Homework 01

Problem 1: Sakurai 1.1

Prove

$$[AB, CD] = -AC\{D, B\} + A\{C, B\}D - C\{D, A\}B + \{C, A\}DB$$

Problem 2: Sakurai 1.4

Using the rules of bra-ket algebra, prove or evaluate the following:
(a) $tr(XY) = tr(YX)$, where X and Y are operators.
(b) $(XY)^\dagger = Y^\dagger X^\dagger$, where X and Y are operators.
(c) $exp[if(A)] =?$ in ket-bra form, where A is a Hermitian operator whose eigenvalues are known.
(d) $\sum_{a'} \psi^*_{a'}(\boldsymbol{x'})\psi_{a'}(\boldsymbol{x''})$, where $\psi_{a'}(\boldsymbol{x'}) = \langle \boldsymbol{x'}|a'\rangle$.

Problem 3: Sakurai 1.6

Suppose $|i\rangle$ and $|j\rangle$ are eigenkets of some Hermitian operator A. Under what condition can we conclude that $|i\rangle + |j\rangle$ is also an eigenket of A? Justify your answer.

Problem 4: Sakurai 1.14

A certain observable in quantum mechanics has a 3×3 matrix representation as follows:

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

(a) Find the normalized eigenvectors of this observable and the corresponding eigenvalues. Is there any degeneracy?
(b) Give a physical example where all this is relevant.

Problem 5: Additional

If operator U satisfies $UU^\dagger = I$ in a certain repersentation, show this is ture for any other representations.