Chapter 2

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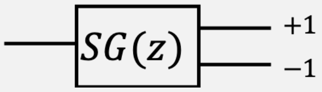
1 Recap

Box 1.1: Postulates of Quantum Mechanics

Postulate 1. At any time t, the state of a physical system is defined by a ket $|\psi\rangle$, or *state* in a relevant Hilbert space H.

Postulate 2. The only possible result of measuring observable A is one of the eigenvalues of A



$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Aside:

1. If A is Hermitian, then the measurement gives a real number.
2. If A's spectrum is discrete, then we only see quantized result.

Postulate 3. Every measurable physical quantity A is described by a Hermitian operator.

Postulate 4. If $A|u_\alpha\rangle = a_\alpha|u_\alpha\rangle$, then for a system in $|\psi\rangle$, when we measure A, then the probability of getting a_α is $P(a_\alpha) = |\langle u_\alpha|\psi\rangle|^2$.
Aside: If we have degenerate a_α's $\{|u_{\alpha,1}\rangle, |u_{\alpha,2}\rangle, ...\}$ share the same eigenvalue, then $P(a_\alpha) = \sum_i |\langle u_{\alpha,i}|\psi\rangle|^2$
Example: $A = I$, all $a_\alpha = 1$

Postulate 5. If a measurement projects $|\psi\rangle$ into a new state $|u_\alpha\rangle$, then a physical new state should be $|u'_\alpha\rangle = \frac{|u_\alpha\rangle}{\sqrt{\langle u_\alpha|u_\alpha\rangle}}$, so that $\langle u'_\alpha|u'_\alpha\rangle = 1$.

Postulate 6. Between measurement the state vector $|\psi(t)\rangle$ evolves in time with time dependent Shrödinger's equation

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = \hat{H}(t)|\psi(t)\rangle$$

here $\hat{H}$ is a Hamiltonian.

Box 1.2: Time evolution and H

If the Hamiltonian of the system is H, then the time evolution operator $U(t)$ is

$$U(t) = e^{-iHt}$$

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