20190305, Ch1 intro to QM

Sunday, March 3, 2019 4:51 PM

recap:

§1 Stern-Gerlach experiment

§2 principles of superposition

§3 kets, bras, operators, $|\psi\rangle$, $\langle\psi|$, $\hat{A}$

§4 Hermitian operator and basis

eigenvalue, eigenket of Hermitian op

HW1. Sakurai 1.1, 1.4, 1.6, 1.14

additional: if operator U satisfies $U \cdot U^{+} = \mathbb{1}$ in a certain representation, show this is true for any other representations.

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operators $\hat{A}$, $\hat{B}$

① commutor $[\ ,\ ]$

$$[\hat{A},\hat{B}] \equiv \hat{A}\hat{B} - \hat{B}\hat{A}$$

$$\hat{C} \equiv \hat{A}\hat{B} - \hat{B}\hat{A}, \quad \hat{C}|\psi\rangle = \hat{A}(\hat{B}|\psi\rangle) - \hat{B}(\hat{A}|\psi\rangle)$$

if $[\hat{A},\hat{B}] = 0$, we call $\hat{A}$ and $\hat{B}$ commutes

$$[[\hat{A},\hat{B}],\hat{C}]$$

② anti commutor $\{\ ,\ \}$

$$\hat{C} = \{\hat{A},\hat{B}\} \equiv \hat{A}\hat{B} + \hat{B}\hat{A}$$

③ trace $tr(\hat{A})$ → trace of op. $\hat{A}$

$$tr(\hat{A}) \equiv \sum_i \langle\alpha_i|\hat{A}|\alpha_i\rangle$$

$\{|\alpha_i\rangle\}$

$$\hat{A} \rightarrow \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & a_{22} & a_{23} & \cdots \\ a_{31} & a_{32} & a_{33} & \cdots \\ & & & \ddots \end{pmatrix}$$

$$\hat{A}|\alpha_i\rangle \rightarrow \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ & \ddots & & \\ & & \ddots & \end{pmatrix} \cdot \begin{array}{c} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{pmatrix} \leftarrow i^{th} \end{array}$$

$$= \begin{pmatrix} a_{1i} \\ a_{2i} \\ \vdots \end{pmatrix}$$

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$|\alpha_i\rangle = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{pmatrix} \leftarrow i^{th}$

$$\langle\alpha_i|\hat{A}|\alpha_i\rangle = \begin{array}{c}(0\ 0 \cdots 1\ 0 \cdots 0) \\ \uparrow \\ i^{th}\end{array} \begin{pmatrix} a_{1i} \\ a_{2i} \\ \vdots \\ a_{ii} \end{pmatrix} = a_{ii}$$

$$\langle \alpha_i | A | \alpha_i \rangle = \begin{array}{c} (0\,0 \cdots 1\,0 \cdots 0) \\ \uparrow \\ i^{th} \end{array} \begin{pmatrix} a_{1i} \\ a_{2i} \\ \vdots \\ a_{ii} \\ \vdots \end{pmatrix} = a_{ii}$$

$$\boxed{\begin{array}{l} tr(\hat{A}) = \sum_i a_{ii} \\ \text{diagonal sum of matrix } \hat{A} \end{array}}$$

④ function of operators.

$\hat{A}$, $f(\hat{A})$ call f of $\hat{A}$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots$$

$$f(\hat{A}) = f(0)\mathbb{1} + f'(0)\hat{A} + \frac{f''(0)}{2!}\hat{A}^2 + \cdots$$

$$\underline{\exp(\hat{A}\cdot\eta) = \mathbb{1} + \hat{A}\cdot\eta + \frac{(\hat{A}\eta)^2}{2!} + \cdots}$$

