3.

$Y = W^T \Phi + e \quad \text{where} \quad e \sim N(0, a)$

$\rightarrow Y \sim N(W^T \Phi, \ a)$

$W \sim N(0, b^{-1} I)$

$$p(y|W) = \frac{1}{(2\pi a)^{\frac{n}{2}}} \exp\left(-\frac{1}{2a}(Y - W^T\Phi)^2\right)$$

$$P(W) = \frac{1}{(2\pi)^{\frac{n}{2}} |b^{-1}I|^{\frac{1}{2}}} \exp\left(-\frac{1}{2} W^T (b^{-1}I) W\right)$$

$$= \frac{1}{(2\pi)^{\frac{n}{2}} b^{-\frac{n}{2}}} \exp\left(-\frac{b}{2} W^T W\right)$$

$p(W|Y) \propto P(Y|W)\,P(W)$

$$\propto \exp\left(-\frac{1}{2a}(Y - W^T\Phi)^2\right) \exp\left(-\frac{b}{2} W^T W\right)$$

$$= \exp\left(-\frac{1}{2a}\left(Y^2 - 2Y(W^T\Phi) + (W^T\Phi)^2\right) - \frac{b}{2} W^T W\right)$$

$$\propto \exp\left(-\frac{1}{2a}\left(-2YW^T\Phi + (W^T\Phi)^2\right) - \frac{b}{2} W^T W\right)$$

$$= -\frac{1}{2} W^T \left(\frac{\Phi\Phi^T}{a} + bI\right) W + W^T \frac{\Phi Y}{a}$$

posterior $\sim N(\mu, \Sigma)$

$$\begin{cases} \Sigma^{-1} = \frac{\Phi\Phi^T}{a} + bI \rightarrow \Sigma = \left(\frac{\Phi\Phi^T}{a} + bI\right)^{-1} \\ \Sigma^{-1}\mu = \frac{\Phi Y}{a} \rightarrow \mu = \Sigma \frac{\Phi Y}{a} \end{cases}$$