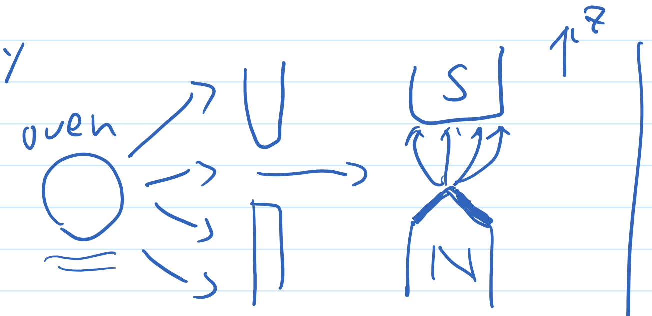
20190226, Ch1 intro to QM

Tuesday, February 26, 2019 4:27 PM

HW: read ch1 of Sakurai
no need to hand in HW in next lecture

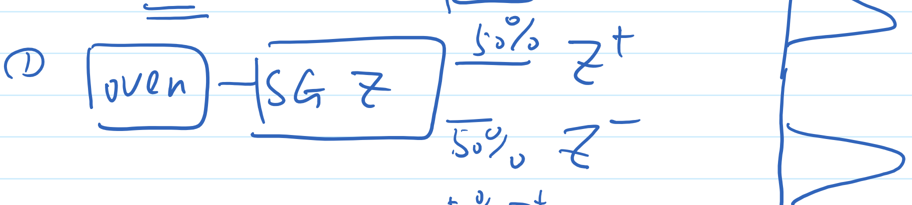
§1 case study

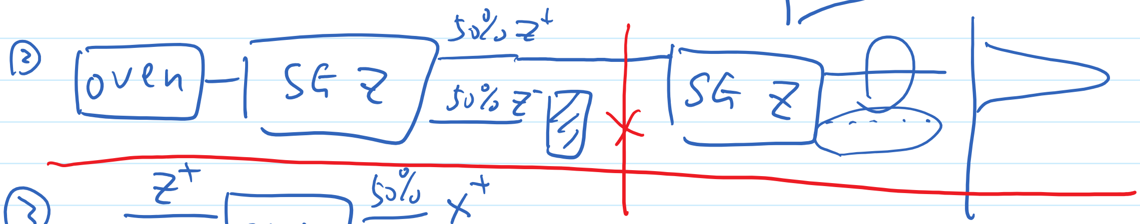


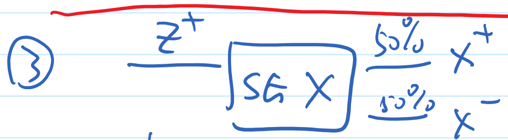
silver atom $\vec{\mu}$ magnetic moment

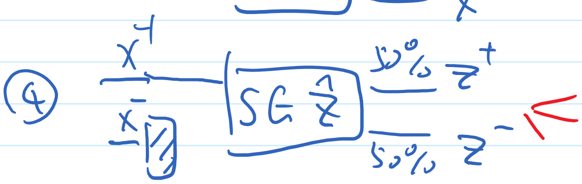
$$V = -\vec{\mu}\cdot\vec{B}$$

$$F = \frac{\partial}{\partial z}(-V) \doteq \mu_z \boxed{\frac{\partial B_z}{\partial z}}$$

① 

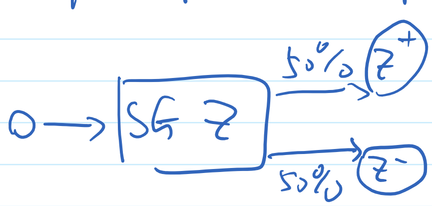
② 

③ 

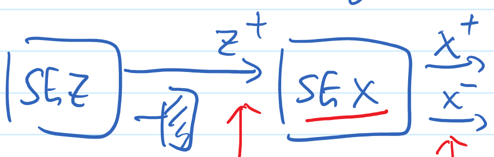
④ 

removes previous info

§2 principle of superposition



"$|\bullet\rangle$" ket



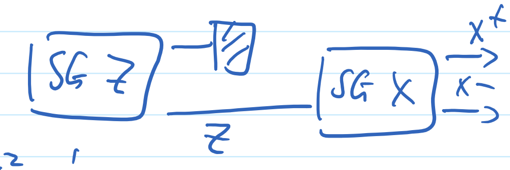
$$|z+\rangle = \frac{1}{\sqrt{2}}|x+\rangle + \frac{1}{\sqrt{2}}|x-\rangle = C_+|x+\rangle + C_-|x-\rangle$$

