

Homework 03

Problem 1

A particle in a one dimensional infinite deep well $V(x) = \begin{cases} 0 & -\frac{a}{2} < x < \frac{a}{2} \\ \infty & \text{other} \end{cases}$, suppose at $t = 0$, $\psi(x, 0) = \frac{4}{\sqrt{a}} \cos \frac{\pi}{a} x \sin^2 \frac{\pi}{a} x$, $-\frac{a}{2} < x < \frac{a}{2}$, try to find $\psi(x, t)$. Sketch $|\psi(x, t)|^2$, discuss its evolution over time.

Problem 2

With $|\alpha\rangle$ a Gaussian wavepacket, $\langle x|\alpha\rangle = \frac{1}{\pi^{1/4}\sqrt{\alpha}} e^{ikx - \frac{x^2}{2\alpha^2}}$, we have proofed $\langle p|\alpha\rangle = \sqrt{\frac{d}{\hbar\sqrt{\pi}}} e^{-\frac{(p-\hbar k)^2 d^2}{2\hbar^2}}$ with $H = \frac{p^2}{2m}$ a free particle with wavefunction $\psi(x, 0) = \langle x|\alpha\rangle$.

1. Find $\psi(p, t)$
2. Find $\psi(x, t)$
3. Sketch $|\psi(x, t)|^2$, discuss its evaluation over time.