作业 08

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题 1

$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, find eigenvalue λ, eigenstates $|\lambda\rangle$ of A. With $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, show

$$\sigma_z \otimes A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 \end{pmatrix},$$

find eigenvalue and eigenvectors of $\sigma_z \otimes A$, show the relation with $\pm\lambda$, and $|0\rangle|\lambda\rangle$, $|1\rangle|\lambda\rangle$.

解. 已知 $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$，则本征值和本征态为

```
In[∘]:= A = {{1, 1}, {1, 1}};

In[∘]:= Grid[Insert[Transpose[{Eigenvalues[A], Normalize /@
          Eigenvectors[A]}], {"Eigenvalue", "Eigenvector"}, 1], Frame → All]
```

格子 插入 转置 特征值 正规化 特征向量 边框 全部

Out[∘]=

Eigenvalue	Eigenvector
2	$\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}$
0	$\left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}$

又 $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$，二者的直积 $\sigma_z \otimes A$ 为

```
In[∘]:= KroneckerProduct[PauliMatrix[3], A] // TraditionalForm
```

克罗内克积 泡利自旋矩阵 传统格式

Out[∘]//TraditionalForm=

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 \end{pmatrix}$$

同理，可以求得 $\sigma_z \otimes A$ 的本征值和本征态，

```
In[∘]:= ZA = KroneckerProduct[PauliMatrix[3], A];
Grid[Insert[Transpose[{Eigenvalues[ZA], Normalize /@
    Eigenvectors[ZA]}], {"Eigenvalue", "Eigenvector"}, 1], Frame → All]
```

Out[∘]=

Eigenvalue	Eigenvector
-2	$\{0, 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\}$
2	$\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0\}$
0	$\{0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\}$
0	$\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0\}$

易知，$\sigma_z \otimes A$ 的本征值和本征态有如下关系：

本征值	本征态
λ	$\|0\rangle\|\lambda\rangle$
$-\lambda$	$\|1\rangle\|\lambda\rangle$

□

题 2

"w-state", for N-spin-$\frac{1}{2}$ particles, one can construct

$$|w_N\rangle \equiv \frac{1}{\sqrt{N}}(|10...0\rangle + |010...0\rangle + ... + |0...01\rangle)$$

with all the permutation of one of the particles at state $|1\rangle$ and the other particles at $|0\rangle$ state. What is the probability of measuring $|1\rangle$ state for the first particle? If we measure $|0\rangle$ for the first particle, find the relation of the remaining state and $|w_{N-1}\rangle$.

解. 由于 $|w_N\rangle = \frac{1}{\sqrt{N}}(|10...0\rangle + |010...0\rangle + ... + |0...01\rangle)$，对 $|w_N\rangle$ 进行测量，有 $\frac{1}{N}$ 的概率得到态 $|10...0\rangle$，即测量到第一个粒子状态为 $|1\rangle$ 的概率为

$$P_{\text{第一个粒子为}|1\rangle} = \frac{1}{N} \tag{1}$$

如果测量第一个粒子得到状态 $|0\rangle$，则测量之后体系的状态变为

$$|w'_N\rangle = \sqrt{\frac{N}{N-1}}\frac{1}{\sqrt{N}}(|010...0\rangle + |0010...0\rangle + ... + |0...01\rangle) \tag{2}$$

$$= \frac{1}{\sqrt{N-1}}(|010...0\rangle + |0010...0\rangle + ... + |0...01\rangle) \tag{3}$$

$$= |0\rangle \otimes \frac{1}{\sqrt{N-1}}(|10...0\rangle + |010...0\rangle + ... + |0...01\rangle) \tag{4}$$

$$= |0\rangle \otimes |w_{N-1}\rangle \tag{5}$$

□

题 3

Evolution of coupled spin-$\frac{1}{2}$ system.

$$H = \Omega(\sigma_z \otimes I + I \otimes \sigma_z),\ |\psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle),$$

find $|\psi(t)\rangle$.

Hint: find the eigenstates and eigenvalues of H first

解. 可以先求出哈密顿量 H 的本征值和本征态，然后利用

$$e^{\hat{A}} = \sum_i e^{A_i}|i\rangle\langle i| \tag{6}$$

来求得时间演化算符 $U(t) = e^{-\frac{i}{\hbar}Ht}$。用 Mathematica 来求解的话，可以直接求得 $U(t)$:

```
In[∘]:= H =
          Ω (KroneckerProduct[PauliMatrix[3], PauliMatrix[0]] +
             |克罗内克积      |泡利自旋矩阵     |泡利自旋矩阵
             KroneckerProduct[PauliMatrix[0], PauliMatrix[3]]);
             |克罗内克积      |泡利自旋矩阵     |泡利自旋矩阵

In[∘]:= Ut = MatrixExp[-(i/ħ) H t];
             |矩阵指数

        TraditionalForm[Ut]
        |传统格式

Out[∘]//TraditionalForm=
```

$$\begin{pmatrix} e^{-\frac{2 i t \Omega}{\hbar}} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{\frac{2 i t \Omega}{\hbar}} \end{pmatrix}$$

代入 $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$，可得 $|\psi(t)\rangle$ 为:

```
In[∘]:= ψ0 = 1/√2 {0, 1, 1, 0};
        ψt = Ut.ψ0
Out[∘]= {0, 1/√2, 1/√2, 0}
```

□