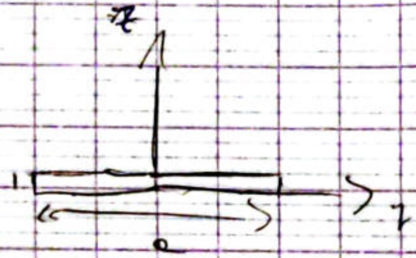
Exo 2 :

Matrice d'inertie d'un solide :

$$I = \begin{pmatrix} \underline{I_{xx}} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & \underline{I_{zz}} \end{pmatrix}$$

avec $I_{xx} = \iiint (y^2+z^2)\,dm$

$dm \to \rho\, dx\, dy\, dz$



$m = \rho a$

$$I_{xx} = \int_0^0 \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_0^0 (y^2+z^2)\rho\, dx\, dy\, dz$$

$$= \cancel{0 + \ell} \, \int_{-\frac{a}{2}}^{\frac{a}{2}} y^2 \rho\, dy = \rho \frac{a^3}{12} = m\frac{a^2}{12}$$

$$I_{yy} = \int_0^0 \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_0^0 (x^2+z^2)\rho\, dx\, dy\, dz$$
$$= 0$$

$I_{zz} = 0$

~~I_{xy}~~

$$I = \frac{m a^2}{12}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

exo 2 suite

Huygens

$$I_{Gxx} = I_{oxx} - OG^2 (S)$$

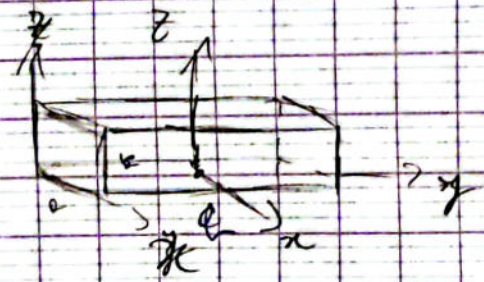
$$= \frac{5 \rho \sqrt{3} a^4}{48} - \left(\frac{\sqrt{3} a}{3}\right)^2 \underbrace{\left(\rho \times \frac{a}{2} \times \frac{\sqrt{3}}{2} a\right)}_{= m}$$

$$= \rho \sqrt{3} a^4 \left(\frac{5}{48} - \frac{3}{9} \times \frac{1}{4}\right)$$

$$= \frac{\rho \sqrt{3} a^4}{48}$$

$$= \frac{m a^2}{12}$$

Exo 3 :



~~I_{xx}~~ S

axe de sym = G connu

$$\begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B \end{pmatrix} ?$$

$$I_{xx} = \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \int_{-c/2}^{c/2} (y^2 + z^2) \, dx \, dy \, dz$$

$$= a \int_{-b/2}^{b/2} \int_{-c/2}^{c/2} (y^2 + z^2) \, dy \, dz$$

$$= a \int_{-b/2}^{b/2} \left[z y^2 + \frac{z^3}{3} \right]_{-c/2}^{c/2} dy$$

$$= a$$

$$I_{xx} = \iiint (y^2 + z^2)\,dm$$

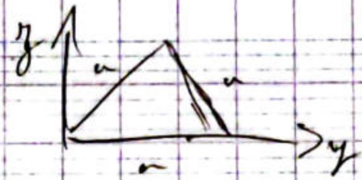
$$= \rho \int_{-a/2}^{a/2} dx \int_{-b/2}^{b/2} y^2\,dy \int_{-c/2}^{c/2} dz$$

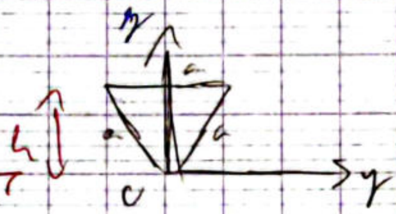
$$+ \rho \int_{-a/2}^{a/2} dx \int_{-b/2}^{b/2} dy \int_{-c/2}^{c/2} z^2\,dz$$

$$= \rho \frac{abc}{12} (b^2 + c^2) \qquad (m = \rho \cdot bc)$$

$$= \frac{m}{12} (b^2 + c^2)$$

$$I_G = \frac{m}{12} \begin{bmatrix} b^2+c^2 & 0 & 0 \\ 0 & a^2+c^2 & 0 \\ 0 & 0 & a^2+b^2 \end{bmatrix}$$





$$z_1 = \frac{a}{\frac{a}{2}} y + 0$$

$$z_2 = -2y$$

$$h = \sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \sqrt{3}\,\frac{a}{2}$$

