CONTENTS

k) **Causal, anti-causal and non-causal signals**

The signals which exist only in the positive time instants are called as causal signals. The signals that exist only in the negative time instants are called as anti-causal signals. And the signals that appear in both the positive and negative time instants are called non-causal signals.

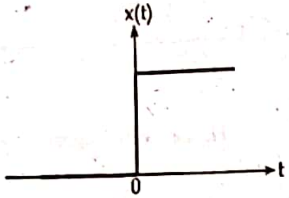

Figure: Causal signal

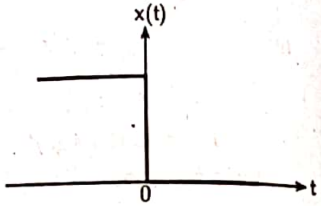

Figure: Anti-causal signal

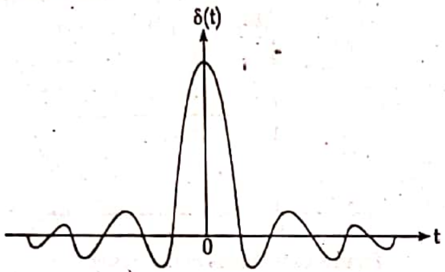

Figure: Non-causal signal

1.4 BOARD EXAM QUESTIONS SOLUTION

1. **Define instrumentation system and also explain the components of generalized instrumentation system in brief with the help of block diagram.** **[2011/S, 2012/F, 2012/S, 2013/S, 2014/S, 2014/F, 2015/S, 2015/F, 2017/F, 2017/S, 2018/F, 2019/F]**

Solution:

An instrumentation system is collection of instruments used to measure, monitor and control a process.

See the definition of 1.1 for components of generalized instrumentation system in brief.

2. **Define signals. Explain the different types of signals used in instrumentation system.** **[2011/F, 2013/F, 2016/S, 2018/S]**

Solution: See the definition of 1.3.

CHAPTER 2

SIGNAL MEASUREMENTS

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2.1 UNITS AND STANDARDS OF MEASUREMENTS

2.1.1 Units

The standard measurement of any physical quantity is known as unit. The number of times the unit occurs in any given amount of the same quantity is the number of measure. *For example;* 100 meters, we know that the meter is the unit of length and that the number of units of length is one hundred. The physical quantity, length, is therefore defined by the unit meter.

Following are the various type of units

i) **Fundamental unit**

The units which are independent and are not related to each other are known as fundamental unit. These units do not vary with time, temperature and pressure etc. There are seven fundamental units, as given below; Fundamental units: length, mass, time, electric current, temperature, luminous intensity and quantity of matter.

ii) **Derived unit**

The units which are derived from fundamental units are called derived unit. Every derived unit originates from some physical law defining that unit. This unit is recognized by its dimension, which can be defined as the complete algebraic formula for the derived unit. *For example;* the area of rectangle is proportional to its length (l) and breadth (b) or $A = l \times b$. If the meter has been chosen as the unit of length, then the unit of area is m^2. The derived unit for area (A) is then the square meter (m^2).

2.1.2 Measurement

Measurement is the act or the result of a quantitative comparison between a given quantity and a quantity of the same kind chosen as a unit. It is a process by which one can convert a physical quantity to meaningful number. It is a means of describing a natural phenomenon in quantitative term.

2.1.2.1 Standards of measurements

A standard is a physical representation of a unit of measurement. The term 'standard' is applied to a piece of equipment having a known measure of physical quantity. They are used for the purpose of obtaining the values of the physical properties of other equipment by comparison methods. In fact, a unit is realized by reference to a material standard or to a natural phenomenon including physical and atomic constants. *For example;* the fundamental unit of mass in the metric system (SI) is the kilogram, defined as the mass of the cubic decimeter of water at its temperature of maximum density of 4°C.

Types of standards of measurements are

i) International standards
ii) Primary standards
iii) Secondary standards
iv) Working standards

i) **International standards**

The international standards are defined on the basis of international agreement. They represent the units of measurements which are closest to the possible accuracy attainable with present day technological and scientific methods. International standards are checked and evalvated regularly against absolute measurements in terms of the fundamental units. The international standards are maintained at the international bereave of weights and measures and are not available to the ordinary user of measuring instruments for the purpose of calibration or comparison. Improvements in the accuracy of absolute measurements have made the international units superfluous and they have been replaced by absolute units. One of the main reason for adopting an absolute system of units is that now wire resistance standards can be constructed which are sufficiently permanent and do not vary appreciably with time.

ii) **Primary standards**

Primary standards are absolute standards of such high accuracy that they can be used as the ultimate reference standards. These standards are maintained by national standards laboratories in different parts of the world. The primary standards which represent the fundamental units and some of the derived electrical and mechanical units are independently calibrated by absolute measurements at each of the national laboratories. The results of these measurements are compared against each other, leading to a world average figure for the primary standards. Primary standards are not available for use outside the national laboratories. One of the main functions of the primary standards is the verification and calibration of secondary standards.

The primary standards are few in number. They must have the highest possible accuracy. Also these standards must have the highest stability *i.e.,* their values should vary as small as possible over long periods of time even if there are environmental and other changes.

iii) **Secondary standards**

The secondary standards are the basic reference standards used in industrial measurement laboratories. The responsibility of maintenance and calibration of these standards lies with the particular industry involved. These standards are checked locally against reference standards available available in the area secondary standards are normally sent periodically to the national standards laboratories for calibration and comparison against primary standards. The secondary standards are sent back to the industry by the national laboratories with a certification as regards their measured values in terms of primary standards.

iv) **Working standards**

The working standards are the major tools of a measurement laboratory. These standards are used to check and calibrate general laboratory instruments for their accuracy and performance. *For example;* a manufacturer of precision resistances, may use a standard resistance (which may be a working standard) in the quality control department for checking the values of resistors that are being manufactured. This way, he verifies that his measurement set up performs within the limits of accuracy that are specified.

v) **IEEE standards**

This standard is maintained by the institute of electrical and electronics engineers, IEEE an engineering society headquartere in New York city. These standards are not physical items that are available for comparison and checking of secondary standards but are standard procedures, nomenclature, definitions etc. It gives the standard test method for testing and evaluating various electronic systems and components. *For example;* there is a standard method for testing and evaluating attenuators.

2.2 MEASURING INSTRUMENTS

It is a device for determining the magnitude of a physical quantity being measured. All measuring instruments are subject to varying degrees of instrument error and measurement uncertainty. There are two main types of the measuring instruments; analog and digital.

The analog instruments indicate the magnitude of the quantity in the form of the pointer movement. The digital measuring instruments indicate the values of the quantity in digital format that is in numbers which can be read easily.

Measuring instruments may be divided into two categories, *i.e.,*

a) Absolute instrument

b) Secondary instrument

a) **Absolute Instrument**

It gives the quantity to be measured in terms of instrument constant and its deflection.

b) **Secondary Instrument**

In secondary instrument, the deflection gives the magnitude of electrical quantity to be measured directly. These instruments are required to be calibrated by comparing with another standard instrument before putting into use secondary instruments can be classified into three types.

i) **Indicating instruments**

It indicates the magnitude of an electrical quantity at the time when it is being measured. The indicators are given by a pointer moving over a graduated dial.

For example; ammeters, watt meters etc.

ii) **Recording instruments**

The instruments which keep a continuous record of the variations of the magnitude of an electrical quantity to be observed over a defined period of time.

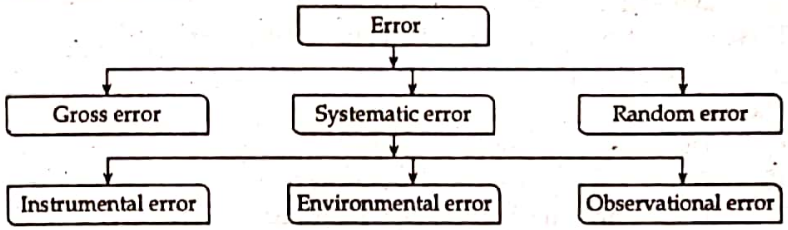
iii) **Integrating instruments**

The instruments which measure the total amount of either quantity of electricity or electrical energy supplied over a period of time.

For example; ampere-hour meter, energy meter etc.

2.2.1 Classification of Error

No measurement can be made with perfect accuracy but it is important to find out what accuracy actually is and how different errors have entered into the measurement. A study of errors is a first step in finding ways to reduce them. Errors may arise from different sources and are usually classified as under.



a) **Gross error**

This class of errors mainly covers human mistakes in reading instruments and recording and calculating measurement results. This responsibility of the mistake normally lies with the experimenter. The experimenter may grossly misread the scales. As long as human beings are involved, some gross wilt definitely be committed. Although complete elimination of gross errors is probably impossible, one should try to anticipate and correct them. Some gross errors are easily detected while others may be very difficult to defect.

Gross errors may be of any amount and therefore their mathematical analysis is impossible. However, they can be avoided by adopting two means. They are:

i) Great care should be taken in reading and recording the data.

ii) Two, three or even more readings should be taken for the quantity under measurement. These readings should be taken preferably by different experimenters and the readings should be taken at a different reading point to avoid re-reading with the same error. It should be understood that no reliance be placed on a single reading. It is always advisable to take a large number of readings as a close agreement between readings assures that no gross error has been committed.

b) **Random (Residual) errors**

It has been consistently found that experimental results show variation from one reading to another, even after all systematic errors have been accounted for. These errors are due to a multitude of small factors which change or fluctuate from one measurement to another and are due surely to change. The quantity being measured is affected by many happenings throughout the universe. We are aware and account for some of the factors influencing the measurement, but about the rest we are unaware. The happenings or disturbance about which we are unaware are lumped together are called 'Random' or 'Residual.' Hence the errors caused by these happenings are called random errors.

Random errors can be eliminated by increasing the number of readings and using statistical means to obtain the best approximate of true value of the quantity under measurement.

c) Systematic errors

These types of errors are divided into three categories:

i) Instrumental Errors

These errors arise due to three main reasons:

- Due to missue of the instruments
- Due to loading effect of instruments
- Due to inherent short comings in the instrument

Elimination Method:

- The procedure of measurement must be carefully planned. Substitution methods or calibration against standards may be used for the purpose.
- The instrument may be re-calibrated carefully.
- Correction factors should be applied after determining the instrumental errors.
- Errors caused by loading effects of the meters can be avoided by using them intelligently.

ii) Observational Errors

There are many sources of observational errors. There are human factors involved in measurement. The sensing capabilities of individual observers effect the accuracy of measurement. No two persons observe the same situation in exactly the same way where small details are concerned. Modern electrical instruments have digital display of output which completely eliminates the errors on account of human observational or sensing powers as the output is in the form of digits.

iii) Environmental Errors

These errors are due to conditions external to the measuring device including conditions in the area surrounding the instrument. These may be effects of temperature, pressure, humidity, dust, vibration or of external magnetic or electrostatic fields.

Elimination Method:

- Arrangements are made to keep the conditions as nearly as constant as possible. *For example;* temperature can be kept constant by keeping the equipment in a temperature controlled enclosure.
- Using equipment which is immune to these effects. *For example;* variation in resistance with temperature can be minimized by using resistance materials which have a very low resistance temperature coefficient.

- Employing techniques which eliminate the effects of these disturbances. *For example;* the effect of humidity dust etc can be entirely eliminated by hermetically sealing the equipment.
- Applying computed corrections: Efforts are normally made to avoid the use of application of computed corrections, but where these corrections are needed and are necessary, they are incorporated for the computations of the results.

2.2.2 Statistical Analysis

a) Arithmetic mean

The arithmetic mean is given by;

$$\bar{x} = \frac{(x_1 + x_2 + x_3 + x_4 + \cdots\cdots + x_n)}{n} = \frac{\Sigma x}{n} = \frac{\text{Sum of values}}{\text{Count of values}}$$

where, $\bar{x}$ = arithmetic mean

n = number of readings

$x_1, x_2, \ldots\ldots, x_n$ = Readings taken

b) Range

The simplest possible measure of dispersion is the range which is the difference between greatest and least values of data.

c) Deviation

Deviation is departure of the observed reading from the arithmetic mean of the group of readings. Let the deviation of reading x_1 be d_1 and that of reading x_2 be d_2, etc, then

$$d_1 = x_1 - \bar{x}$$

$$d_2 = x_2 - \bar{x}$$

$$\ldots = \ldots\ldots$$

$$d_n = x_n - \bar{x}$$

and, $\bar{x} = \dfrac{\Sigma(x_n - d_n)}{n}$

The algebraic sum of deviation is zero.

where, d_n is the deviation of the n^{th} reading from the mean.

d) Average deviation

The average deviation is an indication of the precision of the instruments used in making the measurements. Highly precise instruments yields a low average deviation between readings. It is defined as the sum of the absolute values of deviations divided by the number of readings. It may be expressed as,

$$D = \frac{|d_1| + |d_2| + \cdots\cdots + |d_n|}{n} = \frac{\Sigma|d|}{n}$$

e) Standard Deviation (SD)

The standard deviation of an infinite number of data is defined as the square root of the sum of the individual deviations squared, divided by the number of readings.

$$SD = \sqrt{\frac{d_1^2 + d_2^2 + \dots\dots + d_n^2}{n}} = \sqrt{\frac{\Sigma d^2}{n}}$$

In practice, however, the number of observations is finite. When the number of observations is greater than 20, SD is denoted by symbol σ while if it is less than 20, the symbol used is S. The standard deviation of finite number of data is given by

$$S = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \dots\dots d_n^2}{n-1}}$$

$$\sigma = \sqrt{\frac{\Sigma d^2}{n-1}}$$

f) Variance or mean square deviation

The variance is the mean square deviation which is the same as SD except that square root is not extracted.

Variance, V = (Standard deviation)^2

$$= (SD)^2 = \sigma^2 = \frac{d_1^2 + d_2^2 + d_3^2 + \dots\dots + d_n^2}{n} = \frac{\Sigma d^2}{n}$$

But when the number of observations is less than 20, variance,

$$V = S^2 = \Sigma\frac{d^2}{n-1}$$

Example 1

A set of independent current measurements were taken by six observers and were recorded as 12.8 A, 12.2 A, 12.5 A, 13.1 A, 12.9 A and 12.4 A. Calculate,

i) Arithmetic mean
ii) Deviation from the mean
iii) Standard deviation
iv) Variance
v) Average deviation:

Solution:

i) Arithmetic mean,

$$\bar{x} = \frac{\Sigma x}{n} = \frac{12.8 + 12.5 + 13.1 + 12.9 + 12.4}{6}$$

$$\bar{x} = 12.65 \text{ A}$$

ii) Deviations are,

$$d_1 = x_1 - \bar{x} = 12.8 - 12.65 = 0.15 \text{ A}$$
$$d_2 = x_2 - \bar{x} = 12.2 - 12.65 = -0.45 \text{ A}$$
$$d_3 = x_3 - \bar{x} = 12.5 - 12.65 = -0.15 \text{ A}$$
$$d_4 = x_4 - \bar{x} = 13.1 - 12{,}65 = 0.45 \text{ A}$$

$$d_5 = x_5 - \bar{x} = 12.9 - 12.65 = 0.25 \text{ A}$$
$$d_6 = x_6 - \bar{x} = 12.4 - 12.65 = -0.25 \text{ A}$$

iii) Average deviation,

$$D = \frac{\Sigma|d|}{n} = \frac{0.15 + 0.45 + 0.15 + 0.45 + 0.25 + 0.25}{6} = 0.283$$

For average deviation, absolute value is taken.

iv) Standard deviation,

$$S = \sqrt{\frac{\Sigma d^2}{n-1}}$$

$$= \sqrt{\frac{(0.15)^2 + (-0.45)^2 + (-0.15)^2 + (0.45)^2 + (0.25)^2 + (-0.25)^2}{6-1}}$$

$$= 0.399 \text{ A}$$

v) Variance, $V = S^2 = (0.399)^2 = 0.115 \text{ A}^2$.

2.2.3 Probability of Errors

a) Normal distribution of errors

The normal or Gaussian law of errors is the basis for the major part of study of random effects. This type of distribution is most frequently met in practice. The law of probability states the normal occurrence of deviations from average value of an infinite number of measurements or observations can be expressed by,

$$y = \frac{h}{\sqrt{\pi}} \cdot e^{-h^2x^2} \qquad (1)$$

where, x = Magnitude of deviation
h = A constant called precision index
y = Number of readings at any deviation x, (the probability of occurrence of deviation x).

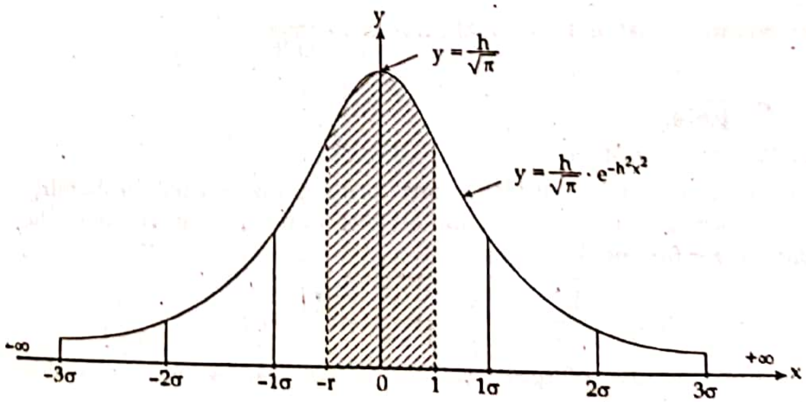


Figure: Normal probability curve

This curve is symmetrical about the arithmetic mean value, and area under the curve is unity. Under the conditions specified here the total number of readings taken is represented by 1.

From equation (1); when x = 0, $y = \frac{h}{\sqrt{\pi}}$

Thus it is clear from above that the maximum value of y depends upon h. The larger the value of h, the sharper the curve. Thus the value of h determines the sharpness of the curve since the curve drops sharply, owing to the term $(-h^2)$ being in the exponent. A sharp curve evidently indicates that the deviations are more closely grouped together around deviation x = 0.

b) Probable error

We know that the most probable or best value of a Gaussian distribution is obtained by taking arithmetic mean of the various values of the variate. In addition, it has been indicated that the confidence in this best value (most probable value) is connected with the sharpness of the distribution curve. A convenient measure of precision is the quantity r. It is called probable error or simple. P.E. The location of point r can be found from equation.

$$n_{1-2} = \frac{h}{\sqrt{\pi}} \int_{x_1}^{x_2} e^{-h^2x^2} dx$$

By putting $\frac{h}{\sqrt{\pi}} \int_{-r}^{+r} e^{-h^2x^2} dx = \frac{1}{2}$

This gives

$$r = \frac{0.4769}{h}$$

The standard deviation is given by, $\sigma^2 = \frac{\Sigma d^2}{n}$

The standard deviation for normal curve is $\sigma = \frac{1}{\sqrt{2}h}$

$$\sigma = \frac{r}{0.6745}$$

Thus, PE = r = ± 0.6745 σ

For an infinite number of deviations forming the normal probability curve, where n is infinite. But for a finite number of deviations, the probable error for one reading is:

$$r_1 = \pm 0.6745\sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \dots + d_n^2}{n-1}} = \pm 0.6745\sqrt{\frac{\Sigma|d|^2}{n-1}}$$

With a finite number of readings, the average reading has a probable error of

$$r_m = \frac{1}{\sqrt{n}} r_1 = \pm 0.6745\sqrt{\frac{\Sigma|d^2|^2}{n(n-1)}}$$

The above equations means that for n finite readings, the probable error is r_m. If we have n > 1, then $n - 1 \simeq 1$ and $r_1 \simeq \pm 0.6745\,\sigma$ and $r_m = \pm 0.6745 \times \frac{\sigma}{\sqrt{n}}$.

