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Exercice 1 :

$M(2, \lambda)$, $A(1,3)$, $L(4, 3-\lambda)$

λ tq MAL rectangle en A

$\Leftrightarrow \overrightarrow{MA} \cdot \overrightarrow{AL} = 0$

$\overrightarrow{MA} = \begin{pmatrix} -1 \\ 3-\lambda \end{pmatrix}$, $\overrightarrow{AL} = \begin{pmatrix} 3 \\ -\lambda \end{pmatrix}$

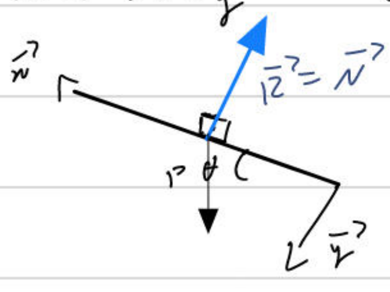
$$\overrightarrow{MA} \cdot \overrightarrow{AL} = -3 - \lambda(3-\lambda)$$
$$= -3 - 3\lambda + \lambda^2$$

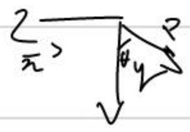
$\Delta = 9 + 12 = 21$

$$\lambda_{1,2} = \frac{3 \pm \sqrt{21}}{2}$$

Exercice 2 :

$m = 500g$ $\theta = 20°$ à $v = 0$ $v_1 = 6 m/s$





a) PFD $\sum \vec{F} = m . \vec{a}$

$\sum F_x = m . a_x$ $-\sin\theta . P = m . a_x$

$\sum F_y = 0$ $-N + \cos\theta . P = 0$

$\sum F_z = 0$ $0 = 0$

$$a_x = -\sin\theta . \frac{P}{m}$$

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$$v_x(t) = v_1 - \sin\theta \cdot \frac{P}{m} \cdot t$$

$$x = v_1 \cdot t - \frac{1}{2} \cdot \frac{P}{m} \cdot \sin\theta \cdot t^2 + 0$$

on cherche t_a tq $v_x(t_a) = 0$

$$0 = v_1 - g \cdot \sin\theta \cdot t_a$$

$$\Rightarrow t_a = \frac{v_1}{g \cdot \sin\theta} = \frac{6}{9{,}81 \cdot \sin(20°)} \approx 1{,}79\ s$$

$$\ell_{(a)} = -\frac{v_{x1}^2}{2 \cdot g \cdot \sin(\theta)} + \frac{v_{x1}^2}{g \cdot \sin(\theta)} = \frac{v_{x1}^2}{2 \cdot g \cdot \sin\theta} = 5{,}86$$

On utilise le TEM :

$\Delta E_m = \Delta E_c + \Delta E_p = 0$ (Pas de frottement)

$$\frac{1}{2} m v_1^2 - \frac{1}{2} m v_2^2 + m \cdot g (h_1 - h_2) = 0$$

$$\frac{1}{2} \cdot m \cdot v_1^2 = m \cdot g \cdot h = m \cdot g \cdot \sin(\theta) \cdot \ell_{(a)}$$

$$\ell_{(a)} = \frac{1}{2} \frac{v_1^2}{g \cdot \sin\theta}$$

$$\ell_{(a)} = \frac{\Delta h}{\sin(\theta)}$$

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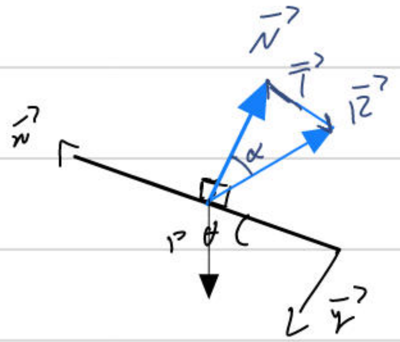
b)



$T = \mu N$

$\frac{T}{N} = \tan \alpha$

PFD) $\sum \vec{F} = m.\vec{a}$

$$\left.\begin{array}{l} \sum F_x = m.a_x \\ \sum F_y = 0 \\ \sum F_z = 0 \end{array}\right) \Rightarrow \begin{array}{l} -m.g.\sin\theta - T = m.a_x \\ m.g.\cos\theta - N = 0 \end{array}$$

