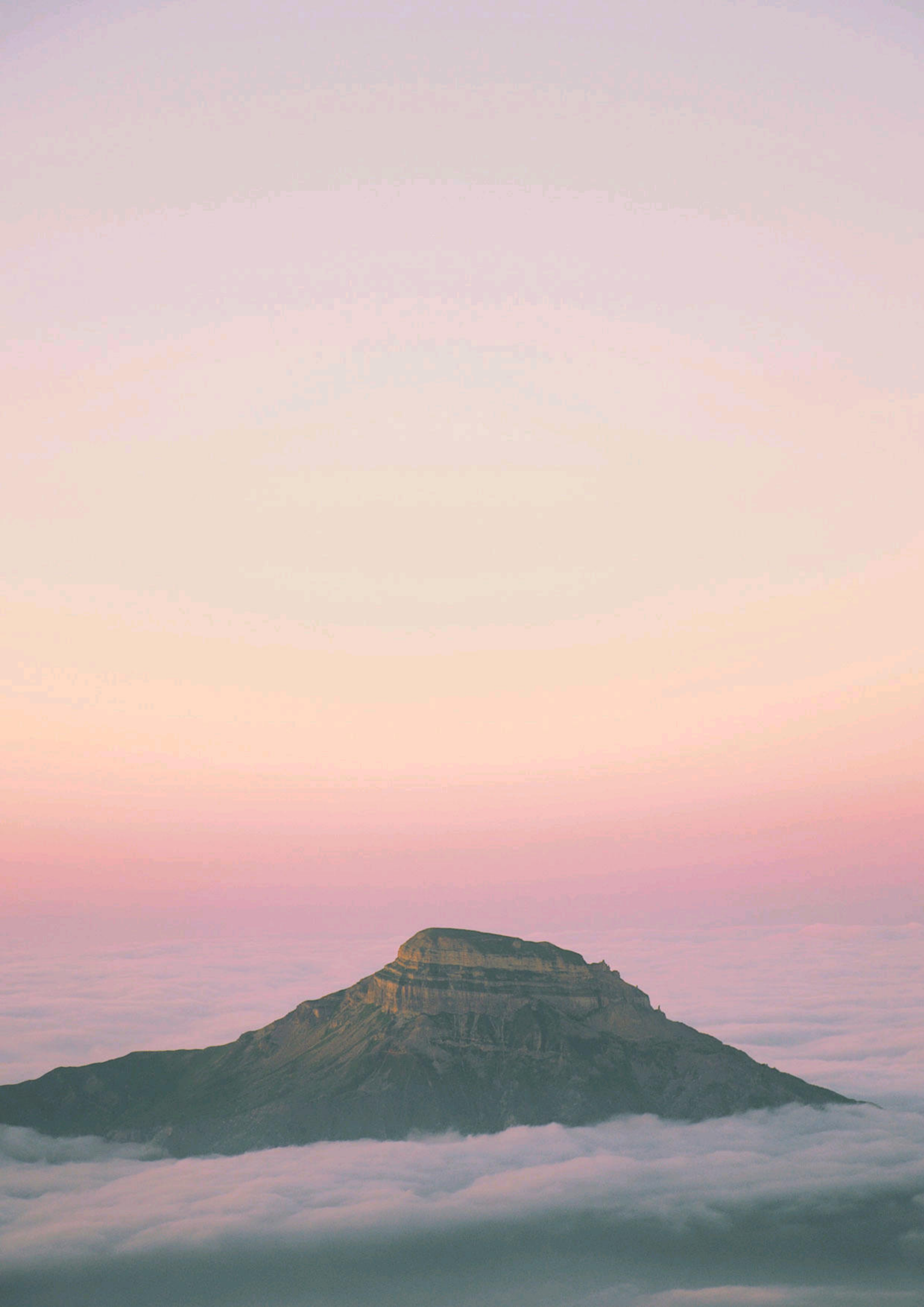
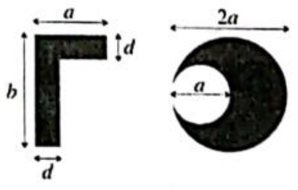


Date: /

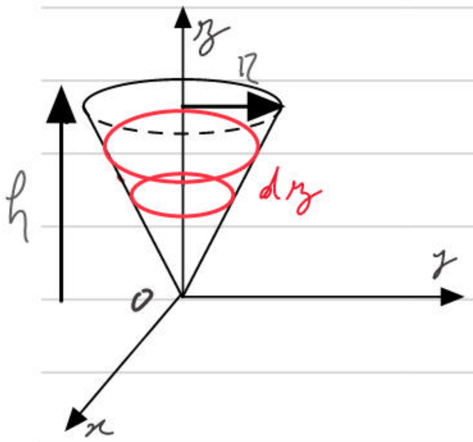
Exercice 1 :