$\hat{C} \equiv \exp(\hat{A}\cdot\eta)$

if $\hat{A} \to \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & a_{22} & a_{23} & \cdots \\ & & & \ddots \end{pmatrix}$, $\hat{C} \to \exp\left(\eta \cdot \begin{pmatrix} a_{11} & a_{12} \\ & \ddots \end{pmatrix}\right)$

if $\hat{A}$ is Hermitian $\hat{A}|\alpha_i\rangle = \underline{\underline{\alpha_i}}\,\underline{\underline{|\alpha_i\rangle}}$, $\underline{\hat{A} = \sum_i \alpha_i |\alpha_i\rangle\langle\alpha_i|} = \begin{pmatrix} \alpha_1 & & 0 \\ & \alpha_2 & \\ 0 & & \alpha_3 \ddots \end{pmatrix}$

in the basis of $\{|\alpha_i\rangle\}$

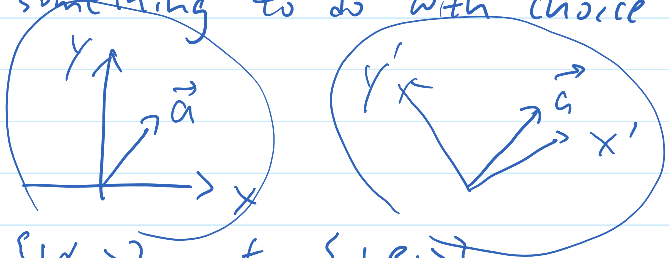
if $\hat{C} \equiv \exp(\hat{A}\eta)$ in the basis of $\{|\alpha_i\rangle\}$ which is the eigenket basis of $\hat{A}$

then $\hat{C}$ can be represented as $\underline{\exp\left(\eta \begin{pmatrix} \alpha_1 & & \\ & \alpha_2 & \\ & & \ddots \end{pmatrix}\right)}$

$$= \begin{pmatrix} \exp(\eta\alpha_1) & & 0 \\ & \exp(\eta\alpha_2) & \\ 0 & & \ddots \end{pmatrix}$$

§5 representation theory

something to do with choice of basis.



$\{|\alpha_i\rangle\}$ of $\{|\beta_i\rangle\}$

① op. $\hat{B}$, a basis $\underline{\{|\alpha_i\rangle\}}$

$$\hat{B} = \begin{pmatrix} b_{11} & b_{12} & b_{13} & \cdots \\ b_{21} & b_{22} & b_{23} & \cdots \end{pmatrix}$$

$$\det\begin{pmatrix} b_{11}-\lambda & b_{12} & b_{13} & \cdots \\ b_{21} & b_{22}-\lambda & b_{23} & \\ & & \ddots & \end{pmatrix} = 0 \Rightarrow \text{eigenvalues } \lambda \text{ of } \hat{B}$$

$$\begin{pmatrix} b_{11}-\lambda & b_{12} & b_{13} & \cdots \\ b_{21} & b_{22}-\lambda & b_{23} & \cdots \\ & & \ddots & \end{pmatrix}\begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \end{pmatrix} = 0 \quad \text{plug in } \lambda_i\text{'s} \Rightarrow \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \end{pmatrix} \in \{|\beta_i\rangle\} \text{ vector}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \left|\begin{pmatrix} -\lambda & 1 \\ 1 & -\lambda \end{pmatrix}\right| = 0 \Rightarrow \lambda^2 = 1, \quad \lambda = +1 \text{ or } -1$$

for $\lambda = 1$, $\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = 0 \Rightarrow \beta_1 = \beta_2$, $|\beta_1|^2 + |\beta_2|^2 = 1 \Rightarrow \beta_1 = \beta_2 = \frac{1}{\sqrt{2}}$

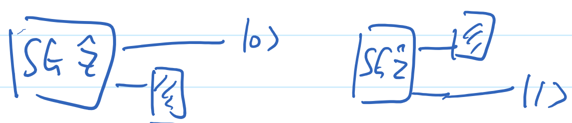
$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ ✓

$\lambda = -1$, $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

$$|+\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \quad |-\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sigma_x = (+1)|+\rangle\langle+| + (-1)|-\rangle\langle-|$$
$$= |+\rangle\langle+| - |-\rangle\langle-|$$

in the basis with $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



→ along $\hat{z}$ direction.