$\sqrt{50\%}$, $\sqrt{50\%}$

probability amplitude

$P_+ = |C_+|^2$, $P_- = |C_-|^2$

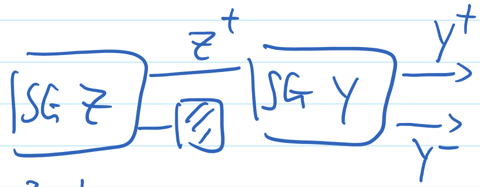
$$\begin{cases} |z+\rangle = \frac{1}{\sqrt{2}}|x+\rangle + \frac{1}{\sqrt{2}}|x-\rangle \\ |z-\rangle = \frac{1}{\sqrt{2}}|x+\rangle - \frac{1}{\sqrt{2}}|x-\rangle \end{cases}$$



$C_- = -\frac{1}{\sqrt{2}}$, $P_- = |C_-|^2 = \frac{1}{2}$

$$\begin{cases} |x+\rangle = \frac{1}{\sqrt{2}}|z+\rangle + \frac{1}{\sqrt{2}}|z-\rangle \\ |x-\rangle = \frac{1}{\sqrt{2}}|z+\rangle - \frac{1}{\sqrt{2}}|z-\rangle \end{cases}$$

$$\begin{cases} |z+\rangle = \frac{1}{\sqrt{2}}|y+\rangle \quad \frac{1}{\sqrt{2}}|y-\rangle \\ |z-\rangle = \frac{1}{\sqrt{2}}|y+\rangle - i\frac{1}{\sqrt{2}}|y-\rangle \end{cases}$$



$C_- = \frac{i}{\sqrt{2}}$, $P_- = |C_-|^2 = \frac{1}{2}$

" $|\ \rangle$ " ket

probability amplitude

superposition

§3 ket, Bras, operators linear algebra

$$\begin{cases} |\psi\rangle = c_0|\psi_0\rangle + c_1|\psi_1\rangle, \quad |\psi_0\rangle \equiv \begin{pmatrix}1\\0\end{pmatrix}, |\psi_1\rangle \equiv \begin{pmatrix}0\\1\end{pmatrix}, |\psi\rangle = \begin{pmatrix}c_0\\c_1\end{pmatrix} \\ c_0, c_1 \in \mathbb{C} \leftarrow \text{complex} \end{cases}$$

$|c_0|^2 + |c_1|^2 = 1$

① linearity

$C|\psi\rangle = |\psi\rangle C$

$|\alpha\rangle + |\beta\rangle = |\beta\rangle + |\alpha\rangle$, $1\cdot|\psi\rangle = |\psi\rangle$

$c_0 c_1 |\psi\rangle = c_1 c_0 |\psi\rangle$, $(c_0 + c_1)|\psi\rangle = c_0|\psi\rangle + c_1|\psi\rangle$

$$\boxed{(1\ \ 0) = \begin{pmatrix}1\\0\end{pmatrix}^T}$$

bra " $\langle\ |$ ", $\langle\psi| \equiv |\psi\rangle^\dagger$ $\dagger$ = transpose T, complex conjugate $*$

$\begin{pmatrix}i\\0\end{pmatrix}^* = \begin{pmatrix}-i\\0\end{pmatrix}$,

$|y+\rangle = \frac{1}{\sqrt{2}}|z+\rangle + \frac{i}{\sqrt{2}}|z-\rangle$, $|z+\rangle = \begin{pmatrix}1\\0\end{pmatrix}$, $|z-\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

$|y+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$

$\begin{cases} |\psi\rangle = c_0|z+\rangle + c_1|z-\rangle, \quad c_0 = \langle z+|\cdot|\psi\rangle = (1\ 0)\cdot\begin{pmatrix}c_0\\c_1\end{pmatrix} \\ \text{inner product.} \qquad\qquad = \langle z+|\psi\rangle \end{cases}$

norm

$\langle\psi|\psi\rangle \geq 0$, $|\psi\rangle = c_0|z+\rangle + c_1|z-\rangle$, $\langle\psi|\psi\rangle = |c_0|^2 + |c_1|^2 = 1$