$N = mg\cos\theta$

$T = \mu.m.g.\cos\theta = \mu.N$

$$\frac{dv_x}{dt} = -g(\sin\theta + \mu\cos\theta)$$

$$v_x(t) = -g(\sin\theta + \mu\cos\theta).t + v_1$$

t_b tq $v_x(t_b) = 0$

$$t_b = \frac{v_1}{g.(\sin\theta + \mu\cos\theta)} =$$

$$x(t) = v_1.t - \frac{1}{2}g(\sin\theta + \mu\cos\theta).t^2$$

$$x(t_b) = \frac{v_1^2}{2.g(\sin\theta + \mu\cos\theta)} = 3{,}80 \text{ m}$$

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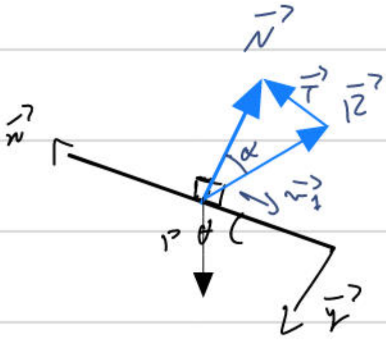
c) Le sens de T change



$\Rightarrow \quad -m.g.\sin\theta + T = m.a_x$

$\Rightarrow \quad -g.(\sin\theta - \mu\cos\theta) = a_x$

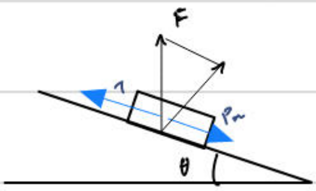
$v_x(t) = -g(\sin(\theta) - \mu\cos(\theta)).t + v_1$

$$t = \frac{v_1}{g.(\sin\theta - \cos\theta)}$$

$$x(t) = -g(\sin\theta - \mu\cos\theta)\frac{t^2}{2} + v_1.t$$

$$l_{(c)} = \frac{v_1^2}{2.g(\sin\theta - \mu.\cos\theta)} = 9,13\ m$$

$\vec{l_{(c)}} = -9,13.\vec{x}$



$P_x \geq T$

$\Leftrightarrow$

$m.g.\sin\theta \leq \mu.m.g.\cos\theta$

$\tan\theta \leq \mu \quad , \quad \theta \leq 8,53°$

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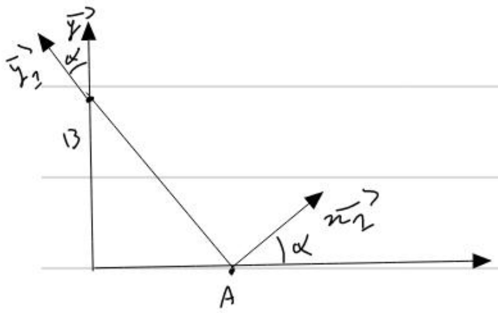
Exercice 3 :



$\vec{AB} = h.\vec{y1}$

Rappel mouvement plan/plan

CIR $\overrightarrow{V_{I \in 1/0}} = \vec{0}$

Base = Trajectoire de I dans $R(o, \vec{x}, \vec{y}, \vec{z})$

Roulante = trajectoire de I dans $S(A ; \vec{x_1}, \vec{y_1}, \vec{z_1})$

1) $\overrightarrow{V_{A \in 1/0}} = \overrightarrow{V_{I \in 1/0}} + \vec{AI} \wedge \overrightarrow{\Omega_{1/0}} = \vec{AI} \wedge \overrightarrow{\Omega_{1/0}}$

$\overrightarrow{V_{I \in 1/0}} = \vec{0}$

$\overrightarrow{V_{B \in 1/0}} = \overrightarrow{V_{I \in 1/0}} + \vec{BI} \wedge \overrightarrow{\Omega_{1/0}} = \vec{BI} \wedge \overrightarrow{\Omega_{1/0}}$