Example 2

The following 10 observations were recorded when measuring a voltage 41.7, 42.0, 41.8, 42.0, 42.1, 41.9, 42.0, 41.9, 42.5 and 41.8 volt. Find the,

i) Mean
ii) Standard deviation
iii) Range
iv) Probable error of one reading
v) Variance

Solution:

i) Mean $(\overline{x}) = \frac{\Sigma x}{n}$

$$= \frac{41.7 + 42 + 41.8 + 42 + 42.1 + 41.9 + 42 + 41.9 + 42.5 + 41.8}{10}$$

$$= 41.97 \text{ V}$$

ii) Standard deviation, $S = \sqrt{\frac{\Sigma d_t^2}{n-1}}$ for n < 20

Measured voltage (x)	Deviation (d)	d^2
41.7	- 0.27	0.0729
42.0	0.03	0.0009
41.8	0.17	0.0289
42.0	0.03	0.0009
42.1	0.13	0.0169
41.9	- 0.07	0.0049
42.0	0.03	0.0009
41.9	- 0.07	0.0049
42.5	0.53	0.2809
41.8	- 0.17	0.0289
Σx = 419.7		$\Sigma d^2 = 0.441$

Thus, $S = \sqrt{\frac{0.441}{10-1}} = 0.22$ volt.

iii) Probable error of one reading,

$r_1 = \pm 0.6745 \times S = \pm 0.6745 \times 0.22 = \pm 0.15$ V

iv) Probable error of mean,

$$r_m = \frac{\pm r_1}{\sqrt{n-1}} = \frac{\pm 0.15}{\sqrt{10-1}} = \pm 0.05 \text{ V}$$

v) Range = (Largest - Smallest) value = 42.5 - 41.7 = 0.8 volt

vi) Variance, $V = S^2 = (0.22)^2 = 0.0484$ volt^2

c) Limiting errors or Guarantee errors

The accuracy and presion of an instrument depends upon its design, the materials used and the workmanship that goes into making the instruments. The choice of an instrument for a particular application depends upon the accuracy desired. If only a fair degree of accuracy is desired, it is not economical to use expensive materials and skill into the manufacture of the instrument. But an instrument used for an application required a high degree of accuracy has to use expensive material and highly skilled workmanship.

The economical production of any instrument requires the proper choice of material, design and skill. Thus, the manufacturer has to specify the deviations from the nominal value of a particular quantity. The limits of these deviations from the specified value are defined as limiting errors.

The magnitude of a quantity having a nominal value as and a maximum error or limiting error of $\pm \delta A$ must have a magnitude Aa between the limits $A_s - \delta A$ and $A_s + \delta A$ or actual value, $A_a = A_s \pm \delta A$.

For example; the nominal magnitude of a resistor is 100 Ω with a limiting error of ± 10 Ω. The magnitude of the resistor will be between the limits:

A = 100 ± 10 Ω or A > 90 and A < 110 Ω. In other words, the manufacturer guarantees that the value of resistance of the resister lies between 90 Ω and 110 Ω.

Relative limiting error, $\epsilon_r = \frac{\delta A}{A_s} = \frac{\epsilon_0}{A_s}$

or, $\epsilon_0 = \delta A = \epsilon_r A_s$

Then,

$$A_a = A_s \pm \delta A = A_s \pm \epsilon_r A_s = A_s (1 + \epsilon_r)$$

∴ Percentage limiting error, $\% \epsilon_r = \epsilon_r \times 100$

∴ Relative limiting error,

$$\epsilon_r = \frac{A_a - A_s}{A_s} = \frac{\text{Actual value - Nominal value}}{\text{Nominal value}}$$

Example 3

A 0 - 150 V voltmeter has a guaranteed accuracy of 1% full scale reading. The voltage measured by this instrument is 83 V. calculate the limiting error in percentage.

Solution:

Magnitude of the limiting error is 1% of 150 V = $\frac{150}{100} = 1.5$ V

% error of a meter indication of 83 V is $\frac{1.5}{83} \times 100\% = 1.81\%$

Example 4

Three resistors have the following ratings,

$R_1 = 37\ \Omega \pm 5\%$

$R_2 = 75\ \Omega \pm 5\%$

$R_3 = 50\ \Omega \pm 5\%$

Determine the magnitude and limiting error in ohm and in percent of the resistance of these resistances connected in series.

Solution:

The values of resistances are;

$$R_1 = 37 \pm \frac{5}{100} \times 37 = 37 \pm 1.85\ \Omega$$

$$R_2 = 75 \pm \frac{5}{100} \times 75 = 75 \pm 3.75\ \Omega$$

$$R_3 = 50 \pm \frac{5}{100} \times = 50 \pm 2.5\ \Omega$$

The limiting value of resultant resistance is,

$$R = (37 + 75 + 50) \pm (1.85 + 3.75 + 2.5) = 162 \pm 8.1\ \Omega$$

∴ Magnitude of resistance = 162 Ω

Error is ohm = ± 8.1 Ω

Percent limiting error = $\pm \frac{8.1}{162} \times 100 = \pm 5\%$

Example 5

The resistance of an unknown resistor is determined by the wheat stone bridge method. The solution for the unknown resistance is stated as

$$R_x = \frac{R_1 \times R_2}{R_3}$$

where, $R_1 = 500\ \Omega \pm 1\%$

$R_2 = 615\ \Omega \pm 1\%$

$R_3 = 100\ \Omega \pm 0.5\%$

Calculate:

i) Nominal value of the unknown resistor

ii) Limiting error in Ω of the unknown resistor

iii) Limiting error in percentage of the unknown resistor

Solution:

$R_1 = 500 \pm 5\ \Omega$

$R_2 = 615 \pm 6.15\ \Omega$

$R_3 = 100 \pm 0.5\ \Omega$

$$R_x = \frac{R_1 \times R_2}{R_3}$$

i) Nominal value of unknown resistor,

$$R_x = \frac{R_1 \times R_2}{R_3} = \frac{500 \times 615}{100} = 3{,}075\ \Omega$$

ii) Limiting error in ohms of the R_x, the value of R_1 and R_2 must be high and the value of R_3 should be low.

$$\text{i.e., } R_x = \frac{(500+5) \times (615+6.15)}{(100-0.5)} = \frac{505 \times 621.5}{99.5} = 3{,}152.57\ \Omega.$$

Now the limiting error is nominal value minus highest value of unknown resistor *i.e.,*

Limiting error $= (3{,}075 - 3{,}152.27)\ \Omega = -77.57\ \Omega$

Hence, the limiting error of unknown resistor is $\pm 77.57\ \Omega$

iii) % limiting error of unknown resistor

$$= \frac{\text{Limiting error}}{\text{Nominal value}} \times 100\% = \frac{77.57}{3{,}075} \times 100\% = \pm 2.52\%$$

2.3 PERFORMANCE PARAMETERS (STATIC AND DYNAMIC)

The treatment of instrument and measurement system characteristics can be divided into two distinct categories viz;

a) Static characteristics
b) Dynamic characteristics

2.3.1 Static Characteristics

The static characteristics of a measurement system are those that must be considered when the system or instrument is used to measure a conditions not varying with time *i.e.,* when steady state condition occurs some of its static performance parameter are as follows;

I) Accuracy

The accuracy of an instrument is a measure of how close the output readings of the instrument is to the correct value. Thus accuracy of a measurement means conformity to truth. In practice, it is more usual to quote the inaccuracy figure rather than the accuracy figure for an instrument. Inaccuracy is the extent to which a reading might be wrong, and is often quoted as a percentage of the full scale reading of an instrument.

II) Precision/repeatability/reproducibility

Precision is a term that describe an instrument's degree of freedom from random errors. If a large number of readings are taken of the same quantity by a high precision instrument, then the spread of readings will be very small. High precision does not imply anything about measurement accuracy. A high precision instrument may have a low accuracy.

Repeatability describes the closeness of output, readings when the same input is applied repetitively over a short period of time, with the same measurement conditions, same instrument and observer, same location and same conditions of use maintained throughout.

Reproducibility describes the closeness of output readings for the same input when there are changes in the method of measurement, observer, measuring instrument, location, conditions of use and time of measurement.

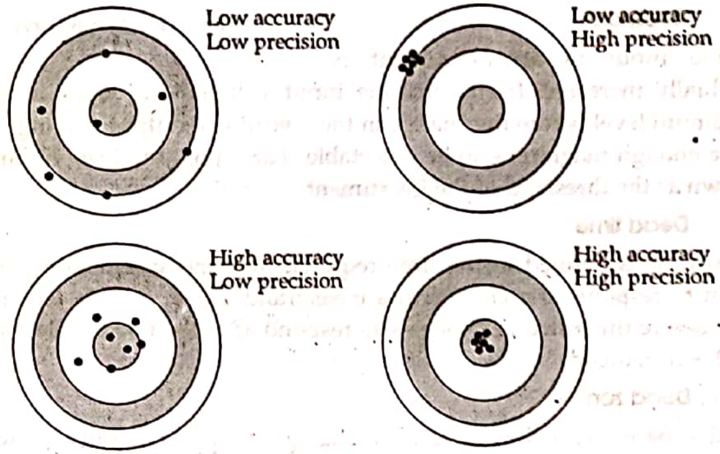


Figure: Accuracy and precision comparison

III) Linearity

One of the best characteristics of an instrument or a measurement system is considered to be linearity, that is, the output is linearly proportional to the input. Most of the system require a linear behavior as it is desirable. This is because the conversion from a scale reading to the corresponding measured value of input quantity is most convenient if one merely has to multiply by a fixed constant rather than consult a non-linear calibration curve or compute from non-linear calibration equations.

It is represented as $y = mx + c$

where, y = output
x = input
m = slope
c = intercept

IV) Hysteresis

Hysteresis effects shows up in any physical, chemical or electrical phenomenon. Hysteresis is a phenomenon which depicts different output effects when loading and unloading whether it is a mechanical system or an electrical system and for that matter any system. Hysteresis is non-incidence of loading and unloading curves. Hysteresis is most commonly found in instruments that contain springs such as the passive pressure gauge.

Hysteresis, in a system, arises due to the fact that all the energy put into the stressed parts when loading is not recoverable upon loading. This is because second law of thermodynamics rules out any perfectly reversible process in the world.

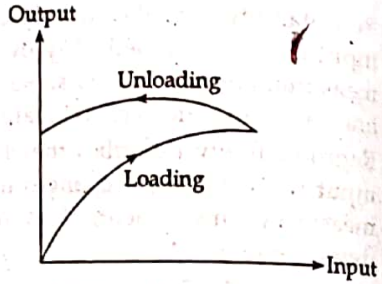


Figure: Hysteresis curve

v) Threshold

If the input to an instrument is gradually increased from zero, the input will have to reach a certain minimum level before the change in the instrument output reading is of a large enough magnitude to be detectable. This minimum level of input is known as the threshold of the instrument.

vi) Dead time

Dead time is defined as the time required by a measurement system to begin to respond to a change in a measurand. Dead time, in fact, is the time before the instrument begins to respond after the measured quantity has been changed.

vii) Dead zone

Dead zone is defined as the largest change of input. Quantity for which there is no output of the instrument. Any instruments that exhibits hysteresis also displays dead space.

viii) Resolution or discrimination

The smallest increment in input (the quantity being measured) which can be detected with certainty by an instrument is its resolution or discrimination. So resolution defines the smallest measurable input changes while the threshold defines the smallest measurable input.

ix) Loading effect

The incapatability of the system to faithfully measure, record or control the input signal (measured) in undistorted form is called the loading effect.

x) Range or span

The range or span of an instrument defines the minimum and maximum values of a quantity that the instrument is designed to measure.

xi) Sensitivity

The sensitivity of measurement is a measure of the change in instrument output that occurs when the quantity being measured changes by a given

amount. Thus, sensitivity is the ratio:

$$\frac{\text{Scale deflection}}{\text{Value of measurand producing deflection}}$$

The sensitivity of measurement is therefore the slope of the straight line drawn on figure below.

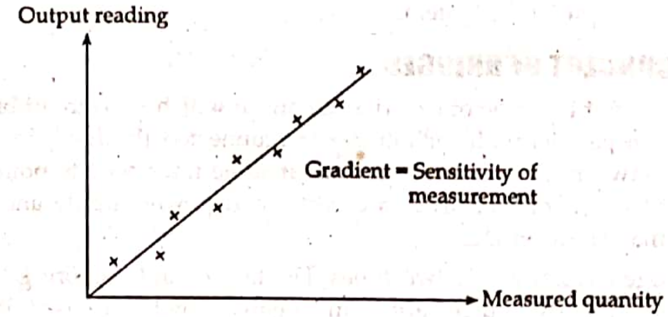


Figure: Instrument output characteristic.

$$\text{Static sensitivity} = \frac{\text{Infinitesimal change in output}}{\text{Infinitesimal change in input}} = \frac{\Delta q_0}{\Delta q_i}$$

Similarly, Inverse sensitivity or deflection factor $= \dfrac{\Delta q_i}{\Delta q_0}$

2.3.2 Dynamic Characteristics

The dynamic characteristics of a measuring instrument describe its behavior between the time a measured quantity changes value and the time when the instrument output attains a steady value in response. The dynamic characteristics of any measurement system are;

i) Speed of response

It is the rapidity with which an instrument responds to changes in the measured quantity.

ii) Response time

It is defined as the time required by instrument or system to settle to its final steady position after the application of the input.

iii) Measuring lag

An instrument does not immediately react to a change. Measuring lag is defined as the delay in the response of an instrument to a change in the measured quantity. This lag is usually quite small but it becomes highly important where high speed measurements are required. In these systems, it becomes essential that the time lag be reduced to minimum.

iv) Fidelity

Fidelity of a system is defined as the ability of the system to reproduce the output in the same form as the input. In the definition of fidelity any time lag or phase difference between output and input is not included.

v) Dynamic error

It is the difference between the true value of the quantity changing with time and the value indicated by the instrument if no static error is assumed. However, the total dynamic error of the instrument is the combination of its fidelity and the time lag or phase difference between input and output of the system.

2.4 CONCEPT OF BRIDGES

A bride circuit is a type of electrical circuit in which two circuit branches (usually in parallel with each other) are connected (bridged) by a third branch between the first two branches at some intermediate point along them. It consists of four arms, a detector and power supply and power supply may be AC or DC.

The bridge circuits are of two types, DC bridge and AC bridge. In DC bridge circuit, a DC source battery and a galvanometer are used. While in the AC bridge circuit, an AC source and a detector sensitive to AC voltage are used. The bridge circuit may be broadly classified into the following categories.

a) DC bridge circuits

Different types of DC bridge circuits are,

i) Wheatstone bridge circuit
ii) Kelvin bridge circuit

b) AC bridge circuits

Different types of AC bridge circuit are,

i) Max well bridge circuit
ii) Hay's bridge circuit
iii) Anderson's bridge circuit
iv) Owen's bridge circuit
v) Schering bridge circuit
vi) Wien bridge circuit
vii) Resonance Bridge circuit
viii) De sauty's bridge circuit

2.4.1 DC Bridge Circuit

2.4.1.1 Wheatstone Bridge

A very important device used in the measurement of medium resistance (1 ohm to about 1,00,000 ohm) is the Wheatstone bridge. It has four resistive arms, together with a source of emf (a battery) and a null detector, usually a galvanometer G or the other sensitive current meter. The current through the galvanometer depends on the potential difference between c and d. The bridge is said to be balanced when there is no current through the galvanometer is zero. This occurs when the voltage from point 'c' to point 'a' equals the voltage from point 'd' to point 'a' or by referring to the other battery terminal, when the voltage from point 'c' to point 'b' equals the voltage from point 'd' to point 'b'.

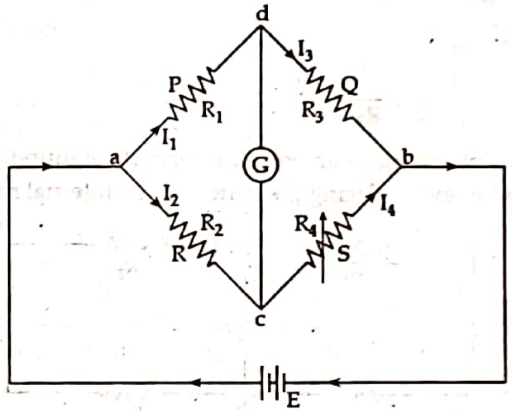


Figure: Wheatstone bridge

For bridge balance, $E_{ac} = E_{ad}$

$$I_1R_1 = I_2R_2 \tag{1}$$

For the galvanometer current to be zero,

$$I_1 = I_3 = \frac{E}{P+Q} \tag{2}$$

and, $$I_2 = I_4 = \frac{E}{R+S} \tag{3}$$

Combining equation (1), (2) and (3);

$$\frac{P}{P+Q} = \frac{R}{R+S}$$

For which QR = PS

i.e., $$R_2R_3 = R_1R_4 \tag{4}$$

Equation (4) is the well known expression for the balance of Wheatstone bridge. If three of the resistance are known, the fourth may be determined from equation (4), and we obtain,

$$R = S \times \frac{P}{Q}$$

i.e., $$R_x = R_2 \times \frac{R_3}{R_1}$$

where, S is called standard arm of the bride P and Q are called the ratio arms.