$$I_{xx} = \rho \iint (y^2+z^2)\rho\, dy\, dz$$

$$= \int_0^{a/2}\int_{2y}^{a} (y^2+z^2)\rho\, dy\, dz + \int_0^{a}\int_{-2y}^{a} (y^2+z^2)\rho\, dy\, dz$$

$$\cancel{= \int_0^{a/2} \left[\frac{y^3}{3} + yz^2\right]_{2y}^{a/2} \rho\, dy\, dy + \int \left[\frac{y^3}{3} + yz^2\right]_{2y}^{a}}$$

$$\cancel{= \int_0^{a} \frac{a^3 - 2y^3}{3} + (a-2y)z^2\, dz}$$

$$= \int_0^{a/2} \left[zy^2 + \frac{z^3}{3}\right]_{2y}^{a} \rho\, dy + \int_0^{-a/2} \left[zy^2 + \frac{z^3}{3}\right]_{-2y}^{a} dy$$

$$= \rho\int_0^{a/2} (a-2y)y^2 + \frac{a^3-8y^3}{3}\, dy + \rho\int_0^{-a/2} (a+2y)y^2 + \frac{a^3+8y^3}{3}\, dy$$

$$= \rho\Bigg(\left[a\cdot\frac{y^3}{3} - 2\cdot\frac{y^4}{4} + \frac{a^3}{3} - \frac{8}{3}\frac{y^4}{4}\right]_0^{a/2}$$

$$+ \left[a\cdot\frac{y^3}{3} - 2\cdot\frac{y^4}{4} + \frac{a^3}{3} - \frac{8}{3}\frac{y^4}{4}\right]_0^{-a/2} \Bigg)$$

$$\cancel{\frac{a}{2h} = \frac{z(y)}{y} \quad z(y) = \pm\frac{ay}{2h}}$$

$$\frac{z(y)}{y} = \pm\frac{h}{-a/2}$$

$$z(y) = \pm\frac{2h\cdot y}{a} = \pm\sqrt{3}\,y$$

Scanné avec CamScanner

$$I_{xx} = \iiint (y^2+z^2)\rho \, \cancel{dx}\, dy\, dz$$

$$= \rho \int_{-a/2}^{0} \int_{z(y)=-\sqrt{3} y}^{h} (y^2+z^2)\,dz\,dy \; + $$

$$\rho \int_{0}^{a/2}\int_{z(y)=\sqrt{3} y}^{h} (y^2+z^2)\,dz\,dy$$

$$= \rho \int_{-a/2}^{0} \Big[zy^2 + \frac{z^3}{3} \Big]_{z(y)=-\sqrt{3} y}^{h} dy + \rho \int_{0}^{a/2} \Big[zy^2+\frac{z^3}{3} \Big]_{z(y)=\sqrt{3} y}^{h} dy$$

$$\cancel{= \rho \int_{-a/2}^{0} \Big[\sqrt{3} y^3 + \frac{3\sqrt{3}}{3} y^3\Big] dy + \rho\int_{0}^{a/2} \sqrt{3} y^3 + \sqrt{3} y^3 \, dy}$$

Il manque les h

$$\cancel{= \rho \Big[-\frac{\sqrt{3} y^4}{4} - \frac{\sqrt{3} y^4}{4}\Big]_{-a/2}^{0} + \rho\Big[\frac{\sqrt{3} y^4}{4}+\frac{\sqrt{3} y^4}{4}\Big]_{0}^{a/2}}$$

$$\cancel{= \rho\Big[-\frac{\sqrt{3} y^4}{2}\Big]_{-a/2}^{0} + \rho\Big[\frac{\sqrt{3} y^4}{2}\Big]_{0}^{a/2}}$$

$$\cancel{= \rho}$$

$$= \rho\int_{-a/2}^{0} \Big(y^2 h + \frac{h^3}{3} + \sqrt{3} y^3 + \sqrt{3} y^3\Big) dy + \rho \int_{0}^{a/2} \Big(y^2h + \frac{h^3}{3} - \sqrt{3} y^3 - \sqrt{3} y^3\Big) dy$$

