement with an interval resistance of 100Ω is table converted into a 0-100mA ammeter. Calculate the value of the shunt resistance required.

Solution:



I_S= I - I_m = 100-1 – 99mA

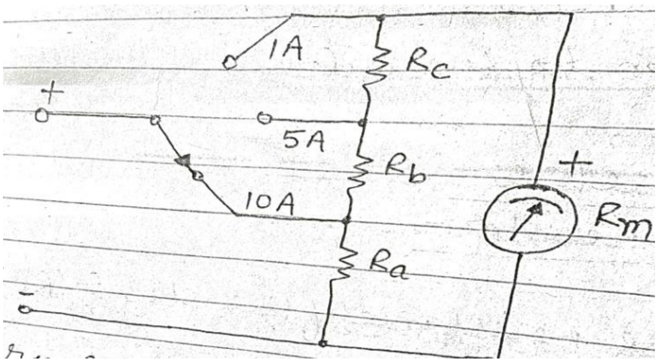
$R_S = \frac{ImRm}{Is} = \left(\frac{1mA*100\Omega}{99mA}\right) = 1.01\Omega$

2. **Ayrton Shunt**

   Example

Design an Ayrton shunt to provide an ammeter with current ranges of 1A, 5A and 10A. A d'Arsonval movement with an internal resistance R_m = 50Ω and full-scale deflection current of 1mA in the configuration.

Solution:



On the 1-A range

$R_a+R_b+R_c$ are in the parallel with the 50Ω movement. Since the movement requires 1mA for full-scale deflection, the shunt will be required to pass a current of 1A-1mA = 999 mA

So, $R_a+R_b+R_c$ = 1*50/999 = 0.05005Ω …………………..(1) [we have $R_s = \frac{ImRm}{Is}$]

On the 5-A range

R_a+ R_b are in parallel with R_c+R_m (50Ω). In this case there will be a 1-mA current through the movement and R_C in series and 4,999 mA through R_a+R_b. So,

$R_a+R_b = \frac{1*(Rc+50)}{4999}$ ………………………(2)'

On the 10-A range

R_a now serves as the shunt and R_b + R_c in series with the movement. The current through the movement again is1mA and the shunt passes the remaining 9,999 mA. So,

$R_a = \frac{1*(Rb+Rc+50)}{9999}$ …………………(3)

Solving the equations (1),(2)and (3)

R_C = 0.04004Ω, R_a= 0.005005 Ω and R_b=0.005005 Ω

This calculation indicates that for larger current, the value of the shunt resistor may become very small.