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Exo 1

$\hat{e}_x = (1,0,0)$ $\hat{e}_y = (0;1;0)$ $\hat{e}_z = (0;0;1)$

a) $\vec{e_x} \cdot \vec{e_x} = 1 \times 1 + 0.0 + 0.0 = 1 = \hat{e_y} \cdot \hat{e_y} = \hat{e_z} \cdot \hat{e_z}$

b) $\hat{e_x} \cdot \hat{e_y} = 1.0 + 0.1 + 0 \times 0 = 0 = \hat{e_y} \cdot \hat{e_z} = \hat{e_z} \cdot \hat{e_x}$

c) $\vec{e_x} \wedge \vec{e_y} = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} = \vec{e_z}$

$\vec{e_y} \wedge \vec{e_z} = \vec{e_x}$

$\vec{e_z} \wedge \vec{e_x} = \vec{e_y}$

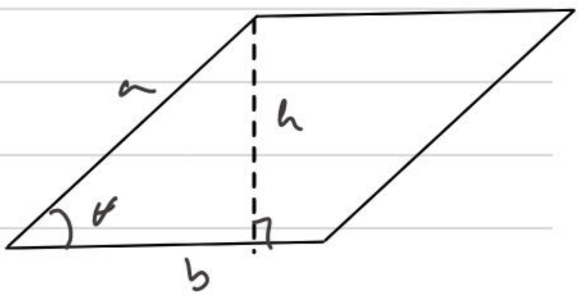
Exo 2 : $\vec{a}\,(a_x ; a_y ; a_z)$

Exo 3 :

a $S_{ab} = ? = b.h$

$= b.a.\sin\theta$

$= \| \vec{a} \wedge \vec{b} \|$



$$S_{ab} = \left\| \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \\ 0 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 0 \\ 0 \\ \frac{\sqrt{3}}{2} \end{pmatrix} \right\| = \frac{\sqrt{3}}{2}$$

$$S_{bc} = \| \vec{b} \wedge \vec{c} \| = \left\| \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 1/2 \\ \sqrt{3}/6 \\ \frac{\sqrt{6}}{3} \end{pmatrix} \right\| = \left| \begin{pmatrix} \frac{\sqrt{6}}{2\sqrt{3}} \\ -1/\sqrt{6} \\ \frac{\sqrt{3}}{12} - \frac{\sqrt{3}}{4} \end{pmatrix} \right|$$

$$= \sqrt{\left(\frac{\sqrt{6}}{2\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{\sqrt{3}}{12} - \frac{\sqrt{3}}{4}\right)^2}$$

$S_{ca} = \frac{\sqrt{3}}{2}$ aussi

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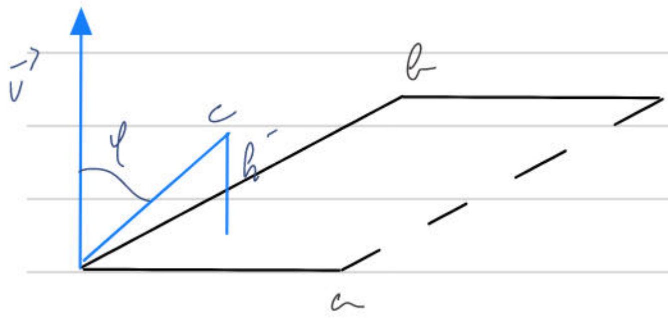
b) calcule du volume V_{abc}

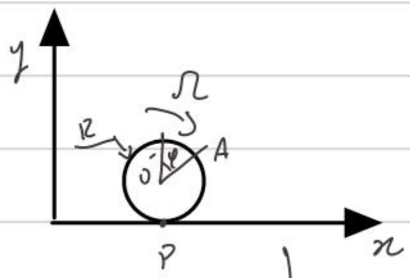


$$V_{abc} = |(a \wedge b).c| = \left| \begin{matrix} 0 \\ 0 \\ \frac{\sqrt{3}}{2} \end{matrix} \right. . \left| \begin{matrix} \frac{1}{2} \\ \frac{\sqrt{3}}{6} \\ \frac{\sqrt{3}}{6} \end{matrix} \right.$$

$$= \frac{\sqrt{2}}{2}$$

$\vec{v}.\vec{c} = v \underbrace{\boxed{c \cos\varphi}}_{h}$

Exo 4 :