Exercice 1 : Centre de gravité

1. Déterminer le centre de gravité d'un cône de hauteur h et de rayon R.
2. Déterminer le centre de gravité des solides plans (homogène) ci dessous :



(a) (b)



$$dm = \rho . \pi . r^2 dz$$

$$\boxed{OG = \frac{1}{m} \iiint \vec{OM}\, dm}$$

⚠!

l'axe oz est axe de symétrie de révolution

$\Rightarrow G \in (o, z)$

centre de gravité

$$OG = \frac{1}{m} \iiint z\, dm$$

$$\frac{r}{z} = \frac{R}{h} \Rightarrow dm = \frac{\rho . \pi . z^2 . R^2}{h^2} dz$$

$$m = \int_0^h \rho . \pi . \frac{R^2}{h^2} . z^2 dz$$

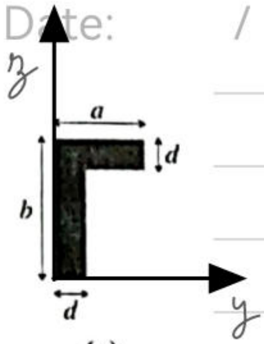
$$= \rho . \pi \frac{R^2}{h^2} \left[\frac{z^2}{3} \right]_0^h = \frac{\rho . \pi . R^2 . h}{3}$$

$$OG = \frac{3}{\rho . \pi . R^2 . h} \int_0^h \frac{\rho . \pi . z^3 . R^2}{h^2} dz$$

$$= \frac{3}{\rho . \pi . R^2 . h} . \frac{\rho . \pi . R^2}{h^2} . \left[\frac{z^4}{4} \right]_0^h$$

$$= \frac{3}{h^3} . \frac{h^4}{4} = \frac{3}{4} h$$

Date: /



(a)

$d = 10 \;,\; b = 100$

$a = 50$

$A = 90 \times 10 + 10 \times 50$

$= 1400 \; m^2$

$Y_G = \frac{1}{S} \iint y \, dS$

$= \frac{1}{900} \int_0^{100}\int_0^{10} y \, dy \, dz + \frac{1}{500} \int_0^{50}\int_{90}^{100} y \, dy \, dz$

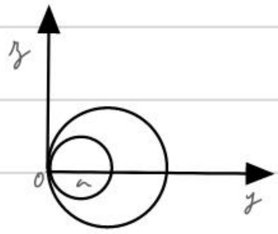
$= \frac{1}{100} \int_0^{100} \left[\frac{y^2}{2} \right]_0^{10} dz + \frac{1}{500} \int_{90}^{100} \frac{2500}{2} \, dz$

$= \frac{5000}{900} + \frac{12500}{500} \approx 5{,}56 + 25$

$\approx 35{,}56$

$$Y_G = \frac{90 \times 10 \times 5 + 50 \times 10 \times 25}{90 \times 10 + 50 \times 10} = 12.142\ldots$$

$$Z_G = \frac{90 \times 10 \times 45 + 50 \times 10 \times (90 + 5)}{90 \times 10 + 50 \times 10} = 62.857\ldots$$



$A = 2\pi a^2 - 2\pi \left(\frac{a}{2}\right)^2$

$= 2\pi a^2 - \frac{\pi \cdot a^2}{2}$

$= a^2 \left(2\pi - \frac{\pi}{2}\right) = \frac{3\pi a^2}{2}$

$Z_G = 0$ Axe de sym

$$Y_G = \frac{\pi a^2 \times a + \left(-\pi \left(\frac{a}{2}\right)^2 \times \frac{a}{2}\right)}{\pi a^2 - \frac{\pi \cdot a^2}{4}}$$

$$= \frac{\pi \cdot a^3 - \frac{\pi a^3}{8}}{\frac{3\pi \cdot a^2}{4}}$$

$$= \frac{\frac{7\pi \cdot a^3}{8}}{\frac{3\pi \cdot a^2}{4}} = \frac{4 \cdot 7 \cdot \pi \cdot a^2}{8 \cdot 3 \cdot \pi \cdot a^2} = \frac{7 \cdot \pi \cdot a^3}{6 \cdot \pi \cdot a^2} = \frac{7 \cdot a}{6}$$