Homework 07

Problem 1

With $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$,
and $\vec{n} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$, $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$, find eigenvalue and eigenstates of $\vec{\sigma}\cdot\vec{n}$

Problem 2

With $\psi(\theta,\varphi) = \frac{1}{\sqrt{3}}\left(\sqrt{2}Y_1^0(\theta,\varphi) + Y_1^1(\theta,\varphi)\right)$, without integrations, find $\langle L^2\rangle, \langle L_z\rangle$

Problem 3

With $\phi(t=0) = \psi(\theta,\varphi)$ as above, $H = \frac{\vec{L}^2}{2mR^2}$, find $\phi(t=T)$

Problem 4

With $J_z|j,m\rangle = m\hbar|j,m\rangle$, $\vec{J}^2|j,m\rangle = \hbar j(j+1)|j,m\rangle$,

$$\langle \Delta A\rangle = \sqrt{\langle l,m|A^2|l,m\rangle - (\langle l,m|A|l,m\rangle)^2}$$

find $\langle\Delta J_x\rangle\langle\Delta J_y\rangle$ and $\langle[J_x,J_y]\rangle$, check $\langle\Delta J_x\rangle\langle\Delta J_y\rangle \geq \frac{1}{2}|\langle[J_x,J_y]\rangle|$.
When $\langle\Delta J_x\rangle\langle\Delta J_y\rangle = \frac{1}{2}|\langle[J_x,J_y]\rangle|$, what's the requirement of m?

Problem 5

Write out J_-J_+ as a matrix in the basis of J_z, with $j=1$, and find the eigenvalue and eigenstates of J_-J_+