P point de contact
$\vec{V(P)} = 0$ pas de glissement
$\vec{\Omega} = -\dot{\varphi}\vec{z}$.

a) $\vec{V}(O') = \vec{V}(P) + \vec{O'P} \wedge \vec{\Omega}$

$$\vec{V(O')} = \left| \begin{matrix} 0 \\ -R \\ 0 \end{matrix} \right. \wedge \left| \begin{matrix} 0 \\ 0 \\ -\dot{\varphi} \end{matrix} \right.$$

$$\vec{V(O')} = R\dot{\varphi}.\vec{x}$$

b) $\vec{V(A)} = \vec{V(O')} + \vec{AO'} \wedge \vec{\Omega}$

$$= \vec{V(P)} + \vec{AP} \wedge \vec{\Omega}$$

$$(\vec{AO'} + \vec{O'P})$$

$$= R\dot{\varphi}\vec{x} + \left| \begin{matrix} -R\sin\varphi \\ -R\cos\varphi \\ 0 \end{matrix} \right. \wedge \left| \begin{matrix} 0 \\ 0 \\ -\dot{\varphi} \end{matrix} \right.$$

$$= R\dot{\varphi}(1+\cos\varphi)\vec{x} - R.\dot{\varphi}\sin\varphi\vec{y}$$

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c) $\underbrace{t = 0 \quad \varphi = 0}_{C.I}$ $\qquad \dfrac{d\varphi}{dt} = \Omega$

$\varphi = \Omega . t + \underset{0}{\overset{\shortparallel}{cte}} \quad C.I$

$\overrightarrow{V_{(A)}}(t) = R.\Omega(1 + \cos(\Omega t))\vec{x} - R\Omega \sin(\Omega . t)\vec{y}$

d) C.I

$t = 0$

$A = (0, 2R)$

on intègre $\overrightarrow{V_{(A)}(t)}$

$$A(t) \left| \begin{array}{l} R\Omega(t + \frac{1}{\Omega}\sin\Omega . t) + cte \\ -R\Omega \times -\frac{1}{\Omega}\cos(\Omega . t) + cste \\ cste \end{array} \right.$$

$$A(t) = \left| \begin{array}{ll} R\Omega(t + \frac{1}{\Omega}\sin(\Omega t)) & t = 0 \quad x_A = 0 \\ R + R.\cos(\Omega . t) & t = 0 \quad y_A = 2R \\ 0 & \end{array} \right.$$

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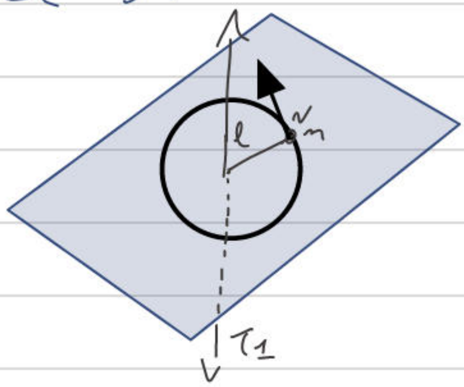
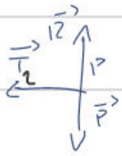
Exercice 5 :





$$\Sigma F_y \Rightarrow -\vec{P} + \vec{R} = 0$$

$$\vec{P} = \vec{R}$$

a) $$\frac{d\vec{\sigma_0}}{dt} \stackrel{?}{=} 0 = \sum_i \vec{OP} \wedge \vec{F_{ext\,i}}$$

$$= \vec{OP} \wedge (\vec{T} + \underbrace{\vec{R} - \vec{P}}_{0}) = \vec{0} \quad \underline{\underline{OK}}$$

$$\Omega \quad \sigma_P = \sum_i \vec{OP} \wedge m_i \vec{v_i} = m \cdot \vec{OP} \wedge \vec{v_{(P)}} = cte$$

$$m \cdot l_1 \cdot v_1 = m \cdot l_2 \cdot v_2$$

$$v_2 = \frac{l_1}{l_2} \cdot v_1$$

b) $$\Delta E_m = W(F_{ext})$$

$$\Delta E_c + \underbrace{\Delta E_p}_{0} = W(F_{ext})$$

$$\frac{1}{2} m (v_2^2 - v_1^2) = W(F_{ext})$$

$$\frac{1}{2} m v_1^2 \left(\frac{l_1^2}{l_2^2} - 1\right) = W(F_{ext})$$

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