$$|z+\rangle = |0\rangle = \frac{1}{\sqrt{2}}|x+\rangle + \frac{1}{\sqrt{2}}|x-\rangle$$
$$|x+\rangle = \frac{1}{\sqrt{2}}|z+\rangle + \frac{1}{\sqrt{2}}|z-\rangle$$

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

in a more general way

we have two basis $\{|\alpha_i\rangle\}$, $\{|\beta_i\rangle\}$

$$\boxed{\hat{B} = \sum_{ij} \langle\alpha_i|\hat{B}|\alpha_j\rangle\, |\alpha_i\rangle\langle\alpha_j|}$$

$$\hat{B} \rightarrow \begin{pmatrix} b_{11} & b_{12} & b_{13} & \cdots \\ b_{21} & b_{22} & b_{23} & \\ & & \ddots & \end{pmatrix} = b_{11}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \\ & & \ddots \end{pmatrix} + b_{12}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ & & \ddots \end{pmatrix} + \cdots$$

i^{th}

$$\hat{B} \rightarrow \begin{pmatrix} b_{11} & b_{12} & b_{13} & \cdots \\ b_{21} & b_{22} & b_{23} & \cdots \\ & & & \ddots \end{pmatrix} = b_{11}\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 0 & \cdots \\ & & \ddots \end{pmatrix} + b_{12}\begin{pmatrix} 0 & 1 & \cdots \\ 0 & 0 & 0 \\ & & \ddots \end{pmatrix} + \cdots$$

$$= \sum_{ij} \underbrace{(0\ 0 \cdots \overset{i\text{th}}{1}\ 0\ 0 \cdots 0) \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ & & \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{pmatrix} \leftarrow j\text{th}}_{b_{ij}} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{pmatrix} \leftarrow i\text{th} \ (0\ 0 \cdots \underset{j\text{th}}{1}\ 0 \cdots 0)$$

$$\hat{B} = \sum_{ij} \langle \beta_i | \hat{B} | \beta_j \rangle\ |\beta_i\rangle\langle\beta_j| = \sum_{ij} \langle \alpha_i | \hat{B} | \alpha_j \rangle\ |\alpha_i\rangle\langle\alpha_j|$$

(I) if we choose $|\alpha_i\rangle \rightarrow \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix} \leftarrow i\text{th}$ we can represent

$$\hat{B} = \begin{pmatrix} b_{11} & b_{12} & b_{13} & \cdots \\ b_{21} & b_{22} & b_{23} & \cdots \\ & & & \ddots \end{pmatrix} \quad \text{where} \quad b_{ij} \equiv \langle \alpha_i | \hat{B} | \alpha_j \rangle$$

(II) if we choose $|\beta_i\rangle = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{pmatrix} \leftarrow i\text{th}$

$$\hat{B} = \begin{pmatrix} b'_{11} & b'_{12} & b'_{13} & \cdots \\ & \ddots & & \\ & & & \ddots \end{pmatrix} \quad \text{where} \quad b'_{ij} \equiv \langle \beta_i | \hat{B} | \beta_j \rangle$$

$\leftarrow$ prime

$\{|0\rangle, |1\rangle\} \longrightarrow \sigma_x$ is represented as $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\{|+\rangle, |-\rangle\} \longrightarrow \hat{\sigma}_x = |+\rangle\langle+| - |-\rangle\langle-|$ can be represented in $|+\rangle, |-\rangle$ basis as $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

not having $|+\rangle\langle-|$ or $|-\rangle\langle+|$ terms.

question: given operator $\hat{B}$, and basis $\{|\alpha_i\rangle\}$

so we have $\hat{B} \longrightarrow \begin{pmatrix} b_{11} & b_{12} & \cdots \\ & & \ddots \end{pmatrix}$ in this basis

then if we have a new basis $\{|\beta_i\rangle\}$

what $\hat{B}$ is going to be represented?