The current through the galvanometer can be found out by finding thevenin equivalent circuit. Thevenin or open circuit voltage appearing between terminals c and d with galvanometer circuit open circuit is,

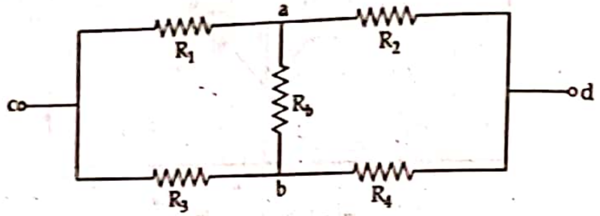
$$E_{th} = E_{cd} = E_{ac} - E_{ad} = I_1R_1 - I_2R_2$$

where, $I_1 = \dfrac{E}{R_1 + R_3}$

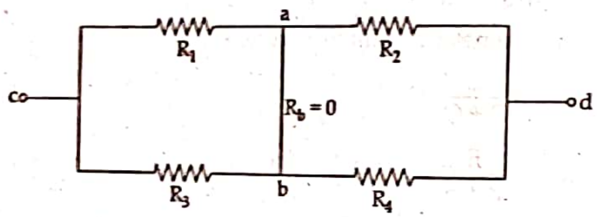
$$I_2 = \frac{E}{R_2 + R_4}$$

$$\therefore \quad E_{th} = E\left(\frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R_4}\right)$$

The resistance of the thevenin equivalent circuit is found by looking into terminal c and d and replacing the battery by its internal resistance.

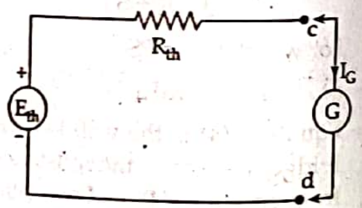


Since in most cases the internal resistance, R_b of the battery is extremely low, it can be neglected *i.e.,* $R_b = 0$, so,



$$\therefore \quad R_{th} = (R_1 \| R_3) + (R_2 \| R_4) = \left(\frac{R_1 \cdot R_3}{R_1 + R_3}\right) + \left(\frac{R_2 \cdot R_4}{R_2 + R_4}\right)$$

Hence, complete thevenin circuit with the galvanometer connected to terminals c and d is,



The galvanometer current, I_G with its internal resistance R_G is

$$I_G = \frac{E_{th}}{R_{th} + R_G}$$

where, R_G is the resistance of galvanometer circuit.

Limitations of Wheatstone bridge

i) The use of Wheatstone bridge is limited to the measurement of resistances ranging from a few ohm to several mega ohm.

ii) The upper limit is set by the reduction in sensitivity to on balance caused by high resistive values.

iii) The lower limit for measurement is set by the resistance of the connecting leads and by contact resistance at the binding posts.

iv) Heating effect due to large current also plays a major role. The excessive currents may generate heat which may cause the permanent change in the resistance.

Advantages of Wheatstone bridge

i) The results are not dependent on the calibration and characteristics of galvanometer as it works on null deflection.

ii) Due to null deflection method used, the accuracy and the sensitivity is higher than direct deflection meters.

iii) The source emf and inaccuracies due to the source fluctuations do not affect the balance of the bridge. Hence the corresponding errors are completely avoided.

Example 6

In the Wheatstone bridge, the values of resistance of various arms are P = 1,000 Ω, Q = 100 Ω, R = 2,005 Ω and S = 200 Ω. The battery has an emf of 5 V and negligible internal resistance. The galvanometer has a current sensitivity of 10 mm/μA and an internal resistance of 100 Ω. Calculate the deflection of galvanometer and the sensitivity of the bridge in terms of deflection per unit change in resistance.

Solution:

Arm P = 1,000 Ω = R_1

Arm Q = 100 Ω = R_3

Arm S = 200 Ω = R_4

Resistance of unknown resistor required for balance,

$$R = \frac{P}{Q} \times S$$

or, $$R_2 = \frac{R_1}{R_3} \times R_4 = \frac{1{,}000}{100} \times 200$$

$$\therefore \quad R_2 = 2{,}000\ \Omega$$

In the actual bridge, the unknown resistor has a value of 2,005 Ω, or the deviation from the balance condition is,

$$\Delta R = 2{,}005\ \Omega - 2{,}000\ \Omega = 5\ \Omega$$

Thevenin source generator emf,

$$E_{th} = E\left[\frac{R}{R+S} - \frac{P}{P+Q}\right]$$

$$= 5\left[\frac{2{,}005}{2{,}005 + 200} - \frac{1{,}000}{1{,}000 + 100}\right]$$

$$\therefore \quad E_{th} = 1.0308 \times 10^{-3}\ \text{V}$$

Internal resistance of bridge looking into terminals c and d,

$$R_{th} = \frac{R \times S}{R + S} + \frac{P \times Q}{P + Q}$$

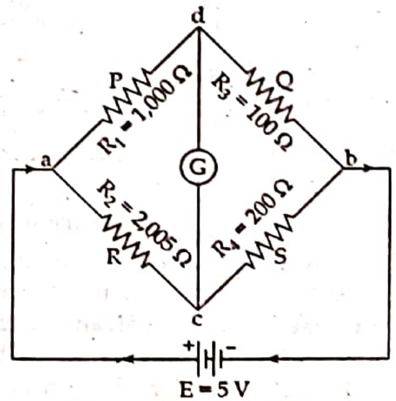


Figure: Wheatstone bridge

$$= \frac{2{,}005 \times 200}{2{,}005 + 200} + \frac{1{,}000 \times 100}{1{,}000 + 100} = 272.4\ \Omega$$

Hence, current through the galvanometer,

$$I_G = \frac{E_{th}}{R_{th} + R_G} = \frac{1.0308 \times 10^{-3}}{272.4 + 100} = 2.77\ \mu A$$

Deflection of galvanometer,

$$\theta = S_i I_G = 10 \times 2.77 = 27.7\ \text{mm}$$

and, Sensitivity of bridge,

$$S_n = \frac{\theta}{\Delta R} = \frac{27.7}{5} = 5.54\ \text{mm}/\Omega$$

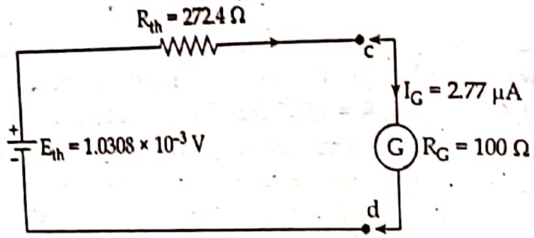


Figure: Thevenin equivalent circuit

2.4.1.2 Kelvin Bridge

The Kelvin bridge is a modification of the Wheatstone bridge and provides greatly increased accuracy in measurement of low value resistances.

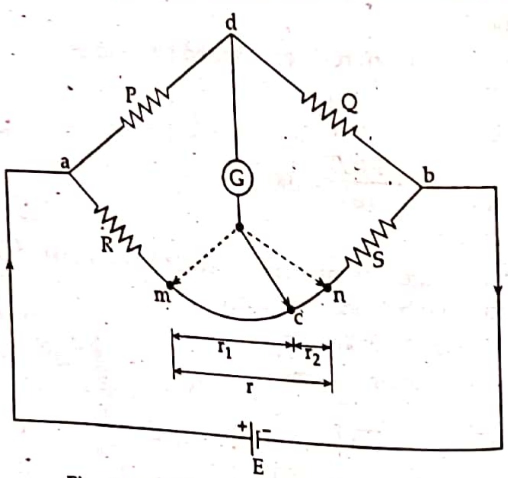


Figure: Illustrating principle of Kelvin bridge

Consider the bridge circuit shown above, where r represents the resistance of the lead that connects the unknown resistance R to standard resistance S. Two galvanometer connections indicated by dotted lines are possible. The connection may be either to point 'm' or to point 'n'. When the galvanometer is connected to point m, the resistance, r, of the connecting leads is added to the standard resistance, S, resulting in too

law an indication for unknown resistance, r, is added to the unknown resistance resulting in too high a value for R.

Suppose that instead of using point m, which gives a low result or n, which makes the result high, we make the galvanometer connection to any intermediate point 'c' as shown by full line in figure. If at point 'c' the resistor r is divided into two parts, r_1 and r_2, such that,

$$\frac{r_1}{r_2} = \frac{P}{Q} \qquad (1)$$

Then the presence of r_1, the resistance of connecting leads causes no error in the result, we have,

$$R + r_1 = \frac{P}{Q} \cdot (S + r_2)$$

But, $$\frac{r_1}{r_2} = \frac{P}{Q}$$

or, $$\frac{r_1}{r_1 + r_2} = \frac{P}{P + Q}$$

or, $$r_1 = \frac{P}{P + Q} \cdot r \qquad [\because r = r_1 + r_2]$$

and, $$R_2 = \frac{Q}{P + Q} \cdot r$$

Hence, $$\left(R + \frac{P}{P + Q} \cdot r\right) = \frac{P}{Q}\left(S + \frac{Q}{P + Q} \cdot r\right)$$

or, $$R = \frac{P}{Q} \cdot S$$

Hence we conclude that making the galvanometer connection as at c, the resistance of leads does not affect the result.

The Kelvin double bridge incorporates the idea of a second set of ratio arms- hence the name double bridge and the use of four terminal resistors for the low resistance arms. The first ratio arms is P and Q. The second set of ratio arms, p and q is used to connect the galvanometer to a point c at the appropriate potential between points m and n to eliminate the effect of connecting lead of resistor r between the known resistance, R and the standard resistance, S.

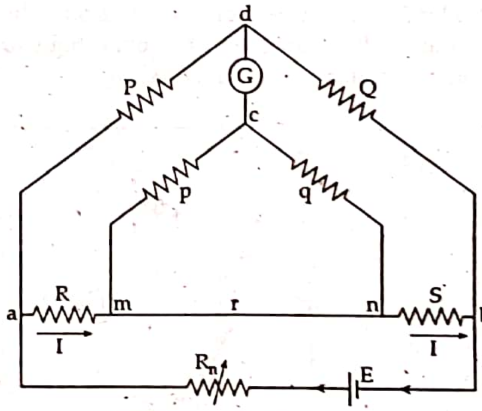


Figure: Kelvin double bridge

The ratio $\frac{p}{q}$ is made equal to $\frac{P}{Q}$. Under balanced conditions, there is no current through the galvanometer which means that the voltage drop between a and d, E_{od} is equal to the voltage drop E_{amo} between a and c.

Now, $E_{ad} = \frac{P}{P+Q} E_{ab}$

$$E_{ab} = I\left[R+S+\frac{(p+q)r}{p+q+r}\right]$$

and, $E_{amo} = I\left[R+\frac{p}{p+q}\left\{\frac{(p+q)\,r}{p+q+r}\right\}\right]$

For zero galvanometer deflection, $E_{ad} = E_{amo}$

or, $\left(\frac{P}{P+Q}\right) I\left[R+S+\frac{(p+q)\,r}{p+q+r}\right] = I\left[R+\frac{p}{p+q}\left\{\frac{(p+q)\times r}{p+q+r}\right\}\right]$

or, $R = \frac{P}{Q}\cdot S + \frac{qr}{p+q+r}\left[\frac{P}{Q}-\frac{p}{q}\right]$

Now, if $\frac{P}{Q} = \frac{p}{q}$

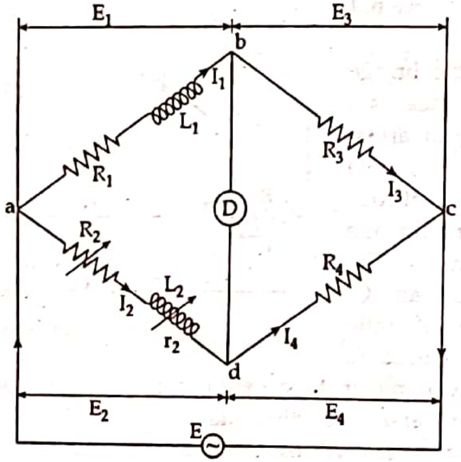
Then, $R = \frac{P}{Q}\cdot S$ (2)

Equation (2) is the usual working equation for the Kelvin bridges. It is desirable to keep r as small as possible in order to minimize the errors in case there is a difference between ratios $\frac{P}{Q}$ and $\frac{p}{q}$.

2.4.2 AC Bridge Circuit

2.4.2.1 Maxwell's Inductance Bridge

This bridge circuit measures an inductance by comparison with a variable standard self-inductance. The connections and the phasor diagrams for balance conditions are shown below,



Let, L_1 = Unknown inductance of resistance R_1

L_2 = Variable inductance of fixed resistance r_2

R_2 = Variable resistance connected in series with inductor L_2 and

R_3, R_4 = known non-inductive resistances

The general equation of the bridge,

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3$$

or, $Z_4 = \frac{Z_2 \cdot Z_3}{Z_1}$

or, $R_4 = \frac{(R_2 + j\omega L_2 + r_2) \times R_3}{R_1 + j\omega L_1}$

or, $R_1R_4 + R_4 j\omega L_1 = R_2R_3 + R_3\, j\omega L_2 + R_3 r_2$

Comparing the real terms and imaginary terms, we have

$$R_1R_4 = R_2\,R_3 + R_3\,r_2$$

or, $R_1 = \frac{R_3}{R_4}(R_2 + r_2)$ (1)

and, $R_4\,L_1 = R_3L_2$

or, $L_1 = \frac{R_3}{R_4}\cdot L_2$ (2)

Resistors R_3 and R_4 normally, a selection of values from 10, 100, 1,000 and 10,000 r_2 is a decade resistance box. In some cases, an additional known resistance may have to inserted in series with unknown coil in order to obtain balance.

The quality factor (Q-factor) of coil is given by,

$$Q = \omega \cdot \frac{L_1}{R_1} = \omega\,\frac{R_3\,L_2 \cdot R_4}{R_4 \cdot R_3\,(R_2 + r_2)} = \omega\,\frac{L_2}{(R_2 + r_2)}$$

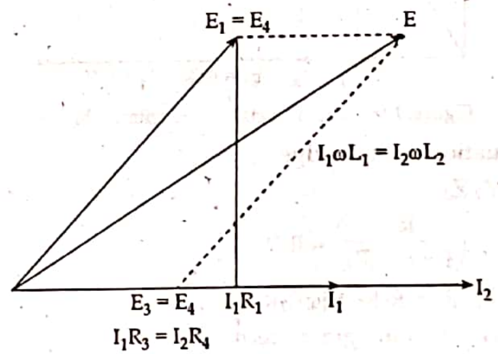


Figure: Phasor diagram on balance conditions

2.4.2.2 Maxwel's Inductance Capacitance Bridge

In this bridge, an inductance is measured by comparison with a standard variable capacitance.

Let, L_1 = Unknown inductance

R_1 = Effective resistance of inductor L_1

C_4 = Variable standard capacitor

R_2, R_3, R_4 = Known non-inductive resistances

The connections and the phasor diagram at the balance conditions are given below.

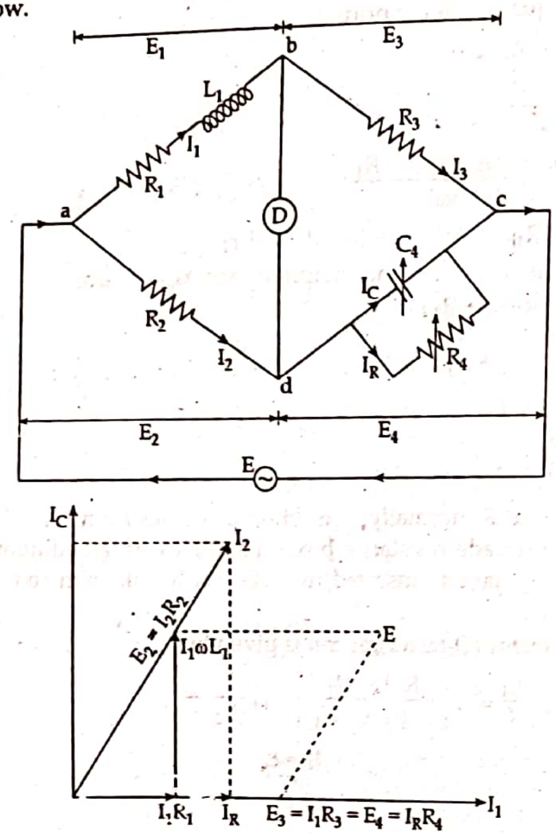


Figure: Max well inductance capacitance bridge

The general equation of the bridge,

$$Z_1 Z_4 = Z_2 Z_3$$

or, $$(R_1 + j\omega L_1)\left(\frac{R_4}{1 + \omega C_4 R_4}\right) = R_2 R_3$$

or, $$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega R_2 R_3 C_4 R_4$$

Separating the real and imaginary terms,

$\therefore$ $$R_1 = \frac{R_2 R_3}{R_4}$$

and, $$L_1 = R_2 R_3 C_4$$

Thus, Q-factor $= \dfrac{\omega L_1}{R_1} = \omega C_4 R_4$

Advantages

i) The two balance equations are independent if we choose R_4 and C_4 as variable elements.

ii) The frequency does not appear in any of the two equations.

iii) This bridge yields simple expression for unknown L_1 and R_1 in terms of known bridge elements.

iv) This bridge is very useful for measurement of a wide range of inductance at power and audio frequencies.

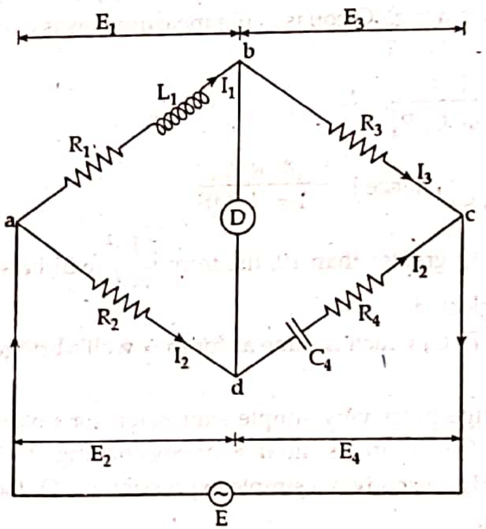
Disadvantages

i) This bridge requires a variable standard capacitor which may be very expensive if calibrated to a high degree of accuracy. Therefore sometimes a fixed standard capacitor is used, either because a variable capacitor is not available or because fixed capacitors have a higher degree of accuracy and are less expensive than the variable ones.

ii) The bridge is limited to measurement of low Q coils, $(1 < Q < 10)$. Measurement of high Q coils demands a large value of resistance R_4. The resistance boxes of such high values are very expensive. Thus for values of $Q > 10$, the max wells bridge is unsuitable.