$\overrightarrow{\Omega_{1/0}} = \dot{\alpha}\, z_1 = \dot{\alpha} . z$

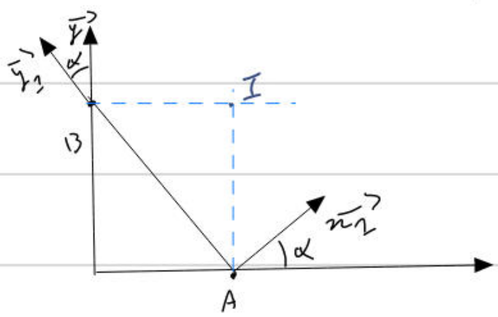
$\overrightarrow{V_{A \in 1/0}} = V_A . \vec{x}$

$\overrightarrow{V_{B \in 1/0}} = V_B . \vec{y}$

$V_A \vec{x} = \underbrace{\vec{AI}}_{\vec{y}} \wedge \dot{\alpha} \vec{z}$

$V_B . \vec{y} = \underbrace{\vec{BI}}_{\vec{x}} \wedge \dot{\alpha} . z$

I est à l'intersection des $\underline{\perp}$ à $\vec{OA}$ et $\vec{OB}$



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2) BASE $\vec{OI}$ dans $R(O, \vec{x}, \vec{y}, \vec{z})$

$$\vec{OI} = \vec{OA} + \vec{AI} = h.\sin\alpha.\vec{x} + h.\cos\alpha.\vec{y}$$
$$= h(\sin\alpha.\vec{x} + \cos\alpha.\vec{y})$$

Cercle de centre o, de rayon h

Roulante $\vec{AI}$ dans $R_1(A, \vec{x_1}, \vec{y_1}, \vec{z_1})$

$$\rightarrow \vec{AI} = h.\cos\alpha(\sin\alpha\,\vec{x_1} + \cos\alpha\,\vec{y_1})$$
$$= h.(\cos\alpha\sin\alpha.\vec{x_1} + \cos^2\alpha\,\vec{y_1})$$
$$= h.\left(\frac{\sin 2\alpha}{2}\vec{x_1} + \frac{1+\cos(2\alpha)}{2}.\vec{y_1}\right)$$
$$= \frac{h}{2}.\vec{y_1} + \frac{h}{2}(\sin 2\alpha.\vec{x_1} + \cos 2\alpha.\vec{y_1})$$

Cercle de centre $(0; \frac{h}{2})$ de rayon $\frac{h}{2}$ dans R_1

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$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$$
$$= r.\overrightarrow{x_1} + h(t).\overrightarrow{z_0} + l.\overrightarrow{y_1} + d.\overrightarrow{z_3}$$
$$\frac{d\overrightarrow{OD}}{dt} = r.\frac{d\overrightarrow{x_1}}{dt}\bigg|_{R} + \dot{h}(t)$$

Exercice 4 :

Les vecteurs vitesses du point D.

$$\overrightarrow{V_{D\in 3/2}} = \overrightarrow{V_{C\in 3/2}} + \overrightarrow{DC} \wedge \overrightarrow{\Omega_{3/2}}$$
$\overrightarrow{V_{C\in 3/2}} = 0$ (rot de 3/2 en C)
$$\overrightarrow{DC} = -d.\overrightarrow{y_3}$$

$$\overrightarrow{\Omega_{1/0}} = \dot{\theta}.\overrightarrow{z_1} = \dot{\theta}\overrightarrow{z_0}$$
$$\overrightarrow{\Omega_{3/0}} = \dot{\phi}\,\overrightarrow{x_3} = \dot{\phi}.\overrightarrow{x_0}$$
$\overrightarrow{AB} = h(t)\overrightarrow{y_1}$ entre 2 et 1

$$\overrightarrow{V_{D\in 3/2}} = -d\,\dot{\phi}\,\overrightarrow{y_3}$$

$$V_{D\in 2/1} = \overrightarrow{V_{B\in 2/1}} + \overrightarrow{DB} \wedge \overrightarrow{\Omega_{2/1}}$$
$\overrightarrow{V_{B\in 2/1}} = \dot{h}(t).\overrightarrow{y_0}$, $\overrightarrow{\Omega_{2/1}} = 0$

$$\overrightarrow{V_{D\in 2/1}} = \dot{h}(t).\overrightarrow{y_0}$$
$$\overrightarrow{V_{D\in 1/0}} = \overrightarrow{V_O} + \overrightarrow{DO} \wedge \overrightarrow{\Omega_{1/0}}$$
$$\overrightarrow{V_O} = \vec{0} \qquad -(\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}) \wedge \dot{\theta}\overrightarrow{y_1}$$