$$\mathbb{1} = \sum_i |\alpha_i\rangle\langle\alpha_i| = \sum_j |\beta_j\rangle\langle\beta_j|$$

$$\hat{B} = \sum_{ij} \langle \alpha_i | \hat{B} | \alpha_j \rangle\ |\alpha_i\rangle\langle\alpha_j|$$

$$\hat{B} = \sum_{lm} \langle \beta_l | \hat{B} | \beta_m \rangle\ |\beta_l\rangle\langle\beta_m|$$

$$\underbrace{\hat{B} = \sum_{lm} \langle \beta_l | \hat{B} | \beta_m \rangle | \beta_l \rangle \langle \beta_m |}$$

$$\Rightarrow \hat{B} = \sum_{ij} \underbrace{\langle \alpha_i | \Big(\underbrace{\sum_{lm} \langle \beta_l | \hat{B} | \beta_m \rangle}_{\text{number}} | \beta_l \rangle \langle \beta_m | \Big) | \alpha_j \rangle}_{\text{a number}} | \alpha_i \rangle \langle \alpha_j |$$

if we have $\langle \varphi | \cdot \text{number} \cdot | \psi \rangle = \text{number} \langle \varphi | \psi \rangle$

$$\Rightarrow \hat{B} = \sum_{ij} \sum_{lm} \underbrace{\langle \alpha_i | \beta_l \rangle \langle \beta_m | \alpha_j \rangle \langle \beta_l | \hat{B} | \beta_m \rangle}_{\langle \alpha_i | \hat{B} | \alpha_j \rangle} | \alpha_i \rangle \langle \alpha_j |$$

so we have

$$\langle \alpha_i | \hat{B} | \alpha_j \rangle = \sum_{lm} \langle \alpha_i | \beta_l \rangle \underbrace{\langle \beta_l | \hat{B} | \beta_m \rangle} \langle \beta_m | \alpha_j \rangle$$

b_{ij} in $\{|\alpha_i\rangle\}$ $\qquad$ b'_{lm} in $\{|\beta\rangle\}$

in $\{|\alpha_i\rangle\}$ $\hat{B}$ as $\begin{pmatrix} b_{11} & b_{12} & \\ & & \ddots \end{pmatrix} = \begin{pmatrix} U^+ \end{pmatrix} \begin{pmatrix} b'_{11} & b'_{12} & b'_{13} & \cdots \\ & & \ddots & \end{pmatrix} \begin{pmatrix} U \end{pmatrix}$

$$\boxed{U = \sum_k |\beta_k\rangle\langle\alpha_k| ,} \quad |\beta_k\rangle = U |\alpha_k\rangle = \sum_{k'} |\beta_{k'}\rangle\langle\alpha_{k'}|\alpha_k\rangle$$

$$= \sum_{k'} |\beta_{k'}\rangle \delta_{k'k} = |\beta_k\rangle \checkmark$$

we can proof this U gives a transformation so that $\begin{pmatrix} b_{11} & b_{12} & \\ & & \ddots \end{pmatrix} = U^+ \begin{pmatrix} b'_{11} & b'_{12} & b'_{13} & \cdots \\ & & \ddots & \end{pmatrix} U$

$$\sum_{lm} |\beta_l\rangle\langle\beta_m| \quad \langle \beta_l | \hat{B} | \beta_m \rangle$$

$$b'_{lm} = \langle \beta_l | \hat{B} | \beta_m \rangle$$

$UU^+ = \mathbb{1}$

$$\mathbb{1} = \sum_i |\alpha_i\rangle\langle\alpha_i| = \sum_l |\beta_l\rangle\langle\beta_l|$$

$$\hat{B} = \hat{B} \cdot \underline{\mathbb{1}}$$

$$\rightarrow \sum_{ij} |\alpha_i\rangle\langle\alpha_j| \langle\alpha_i|B|\alpha_j\rangle$$

$$= \sum_{ij} \underset{\uparrow}{\mathbb{1}} \cdot |\alpha_i\rangle\langle\alpha_j| \underset{\uparrow}{\mathbb{1}} \; \underline{\langle\alpha_i|\hat{B}|\alpha_j\rangle}$$

$$\underline{\mathbb{1} = \sum_l |\beta_l\rangle\langle\beta_l| = \sum_m |\beta_m\rangle\langle\beta_m|}$$