$|\tilde{\psi}\rangle = \frac{1}{\sqrt{\langle\psi|\psi\rangle}}|\psi\rangle$, $\langle\psi|\psi\rangle \neq 1 \Rightarrow \langle\tilde{\psi}|\tilde{\psi}\rangle = 1$

outer product

$\boxed{|\varphi\rangle\langle\psi|}$, $|\varphi\rangle = \begin{pmatrix}d_0\\d_1\end{pmatrix}$, $|\psi\rangle = \begin{pmatrix}c_0\\c_1\end{pmatrix}$

$\begin{pmatrix}d_0\\d_1\end{pmatrix} \times (c_0^*\ c_1^*) \equiv \begin{pmatrix}d_0c_0^* & d_0c_1^* \\ d_1c_0^* & d_1c_1^*\end{pmatrix}$

$U = |\varphi\rangle\langle\psi|$, $U\cdot|\psi\rangle = |\varphi\rangle\underbrace{\langle\psi|\cdot|\psi\rangle} \sim |\varphi\rangle$

$(|\beta\rangle\langle\alpha|)\cdot|\gamma\rangle = |\beta\rangle\cdot(\underbrace{\langle\alpha|\gamma\rangle}_{\text{number}})$

bra space $\longleftrightarrow$ ket space

$\langle\psi| \equiv |\psi\rangle^+$

$\{|\psi\rangle\}$, $\underline{|z+\rangle}, \underline{|z-\rangle}$, $\{|z\rangle\}$

② basis

$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$, $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ $\begin{cases} \underline{\langle 0|0\rangle = (1\ 0)\begin{pmatrix}1\\0\end{pmatrix} = 1}, \ \underline{\langle 1|1\rangle = 1} \rightarrow \text{noral} \\ \langle 0|1\rangle = 0 \ \rightarrow \text{orthogonal 正交} \\ \langle 1|0\rangle = 0 \end{cases}$

$\{|0\rangle, |1\rangle\} \rightarrow$ basis. $\boxed{\text{orthogonal}}$ $\boxed{\text{noral}}$

③ operator 算符

$$\boxed{\hat{A}|\alpha\rangle = C\cdot|\beta\rangle}$$

$\begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix}\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix} = C\begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix}$ $\hat{A}$ in matrix only true for discrete system.

$a_{ij} \in$ complex numbers.

$\begin{cases} \hat{A} = \hat{B} \iff \text{for any } |\psi\rangle,\ \hat{A}|\psi\rangle = \hat{B}|\psi\rangle \\ \hat{A} + \hat{B} = \hat{B} + \hat{A} \end{cases}$

$\hat{A} = \hat{B} \iff$ for any $|\psi\rangle$, $\hat{A}|\psi\rangle = \hat{B}|\psi\rangle$

$\hat{A} + \hat{B} = \hat{B} + \hat{A}$

$\hat{A} + (\hat{B} + \hat{C}) = (\hat{A} + \hat{B}) + \hat{C}$

$\hat{A}\,(c_\alpha|\alpha\rangle + c_\beta|\beta\rangle) = c_\alpha(\hat{A}|\alpha\rangle) + c_\beta(\hat{A}\cdot|\beta\rangle)$

$(\langle\alpha|)\hat{A} \longleftrightarrow (\alpha_1^* \;\; \alpha_2^*)\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

$\hat{A}^\dagger$, if $\hat{A} \to \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $\hat{A}^\dagger \to \begin{pmatrix} a_{11}^* & a_{21}^* \\ a_{12}^* & a_{22}^* \end{pmatrix}$

$\dagger = T$ then $*$

$\hat{A}^T \to \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{pmatrix}$

$\underline{\langle\alpha|\hat{A}|\beta\rangle} = (\langle\alpha|)\cdot(\hat{A}|\beta\rangle) = (\langle\alpha|\hat{A})\cdot|\beta\rangle$

important variations.

① identity operator. $\hat{\mathbb{1}}$, for any $|\psi\rangle$, $\mathbb{1}|\psi\rangle = |\psi\rangle$

② Hermitian operator $\hat{A} = \hat{A}^\dagger$, for any $|\psi\rangle$, $\langle\psi|A|\psi\rangle = \langle\psi|A^\dagger|\psi\rangle$

any $|\varphi\rangle, |\psi\rangle$ $\langle\varphi|A|\psi\rangle = \langle\varphi|A^\dagger|\psi\rangle$.