2.4.2.3 Hay's Bridge

The Hay's bridge is a modification of max well's bridge. The connection diagram is shown below,



Let, R_1, R_3, R_4 = Known non-inductive resistances

C_4 = Standard capacitor

L_1 = Unknown inductance having a resistance R_1

At balance condition,

$$Z_1 Z_4 = Z_2 Z_3$$

or, $$(R_1 + j\omega L_1)(R_4 - j/\omega C_4) = R_2 R_3$$

or, $R_1 R_4 + \frac{L_1}{C_4} + j\omega L_1 R_4 - j\frac{R_1}{\omega C_4} = R_2 R_3$

Separating the real and imaginary terms,

$$R_1 R_4 + \frac{L_1}{C_4} = R_2 R_3$$

and, $L_1 = \frac{R_1}{\omega^2 R_4 C_4}$

On solving,

$$L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 C_4^2 R_4^2}$$

and, $R_1 = \frac{\omega^2 R_2 R_3 R_4 C_4^2}{1 + \omega^2 C_4^2 R_4^2}$

The Q-factor of the coil is,

$$Q = \frac{\omega L_1}{R_1} = \frac{1}{\omega R_4 C_4}$$

The expression for the unknown inductance and resistance contain the frequency term. Therefore it appears that the frequency of the source of supply to the bridge must be accurately known. This is not true for the inductance when a high Q coil is being measured, as is explained below,

Now,

$$L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 C_4^2 R_4^2}$$

But, $Q = \frac{1}{\omega R_4 C_4}$, hence $L_1 = \frac{R_2 R_3 C_4}{1 + (1/Q)^2}$

For a value of Q greater than 10, the term $\left(\frac{1}{Q}\right)^2$ will be smaller than $\frac{1}{100}$ and can be neglected.

Hence, $L_1 = R_2 R_3 C_4$ which is same as for max well's bridge.

Advantages

i) The bridge gives very simple expression for unknown inductance for high Q coils and is suitable for coils having $Q > 10$.

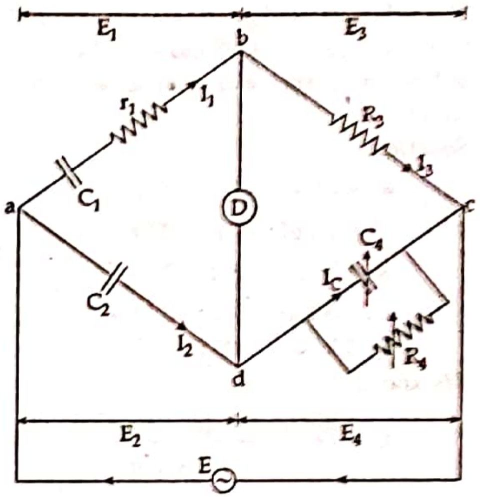
ii) This bridge also gives a simple expression for Q- factor.

Disadvantages

i) The Hay's bridge is suited for the measurement of high Q inductors, especially those inductors having a Q greater than 10. For inductors having Q values smaller than 10, the term $\left(\frac{1}{Q}\right)^2$ becomes rather important and thus cannot be neglected. Hence this bridge is not suited for measurement of coils having Q less than 10 and for these applications a max well's bridge is more suited.

2.4.2.4 Schering Bridge

The Schering bridge is one of the most important and useful circuits available for measurement of the capacitance, dielectric loss and power factor. It is widely used both for precision measurements of capacitors on low voltage and for study of insulation and insulating structures at high voltages. The connection diagram of the bridge under balance condition is shown in figure.



Let, C_1 = Capacitor whose capacitance is to be determined

r_1 = a series resistance corresponding the loss in the capacitor C_1

R_3 = a non-inductive resistance

C_4 = a variable capacitor

R_4 = a variable non-inductive resistance in parallel with variable capacitor C_4

C_2 = a standard capacitor

The general equation for ac bridge balance is,

$$Z_1 Z_4 = Z_2 Z_3$$

or, $\left(r_1 + \frac{1}{j\omega C_1}\right)\left(\frac{R_4}{1 + j\omega R_4 C_4}\right) = \frac{1}{j\omega C_2} \cdot R_3$

or, $\left(r_1 + \frac{1}{j\omega C_1}\right) R_4 = \frac{R_3}{j\omega C_2}(1 + j\omega C_4 R_4)$

or, $r_1 R_4 - j\frac{R_4}{\omega C_1} = -j\frac{R_3}{\omega C_2} + \frac{R_3 R_4 C_2}{C_2}$

Equating the real and imaginary terms, we obtain,

$$r_1 = \frac{C_4}{C_2} \cdot R_3$$

and, $C_1 = \frac{R_4}{R_3} \cdot C_2$

Two independent balance equations are obtained if C_4 and R_4 are chosen as the variable elements.

Dissipation factor,

$$D_1 = \tan\theta = \omega C_1 r_1 = \frac{\omega R_4 C_2 \cdot C_4 \cdot R_3}{R_3 C_2} = \omega C_4 R_4$$

Hence values of capacitance C_1 and its dissipation factor are obtained from the values of bridge elements at balance.

Example 7

The Schering bridge shown has the following constants $R_1 = 1.5\ K\Omega$, $C_1 = 0.4\ \mu F$, $R_2 = 3\ K\Omega$ and $C_3 = 0.4\ \mu F$ at frequency 1 kHz. Determine the unknown resistance and capacitance of the bridge circuit and dissipation factor.

Solution:

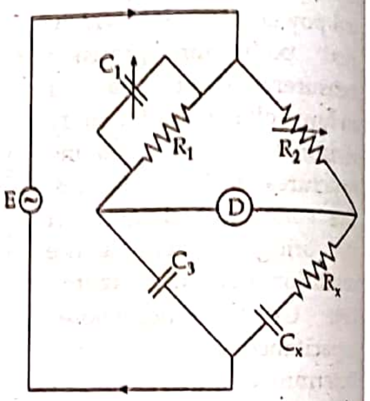
$R_1 = 1.5\ k\Omega = 1.5 \times 10^3\ \Omega$

$C_1 = 0.4\ \mu F = 0.4 \times 10^{-6}\ F$

$R_2 = 3\ k\Omega = 3 \times 10^3\ \Omega$

$C_3 = 0.4\ \mu F = 0.4 \times 10^{-6}\ F$

$f = 1\ kHz = 1000\ Hz$



We know,

$$R_x = \frac{R_2 C_1}{C_3} = \frac{3 \times 10^3 \times 0.4 \times 10^{-6}}{0.4 \times 10^{-6}}$$

$$= 3 \times 10^3\ \Omega = 3\ k\Omega$$

and, The unknown capacitance

$$C_x = \frac{R_1 C_3}{R_2} = (1.5 \times 10^3) \times \frac{(0.4 \times 10^{-6})}{3 \times 10^3} = 0.2 \times 10^{-6} = 0.2\ \mu F$$

and, Dissipation factor,

$$D = \frac{R_x}{X_x} = wC_x R_x = 2\pi f C_x R_x = 2\pi \times 1{,}000 \times 0.2 \times 10^{-6} \times 3 \times 10^3 = 3.77$$

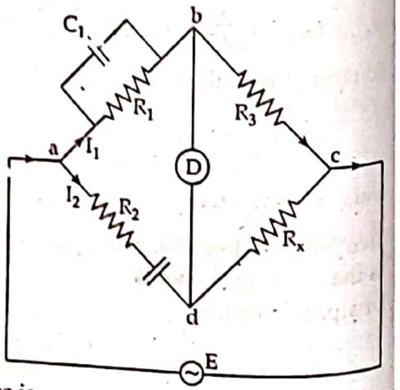
Advantages of Schering bridge

i) The balanced equation is independent of frequency.

ii) The arrangement of the bridge is less costly as compared to the other bridges.

2.4.2.5 Wien Bridge

The Wien bridge is primary known as a frequency determining bridge. A Wien's bridge. *For example;* may be employed in a harmonic distortion analyzer, where it is used as notch filter, discriminating against one specific frequency. The Wien's bridge also finds application in audio and high frequency oscillators as the frequency determining device primarily used in ac bridges to measure frequency.



The general equation for the AC bridge is,

$$Z_1 Z_4 = Z_2 Z_3$$

or, $$\frac{1}{Y_1} \cdot Z_4 = Z_2 \cdot Z_3$$

or, $$\frac{1}{\left(\frac{1}{R_1} + j\omega C_1\right)} R_4 = \left(R_2 - \frac{j}{\omega C_2}\right) \cdot R_3$$

or, $$\frac{R_1}{1 + j\omega C_1 R_1} \cdot R_4 = \left(R_2 - \frac{j}{\omega C_2}\right) R_3$$

or, $$\frac{R_4}{R_3} = \frac{R_3}{R_1} + \frac{C_1}{C_2} + j\left(\omega C_1 R_2 - \frac{1}{\omega C_2 R_1}\right)$$

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2}$$

and, $$\omega C_1 R_2 - \frac{1}{\omega C_2 R_1} = 0$$ from which

$$\omega = \frac{1}{\sqrt{R_1 \cdot R_2 \cdot C_1 \cdot C_2}}$$

and, Frequency, $$f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

In most wien bridge, the components are so chosen that,

$$R_1 = R_2 = R \text{ and } C_1 = C_2 = C$$

Hence, $$\frac{R_4}{R_3} = \frac{R}{R} + \frac{C}{C} = 1 + 1 = 2$$

Thus, $$f = \frac{1}{2\pi RC}$$

This bridge is stable for measurement of frequencies from 100 Hz to 100 kHz.

Example 8

Determine the equivalent parallel resistance and capacitance that causes a Wien bridge to null with the following component values.

$R_2 = 2.8\ k\Omega$ $\qquad$ $C_2 = 4.8\ \mu F$

$R_1 = 20\ k\Omega$ $\qquad$ $R_3 = 80\ k\Omega$

$f = 2\ kHz$

Solution:

$R_2 = 2.8 \times 10^3\ \Omega$ $\qquad$ $C_2 = 4.8 \times 10^{-6}\ F$

$R_1 = 20 \times 10^3\ \Omega$ $\qquad$ $R_3 = 80 \times 10^3\ \Omega$

$f = 2 \times 10^3\ Hz$

We know that,

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_1}$$

and, $$C_1 = \frac{1}{\omega^2 R_1 R_2 C_2} = \frac{1}{(2\pi f)^2 R_1 R_2 C_2}$$

Putting value of C_1,

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{1}{\omega^2 R_1 R_2 C_2^2} = \frac{R_2}{R_1} + \frac{1}{(2\pi f)^2 R_1 R_2 C_2^2}$$

or, $$R_4 = \frac{(80 \times 10^3)}{(20 \times 10^3)}\left[2.8 \times 10^3 + \frac{1}{(2\pi \times 2 \times 10^3)^2 \times (4.8 \times 10^{-6})^2 \times (2.8 \times 10^3)}\right]$$

∴ $R_4 = 11.20\ K\Omega$

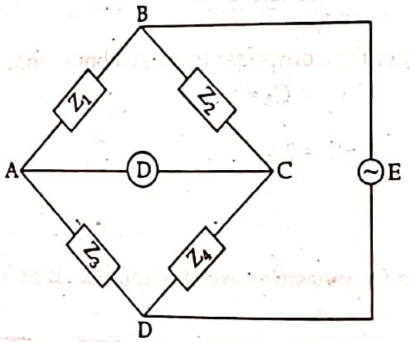
and, $$C_1 = \frac{1}{(2\pi \times 2 \times 10^3)^2 \times (4.8 \times 10^{-6})^2 \times (2.8 \times 10^3) \times (1.10 \times 10^3)}$$

∴ $C_1 = 42.04\ pF$

Example 9

An ac bridge circuit working at 1,000 Hz is shown. Arm AB is a 0.2 μF pure capacitance, Arm BC is a 500 Ω pure resistance, arm CD contains an unknown impedance and arm DA has a 300 Ω resistance in parallel with 0.1 μF capacitor. Find the R and C or L contains of arm CD considering it as a series circuit.

Solution:



$$Z_1 = \frac{1}{2\pi fC_1} = \frac{1}{2\pi \times 1000 \times 0.2 \times 10^{-6}} = 795.77\ \Omega$$

$Z_1 = 795.77\ \angle{-90°}$ since it is a pure capacitance

$Z_2 = 500\ \Omega\ \angle 0°$, since it is a pure resistance

$Z_4 = ?$

$Z_3 = R_3 \parallel X_3$

or, $$Y_3 = \frac{1}{R_3} + j\omega X_3 = \frac{1}{300} + j\,2\pi \times 1{,}000 \times 0.1 \times 10^{-6} = 3.33 \times 10^{-3} + j\,6.28 \times 10^{-4}$$

or, $Y_3 = 3.39 \times 10^{-3}\ \angle{-10.68°}$

or, $$Z_3 = \frac{1}{Y_3} = 294.99\ \angle{-10.68°}$$

Now, for bridge balance,

$Z_1 Z_4 = Z_2 Z_3$

or, $$Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{500\ \angle 0° \times 294.99\ \angle{-10.68°}}{795.77\ \angle{-90°}}$$

∴ $Z_4 = 185.35\ \angle 0° - 10.68° + 90°$

The positive angle for impedance indicates that the branch consists of a series R – L circuit.

For resistance, $R_4 = |Z_4| \cos\theta_4 = 185.35 \times \cos(79.32°) = 34.35\ \Omega$

For inductive reactance,

$X_{L4} = |Z_4| \sin\theta_4 = 185.35 \times \sin(79.32°) = 182.14\ \Omega$

so, $X_{L4} = \omega L_4$

or, $$L_4 = \frac{182.14}{\omega} = \frac{182.14}{2\pi \times 1{,}000}$$

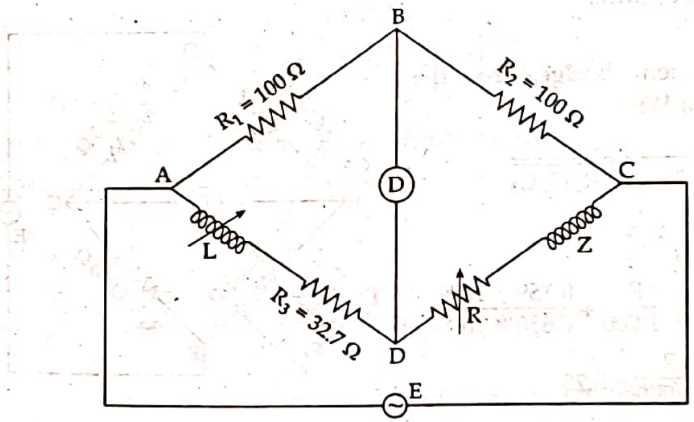
∴ $L_4 = 29\ mH$

Hence, $R_4 = 34.35\ \Omega$ and $L_4 = 29\ mH$.

Example 10

Four arms of the bridge are as follows. AB and BC are non-reactive resisters of 100 Ω each DA is standard variable inductor L in series with a resistance 32.7 Ω and CD comprises a standard variable resistor R in series with the coil of unknown impedance. Balance is obtained when L = 47.8 mH and R = 1.36 Ω, find the resistance and reactance of the coil.

Solution:



$Z_1 = R_1 = 100\ \Omega$

$Z_2 = R_2 = 100\ \Omega$

$Z_3 = R_3 + j\omega L_3 = (32.7 + j\omega \times 47.8)\ \Omega$

$Z_4 = R_4 + R + j\omega L_4$

For balanced condition,

$Z_1 Z_4 = Z_2 Z_3$

or, $R_1 \times (R_4 + R + j\omega L_4) = R_2 \times (R_3 + j\omega L)$

or, $R_1R_4 + RR_1 + j\omega L_4 R_1 = R_2R_3 + j\omega L R_2$

Equating real terms

$$R_1R_4 + RR_1 = R_2 R_3$$

or, $R_4 = \dfrac{R_2R_3 - R_1R}{R_1}$

or, $R_4 = \dfrac{100 \times 32.7 - 100 \times 1.36}{100}$

$\therefore$ $R_4 = 31.34\ \Omega$

Now, Equating the imaginary parts,

$$L_4R_1 = LR_2$$

or, $L_4 = L\dfrac{R_2}{R_1}$

or, $L_4 = 47.8 \times \dfrac{100}{100}$

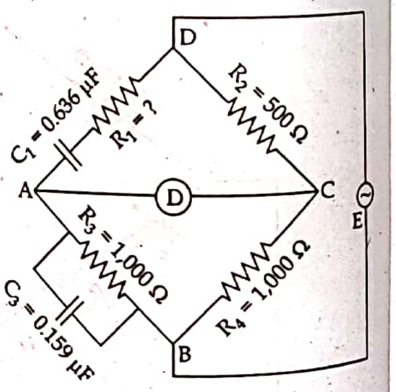
$\therefore$ $L_4 = 47.8$ mH.

Example 11

An AC bridge has the following constants arm AB, R = 1,000 Ω in parallel with C = 0.159 μF; BC, R = 1,000 Ω; CD = 500 Ω; DA, C = 0.636 μF in series with an unknown resistance. Find the frequency for which this bridge is in balance and determine the value of the resistance in arm DA to produce this balance.

Solution:

This is Wien bridge, so the frequency for Wien



Bridge is $f = \dfrac{1}{2\pi\sqrt{R_1 R_2 C_1 C_3}}$

and, $\dfrac{R_2}{R_4} = \dfrac{R_1}{R_3} + \dfrac{C_3}{C_1}$

or, $\dfrac{500}{1{,}000} = \dfrac{R_1}{1{,}000} + \dfrac{0.159 \times 10^{-6}}{0.636 \times 10^{-6}}$

or, $0.5 = \dfrac{R_1}{1{,}000} + 0.25$

or, $R_1 = 0.25 \times 1{,}000$

$\therefore$ $R_1 = 250\ \Omega$

and, $f = \dfrac{1}{2\pi\sqrt{0.636 \times 10^{-6} \times 0.159 \times 10^{-6} \times 250 \times 1{,}000}}$

$\therefore$ $f = 1{,}000\ \text{Hz} = 1\ \text{kHz}$

Hence the frequency for which this bridge is in balance is 1,000 Hz. And the value f resistance in arm DA to produce this balance is 250 Ω.

2.5 BOARD EXAM QUESTIONS SOLUTION

1. **Describe about various performance parameters of instrumentation system. [2011/S, 2012/S, 2014/F, 2013/F, 2015/S, 2016/S, 2017/F]**

Solution: See the definition of 2.3.

2. **Write short notes on dynamic parameters. [2011/F]**

Solution: See the definition of 2.3.2.

3. **Explain static characteristic of measurement system. [2014/F, 2019/F]**

Solution: See the definition of 2.3.1.

4. **Write short notes on environmental error. [2011/F, 2012/F, 2012/S]**

Solution: See the definition of 2.2.1. C (iii).

5. **Write short notes on IEEE standard. [2014/S, 2018/S]**

Solution: See the definition of 2.1.2.1 (v).

6. **Write short notes on standards of measurements. [2013/F]**

Solution: See the definition of 2.1.2.1.

7. **Write short notes on errors in instrumentation. [2015/S]**

Solution: See the definition of 2.2.1.

8. **Write short notes on probability of errors. [2016/F]**

Solution: See the definition of 2.2.3.

9. **Write short notes on probable error. [2016/S]**

Solution: See the definition of 2.2.3 (b).

10. **Write short notes on types of instruments. [2017/S]**

Solution: See the definition of 2.2.

11. **Define measurement system. [2018/S]**

Solution:

Measurement system is the complete process to obtain measurements. A system of measurement is a collection of units of measurement and rules relating then to each other.

12. **How can we measure the value of unknown inductance by using max well's bridge circuit? Explain with limitation. [11/F, 13/F, 18/S]**

Solution: See the definition of 2.4.2.2.