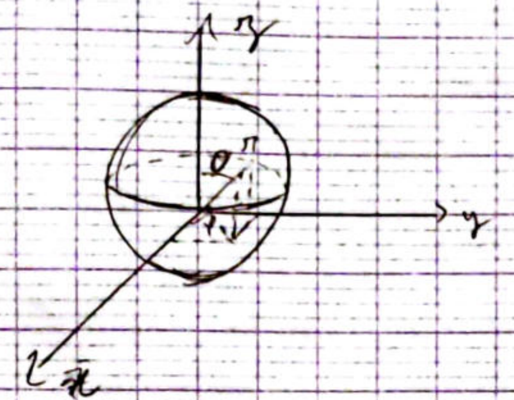
$$= \rho\Big[\frac{y^3 h}{3} + \frac{h^3 y}{3} + \frac{2\sqrt{3}}{4} y^4\Big]_{-a/2}^{0} + \rho\Big[\frac{y^3h}{3}+\frac{h^3 y}{3} - \frac{2\sqrt{3}}{4}y^4\Big]_{0}^{a/2}$$

$$= \rho\Big(\frac{a^3h}{24} + \frac{h^3 a}{6} - \frac{\sqrt{3} a^4}{32} + \frac{a^3h}{24} + \frac{h^3 a}{6} - \frac{\sqrt{3} a^4}{32}\Big)$$

$$= \rho a\Big(\frac{ha^2}{12} + \frac{h^3}{3} - \frac{\sqrt{3} a^3}{16}\Big) = \rho\sqrt{3}\, a^4\Big[\frac{1}{24} + \frac{3}{3\times 8} - \frac{1}{16}\Big]$$

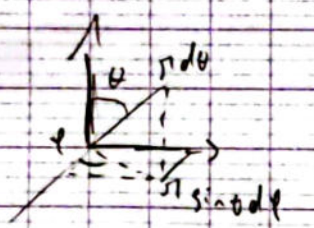
$$= \frac{5\rho\sqrt{3}\, a^4}{48}$$

Exercice 3 :



∞ d'axe de symétrie
=> Matrice diagonale
et $I_{xx} = I_{yy} = I_{zz}$

$x^2 + y^2 + z^2 = r^2$



$dv = r \sin\theta \, d\varphi \;\; r\,d\theta \;\; dr$

$[0, 2\pi] \quad [0, \pi] \quad [0, R]$

$I_{xx} = \iiint_V (y^2 + z^2)\, dm$

$$\cancel{\quad} = I_{xx} + I_{yy} + I_{zz} =$$
$$\iiint_V 2y^2 + 2z^2 + 2x^2 \; dm$$
$$= 2 \iiint_V r^2 \, dm$$
$$= 2\iiint_V \rho r^4 \sin\theta \, d\varphi \, d\theta \, dr$$
$$= 2\int_0^{2\pi}\int_0^{\pi}\int_0^{R} \rho r^4 \sin\theta \, d\varphi \, d\theta \, dr$$
$$= 2\cdot 4\pi\rho \left[\frac{r^5}{5}\right]^R = \frac{8\pi\rho R^5}{5} = 3 I_{xx}$$

$$\Rightarrow I_{xx} = \frac{8\pi\rho R^5}{3\cdot 5}$$

Soit $I_{xx} = \dfrac{8\pi\rho R^5}{15} = \dfrac{2}{5} m R^2 = I_0$

$m = 2\pi \times 2 \times \dfrac{R^3}{3}\rho = \dfrac{4}{3}\pi R^3 \rho$

3) Sphère creuse

$$I_0 = \cancel{8\pi}\ \frac{8\pi}{15}\rho\,(R^5 - (R-e)^5)$$

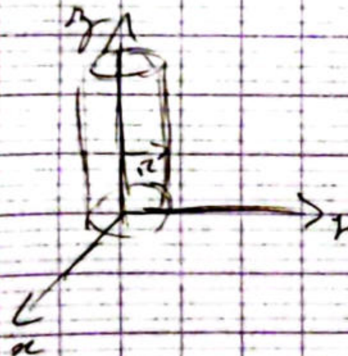
$$m = \frac{4}{3}\pi\rho\,(R^3 - (R-e)^3)$$

CIR

Inertie / axe tangent à la sphère Δ

Huygens

$$I_\Delta = I_{GG} + R^2 \times m = \frac{2}{5} m R^2 + m R^2$$
$$= \frac{7}{5} m . R^2$$