$$= \sum_{ij} \sum_l |\beta_l\rangle \underbrace{\langle\beta_l| \cdot |\alpha_i\rangle} \langle\alpha_j| \cdot \sum_m |\beta_m\rangle\langle\beta_m| \langle\alpha_i|\hat{\beta}|\alpha_j\rangle$$

$$= \sum_{lm} |\beta_l\rangle\langle\beta_m| \Big(\underbrace{\sum_{ij} \langle\beta_l|\alpha_i\rangle \langle\alpha_i|\hat{B}|\alpha_j\rangle \langle\alpha_j|\beta_m\rangle}_{\langle\beta_l|\hat{B}|\beta_m\rangle} \Big)$$

- compatible observable

$$[\hat{A}, \hat{B}] \equiv \hat{A}\hat{B} - \hat{B}\hat{A}$$

we call $\hat{A}, \hat{B}$ are compatible, if $[\hat{A}, \hat{B}] = 0$

Theorem: if $[\hat{A}, \hat{B}] = 0$, then $\hat{A}, \hat{B}$ can be simultaneously diagonalized in a certain basis $\{|\alpha\rangle\}$. <u>if $\hat{A}$ and $\hat{B}$ are Hermitian.</u>

proof: we can choose $\{|\alpha\rangle\}$ to be eigenkets of $\hat{A}$

$$\big(\underline{\hat{A}|\alpha_i\rangle = \alpha_i |\alpha_i\rangle}\big)^{+}, \quad \langle\alpha_i|\hat{A} = \alpha_i \langle\alpha_i|$$

① $$\langle\alpha_i| \underline{[\hat{A}, \hat{B}]} |\alpha_j\rangle = \langle\alpha_i|\hat{A}\hat{B} - \hat{B}\hat{A}|\alpha_j\rangle$$

$$= \alpha_i \langle\alpha_i|\hat{B}|\alpha_j\rangle - \langle\alpha_i|\hat{B}|\alpha_j\rangle \alpha_j$$

$$= (\alpha_i - \alpha_j) \langle\alpha_i|\hat{B}|\alpha_j\rangle = 0$$

in the case $\alpha_i \neq \alpha_j$ we have $\langle\alpha_i|\hat{B}|\alpha_j\rangle = 0$

$$\langle\alpha_i|\alpha_j\rangle = 0$$

② if $\alpha_i = \alpha_j \leftarrow$ degenerate eigenvalue

$$\hat{A} \rightarrow \begin{pmatrix} \alpha_1 & & & & & \\ & \alpha_2 & & & & \\ & & \ddots & & & \\ & & & \alpha_i & & \\ & & & & \ddots & \\ & & & & & \alpha_j \\ & & & & & & \ddots \end{pmatrix} \qquad \mathbb{1}$$

$$B = \begin{pmatrix} \beta_1 & & & & \\ & \beta_2 & & & \\ & & \ddots & & \\ & & & \beta_i & \cdots \; \beta_{ij} \\ & & & \vdots & \ddots \end{pmatrix}$$

$$B = \begin{pmatrix} \beta_1 & & & & \\ & \ddots & & & \\ & & \beta_i & \cdots & \beta_{ij} \\ & & \vdots & \ddots & \\ & & \beta_{ji} & \cdots & \beta_j \end{pmatrix}$$

treating a matrix in BLOCKS ← 分块矩阵

- Pauli matrix

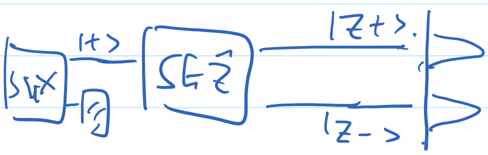
$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

show $\sigma_i \cdot \sigma_i = \mathbb{1}$, $\{\sigma_i, \sigma_j\} = 0$, if $i \neq j$

$$[\sigma_x, \sigma_y] = 2i\sigma_z$$
$$[\sigma_y, \sigma_z] = 2i\sigma_x$$
$$[\sigma_z, \sigma_x] = 2i\sigma_y$$

$$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0$$

§6. measurement.