13. **How can we measure the self-inductance by comparing with a standard variable capacitance? Derive the relationship. [2012/F, 2014/F]**

Solution: See the definition of 2.4.2.2 (same as Q. No. 12).

14. **Explain Kelvin's bridge with its necessary diagram.** **[2013/S, 2014/S]**

Solution: See the definition of 2.4.1.2.

15. **What is the use of Wien bridge? Derive the expression for unknown components in Wien bridge. What are the limitations of Wien bridge?** **[2012/S, 2015/S, 2019/F]**

Solution: See the definition of 2.4.2.5.

Limitations:

i) Because of frequency sensitivity, Wien bridge is difficult to balance (unless wave form of applied voltage is purely sinusoidal.)

Use of Wien bridge:

i) To measure frequency

16. **Name the commonly used detectors for AC bridge. Derive the expression for unknown components in Schering bridge.** **[2016/F]**

Solution: See the definition of 2.4.2.4.

Commonly used detectors for AC bridges are:

i) An oscilloscope which is suitable for use with a very wide range of frequencies.
ii) Vibration galvanometer
iii) Head phones
iv) Various electronic detectors which use tuned circuit to detect current at current frequencies.

17. **Describe the Hay's bridge for measurement of inductance. Why is this bridge suited for measurement of inductance of high Q coils?** **[2017/F, 2018/F]**

Solution: See the definition of 2.4.2.3.

18. **Explain the conditions for DC bridge balance with necessary diagram.** **[2015/F, 2017/S]**

Solution:

Let us consider that the bridge circuit construct from the resistor element (*i.e.*, for dc bridge circuit), one of which has an unknown value, R_x and known resistance R_1, R_2 and R_3, where R_3 is the variable resistance.

The two opposite corners of the square are connected to a source of electric current, such as battery, while the galvanometer is connected across the other two

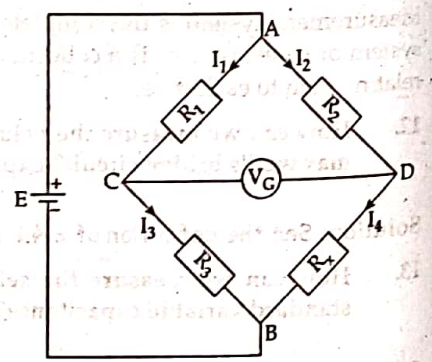


Figure: Simple bridge circuit

Solution:

Magnitude of the limiting error is 1.5% of 10 A = $\pm\frac{1.5}{100} \times 10 = \pm 0.15$ A

and, % error of a meter indication of 2.5 A

$$\% e = \frac{0.15}{2.5} \times 100\% = 6\%$$

Hence, limiting values of current are

$I = 2.5 \pm 0.15$ A = 2.35 A, 2.65 A

Hence, limiting values of current is 2.35 A, 2.65 A and percentage limiting error is 6%.

21. **A bridge is balanced at 1,000 Hz and has the following constants; AB = 0.2 μF pure capacitances, BC = 500 Ω pure resistance, CD = unknown, DA = 300 Ω in parallel with C = 0.1 μF. Find R and C or L constants of arm CD, considered as a series circuit.** **[2012/S]**

Solution: See the solution of example number 9.

22. **The measurements of current in a branch yield values of 50.2, 50.6, 49.7, 49.2, 48.9, 51.1, 50.3, 49.9, 50.3 and 51.0 mA. Assuming only the random errors are present in the measurement system, calculate,**
   i) **Average value**
   ii) **Standard deviation**
   iii) **Probable error of the reading**

Solution:

n = 10

i) Average value or mean $(\bar{x})$

$$= \frac{50.2 + 50.6 + 49.7 + 49.2 + 48.9 + 51.1 + 50.3 + 49.9 + 50.3 + 51}{10}$$

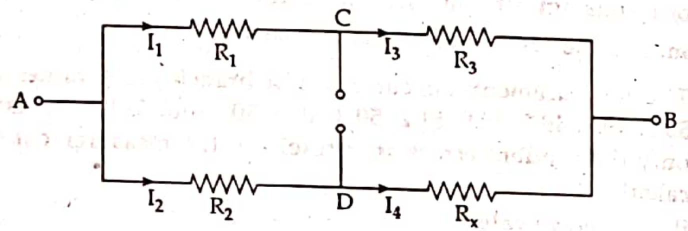
$$= \frac{501.20}{10} = 50.12 \text{ mA}$$

Measured value (x)	Deviation, $d = x - \bar{x}$	d^2
50.2	0.08	0.0064
50.6	0.48	0.2304
49.7	- 0.42	0.1764
49.2	- 0.920	0.8464
48.9	- 1.22	1.4884
51.1	0.98	0.9604
50.3	0.18	0.0324
49.9	- 0.22	0.0484
50.3	0.18	0.0324
51.0	0.88	0.7744
	$\Sigma\lvert d\rvert = 5.56$	$\Sigma d^2 = 4.5960$

opposite corners. The current through the galvanometer depends on the potential difference between the two point C and D. To find the value of unknown resistor, we have to first balance to bridge circuit. The bridge is balanced when there is no current flow through the galvanometer. The potential difference across the galvanometer is zero.

The variable resistor is adjusted until the galvanometer reads zero. Then the ratio between the variable resistor and its neighbor is equal to the ratio between the unknown resistor and its neighbor and this enables the value of the unknown resistor to be calculated. The current through the galvanometer becomes 0 volt when the voltage from point C to point A equals the voltage from point D to point A or by referring to the other battery terminal, when the voltage from the point C to B equals voltage from point D to B. Hence, bridge is balanced when

$$I_1 R_1 = I_2 R_2 \qquad (1)$$



Hence,

$$I_1 = I_3 = \frac{E}{R_1 + R_3} \qquad (2)$$

and, $$I_2 = I_4 = \frac{E}{R_2 + R_x} \qquad (3)$$

From (1), (2) and (3); we get,

or, $$\frac{R_1}{R_1 + R_3} = \frac{R_1}{R_2 + R_x}$$

or, $$R_1R_2 + R_1 R_x = R_1R_2 + R_2R_3$$

or, $$R_1 R_x = R_2 R_3$$

$\therefore$ $$\frac{R_1}{R_2} = \frac{R_3}{R_x}$$

This is the condition for balance of DC bridge *i.e.,* the DC bridge is balanced if product of resistance of any two opposite arms is equal to that of other two.

19. Write short notes on Wheatstone bridge. **[2017/F]**

Solution: See the definition of 2.4.1.1.

20. A (0 – 10) A ammeter has a guaranteed accuracy of 1.5% full scale reading. The current measured by this instrument is 2.5 A. Calculate the limiting values of current and percentage limiting error. **[2018/F]**

ii) Standard deviation

$$s = \sqrt{\frac{\Sigma d^2}{n-1}} = \sqrt{\frac{4.5960}{10-1}} = \sqrt{0.520667}$$

$\therefore$ $s = 0.7147$ mA

iii) Probable error of one reading,

$$r_1 = \pm 0.6745 \times s = \pm 0.6745 \times 0.7147 = \pm 0.4820 \text{ mA}.$$

23. Ten measurements of resistance of resistors are 101.2 Ω, 101.7 Ω, 101.8 Ω, 101.5 Ω, 101.3 Ω, 101.2 Ω, 101.4 Ω, 101.3 Ω and 101.2 Ω. Assuming that only random errors are present, calculate;

i) Arithmetic mean

ii) Standard deviation

iii) Probable error **[2011/F, 2016/F (similar)]**

Solution:

Here, n = 10

i) Arithmetic mean

$$\bar{x} = \frac{101.2+101.7+101.8+101.5+101.3+101.2+101.4+101.3+101.2+101.0}{10}$$

$$= \frac{1013.6}{10} = 101.36\ \Omega$$

ii)

Measured value, x	$d = x - \bar{x}$	d^2
101.2	-0.16	0.0256
101.7	0.34	0.1156
101.8	0.44	0.1936
101.5	0.14	0.0196
101.3	-0.06	0.0036
101.2	-0.16	0.0256
101.4	0.04	0.0016
101.3	-0.06	0.0036
101.2	-0.16	0.0256
101.0	-0.36	0.1296
	$\Sigma\lvert d\rvert = 1.92$	$\Sigma d^2 = 0.5440$

$$\text{Standard deviation (s)} = \sqrt{\frac{\Sigma d^2}{n-1}} = \sqrt{\frac{0.5440}{10-1}} = 0.2459\ \Omega$$

iii) Probable error $= \pm 0.6745 \times s = \pm 0.6745 \times 0.2459 = \pm 0.1658\ \Omega$.

24. Ten measurements of resistance of resistors are 50 Ω, 50.1 Ω, 50.0 Ω, 50.3 Ω, 50.1 Ω, 50.2 Ω, 50.0 Ω, 50.3 Ω, 50 Ω, 50.2 Ω. Assume only random errors are present, calculate: **[2014/S, 2017/F]**

i) Arithmetic mean

ii) Standard deviation

iii) Probable error

Solution:

i) Arithmetic mean

$$\bar{x} = \frac{50 + 50.1 + 50 + 50.3 + 50.1 + 50.2 + 50 + 50 + 50.3 + 50.2}{10}$$

$$= \frac{501.20}{10} = 50.12\ \Omega$$

ii)

Measured value, x	$d = x - \bar{x}$	d^2
50	-0.120	0.0144
50.1	-0.020	0.0004
50	-0.120	0.0144
50.3	0.180	0.0324
50.1	-0.020	0.0004
50.2	0.080	0.0064
50	-0.120	0.0144
50	-0.120	0.0144
50.3	0.180	0.0324
50.2	0.080	0.0064
	$\Sigma\|d\| = 1.040$	$\Sigma d^2 = 0.1360$

Standard deviation (s) $= \sqrt{\frac{\Sigma d^2}{n-1}} = \sqrt{\frac{0.1360}{9}} = 0.1229\ \Omega$

iii) Probable error $= \pm 0.6745 \times 0.1229 = \pm 0.08291\ \Omega$

25. **The table given below lists a sample of experimental data:**

Value	3	4	5	6	7	8	9	10	11
Frequency of occurrence	1	2	3	6	7	6	4	2	1

Calculate:

i) Mean

ii) Standard deviation

iii) Probable error of one reading

iv) Probable error of mean [2017/S]

Solution:

$N = 1 + 2 + 3 + 6 + 7 + 6 + 4 + 2 + 1 = 32$

x	f	x = X – A	fx	fx²
3	1	-3	-3	9
4	2	-2	-4	8
5	3	-1	-3	3
6	6	0	0	0
7	7	1	7	7
8	6	2	12	24
9	4	3	12	36
10	2	4	8	32
11	1	5	5	25
	N = 32		Σfx = 34	Σfx² = 144

Let, A = 6

i) Mean, $\bar{x} = A + \frac{\Sigma fx}{N} = 6 + \frac{34}{32} = 7.0625$

ii) Standard deviation,

$$s = \sqrt{\frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2} = \sqrt{\frac{144}{32} - \left(\frac{34}{32}\right)^2} = \sqrt{4.5 - 1.1289}$$

$\therefore\ s = 1.8360$

iii) Probable error of one reading,

$r_1 = \pm 0.6745 \times s = \pm 0.6745 \times 1.8360$

$\therefore\ r_1 = \pm 1.2384$

iv) Probable error of mean,

$$r_m = \frac{r_1}{\sqrt{n-1}} = \pm\frac{1.2384}{\sqrt{32-1}} = \pm 0.2224$$

26. **If $R_x = \frac{R_1 . R_2}{R_3}$ Where $R_1 = 100 \pm 1\%$, $R_2 = 200 \pm 2.5\%$, $R_3 = 100 \pm 2\%$, find,**

i) The nominal value ii) The limiting error

iii) The percentage limiting error of R_x. [2018/S]

Solution:

$$R_x = \frac{R_1 \cdot R_2}{R_3}$$

$R_1 = 100 \pm 1\%$

$R_2 = 200 \pm 2.5\%$

$R_3 = 100 \pm 2\%$

i) Nominal value of

$$R_x = \frac{R_1 \cdot R_2}{R_3} = \frac{100 \times 200}{100} = 200\ \Omega$$

ii) The limiting error in ohms of R_x:

The value of R_1 and R_2 must be high and the value of R_3 must be low.

i.e., $R_x = \frac{(100 + 1) \times (250 + 6.25)}{(100 + 2)} = \frac{101 \times 256.25}{102} = 253.738\ \Omega$

Now, the limiting error is nominal value minus highest value of unknown resistor, *i.e.,*

Limiting error = $(200 - 253.738)\ \Omega = -53.738\ \Omega$

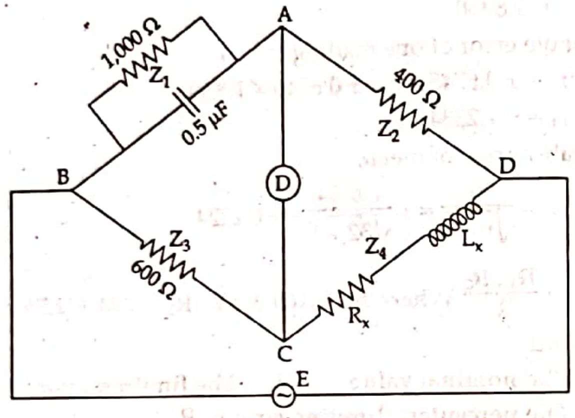
Hence, limiting error of R_x is $\pm 53.738\ \Omega$

iii) % limiting error of $R_x = \frac{\text{Limiting error}}{\text{Nominal value}} \times 100\%$

$$= \pm\frac{53.738}{200} \times 100\% = \pm 26.869\%$$

27. The arms of max well's ABCD bride are as follows: AB is an non inductive resistance 1,000 Ω in parallel with a capacitance of 0.5 μF, BC is an non inductive resistance of 600 Ω, CD is an inductive impedance of unknown value and DA is non inductive resistance of 400 Ω. If the balance is obtained under these conditions, find the values of unknown of arm CD. [2011/S]

Solution:



$Z_2 = 400\ \Omega$

$Z_3 = 600\ \Omega$

$Z_4 = R_x + j\omega L_x$

$$Z_1 = \frac{R_1}{1 + R_1 j\omega C_1} = \frac{1{,}000}{1 + 1{,}000\, j\omega \times 0.5 \times 10^{-6}}$$

Under balanced condition,

$Z_1 Z_4 = Z_2 Z_3$

or, $Z_4 = \dfrac{Z_2 \cdot Z_3}{Z_1}$

or, $Z_4 = Z_2 \cdot Z_3 \cdot y_1$

or, $Z_4 = 400 \times 600 \times \left(\dfrac{1}{R_1} + j\omega C_1\right)$

or, $Z_4 = 240000 \times \left(\dfrac{1}{1000} + j\omega \times 0.5 \times 10^{-6}\right)$

or, $R_x + j\omega L_x = \dfrac{2{,}40{,}000}{1{,}000} + j\omega \times 0.5 \times 10^{-6} \times 2{,}40{,}000$

or, $R_x + j\omega L_x = 240 + j\omega \times 0.5 \times 10^{-6} \times 2{,}40{,}000$

Comparing real and imaginary terms,

or, $R_x = 240\ \Omega$

and $j\omega L_x = j\omega \times 0.5 \times 10^{-6} \times 2{,}40{,}000$

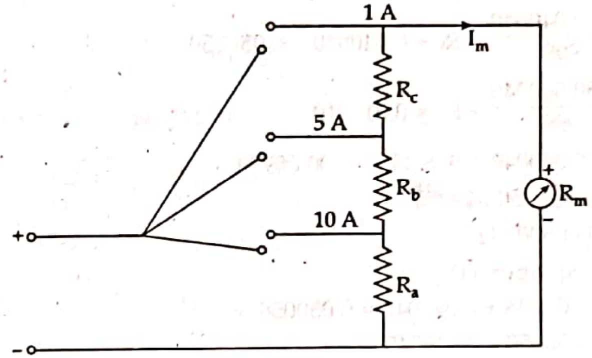
∴ $L_x = 0.12$ H

Hence, value of unknown of arm CD is 240 Ω and 0.12 H.