$$R^2 = y^2 + x^2$$

$$dV = r\, d\theta\, dr\, dz$$

$$I_{xx} = I_{yy} =$$
$$I_{zz} = \iiint (y^2 + x^2)\, dm$$

$$\Rightarrow \iiint_V (y^2 + z^2)\, dm = \iiint_V (x^2 + z^2)\, dm$$

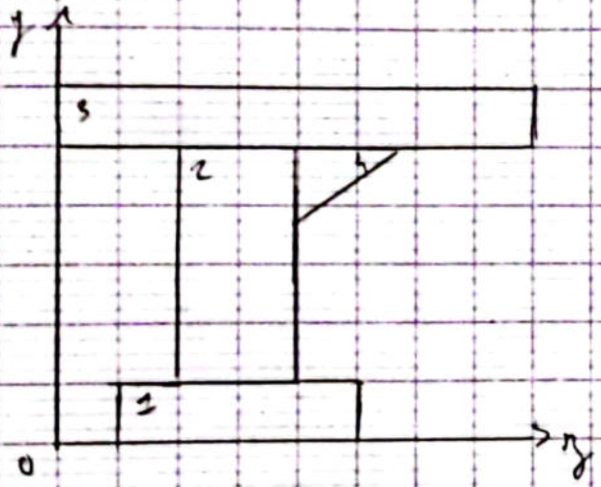
$$\Rightarrow \iiint_V ((y^2 + x^2) + 2z^2)\, dm = 2 I_{xx}$$

$$2 I_{xx} = \iiint_V (r^2 + 2z^2)\, dm$$
$$= \rho \int_0^R \int_0^{2\pi} \int_0^h (r^2 + 2z^2)\, r\, dr\, d\theta\, dz$$
$$= 2\pi\rho \left(\int_0^R \int_0^h r^3\, dr\, dz + 2 \int_0^R \int_0^h z^2 r\, dr\, dz \right)$$
$$= 2\pi\rho \left(\frac{h R^4}{4} + 2 \int_0^h \frac{z^2 R^2}{2}\, dz \right)$$
$$= 2\pi\rho \left(\frac{h R^4}{4} + \frac{2h^3 R^2}{6} \right)$$
$$= 2\pi\rho \left(\cancel{\frac{h R^4}{2} + 2h}\frac{^3 R^4}{3} \right)$$

$I_{xx} = I_{Gy} + \iint y^2 \, dm$

$= \frac{m}{4} \left(R^2 + \frac{e^2}{3} \right)$

Exercice 4



N°	y_i	z_i	S_i	y_{Gi}	z_{Gi}	$S_i y_{Gi}$ ($S_i \times y_{Gi}$)	$S_i z_{Gi}$ ($S_i \times z_{Gi}$)	I
1	0,3	0,7	0,21	0,15	0,65	0,0315	0,1365	
2	1,5	0,3	0,45	1,05	0,65	0,4725	0,2925	
3	0,2	2	0,4	1,9	1	0,76	0,4	
4	0,3	0,3	0,045	1,7	0,9	0,0765	0,0405	
	X	X	1,105	X	X	1,3405	0,8695	

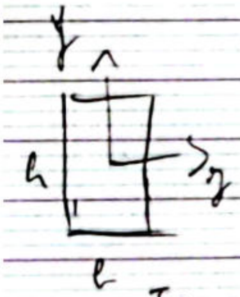
$Y_G = \frac{\sum_i S_i y_i}{\sum_i S_i}$ $\quad Y_G = \frac{\sum_i S_i y_i}{\sum_i S_i} = \cancel{0,787}\ 1,213 = 1,213$

~~$Z_G = \sum_i S_i z_i = 0,78$~~

$Z_G = \cancel{1,213}\ 0,787 = 0,787$

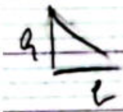
Voir cours

	$I_{Ci,y}$	$I_{Ci,z}$	$I_{Ci,yz}$	$I_{G,y}$	$I_{G,z}$	$I_{G,yz}$
1	$0,3\times0,7^3/12$	$8,7.10^{-4}$	0	0,0125	0,2389	0,03
2	3.10^{-3}	$8,4.10^{-2}$	0	0,0118	0,0963	0,010
3	1.10^{-3}	$1,3.10^{-1}$	0	0,1515	0,190	0,058
4	$2,25.10^{-4}$	$2,25.10^{-4}$	$1,13.10^{-4}$	0,0008	0,010	0,002
	X	X	X	0,1766	0,5362	0,1018



$I_{Gz} = \frac{bh^3}{12}$

$I_{Gy} = \frac{hb^3}{12}$



$I_{Gz} = \frac{bh^3}{36}$

$I_{Gy}\ \frac{hb^3}{36}$

$I_{Gyz} = \frac{h^2b^2}{72}$

1 $b = 0,7$
$h = 0,3$

2 $b = 0,3$
$h = 1,5$

3 $b = 2$
$h = 0,2$

4 $b = h = 0,3$

$$I_{Gz} = I_{Giz} + A_i \times (z_{Gi} - z_G)$$