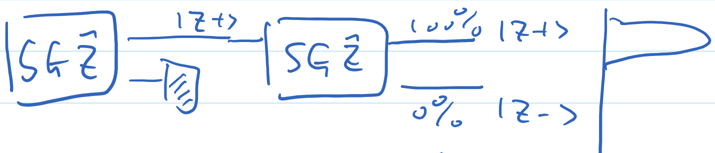
$$\underline{|x+\rangle = \frac{1}{\sqrt{2}}|z+\rangle + \frac{1}{\sqrt{2}}|z-\rangle} \longrightarrow \begin{matrix} |z+\rangle \\ \text{or} \\ |z-\rangle \end{matrix}$$ collapsed 坍缩

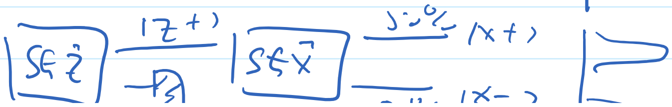
① formal description.

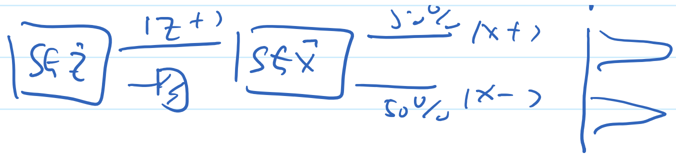
$|\psi\rangle = \sum_i c_i |\alpha_i\rangle$, if we measure in basis of $\{|\alpha_i\rangle\}$

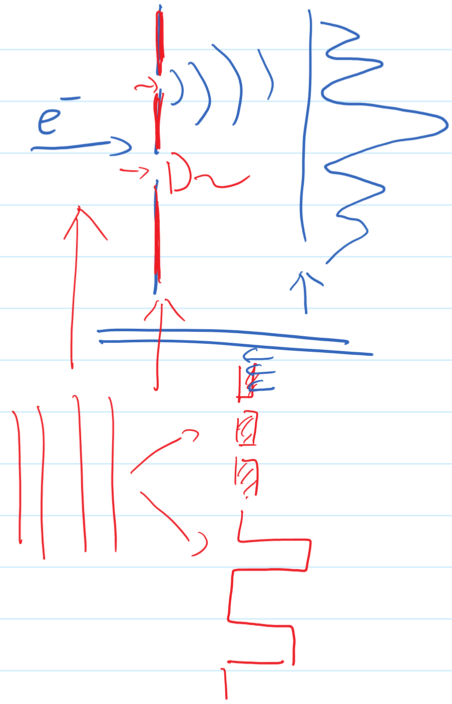
then we have outcome state of $|\alpha_i\rangle$ with a probability of $\underline{P_i} = |c_i|^2 = |\underline{\langle\alpha_i|\psi\rangle}|^2$, $0 \sim 1$

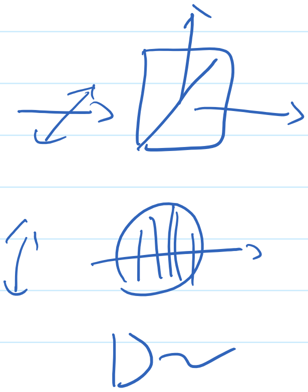
$$|\psi\rangle = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \end{pmatrix},\quad \begin{array}{c} (0\,0 \cdots 1\, 0\, 0) \\ \uparrow \\ i^{th} \end{array} \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_i \\ \vdots \end{pmatrix} = c_i$$