28. Design on Ayrton shunt to provide an ammeter with current ranges of 1 A, 5 A and 10 A. D' Arsonval movement with internal resistance R_m = 50 Ω and full deflection of 1 mA current is used in the configuration. [2011/F, 2013/S]

Solution:

$R_m = 50\ \Omega$



On 1 A range

$R_a + R_b + R_c$ is in parallel with 50 Ω, since full scale deflection current is 1 mA. The shunt requires to pass current of value,

∴ 1 A − 1 mA = 999 mA

so, $R_a + R_b + R_c = 1 \times \dfrac{50}{999} = 0.050050\ \Omega$ (1)

On 5 A range,

$(R_b + R_a) \parallel (R_m + R_c)$

or, $R_a + R_b = 1 \times \dfrac{(R_c + 50)}{5 - 1\text{ mA}} = \dfrac{R_c + 50}{4999}$ (2)

On 10 A range,

$R_a \parallel (R_b + R_c + R_m)$

or, $Ra = 1 \times \dfrac{(R_b + R_c + R_m)}{10\text{ A} - 1\text{ mA}}$

∴ $R_a = \dfrac{R_b + R_c + 50}{9{,}999}$ (3)

From equation (1) and (2);

$\dfrac{(R_c + 50)}{4{,}999} + R_c = 0.050050$

or, $R_c + 50 + 4{,}999\, R_c = 250.1999$

or, $5{,}000\, R_c = 200.1999$

or, $R_c = \dfrac{200.1999}{5{,}000}$

∴ $R_c = 0.040040\ \Omega$

From equation (3);

$$R_a = \frac{R_b + 0.040040 + 50}{9,999}$$

or, $$R_a = \frac{R_b + 50.040040}{9,999}$$

Putting value of R_a in equation (1),

or, $$\frac{R_b + 50.040040}{9,999} + R_b + 0.040040 = 0.050050$$

or, $$\frac{R_b + 50.040040}{9,999} + R_b = 0.010010$$

or, $R_b + 50.040040 + 9,999\, R_b = 100.08999$

or, $10,000\, R_b = 50.049950$

$\therefore$ $R_b = 0.005005\ \Omega$

Hence, from equation (1),

$R_a + 0.005005 + 0.040040 = 0.050050$

or, $R_a = 0.050050 - 0.045045$

$\therefore$ $R_a = 0.005005\ \Omega$

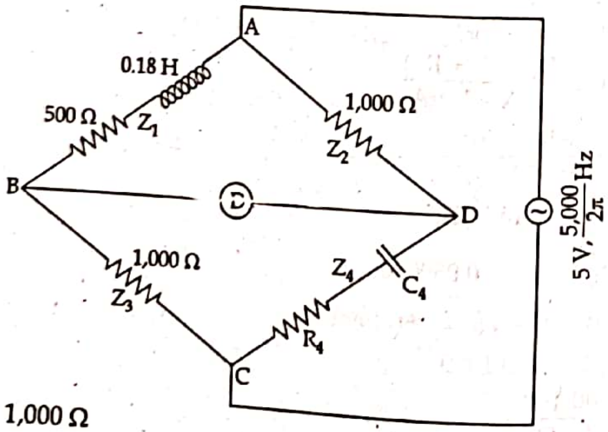
Thus, $R_a = 0.005005\ \Omega$

$R_b = 0.005005\ \Omega$

$R_c = 0.04004\ \Omega$

29. **In a balanced network, AB is a resistance of 500 Ω in series with an inductor of 0.18 H, BC and DA are non-inductive resistances of 1000 Ω each and CD consists of a resistance R in series with a capacitor C. A potential difference of 5 V at a frequency 5000/2π is applied between points A and C. Determine the values of R and C.** **[2012/F, 2014/F]**

Solution:



$Z_2 = 1,000\ \Omega$

$Z_3 = 1,000\ \Omega$

$Z_1 = (500 + j\omega\, 0.18)\ \Omega$

$$Z_4 = R_4 + \frac{1}{j\omega C_4}$$

Under balanced condition,

$Z_1 \cdot Z_4 = Z_2 Z_3$

or, $$(500 + j\omega\, 0.18) \times \left(R_4 + \frac{1}{j\omega C_4}\right) = 1,000 \times 1,000$$

or, $$500\, R_4 + \frac{500}{j\omega C_4} + j\omega\, R_4\, 0.18 + \frac{j\omega\, 0.18}{j\omega\, C_4} = 10,00,000$$

or, $$500\, R_4 - \frac{500\, j}{\omega\, C_4} + j\omega\, R_4\, 0.18 + \frac{0.18}{C_4} = 10,00,000$$

Comparing real and imaginary terms,

or, $$500\, R_4 + \frac{0.18}{C_4} = 10,00,000 \quad (1)$$

and, $$-\frac{500\, j}{\omega C_4} + j\omega R_4\ 0.18 = 0$$

Since, $$\omega = 2\pi f = 2\pi \times \frac{5,000}{2\pi} = 5,000 \text{ rad/sec}$$

or, $$-\frac{500}{5,000 \times C_4} \times 5,000\, R_4 \times 0.18 = 0$$

or, $$-\frac{0.10}{C_4} + 900\, R_4 = 0$$

or, $$900\, R_4 = \frac{0.10}{C_4}$$

or, $$R_4 = \frac{0.10}{900\, C_4}$$

Hence,

or, $$500 \times \frac{0.10}{900 \times C_4} + \frac{0.18}{C_4} = 10,00,000$$

or, $$\frac{0.0556}{C_4} + \frac{0.18}{C_4} = 10,00,000$$

or, $0.0556 + 0.18 = 10,00,000 \times C_4$

or, $$\frac{0.2355}{10,00,000} = C_4$$

$\therefore$ $C_4 = 2.356 \times 10^{-7}$ F

and, $$R_4 = \frac{0.10}{900 \times 2.356 \times 10^{-7}}$$

$\therefore$ $R_4 = 471.60\ \Omega$

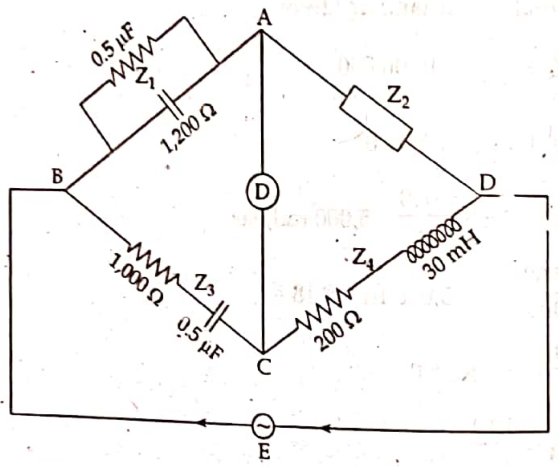
Hence, value of R is 471.60 Ω and C is 0.2356 μF of arm CD.

30. Explain the following with reference to instrumentation system.
   i) Accuracy and precision
   ii) Sensitivity and resolution
   iii) Linearity and hysteresis [2013/S]

Solution: See the definition of 2.3.1 i.e., Static Characteristics.

31. A 1,000 Hz bridge has following constants.
   Arm AB: R = 1200 Ω in parallel with C = 0.5 μF,
   Arm BC: R = 1,000 Ω in series with C = 0.5 μF,
   Arm CD: L = 30 mH in series with R = 200 Ω.
   Find the constants of arm DA to balance the bridge. [2013/F, 2015/F, 2017/F]

Solution:



Let, frequency (f) = 1,000 Hz

$$Z_4 = R_4 + j\omega L_4 = (200 + j\omega \times 30 \times 10^{-3})\ \Omega$$

$$Z_2 = ?$$

$$Z_1 = \frac{R_1}{1 + j\omega R_1 C_1} \qquad \left(\because y_1 = \frac{1}{Z_1} = \left(\frac{1}{R_1} + j\omega C_1\right)\right)$$

so, $$Z_1 = \frac{1{,}200}{1 + j\omega \times 1{,}200 \times 0.5 \times 10^{-6}} = \left(\frac{1{,}200}{1 + j\omega \times 0.0006}\right)\Omega$$

$$Z_3 = R_3 + \frac{1}{j\omega C_3}$$

$$= \left(1{,}000 + \frac{1}{j\omega \times 0.5 \times 10^{6}}\right)\Omega = \left(1{,}000 - \frac{j}{\omega\ 0.5 \times 10^{-6}}\right)\Omega$$

Under balanced condition,

$$Z_1 Z_4 = Z_2 Z_3$$

or, $$\frac{1{,}200}{1 + j\omega \times 0.0006} \times (200 + j\omega \times 30 \times 10^{-3}) = Z_2 \times \left(1{,}000 - \frac{j}{\omega \times 0.5 \times 10^{-6}}\right)$$

Since, ω = 2πf = 2π × 1,000 = 6283.185 Hz

so, $$\frac{1{,}200}{1 + j \times 6283.185 \times 0.0006} \times (200 + j \times 6283.185 \times 30 \times 10^{-3}$$

$$= Z_2 \times \left(1{,}000 - \frac{j}{6{,}283.185 \times 0.5 \times 10^{-6}}\right)$$

or, $$\frac{1200}{1 + j\,3.7699} \times (200 + j\,188.50) = Z_2 \times \left(1{,}000 - \frac{j}{0.003142}\right)$$

or, $$1{,}200 \times (59.861 - 37.172\,j) = Z_2 \times (1{,}000 - j\,318.309)$$

or, $$1{,}200 \times \frac{(59.861 - 37.172\,j)}{1{,}000 - j\,318.309}) = Z_2$$

or, $$Z_2 = 1{,}200 \times (0.06509 - j\,0.01645)$$

or, $$Z_2 = (78.12 - j\,19.741)\ \Omega$$

This result indicates that Z_2 has a resistor 78.12 Ω in series with capacitor at a frequency of 1000 Hz. So,

$$X_2 = \frac{1}{2\pi f\,C_2}$$

or, $$19.741 = \frac{1}{2\pi \times 1{,}000 \times C_2}$$

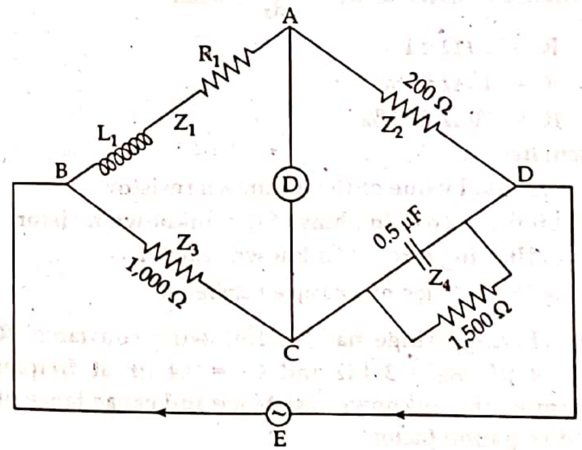
or, $$C_2 = \frac{1}{2\pi \times 1{,}000 \times 19.741}$$

∴ $$C_2 = 0.00008\ \text{F}$$

Hence, constants of arm DA are R_2 = 78.12 Ω and C_2 = 8 μF.

32. **The four arms of an ac bridge at balance are: Arm AB-an unknown inductance L_1 having an inherent resistance R_1; arm BC-a non-inductive resistance of 1000 Ω, Arm CD-a capacitor of 0.5 μF in parallel with a resistance of 1500 Ω; arm DA - a resistance of 200 Ω. Find the value of unknowns.** **[2014/S]**

Solution:



$$Z_1 = R_1 + j\omega L_1$$
$$Z_2 = 200\ \Omega$$
$$Z_3 = 1{,}000\ \Omega$$
$$Y_4 = \frac{1}{R_1} + j\omega C_1$$

or, $$Y_4 = \frac{1}{1{,}500} + j\omega \times 0.5 \times 10^{-6}$$

or, $$Z_4 = \frac{1}{Y_4} = \frac{1{,}500}{1 + j\omega \times 0.5 \times 10^{-6} \times 1{,}500}$$

∴ $$Z_4 = \frac{1{,}500}{1 + j\omega \times 0.000750}$$

Under balance condition,

$$Z_1 Z_4 = Z_2 Z_3$$

or, $$(R_1 + j\omega L_1) \times \frac{1{,}500}{1 + j\omega \times 0.00750} = 200 \times 1{,}000$$

or, $$R_1 + j\omega L_1 = \frac{200 \times 1{,}000}{1{,}500} \times (1 + j\omega \times 0.00750)$$

or, $$R_1 + j\omega L_1 = 133.333 \times (1 + j\omega \times 0.00750)$$

or, $$R_1 + j\omega L_1 = 133.333 + j\omega \times 1$$

Comparing real and imaginary terms,

∴ $$R_1 = 133.33\ \Omega$$

and, $$j\omega\, L_1 = j\omega .\ 1$$

∴ $$L_1 = 1\ \text{H}$$

Hence, unknown resistance is 133.33 Ω and inductance is 1 H of arm AB.

33. The resistance of an unknown resistor is determined by the Wheatstone bridge method. The solution for the unknown resistance is stated as $R_x = \frac{R_1 \cdot R_2}{R_3}$, where [2016/F]

$R_1 = 500\ \Omega \pm 1\%$

$R_2 = 615\ \Omega \pm 1\%$

$R_3 = 100\ \Omega \pm 0.5\%$

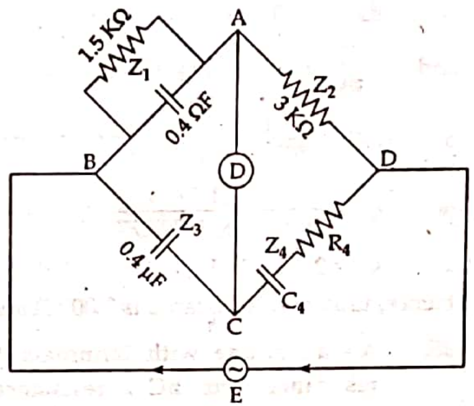
Calculate:

i) Nominal value of the unknown resistor
ii) Limiting error in ohms of the unknown resistor
iii) % limiting error of unknown resistor

Solution: *See the solution of example number 5.*

34. The Schering bridge has the following constants, R_1 = 1.5 kΩ, C_1 = 0.4 μF, R_2 = 3 kΩ and C_3 = 0.4 μF at frequency 1 kHz. Determine the unknown resistance and capacitance of the bridge and dissipation factor. [2015/S]

Solution:



$$R_1 = 1.5\ \text{k}\Omega = 1{,}500\ \Omega$$
$$C_1 = 0.4\ \mu\text{F} = 0.4 \times 10^{-6}\ \text{F}$$
$$R_2 = 3\ \text{k}\Omega = 3{,}000\ \Omega$$
$$C_3 = 0.4\ \mu\text{F} = 0.4 \times 10^{-6}\ \text{F}$$
$$f = 1\ \text{kHz} = 1{,}000\ \text{Hz}$$

Now,

∴ $$Z_2 = R_2 = 3{,}000\ \Omega$$

∴ $$Z_3 = \frac{1}{j\omega C_3}$$

$$= \frac{-j}{\omega C_3} = \frac{-j}{\omega \times 0.4 \times 10^{-6}}$$

$$= -j \times \frac{2.50 \times 10^6}{\omega}$$

∴ $$Z_1 = \frac{1}{Y_1}$$

or, $$Y_1 = \frac{1}{R_1} + j\omega C_1$$

or, $$Y_1 = \frac{1 + j\omega\, C_1 R_1}{R_1}$$

or, $$Z_1 = \frac{R_1}{1 + j\omega\, C_1 R_1} = \frac{1{,}500}{1 + j\omega \times 1{,}500 \times 0.4 \times 10^{-6}} = \frac{1{,}500}{1 + j\omega \times 6 \times 10^{-4}}$$

or, $$Z_4 = R_4 + \frac{1}{j\omega\, C_4} = R_4 - \frac{j}{\omega C_4}$$

Under balanced condition,

$$Z_1 Z_4 = Z_2\, Z_3$$

or, $$Z_4 = \frac{Z_2\, Z_3}{Z_1}$$

or, $$Z_4 = 3{,}000 \times \frac{\left(\frac{-j \times 2.50 \times 10^6}{\omega}\right)}{\left(\frac{1500}{1 + j\omega \times 6 \times 10^{-4}}\right)}$$

or, $$Z_4 = 3{,}000 \times \frac{2.50 \times 10^6}{2\pi \times 1{,}000} \times \frac{-j \times (1 + j \times 1{,}000 \times 6 \times 10^{-4} \times 2\pi)}{1{,}500}$$

or, $$Z_4 = 795.774 \times (-j + j^2 \times 3.769)$$

or, $$Z_4 = 795.774 \times (-j + 3.769)$$

or, $$R_4 - \frac{j}{\omega C_4} = 3{,}000 - j795.774$$

Comparing real and imaginary terms,

$\therefore \quad R_4 = 3{,}000\ \Omega$

and, $-\dfrac{j}{\omega C_4} = -j\,795.774$

or, $\omega C_4 = \dfrac{1}{795.774}$

or, $C_4 = \dfrac{1}{2\pi \times 1{,}000 \times 795.774}$

$\therefore \quad C_4 = 2.0 \times 10^{-7}$ F

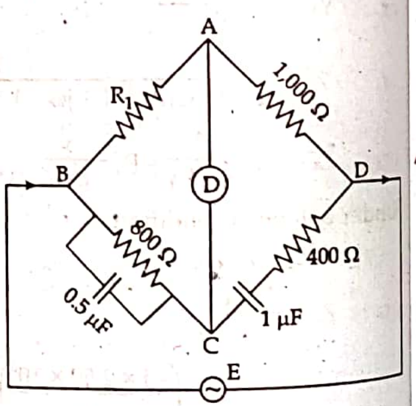
Hence, unknown resistance is 3,000 Ω and capacitance is 0.2 μF.

35. **An ac bridge with terminals ABCD has in arm AB, a pure resistance, arm BC a resistance of 800 Ω in parallel with a capacitor of 0.5 micro farad; arm CD, a resistance of 400 ohm in series with a capacitor of 1 micro farad; and arm DA, a resistance of 1,000 ohm.**
   i) **Obtain the value of frequency for which the bridge can be balanced.**
   ii) **Calculate the value of resistance in arm AB to produce balance.** **[2016/F]**

Solution:

$Z_1 = R_1 = ?$

$Z_2 = 1000\ \Omega$



$Z_3 = \dfrac{1}{Y_3}$

$Y_3 = \dfrac{1}{R_3} + j\omega C_3$

$= \dfrac{1}{800} + j\omega \times 0.5 \times 10^{-6}$

$= \dfrac{1 + j\omega \times 0.5 \times 10^{-6} \times 800}{800}$

$\therefore \quad Z_3 = \dfrac{800}{1 + j\omega \times 4 \times 10^{-4}}$

$Z_4 = R_4 - \dfrac{j}{\omega C_4} = 400 - \dfrac{j}{\omega \times 1 \times 10^{6}} = 400 - \dfrac{j \times 10^{-6}}{\omega}$

Under balanced condition,

$Z_1 Z_4 = Z_2 Z_3$

or, $Z_1 \times \left(400 - \dfrac{j \times 10^{6}}{\omega}\right) = 1000 \times \left(\dfrac{800}{1 + j\omega \times 4 \times 10^{-4}}\right)$

or, $R_1 \times \left(400 - \dfrac{j \times 10^{6}}{2\pi \times f}\right) = \dfrac{1000 \times 800}{1 + j \times 2\pi \times 4 \times 10^{-4} \times f}$

or, $R_1 \times \left(400 - \dfrac{j \times 1.595 \times 10^{5}}{f}\right) = \dfrac{800000}{1 + j \times f \times 2.513 \times 10^{-3}}$

or, $\left(\dfrac{400f - 1.595 \times 10^{5}\,j}{f}\right) \times (1 + j \times f \times 2.513 \times 10^{-3}) = \dfrac{8{,}00{,}000}{R_1}$

or, $\dfrac{400f + j\,1.0052 \times f^2 - 1.595 \times 10^5 j - j^2 \times 400.82\,f}{f} = \dfrac{8{,}00{,}000}{R_1}$

or, $400\,f + j\,1.0052\,f^2 - 1.595 \times 10^5\,j + 400.82\,F = \dfrac{8{,}00{,}000\,f}{R_1}$

Comparing real and imaginary terms,

$j\,1.0052\,f^2 - 1.595 \times 10^5\,j = 0$

or, $f^2 = \dfrac{1.595 \times 10^5}{1.0052}$

or, $f = \sqrt{158674.890}$

or, $f = 398.34$ Hz

or, $f = 400$ Hz

and, $400\,f + 400.82\,f = 8{,}00{,}000\,\dfrac{f}{R_1}$

or, $R_1 = \dfrac{8{,}00{,}000\,f}{800.82\,f}$

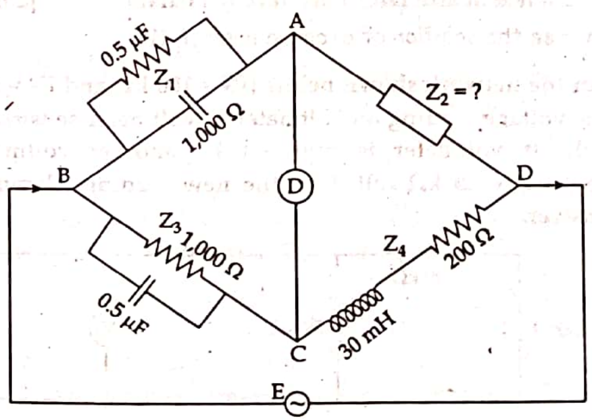
$\therefore \quad R_1 = 998.97\ \Omega \simeq 1{,}000\ \Omega$

i) The bridge can be balanced at 400 Hz frequency.

ii) Value of resistance in arm AB is 1,000 Ω to produce balance.