* Determiner le vecteur Position

$$\left[\frac{d\,\vec{OD}}{dt}\right]_{R_0} = \left[\frac{d\,(r.\vec{x_1} + h(t).\vec{z_1} + l.\vec{y_1} + d\,\vec{z_3})}{dt}\right]_0$$

⚠ $$\left[\frac{d\vec{x_1}}{dt}\right]_{R_0} = \vec{\Omega}_{1/0} \wedge \vec{x_1} = \dot{\theta}\,\vec{z_1} \wedge \vec{x_1} = \dot{\theta}.\vec{y_1}$$

$$\left[\frac{d\vec{z_1}}{dt}\right]_{R_0} = \vec{\Omega}_{1/0} \wedge \vec{z_1} = \dot{\theta}\,\vec{z_1} \wedge \vec{z_1} = \vec{0}$$

$$\left[\frac{d\vec{y_1}}{dt}\right]_{R_0} = \vec{\Omega}_{1/0} \wedge \vec{y_1} = \dot{\theta}\,\vec{z_1} \wedge \vec{y_1} = -\dot{\theta}\,\vec{x_1}$$

$$\begin{aligned}\left[\frac{d\vec{z_3}}{dt}\right]_{R_0} = \vec{\Omega}_{3/0} \wedge \vec{z_3} &= (\dot{\phi}\,\vec{x_3} + \vec{0} + \dot{\theta}\,\vec{z_1}) \wedge \vec{z_3} \\ &= -\dot{\phi}\,\vec{y_3} + \dot{\theta}\,\vec{z_1} \wedge (\cos\phi\,\vec{z_1} - \sin\phi\,\vec{y_1}) \\ &= -\dot{\phi}\,\vec{y_3} + \sin\phi\,\vec{x_1}\end{aligned}$$

$$\left[\frac{d\,\vec{OP}}{dt}\right]_{R_0} = r.\dot{\theta}\,\vec{y_1} + \dot{h}(t)\,\vec{z_1} + \cancel{h(t).\vec{0}} - l.\dot{\theta}.\vec{x_1} - d.\dot{\phi}\,\vec{y_3} + d.\dot{\theta}.\sin\phi.\vec{x_1}$$

Exercice 3 :

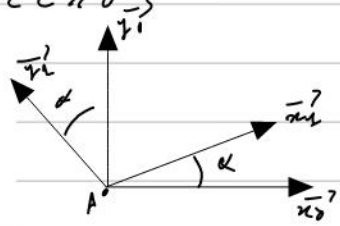
S_0, S_1, S_2, S_3

on cherche : $V_{C\in 3/2}$, $V_{C\in 2/1}$, $V_{C\in 1/0}$ et $V_{C\in 3/0}$

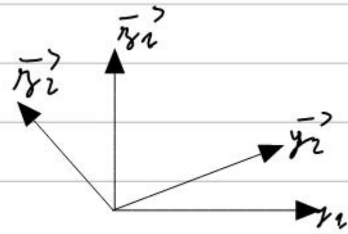
Déterminer les vecteurs vitesse $\underline{V}_{C\in3/2}$, $\underline{V}_{C\in2/1}$, $\underline{V}_{C\in1/0}$ et , $\underline{V}_{C\in3/0}$.

Pour paramétrer les 2 rotations et la translation, on utilise 2 paramètres angulaires et 1 paramètre linéaire :

Soit : $\alpha = (\vec{x_0},\vec{x_1}) = (\vec{y_0},\vec{y_1})$, $\beta = (\vec{y_1},\vec{y_2}) = (\vec{z_1},\vec{z_2})$ et $\vec{BC} = \lambda.\vec{z_2}$.



translation du solide 3/r au 2

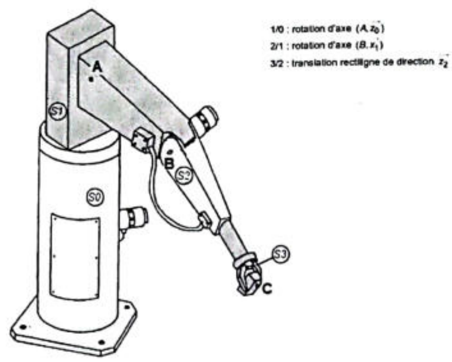


1) $\vec{V_{C\in 3/2}} = \dot{\lambda}\,\vec{z_2}$

2) $\vec{V_{C\in 2/1}} = \underbrace{\vec{V_{B\in 2/1}}}_{\vec{0}} + \vec{CB}\wedge\vec{\Omega_{2/1}}$

