Chapter 3

Yuquan Chen

2019/05/07

1 Coupled spin-$\frac{1}{2}$ system

Suppose we have an electron orbiting around an In the electron's frame, the positive charge orbits around e^- and create a magnetic field

$$\vec{L} \Rightarrow \vec{B} \propto \vec{L} \tag{1}$$

similar to $\vec{\mu} \cdot \vec{B}$ spin under of a magnetic field. Here we have

$$H \propto \vec{S} \cdot \vec{L}, \text{ where} \begin{cases} \vec{S} \text{ is spin of } e \\ \vec{L} \text{ is} \end{cases} \tag{2}$$

generalize to quantum,

$$H = \Omega \vec{S} \cdot \vec{L} = \Omega \left(\hat{S}_x \otimes \hat{L}_x + \hat{S}_y \otimes \hat{L}_y + \hat{S}_z \otimes \hat{L}_z \right) \tag{3}$$

1.1 Recap on angular momentum

With total spin $\vec{S}$, projection along z to be S_z, we can define eigenstate $|s, m_s\rangle$

$$\begin{cases} \vec{S}^2 |s, m_s\rangle = \hbar^2 s(s+1) |s, m_s\rangle \\ S_z |s, m_s\rangle = \hbar m_s |s, m_s\rangle \end{cases} \tag{4}$$

with atomic physics, we know

$$|l - s| \leq j \leq s + l \tag{5}$$

where j is the quantum number of $\vec{J}$. The possible choice of j is

$$|l - s|, |l - s| + 1, |l - s| + 2, ..., s + l - 1, s + l \tag{6}$$

Box 1.1: Examples

1. The ground orbital of H-atom, $l = 0, s = \frac{1}{2}$, ignore nuclei for now. Then

$$j = \frac{1}{2}, \ j_z = -\frac{1}{2} \text{ or } \frac{1}{2}$$

2. H-atom, first excited orbital $l = 1,\ s = \frac{1}{2}$. Then

$$j = \frac{1}{2} \text{ or } \frac{3}{2}$$

1.2 Termsymbol