36. **An ac bridge has the following components: arm AB, R = 1000 Ω in parallel with C = 0.5 μF; BC, R = 1000 Ω in parallel with C = 0.5 μF; CD, L = 30 mH in series with R = 200 Ω. Name the unknown components and its value of arm DA to balance the bridge. Assume frequency, f = 1000 Hz.** **[2016/S, 2018/F]**

Solution:



$Z_2 = ?$

$Z_4 = R_4 + j\omega L_4 = (200 + j\omega \times 30 \times 10^{-3})\ \Omega$

$$Z_1 = \frac{1}{y_1} = \frac{1,000}{1 + j\omega \times 0.5 \times 10^{-6} \times 1,000} = \frac{1,000}{1 + j\omega \times 0.00050}$$

$$Z_3 = \frac{1,000}{1 + j\omega \times 0.00050}$$

$f = 1,000$ Hz

Under balanced condition,

$Z_1 Z_4 = Z_2 Z_3$

or, $$\left(\frac{1,000}{1 + j\omega \times 0.0005}\right) \times (200 + j\omega \times 30 \times 10^{-3}) = \left(\frac{1,000}{1 + j\omega \times 0.0005}\right) \times Z_2$$

or, $Z_2 = 200 + j\omega \times 30 \times 10^{-3}$

NOTE

$(A \pm jB)$; if $+jB \rightarrow$ Inductance

If $-jB \rightarrow$ Capacitance

Hence the arm AD consists of resistance and inductance *i.e.,* R – L circuit.

or, $R_2 + j\omega L_2 = 9200 + j\omega \times 30 \times 10^{-3})\ \Omega$

$\therefore$ $R_2 = 200\ \Omega$

and, $j\omega L_2 = j\omega \times 30 \times 10^{-3}$

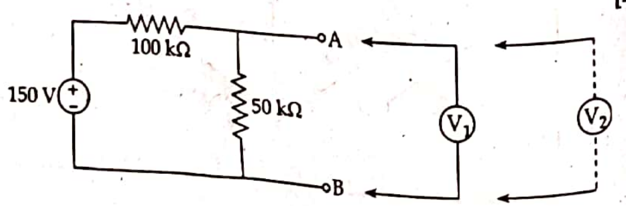
$\therefore$ $L_2 = 30 \times 10^{-3}$ H = 30 mH

Hence, $Z_2 = (200 + j \times 2p \times 1000 \times 30 \times 10^{-3})\ \Omega = (200 + j\, 60\, \pi)\ \Omega$

37. An ac bridge has the following constants: arm AB, R = 1,000 Ω in parallel with C = 0.159 μF, BC, R = 1,000 Ω, CD, R = 500 Ω; DA, C = 0.636 μF In series with an unknown resistance. Find the frequency for which this bridge is in balance and determine the value of the resistance in arm DA to produce his balance. [2017/S, 2018/F]

Solution: See the solution of example number 11.

38. For the network shown below (R_1 = 100 kΩ and R_2 = 50 kΩ), find the voltage reading on voltmeter, if voltmeter sensitivity is 1 kΩ/volt. If voltmeter is replaced by another voltmeter having sensitivity 25 kΩ/volt, find the new reading. Comment on the answer. [2017/S]



Solution:

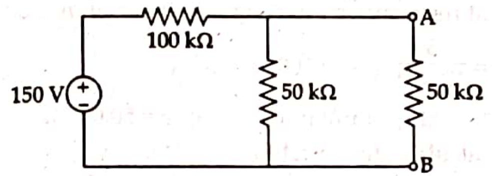
By voltage divider, the voltage across the points A – B is,

$$V_{AB} = 50\ k\Omega \times \frac{150\ V}{(100\ k\Omega + 50\ k\Omega)} = 50\ V$$

Now voltmeter one has sensitivity 1 kΩ/ volt, hence resistance offered by the voltmeter is,

$R = (1\ k\Omega/\text{volt}) \times 50 = 50\ k\Omega$

Hence, circuit becomes,



$$\therefore \quad V_{AB} = (50\ k\Omega \parallel 50\ k\Omega) \times \frac{150}{100\ k\Omega + [50\ k\Omega \parallel 50\ k\Omega]} = \frac{25 \times 150}{125} = 30\ V$$

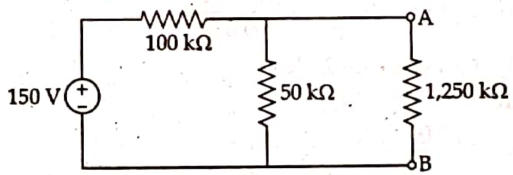
Thus the percentage error in the reading is,

$$\%\ e = \frac{\text{True value} - \text{Measured value}}{\text{True value}} = \left(\frac{50 - 30}{50}\right) \times 100\% = +40\%$$

Now the voltmeter is replaced by another one having sensitivity 25 kΩ/ volt. Thus it will offer the resistance,

$R = 25\ k\Omega/\text{volt} \times 50 = 1,250\ k\Omega$

Hence, circuit becomes,



$$\therefore \quad V_{AB} = (50\ k\Omega \parallel 1,250\ k\Omega) \times \frac{150}{100\ k\Omega + [50\ k\Omega \parallel 1,250\ k\Omega]}$$

$$= 48.077 \times \frac{150}{100 + 48.077} = 48.701\ V$$

Thus the percentage error in the reading is

$$\%\ e = \left(\frac{50 - 48.701}{50}\right) \times 100\% = +2.59\%$$

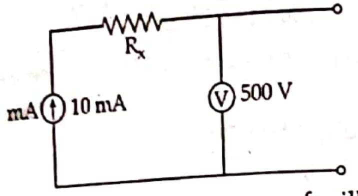
Thus the voltmeter with low sensitivity shows more error while the voltmeter with high sensitivity shows less error.

39. A voltmeter having a sensitivity of 2,000 Ω/V reads on its 500 V scale when connected across an unknown resistor in serial with milli ammeter. When milli ammeter reads 10 mA. Compute:

i) The apparent resistance of unknown resistor

ii) The actual resistance of unknown resistor
iii) The error due to loading effect of the voltmeter. [2016/S]

Solution:



The total circuit resistance neglecting resistance of milliammeter is,

$$R_T = \frac{V}{I} = \frac{500}{10 \times 10^{-3}} = 50{,}000\ \Omega$$

i) Apparent value of unknown resistor = 50,000 Ω

ii) Let us calculate the actual R_x

The resistance of the voltmeter be R_v

or, $R_V = 2{,}000\ \Omega/V \times 500$ as 500 V is full scale reading

$\therefore$ $R_V = 1{,}00{,}0000\ \Omega$

Thus,

$$R_T = R_x \parallel R_V$$

or, $$R_T = \frac{R_x \times R_v}{R_x + R_v}$$

or, $$50{,}000 = \frac{R_x \times 10{,}00{,}000}{R_x + 10{,}00{,}000}$$

or, $$R_x + 10{,}00{,}000 = R_x \times \frac{10{,}00{,}000}{50{,}000}$$

or, $$R_x + 10{,}00{,}000 = 20\ R_x$$

or, $$10{,}00{,}000 = 19\ R_x$$

or, $$R_x = \frac{10{,}00{,}000}{19}$$

$\therefore$ $R_x = 52{,}631.58\ \Omega$

This is the actual value of the unknown resistance.

iii) Error due to loading effect of the voltmeter:

$$\%\ \text{error} = \left(\frac{A_t - A_m}{A_t}\right) \times 100 = \left(\frac{52{,}631.58 - 50{,}000}{52{,}631.58}\right) \times 100$$

$$= 0.050 \times 100 = 5\%$$

40. Three resistances have the following ratings;

$R_1 = (20\ \Omega \pm 0.1)\ \Omega$

$R_2 = (20\ \Omega \pm 0.1)\ \Omega$

$R_3 = (60\ \Omega \pm 0.25)\ \Omega$

Determine the magnitude and limiting errors in ohms, if the resistances are connected in series. Also obtain percentage relative error in the resultant. [2019/F]

Solution:

$R_1 = 20 \pm 0.1\ \Omega$, $R_2 = 20 \pm 0.1\ \Omega$, $R_3 = 60 \pm 0.25\ \Omega$

Total resistances when connected in series,

$$R_T = R_1 + R_2 + R_3 = 20 + 20 + 60 = 100\ \Omega$$

$e_1 = 0.1\ \Omega = 0.5\%$

$e_2 = 0.1\ \Omega = 0.5\%$

$e_3 = 60\ \Omega = 0.417\%$

Now, $$\%\ e_T = \pm\left[\frac{R_1}{R_T} \times e_1 + \frac{R_2}{R_T} \times e_2\ \frac{R_3}{R_T} \times e_3\right]$$

$$= \pm\left[\frac{20}{100} \times 0.5\% + \frac{20}{100} \times 0.5\% + \frac{60}{100} \times 0.417\%\right]$$

$$= \pm\left[0.10\% + 0.10\% + 0.2520\%\right]$$

$$= \pm 0.450\%$$

Thus the limiting error in percentage = ± 0.450%

i.e., $e_T = \pm 0.45\%$

Thus, $$\%e_T = \pm \frac{\delta R_T}{R_T} \times 100$$

or, $$0.45 = \pm \frac{\delta R_T}{100} \times 100$$

$\therefore$ $\delta R_T = \pm 0.45\ \Omega$

i) Hence magnitude of resultant resistance is 100 Ω. If the resistance are connected in series and magnitude of limiting error is ± 0.45 Ω.

ii) Percentage relative error in the resultant is % e_T = ± 0.45%.

41. The AC bridge has the following constants. The arm AB is unknown. The arm BC has a resistance of 600 Ω. The arm AD is non-reactive and has a resistance of 400 Ω. The arm CD has a resistance of 1,000 Ω in parallel with capacitance of 0.5 µF. Calculate the values of unknown by using balancing the bridge. Also calculate the Q-factor of unknown. [2013/S]

Solution:

$Z_1 = ?$

$Z_2 = 400\ \Omega$

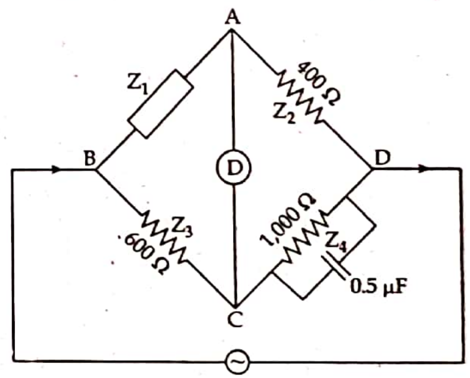
$Z_3 = 600\ \Omega$

$$Z_4 = \frac{1}{Y_1}$$

$$Y_1 = \frac{1}{R_1} + j\omega C_1$$

$$= \frac{1 + j\omega\ C_1R_1}{R_1}$$

$\therefore$ $$Z_4 = \frac{R_1}{1 + j\omega\ C_1R_1}$$



For bridge balancing,

$$Z_1 Z_4 = Z_2 Z_3$$

or, $$Z_1 = \frac{400 \times 600}{\left(\frac{1,000}{1 + j\omega \times 1,000 \times 0.5 \times 10^{-6}}\right)}$$

or, $$Z_1 = \frac{2,40,000}{\left(\frac{1,000}{1 + j\omega \times 0.0005}\right)}$$

or, $$Z_1 = \frac{2,40,000}{1,000} \times 1 + j\omega \times 0.0005$$

or, $$Z_1 = 240 \times (1 + j\omega \times 0.0005)$$

$\therefore$ $$Z_1 = (240 + j\omega\, 0.12)\ \Omega$$

Hence the components are resistance and inductance *i.e.*,

$$R_1 + j\omega L_1 = 240 + j\omega\, 0.12$$

Comparing real and imaginary terms,

$\therefore$ $$R_1 = 240\ \Omega$$

and, $$L_1 = 0.12\ H = 120\ mH$$

Then,

$$\text{Q-factor, } Q = \frac{\omega L_1}{R_1} = 2\pi f \times \frac{L_1}{R_1} = 2\pi \times 50 \times \frac{0.12}{240}$$

$\therefore$ $$Q = 0.157080 = \frac{\pi}{20}$$

2.6 ADDITIONAL QUESTIONS SOLUTION

1. **A voltmeter reads 111.15 V. The error taken from an error curve is 5.3%. Find the true value of the voltage.**

Solution:

$$A_m = 111.15\ V$$
$$\%e = 5.3\%$$

Now,

$$\%\ e = \frac{A_t - A_m}{A_t} \times 100$$

or, $$5.3 = \frac{A_t - 111.5}{A_t} \times 100$$

or, $$0.053\ A_t = A_t - 111.5$$

$\therefore$ $$A_t = 117.74\ V$$

2. **A 0 – 100 V voltmeter has 200 scale divisions which can be read to 1/2 division. Determine the resolution of the meter in volt.**

Solution:

$$1 \text{ scale division} = \frac{\text{Full scale division}}{\text{Number of division}} = \frac{100}{200} = 0.5$$

$\therefore$ $$\text{Resolution} = \frac{1}{2} \times \text{Scale division} = \frac{1}{2} \times 0.5 = 0.25\ V$$

3. **Three resistances are specified as**

$$R_1 = 200\ \Omega \pm 5\%$$
$$R_2 = 100\ \Omega \pm 5\%$$
$$R_3 = 50\ \Omega \pm 5\%$$

Determine the magnitude of the resultant resistance and the limiting error in percentage and in ohms if the resistance are connected in series.

Solution:

The resistances are in series,

$$R_T = R_1 + R_2 + R_3 = 200 + 100 + 50 = 350\ \Omega$$
$$e_1 = e_2 = e_3 = 5\%$$
$$a_1 = R_1 = 200\ \Omega$$
$$a_2 = R_2 = 100\ \Omega$$
$$a_3 = R{=}3 = 50\ \Omega$$

Now,

$$\%\ e_T = \pm\left[\frac{R_1}{R_T} \cdot e_1 + \frac{R_2}{R_T} \cdot e_2 + \frac{R_3}{R_T} \cdot e_3\right]$$

$$= \pm\left[\frac{200}{350} \times 5\% + \frac{100}{350} \times 5\% + \frac{50}{350} \times 5\%\right] = \pm 5\% \times \left[\frac{350}{350}\right]$$

$$= \pm 5\%$$

Thus the limiting error in percentage is,

$e_T = \pm 5\%$

Then,

$$\% \, e_T = \pm \frac{\delta R_T}{R_T} \times 100$$

or, $$5 = \pm \frac{\delta R_T}{350} \times 100$$

or, $$\delta R_T = \pm 5 \times \frac{350}{100}$$

$\therefore$ $$\delta R_T = \pm 17.5 \, \Omega$$

4. **The two resistances are specified as,**

$R_1 = 36 \, \Omega \pm 5\%$ and

$R_2 = 75 \, \Omega \pm 5\%$

Calculate the magnitude of limiting error in ohms and in percent if the two resisters are connected in parallel.

Solution

When two resistances are in parallel,

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{36 \times 75}{36 + 75} = 24.324 \, \Omega$$

Now to calculate the limiting error, consider

$$R_T = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

For the product, the limiting errors get added.

For R_1R_2, $e_1 = 5\% + 5\% = 10\%$

For summation $R_1 + R_2$, the error is,

$$e_1 = \pm \left[\frac{R_1}{R_1 + R_2} \times \text{error of } 1 + \frac{R_2}{R_1 + R_2} \times \text{error of } 2\right]$$

$$= \pm \frac{36}{111} \times 5\% + \frac{75}{111} \times 5\%$$

$$= \pm 5\%$$

Now R_T which is the quotient of $(R_1 R_2)$ and $(R_1 + R_2)$, the resultant error is addition of two errors.

$\therefore$ $$e_T = 10\% + 5\% = 15\%$$

Thus the resultant limiting error is $\pm$ 15% and the magnitude of the limiting error is,

$$e_T = \pm \frac{\delta R_T}{R_T}$$

or, $$0.15 = \pm \frac{\delta R_T}{24.324}$$

or, $$\delta R_1 = \pm 0.15 \times 24.324$$

$\therefore$ $$\delta R_T = \pm 3.65 \, \Omega$$

5. **The voltmeter reads 40 V on its 50 V range while an ammeter reads 50 mA on its 125 mA range, while used in a circuit. Both the instruments are guaranteed to be accurate within ± 2% at full scale deflection. Determine the limiting error of the power calculated.**

Solution:

For a voltmeter, the limiting error is 2% at full scale.

i.e., 50 V

$\therefore$ $$\delta a_1 = \frac{2}{100} \times 50 = 1 \text{ V}$$

For an ammeter, the limiting error is 2% at full scale.

i.e., 125 mA

$\therefore$ $$\delta a_2 = \frac{2}{100} \times 125 = 2.5 \text{ mA}$$

Hence the relative limiting error in voltmeter reading is,

$$e_1 = \frac{\delta a_1}{A_1} = \frac{1}{40}$$

While the relative limiting error in ammeter reading is,

$$e_2 = \frac{\delta a_2}{A_2} = \frac{2.5}{125}$$

Now,

$$P = IV$$

It is the product of two measurements a_1 and a_2,

Hence the resultant error is,

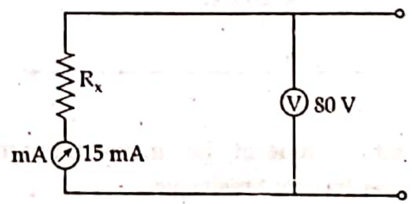
$$e_T = \pm [e_1 + e_2] \times 100 = \pm \left[\frac{1}{40} + \frac{2.5}{125}\right] \times 100$$

$\therefore$ $$e_T = \pm 4.5\%.$$

6. **A voltmeter having a sensitivity of 1.5 kΩ/volt reads 80 V on its 150 V range, when connected across an unknown resistor in series with a milli ammeter. The ammeter reads 15 mA. Calculate,**

   i) **Apparent resistance**
   ii) **Percentage relative accuracy**
   iii) **Actual resistance of unknown resistor**
   iv) **Error due to loading effect of voltmeter**

Solution:



The total circuit resistance, neglecting resistance of milliammeter is,

$$R_T = \frac{V}{I} = \frac{80}{15 \times 10^{-3}} = 5.333\ k\Omega$$

i) Apparent value of the resistance is 5.333 kΩ

ii) Le us calculate the actual R_x

The resistance of the voltmeter be R_v.