$\hat{A} \to \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \underline{\underline{\text{if Hermitian}}} \begin{pmatrix} a_{11}^* & a_{21}^* \\ a_{12}^* & a_{22}^* \end{pmatrix} \Rightarrow \begin{cases} a_{11} = a_{11}^*, & a_{22} = a_{22}^* \\ a_{21}^* = a_{12} \end{cases}$

operator multiplication

$\hat{C} = \hat{A}\hat{B}$

$|\psi\rangle$, $\hat{B}|\psi\rangle = |\psi'\rangle$, $\hat{C}|\psi\rangle = \hat{A}\,\underline{\hat{B}|\psi\rangle} = \hat{A}|\psi'\rangle$.

$C = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$, $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

$\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

$|\psi\rangle$ — $\boxed{B}$ — $\boxed{A}$ — $|\psi''\rangle$ not go to exams here.

$|\psi\rangle$ — $\boxed{C}$ — $|\psi''\rangle$

$\hat{C}|\psi\rangle = |\psi''\rangle$

$\hat{C} = \underline{\hat{A}\cdot\hat{B}}$

③ unitary operators $\hat{U}\hat{U}^\dagger = \mathbb{1}$, $\hat{U}^\dagger\hat{U} = \mathbb{1}$

③ unitary operator $\hat{U}$ $\hat{U}\hat{U}^{+} = \mathbb{1}$, $\hat{U}^{+}\hat{U} = \mathbb{1}$

$\hat{U}^{-1} = \hat{U}^{+}$, $U|\psi\rangle = |\psi'\rangle$. $\underline{U^{+}U}|\psi\rangle = U^{+}|\psi'\rangle = |\psi\rangle$

$\hat{A}\cdot\hat{B} = \mathbb{1}$

§4 Hermitian operator and basis.

$$\hat{A}|\alpha\rangle = c|\beta\rangle$$

if $\hat{A}$ is operator, if we can find a ket $|\alpha\rangle$

$\hat{A}|\alpha\rangle = C_\alpha|\alpha\rangle$, then we call $\begin{cases} C_\alpha \text{ is eigenvalue of } \hat{A} \\ |\alpha\rangle \text{ is eigenket of } \hat{A} \end{cases}$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = C_\alpha\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \leftarrow$$

$\boxed{\text{Theorem}}$ if $\hat{A}$ is Hermitian, then its eigenvalue of alway real.

$\hat{A}^{+} = \hat{A}$, $\hat{A}|\alpha\rangle = C_\alpha|\alpha\rangle$, $\hat{A}^{+}|\alpha\rangle = ?$

$$\begin{cases} \langle\alpha|\hat{A}|\alpha\rangle = \langle\alpha|(C_\alpha|\alpha\rangle) = C_\alpha\langle\alpha|\alpha\rangle \\ \langle\alpha|\hat{A}^{+}|\alpha\rangle = (\langle\alpha|\hat{A}^{+})|\alpha\rangle = C_\alpha^{*}\langle\alpha|\alpha\rangle \end{cases}$$

$\langle\alpha|\alpha\rangle \geq 0$, $C_\alpha = C_\alpha^{*} \longrightarrow$ real.

$\hat{A}|\alpha\rangle = C_\alpha|\alpha\rangle \rightarrow (\hat{A}|\alpha\rangle)^{+} = (C_\alpha|\alpha\rangle)^{+}$

$\langle\alpha|A^{+} = \langle\alpha|C_\alpha^{*}$

$$\left[\begin{pmatrix} & \end{pmatrix}\begin{pmatrix} \\ \end{pmatrix}\right]^{+}$$

$$= (\quad)(\quad)^{+}$$

- diagonalize of A

$$A = \begin{pmatrix} a_{11} & a_{12} & \\ & \ddots & \\ & & \ddots \end{pmatrix}_{n\times n}$$

try to find eigen value and eigen state of A

$\hat{A}|\alpha\rangle = C_\alpha|\alpha\rangle$

$$|\alpha\rangle = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \end{pmatrix} \Longrightarrow \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & a_{22} & a_{23} & \cdots \\ & & \ddots & \end{pmatrix}\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \end{pmatrix} = C_\alpha\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \end{pmatrix}$$

