

3. outcome : {HHH, HHT, TTT}

initial value $k = 0.5$, $p_0 = 0.6$, $p_1 = 0.1$

(i) (ii) E-step

$$\text{HHH} : w_i = P(C_0 | \text{HHH}) = \frac{0.5 \times 0.6^3}{0.5 \times 0.6^3 + 0.5 \times 0.1^3} = 0.995$$

$$1 - w_i = P(C_1 | \text{HHH}) = 1 - 0.995 = 0.05$$

$$\text{HHT} : w_i = P(C_0 | \text{HHT}) = \frac{0.5 \times 0.6^2 \times 0.4}{0.5 \times 0.6^2 \times 0.4 + 0.5 \times 0.1^2 \times 0.9} = 0.941$$

$$1 - w_i = P(C_1 | \text{HHT}) = 1 - 0.941 = 0.059$$

$$\text{TTT} : w_i = P(C_0 | \text{TTT}) = \frac{0.5 \times 0.4^3}{0.5 \times 0.4^3 + 0.5 \times 0.9^3} = 0.081$$

$$1 - w_i = P(C_1 | \text{TTT}) = 1 - 0.081 = 0.919$$

(iii) M-step

$$k = \frac{0.995 + 0.941 + 0.081}{3} = 0.672$$

$$p_0 = \frac{3 \times 0.995 + 2 \times 0.941 + 0 \times 0.081}{3(0.995 + 0.941 + 0.081)} = 0.804$$

$$p_1 = \frac{3 \times 0.005 + 2 \times 0.059 + 0 \times 0.919}{3(0.005 + 0.059 + 0.919)} = 0.045$$

(2) (i) E-step

$$\text{HHH} : \quad w_0 = P(C_0 | \text{HHH}) = \frac{0.672 \times (0.804)^3}{0.672 \times (0.804)^3 + 0.328 \times (0.196)^3} = 0.999$$

$$1 - w_0 = P(C_1 | \text{HHH}) = 1 - 0.999 = 0.001$$

$$\text{HTT} : \quad w_0 = P(C_0 | \text{HTT}) = \frac{0.672 \times (0.804)^3 \times (0.196)}{0.672 \times (0.804)^3 \times (0.196) + 0.328 \times (0.048)^3 \times 0.955} = 0.995$$

$$1 - w_0 = P(C_1 | \text{HTT}) = 1 - 0.995 = 0.005$$

$$\text{TTT} : \quad w_0 = P(C_0 | \text{TTT}) = \frac{0.672 \times (0.196)^3}{0.672 \times (0.196)^3 + 0.328 \times 0.955} = 0.020$$

$$1 - w_0 = P(C_1 | \text{TTT}) = 0.980$$

(ii) M-step

$$k = \frac{0.999 + 0.995 + 0.020}{3} = 0.671$$

$$P_0 = \frac{3 \times 0.999 + 2 \times 0.995 + 0 \times 0.020}{3(0.999 + 0.995 + 0.020)} = 0.826$$

$$P_1 = \frac{3 \times 0.001 + 2 \times 0.005 + 0 \times 0.980}{3(0.001 + 0.005 + 0.020)} = 0.004$$

(3) (i) E-step

$$\text{HHH} : w_i = P(C_0 | \text{HHH}) = \frac{0.691 \times 0.826^3}{0.691 \times 0.826^3 + 0.329 \times 0.004^3} = 0.9999$$

$$1 - w_i = P(C_1 | \text{HHH}) = 1 - 0.9999 = 0.0001$$

$$\text{HHT} : w_i = P(C_0 | \text{HHT}) = \frac{0.691 \times 0.826^2 \times 0.194}{0.691 \times 0.826^2 \times 0.194 + 0.329 \times 0.004 \times 0.996} = 0.997$$

$$1 - w_i = P(C_1 | \text{HHT}) = 1 - 0.997 = 0.003$$

$$\text{TTT} : w_i = P(C_0 | \text{TTT}) = \frac{0.691 \times 0.194^3}{0.691 \times 0.194^3 + 0.329 \times 0.996^3} = 0.010$$

$$1 - w_i = P(C_1 | \text{TTT}) = 1 - 0.01 = 0.99$$

(ii) M-step

$$k = \frac{0.9999 + 0.997 + 0.01}{3} = 0.669$$

$$p_0 = \frac{3 \times 0.9999 + 2 \times 0.997 + 0.010}{3(0.9999 + 0.997 + 0.010)} = 0.83$$

$$p_1 = \frac{3 \times 0.0001 + 2 \times 0.003 + 0 \times 0.990}{3(0.0001 + 0.003 + 0.990)} = 0.002$$

(4) (i) E-step

$$\text{HHH} : W_0 = P(C_0 | \text{HHH}) = \frac{0.669 \times 0.83^3}{0.669 \times 0.83 + 0.331 \times 0.002} = 0.9999$$

$$1 - W_0 = P(C_1 | \text{HHH}) = 1 - 0.9999 = 0.0001$$

$$\text{HHT} : W_0 = P(C_0 | \text{HHT}) = \frac{0.669 \times 0.83^2 \times 0.17}{0.669 \times 0.83^2 \times 0.17 + 0.331 \times 0.002^2 \times 0.998} = 0.998$$

$$1 - W_0 = P(C_1 | \text{HHT}) = 1 - 0.998 = 0.002$$

$$\text{TTT} : W_0 = P(C_0 | \text{TTT}) = \frac{0.669 \times 0.17^3}{0.669 \times 0.17^3 + 0.331 \times 0.998^2} = 0.01$$

$$1 - W_0 = P(C_1 | \text{TTT}) = 1 - 0.01 = 0.99$$

(ii) M-step

$$k = \frac{3 \times 0.9999 + 2 \times 0.998 + 0.01}{3} = 0.669$$

$$P_0 = \frac{3 \times 0.9999 + 2 \times 0.998 + 0.01}{3(0.9999 + 0.998 + 0.01)} = 0.83$$

$$P_1 = \frac{3 \times 0.0001 + 2 \times 0.002 + 0.98}{3(0.0001 + 0.002 + 0.998)} = 0.001$$

(5) (i) E-step

$$HHH : w_i = P(C_0 | HHH) = \frac{0.669 \times 0.83^3}{0.669 \times 0.83^3 + 0.331 \times 0.001} = 0.9999$$

$$1 - w_i = P(C_1 | HHH) = 1 - 0.9999 = 0.0001$$

$$HTT : w_i = P(C_0 | HTT) = \frac{0.669 \times 0.83^2 \times 0.17}{0.669 \times 0.83^2 \times 0.17 + 0.331 \times 0.001 \times 0.999} = 0.998$$

$$1 - w_i = P(C_1 | HTT) = 1 - 0.998 = 0.002$$

$$TTT : w_i = P(C_0 | TTT) = \frac{0.669 \times 0.17^3}{0.669 \times 0.17^3 + 0.331 \times 0.999^3} = 0.01$$

$$1 - w_i = P(C_1 | TTT) = 1 - 0.01 = 0.99$$

(ii) M-step

$$k = \frac{3 \times 0.9999 + 2 \times 0.998 + 0.010}{3} = 0.669$$

$$p_0 = \frac{3 \times 0.9999 + 2 \times 0.998 + 0.010}{3(0.9999 + 0.998 + 0.01)} = 0.83$$

$$p_1 = \frac{3 \times 0.0001 + 2 \times 0.002 + 0.999}{3(0.0001 + 0.002 + 0.999)} = 0.001$$

跟第4次結果相同 → 收斂