$\therefore$ $R_v = 1.5\ k\Omega/\text{volt} \times 150$ as 150 V is full scale reading

$= 225\ k\Omega$

Thus,

$$R_T = R_x \parallel R_v$$

$$\therefore \quad R_T = \frac{R_x \cdot R_v}{R_x + R_v}$$

$$\text{or,} \quad 5.33 = \frac{R_x \times 225}{R_x + 225}$$

$$\text{or,} \quad R_x + 225 = 42.19\ R_x$$

$$\therefore \quad R_x = 5.462\ k\Omega$$

This is the actual value of the unknown resistance.

iii) $$\%\ \text{error} = \frac{A_t - A_m}{A_t} \times 100 = \left(\frac{5.462 - 5.333}{5.462}\right) \times 100 = 2.36\%$$

iv) Relative accuracy, $\%\ A = (1 - |\text{error}|) \times 100$

$= (1 - 0.0236) \times 100$

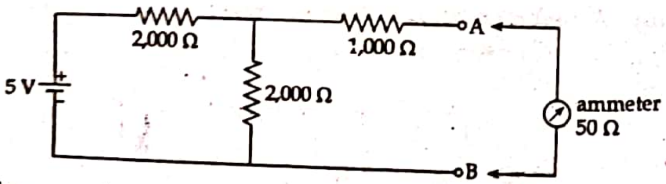
$= 97.63\%$

7. It is desired to measure the value of current in the 1000 Ω resistor as shown in figure below by connecting 50 Ω ammeter calculate:

i) Actual value of current

ii) % error and accuracy

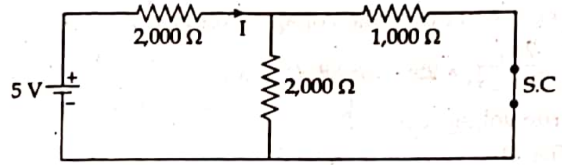
iii) Measured value of current



Solution:

i) To find the actual value of the current through 1,000 Ω, assume that the ammeter has zero resistance

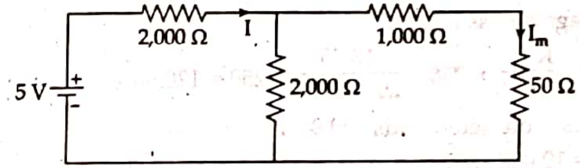
Hence shorting A – B, we get,



$$I = \frac{5}{2{,}000 + [2{,}000 \parallel 1{,}000]} = \frac{5}{2{,}000 + 666.66} = 1.875\ mA$$

This is the actual value of the current

ii) Now due to ammeter resistance of 50 Ω, the circuit reduces to



Now,

$$I = \frac{5}{2{,}000 + [2{,}000 \parallel 1{,}050]} = \frac{5}{2{,}000 + 688.525} = 1.859\ mA$$

$$\therefore \quad I_m = 1.859 \times \frac{2{,}000}{2{,}000 + 1{,}050} = 1.219\ mA$$

This is the measured value of the current

iii) $$\%\ \text{error} = \left(\frac{I_{true} - I_m}{I_{true}}\right) \times 100 = \left(\frac{1.25 - 1.219}{1.25}\right) \times 100 = 2.48\%$$

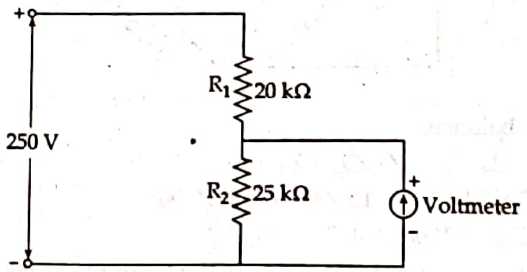
$\%$ accuracy $= (100 - 2.48)\% = 97.52\%$.

8. Figure shows a simple series circuit of R_1 and R_2 connected to a 250 V dc source. If the voltage across R_2 is to be measured by the voltmeters having

i) A sensitivity of 500 Ω/V

ii) A sensitivity of 10,000 Ω/V

Find which voltmeter will read more accurately. Both the meters are used on the 150 V range.



Solution:

By the voltage divider rule, the voltage across R_2,

$$V = \left(\frac{250}{20 + 25}\right) \times 25 = 138.88 \text{ V}$$

This is the true voltage across R_2.

Case i) $S = 500\ \Omega/V$

The voltmeter resistance,

$$R_v = S \times V = 500 \times 150 = 75 \text{ k}\Omega$$

$$\therefore \quad R_{eq} = R_2 \parallel R_v = \frac{25 \times 75}{25 + 75} = 18.75 \text{ k}\Omega$$

Hence voltage across R_{eq},

$$V = \frac{R_{eq}}{R_{eq} + R_1} \times 250 = \frac{18.75}{20 + 18.75} \times 250 = 120.96 \text{ V}$$

Thus the first voltmeter reads 120.96 V

Case ii) $S = 10{,}000\ \Omega/V$

The voltmeter resistance, $R_v = 5\text{ V} = 10{,}000 \times 150 = 1.5 \text{ m}\Omega$

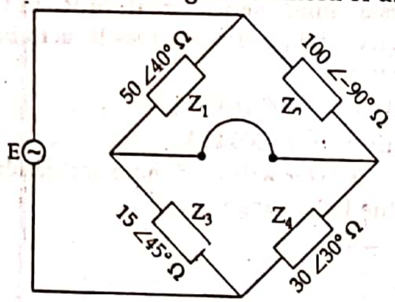
$$\therefore \quad R_{eq} = R_2 \parallel R_v = \frac{25 \times 1.5 \times 10^6 \times 10^3}{(25 \times 10^3 + 1.5 \times 10^6)} = 24.59$$

Hence the voltage across R_{eq} is

$$V = \left(\frac{R_{eq}}{R_{eq} + R_1}\right) \times 250 = \left(\frac{24.59}{24.59 + 20}\right) \times 250 = 137.86 \text{ V}$$

Thus the second voltmeter reads more accurately. Thus the high sensitivity voltmeter gives more accurate reading though the voltage range for both the meter is same.

9. **The arms of an ac bridge have impedances as shown in figure. Determine whether the bridge is balanced or unbalanced.**



Solution:

For the bridge balance,

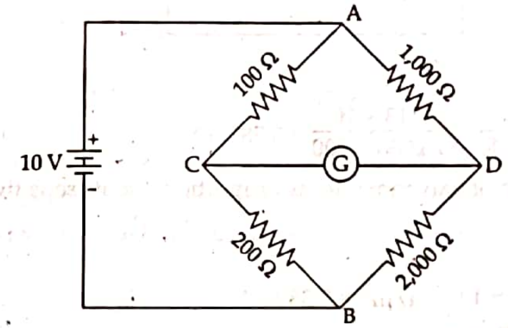
$$Z_1 \angle\theta_1 \times Z_4 \angle\theta_4 = Z_2 \angle\theta_2 \times Z_3 \angle\theta_3$$

or, $\quad 50 \angle 40° \times 30 \angle 30° = 15 \angle 45° \times 100 \angle -90°$

or, $\quad 1{,}500\ \angle 70° \neq 1500\ \angle -45°$

Since, $\angle\theta_1 + \angle\theta_4 \neq \angle\theta_2 + \angle\theta_3$, the given bridge is unbalanced.

10. **The Wheatstone bridge is shown below. The galvanometer has a current sensitivity of 12 mm/μA. The internal resistance of galvanometer is 200 Ω. Determine the deflection of the galvanometer caused due to 5 Ω unbalance in the arm BD.**



Solution:

From the given bridge,

$$R_1 = 100\ \Omega$$
$$R_2 = 1{,}000\ \Omega$$
$$R_3 = 200\ \Omega$$
$$R_4 = 2{,}000\ \Omega$$

Now,

$$R_1 R_4 = 100 \times 2{,}000 = 2{,}00{,}000$$
$$R_2 R_3 = 200 \times 1{,}000 = 2{,}00{,}000$$

For $R_4 = 2{,}000\ \Omega$, the bridge is balanced. But there is unbalance of 5 Ω in the resistance of arm BD *i.e.*, R_4

$$R_4 = 2{,}000 + 5 = 2{,}005\ \Omega$$

Due to this impalance, current will flow through the galvanometer.

By thevenin's equivalent,

$$V_{th} = E\left[\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4}\right]$$

$$= 10\left[\frac{200}{100 + 200} - \frac{2{,}005}{1{,}000 + 2{,}005}\right]$$

$$= 10\ [0.6667 - 0.6672]$$

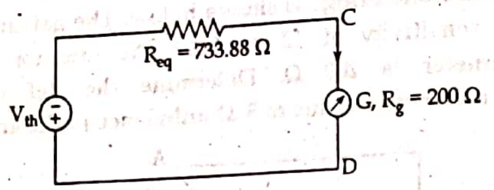
$$\therefore \quad V_{th} = -5.213 \text{ mV}$$

The negative sign indicates that D is more positive than

$$R_{eq} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

$$= \frac{100 \times 200}{100 + 200} + \frac{1{,}000 \times 2{,}005}{1{,}000 + 2{,}005}$$

$$= 733.88\ \Omega$$



Hence,

$$I_g = \frac{V_{th}}{R_{th} + R_g} = \frac{5.213 \times 10^{-3}}{733.88 + 200} = 5.582\ \mu A$$

Now, deflection of galvanometer is proportional to its sensitivity

$$S = \frac{D}{I_g}$$

or, $D = S.\ I_g = 12\ mm/\mu A \times 5.582$

∴ $D = 66.98\ mm$

11. Consider the basic ac bridge with four arms as,

$Z_2 = 400\ \angle{-60°}\ \Omega$

$Z_1 = 200\ \angle 60°\ \Omega$

$Z_3 = 300\ \angle 0°\ \Omega$

$Z_4 = 600\ \angle 30°\ \Omega$

Determine whether it is possible to balance the bridge under above conditions.

Solution:

The balance conditions for the magnitude and phases are given as follows,

$Z_1 \cdot Z_4 = Z_2 \cdot Z_3$ and

$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$

Applying condition of balance for magnitudes,

$Z_1Z_4 = Z_2Z_3$

or $200 \times 600 = 400 \times 300$

or, $1{,}20{,}000 = 1{,}20{,}000$

As the values of LHS and RHS are equal, the condition of balance for magnitude is satisfied.

Applying condition of balance for phases,

$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$

or, $60° + 30° = -60° + 0°$

or, $90° \neq -60°$

As values of LHS and RHS are not equal, the condition of balance for phase is not satisfied. Thus, for above given conditions, the bridge is in unbalanced condition, because even though the condition of balance for the magnitude is satisfied, the condition of balance for phase is not satisfied.

12. **Condition for AC Bridge balance**

Condition 1:

The products of the magnitude of impedances of the opposite arms must be equal.

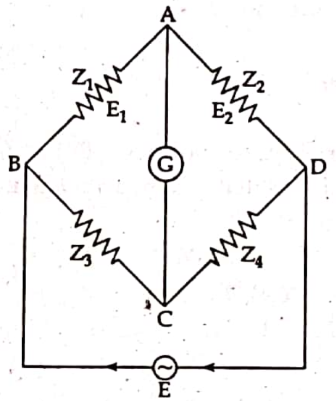
i.e., $Z_1 \cdot Z_4 = Z_2 \cdot Z_3$

Condition 2:

The sum of phase angles impedances in the opposite arms must be equal.

i.e., $\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$

It is not necessary for the four impedances to have identical phase angles or even for the impedances to be of the same kind, so long as the phase angle differences satisfy the above condition.



$$I_1 = I_3 = \frac{E}{Z_1 + Z_3} \quad (1)$$

and, $$I_2 = I_4 = \frac{E}{Z_2 + Z_4} \quad (2)$$

Also,

$E_1 = E_2$

$I_1Z_1 = I_2Z_2 \quad (3)$

So, on solving equation (1), (2) and (3);

$Z_1 \cdot Z_4 = Z_2 \cdot Z_3$

In the admittance form,

$Y_1 \cdot Y_4 = Y_2 \cdot Y_3$

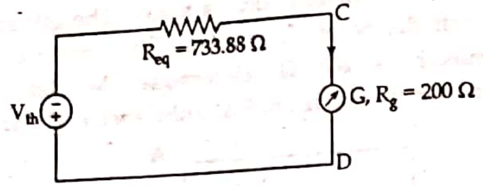
Writing impedance in phasor form,

$Z = Z\ \angle\theta$

so, $(Z_1 \angle\theta_1)\ (Z_4 \angle\theta_4) = (Z_2 \angle\theta_2)\ (Z_3 \angle\theta_3)$

or, $Z_1 Z_4\ (\angle\theta_1 + \theta_4) = Z_2Z_3\ (\angle\theta_2 + \angle\theta_3)$

The above equation indicates that two conditions must be satisfied simultaneously to balance the AC bridge.



Hence,

$$I_g = \frac{V_{th}}{R_{th} + R_g} = \frac{5.213 \times 10^{-3}}{733.88 + 200} = 5.582\ \mu A$$

Now, deflection of galvanometer is proportional to its sensitivity

$$S = \frac{D}{I_g}$$

or, $D = S.\ I_g = 12\ mm/\mu A \times 5.582$

$\therefore$ $D = 66.98\ mm$

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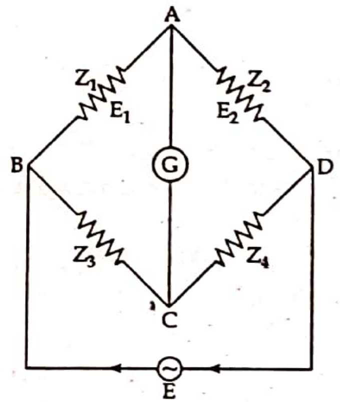

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$Z = Z\ \angle\theta$

so, $(Z_1 \angle\theta_1)(Z_4 \angle\theta_4) = (Z_2 \angle\theta_2)(Z_3 \angle\theta_3)$

or, $Z_1 Z_4 (\angle\theta_1 + \theta_4) = Z_2Z_3 (\angle\theta_2 + \angle\theta_3)$

The above equation indicates that two conditions must be satisfied simultaneously to balance the AC bridge.

13. The impedance of the basic AC bridge are,

$Z_1 = 50\,\Omega \angle 80^\circ$

$Z_2 = 250\,\Omega \angle 0^\circ$

$Z_3 = 200\,\Omega \angle 30^\circ$

Calculate the constants of the unknown impedance

Solution:

The bridge balance equation is,

$$Z_1 Z_4 = Z_2 Z_3$$

or, $50 \times Z_4 = 250 \times 200$

$\therefore$ $Z_4 = 1{,}000\,\Omega$

Thus the magnitude of Z_4 is $1{,}000\,\Omega$

While the phase angle condition is,

$$\theta_1 + \theta_4 = \theta_2 + \theta_3$$

or, $80^\circ + \theta_4 = 0^\circ + 30^\circ$

$\therefore$ $\theta_4 = -50^\circ$

Hence the unknown impedance is $1{,}000 \angle -50^\circ\,\Omega$

The negative angle indicates that it is capacitive in nature.

$$Z_4 = 1{,}000 \angle -50^\circ\,\Omega$$
$$= 1{,}000 \cos(-50^\circ) + j\,1{,}000 \sin(-50^\circ)$$
$$= (642.79 - j\,766.1)\,\Omega$$

Comparing with $(R - jX_C)$,

$\therefore$ $R_4 = 642.79\,\Omega$

$\therefore$ $X_{C4} = 766.1\,\Omega$

14. Calculate the unknown inductance and resistance measured by Hay's bridge. The bridge elements at the balancing condition are $R_4 = 5.1\,k\Omega$, $C_4 = 2\,\mu F$, $R_2 = 7.9\,k\Omega$, $R_3 = 790\,\Omega$
The supply frequency is 1,000 Hz

Solution:

Given that:

$C_4 = 2\,\mu F = 2 \times 10^{-6}\,F$

$R_4 = 5.1\,k\Omega = 5.1 \times 10^3\,\Omega$

$R_2 = 7.9\,k\Omega = 7.9 \times 10^3\,\Omega$

$R_3 = 790\,\Omega$

$L_1 = ?$

$R_1 = ?$

We know,

From Hay's bridge balance equation,

$$R_1 = \frac{\omega^2 R_2 R_3 R_4 C_4^2}{1 + \omega^2 C_4^2 R_4^2}$$

Since $\omega = 2\pi f = 2\pi \times 1{,}000 = 2{,}000\,\pi$ rad/sec

so, $$R_1 = \frac{(2{,}000\pi)^2 \times 7{,}900 \times 790 \times (2 \times 10^{-6})^2 \times 5{,}100}{1 + (2{,}000\,\pi)^2 \times (2 \times 10^{-6})^2 \times 5{,}100^2}$$

$$= \frac{50{,}26{,}250}{1 + 4{,}107.34}$$

$\therefore$ $R_1 = 1{,}223.42\,\Omega$

and, $$L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 C_4^2 R_4^2}$$

or, $$L_1 = \frac{7{,}900 \times 790 \times 2 \times 10^{-6}}{1 + (2{,}000\,\pi)^2 \times (2 \times 10^{-6})^2 \times (5{,}100)^2}$$

or, $$L_1 = \frac{12.482}{1 + 4{,}107.34}$$

or, $L_1 = 0.003038$ H

$\therefore$ $L_1 = 3.038$ mH

15. Difference between AC bridge and DC bridge.

	AC Bridge	DC Bridge
i)	Ac supply used as a excitation voltage.	DC supply is used as a excitation voltage.
ii)	Wagner's earthing device is remove the earth capacitance from the bridge.	There is no need for wagner's earthing device.
iii)	Resistive and reactive components are used.	Only resistive components are used.
iv)	Balancing time is high.	Balancing time is relatively less.
v)	There are 7 types of AC bridge. *i.e.,* • Wien bridge • Schering bridge • Anderson bridge • Hay's bridge • Max well's bridge • Owen's bridge • De sanity's bridge	There are 2 types of DC bridge. *i.e.,* • Wheatstone bridge • Kelvin bridge
vi)	Detector used is AC detector.	Detector used is DC detector.
vii)	AC bridge balance condition: magnitude balance: $Z_1\ Z_4 = Z_2\ Z_3$ and phase balance: $\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$	DC bridge balance condition: $R_1R_4 = R_2R_3$

16. **Difference between accuracy and precision.**

	Accuracy	Precision
i)	It is closeness with the true value of the quantity being measured.	It is the measure of the reproducibility of the measurement.
ii)	The accuracy of measurement means conformity to truth.	The term precise means clearly or sharply defined.
iii)	Accuracy can be improved.	Precision cannot be improved.
iv)	Accuracy is necessary but not sufficient condition for precision.	Precision is necessary but not a sufficient condition for accuracy.
v)	Accuracy = Mean value - True value	Precision = Individual value - Arithmetic mean value.
vi)	Concerned with systematic error.	Concerned with random error.
vii)	*Example:* If you are playing football and you always hit the right goal post instead of scoring, then you are not accurate but you are precise.	