← similar to finding eigenvalue eigenvector of a matrix

$$\boxed{\begin{pmatrix} a_{11}-C_\alpha & a_{12} & a_{13} & \\ a_{21} & a_{22}-C_\alpha & a_{23} & \cdots \\ & & \ddots & \\ & & & \ddots \end{pmatrix}\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \vdots \end{pmatrix} = 0} \Longrightarrow \alpha's$$

$$\begin{vmatrix} a_{11}-C_\alpha & a_{12} & a_{13} & \cdots \\ a_{21} & a_{22}-C_\alpha & a_{23} & \cdots \\ & & \ddots & \\ & & & \ddots \end{vmatrix} = 0 \longrightarrow$$

n linear equation

n solutions of C_α

$$\left|\left(\quad \ddots \quad \right)_{n\times n}\right| \qquad n \text{ solutions of } c_\alpha$$

example 1. $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$, $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

eigenvalue and eigenstate $\begin{pmatrix}0&1\\1&0\end{pmatrix} = \sigma_x$ operator

$\begin{pmatrix}0&1\\1&0\end{pmatrix}^\dagger = \begin{pmatrix}0&1\\1&0\end{pmatrix}$ Hermitian.

$$\left|\begin{pmatrix}0-c_\alpha & 1\\ 1 & 0-c_\alpha\end{pmatrix}\right| = 0 \Rightarrow c_\alpha^2 - 1 = 0 \Rightarrow c_\alpha = \pm 1$$

eigenvalue of σ_x is $+1$ and -1

eigen state
if $c_\alpha = +1$ $\quad \begin{pmatrix}0-1 & 1\\ 1 & 0-1\end{pmatrix}\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix} = 0 \Rightarrow \begin{pmatrix}-1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix} = 0$

$$\Rightarrow \underline{-\alpha_1 + \alpha_2 = 0} \Rightarrow \alpha_1 = \alpha_2 = \alpha$$

for eigenvalue of $+1$, we have eigenstate $\begin{pmatrix}\alpha\\ \alpha\end{pmatrix}$

physical: $|\alpha_1|^2 + |\alpha_2|^2 = 1 \Rightarrow 2|\alpha|^2 = 1$, $|\alpha| = \frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{1}{\sqrt{2}} e^{i\phi}$

example 2. $\mathbb{1}$, if we chose basis of $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$, $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

$$\mathbb{1} = \begin{pmatrix}1&0\\0&1\end{pmatrix} \qquad \begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix} = \begin{pmatrix}c_1\\c_2\end{pmatrix}$$

eigenvalue and eigenstates of $\mathbb{1}$

$$\left|\begin{pmatrix}1-c_\alpha & 0\\ 0 & 1-c_\alpha\end{pmatrix}\right| = 0 \Rightarrow c_\alpha = 1$$

$$\underbrace{\begin{pmatrix}1-c_\alpha & 0\\ 0 & 1-c_\alpha\end{pmatrix}}_{\begin{pmatrix}0&0\\0&0\end{pmatrix}}\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix} = 0$$

any states are eigenstates of identity operator

$$\mathbb{1}|\psi\rangle = |\psi\rangle, \quad \mathbb{1}|\psi\rangle = 1\cdot|\psi\rangle$$

in general, we can express $\boxed{\hat{A} = \sum_\alpha c_\alpha |\alpha\rangle\langle\alpha|}$

∘ basis. $\hat{B} = \begin{pmatrix}b_{11} & b_{12} & \cdots\\ b_{21} & b_{22} & \cdots\end{pmatrix} \xrightarrow{\text{for example}} \begin{pmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{pmatrix} = b_{11}\begin{pmatrix}1&0\\0&0\end{pmatrix} + b_{12}\begin{pmatrix}0&1\\0&0\end{pmatrix}$

basis: $B = \begin{pmatrix} \dots \\ b_{21} & b_{22} & \dots \end{pmatrix} \rightarrow \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = b_{11}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b_{12}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$+ b_{21}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + b_{22}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$|0\rangle\langle 1| = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad |1\rangle\langle 0| = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$|0\rangle\langle 0| = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$|1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$B \rightarrow \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = b_{11}|0\rangle\langle 0| + b_{12}|0\rangle\langle 1| + b_{21}|1\rangle\langle 0| + b_{22}|1\rangle\langle 1|$

$\hat{B} = \sum_{ij} C_{ij} |\alpha_i\rangle\langle\alpha_j|, \quad |\alpha_i\rangle, |\alpha_j\rangle$ within $\{|\alpha\rangle\}$