$\vec{V_{C\in 2/1}} = -\lambda.\vec{z_2}\wedge\beta.\vec{x_1}$

$= -\lambda.\beta.\vec{y_2}$



On pose $\vec{AB} = a\,\vec{y_1}$ (a étant une constante).

3) $\vec{V_{C\in 1/0}} = \underbrace{\vec{V_{A\,1/0}}}_{\vec{0}} + \underbrace{\vec{CA}}_{\vec{CB}+\vec{BA}}\wedge\vec{\Omega_{1/0}}$

$= -\lambda.\vec{z_2} - a.\vec{y_1}$

$\vec{z_2} = \cos(\beta).\vec{z_1} - \sin(\theta)\,\vec{y_1}$

$\vec{V_{C\in 1/0}} = \left(-\lambda\cos(\beta).\vec{z_1} + (\lambda.\sin(\beta) - a)\,\vec{y_1}\right)\wedge\dot{\alpha}.\vec{z_1}$

$= (\lambda.\sin(\beta) - a)\,\dot{\alpha}.\vec{x_1}$

$\vec{V_{C\in 3/0}} = \vec{V_{C\in 3/2}} + \vec{V_{C\in 2/1}} + \vec{V_{C\in 1/0}}$

$= (\lambda.\sin\beta - a)\,\dot{\alpha}.\vec{x_1} - \lambda.\dot{\beta}.\vec{y_2} + \dot{\lambda}.\vec{z_2}$

$$\overrightarrow{V_{C\in 3/0}} = \left[\frac{d\overrightarrow{A_C}}{dt}\right]_{R_0}$$

$$= \left[\frac{d}{dt}\, a\cdot\overrightarrow{y_1} + \lambda\overrightarrow{z_2}\right]_{R_0}$$

$$= a\left[\frac{d\overrightarrow{y_1}}{dt}\right]_{R_0} + \dot{\lambda}\cdot\overrightarrow{z_2} + \lambda\left[\frac{d\overrightarrow{z_2}}{dt}\right]_{R_0}$$

$$\left[\frac{d\overrightarrow{y_1}}{dt}\right]_{R_0} = \overrightarrow{\Omega_{1/0}}\wedge\overrightarrow{y_1} = \dot{\alpha}\,\overrightarrow{z_1}\wedge\overrightarrow{y_1} = -\dot{\alpha}\cdot\overrightarrow{x_1}$$

$$\left[\frac{d\overrightarrow{z_2}}{dt}\right]_{R_0} = \overrightarrow{\Omega_{2/0}}\wedge\overrightarrow{z_2} = \Big(\overbrace{\dot{\beta}\cdot\underset{\overrightarrow{x_0}}{\underset{\shortparallel}{\overrightarrow{x_1}}}}^{\overrightarrow{\Omega_{2/1}}} + \overbrace{\dot{\alpha}\,\overrightarrow{z_1}}^{+\overrightarrow{\Omega_{1/0}}}\Big)\wedge\overrightarrow{z_2}$$

$$= -\dot{\beta}\cdot\overrightarrow{y_2} + \dot{\alpha}\left(\cos(\beta)\overrightarrow{z_2} + \sin(\beta)\overrightarrow{y_2}\right)\wedge\overrightarrow{z_2}$$

$$= -\dot{\beta}\,\overrightarrow{y_2} + \dot{\alpha}\cdot\sin\beta\cdot\overrightarrow{x_2}$$

$$\overrightarrow{V_{C\in 3/0}} = -a\cdot\dot{\alpha}\cdot\overrightarrow{x_1} + \dot{\lambda}\cdot\overrightarrow{z_2} - \dot{\beta}\cdot\lambda\cdot\overrightarrow{y_2} + \lambda\cdot\dot{\alpha}\sin\beta\,\overrightarrow{x_2}$$