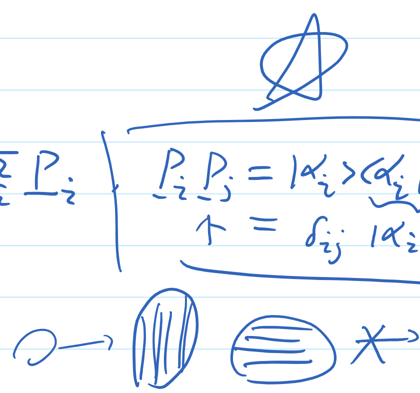


- projection operator $P_i = |\alpha_i\rangle\langle\alpha_i|$

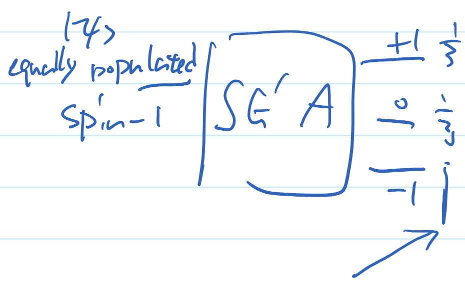
$|\psi\rangle = \sum_i c_i |\alpha_i\rangle$

$P_i|\psi\rangle = c_i|\alpha_i\rangle$

$P_i^2 = P_i$, $\quad \mathbb{1} = \sum_i |\alpha_i\rangle\langle\alpha_i| = \sum_i P_i$, $\quad P_i P_j = |\alpha_i\rangle\langle\alpha_i|\cdot|\alpha_j\rangle\langle\alpha_j| = \delta_{ij}|\alpha_i\rangle\langle\alpha_j|$.



example



$|\psi\rangle = \frac{1}{\sqrt{3}}|+1\rangle + \frac{1}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|-1\rangle$

$|\psi\rangle \to \frac{1}{\sqrt{3}}|-1\rangle$

$\to \frac{1}{\sqrt{3}}|+1\rangle + \frac{1}{\sqrt{3}}|0\rangle$

① if we get a click

$\to \frac{1}{\sqrt{3}}|+1\rangle + \frac{1}{\sqrt{3}}|0\rangle$

① if we get a click
applying $P_{-1} \to \frac{1}{\sqrt{3}}|-1\rangle$

② if we don't get a click
applying $\mathbb{1} - P_{-1} = P_{+1} + P_0$ to $|\psi\rangle \to (P_{+1}+P_0)|\psi\rangle$

$= \frac{1}{\sqrt{3}}|+1\rangle + \frac{1}{\sqrt{3}}|0\rangle$

② measurement

observable $\to$ Hermitian operator A

$\langle A \rangle$ is a measurement of A

$\equiv \langle \psi | A | \psi \rangle$

$P_i = |C_i|^2$

$|\psi\rangle = \sum_i C_i |\alpha_i\rangle$, where $\hat{A}|\alpha_i\rangle = \alpha_i |\alpha_i\rangle$

$\langle \psi | = (|\psi\rangle)^\dagger = \sum_i C_i^* \langle \alpha_i |$ dummy variable

wrong

$\cancel{\langle A \rangle = \sum_i C_i^* \langle \alpha_i | A | \alpha_i \rangle C_i}$

$\langle A \rangle = \sum_{ij} C_i^* \langle \alpha_i | A | \alpha_j \rangle C_j$

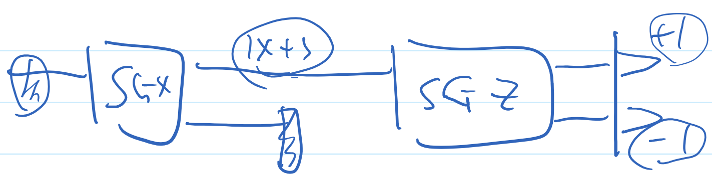
$= \sum_{ij} C_i^* C_j \alpha_j \underbrace{\langle \alpha_i | \alpha_j \rangle}_{\delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}}$

$\boxed{\sum_{ij} \xi_j \delta_{ij} = \xi_i}$

$= \sum_{ij} C_i^* C_j \alpha_j \delta_{ij} = \sum_i C_i^* C_i \alpha_i$

$= \sum_i |C_i|^2 \alpha_i$

$= \sum_i \alpha_i P_i$



$(+1)\times 50\% + (-1)\times 50\% = 0$

$|z+\rangle$

$(+1)\times 100\% = +1$

$\hat{z} = \sigma_z$