Homework 08

Problem 1

$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, find eigenvalue λ, eigenstates $|\lambda\rangle$ of A. With $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, show

$$\sigma_z \otimes A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 \end{pmatrix},$$

find eigenvalue and eigenvectors of $\sigma_z \otimes A$, show the relation with $\pm\lambda$, and $|0\rangle|\lambda\rangle$, $|1\rangle|\lambda\rangle$.

Problem 2

"w-state", for N-spin-$\frac{1}{2}$ particles, one can construct

$$|w_N\rangle \equiv \frac{1}{\sqrt{N}}(|10...0\rangle + |010...0\rangle + ... + |0...01\rangle)$$

with all the permutation of one of the particles at state $|1\rangle$ and the other particles at $|0\rangle$ state. What is the probability of measuring $|1\rangle$ state for the first particle? If we measure $|0\rangle$ for the first particle, find the relation of the remaining state and $|w_{N-1}\rangle$.

Problem 3

Evolution of coupled spin-$\frac{1}{2}$ system.

$$H = \Omega(\sigma_z \otimes I + I \otimes \sigma_z),\ |\psi(t=0)\rangle = \frac{1}{2}(|01\rangle + |10\rangle),$$

find $|\psi(t)\rangle$.
Hint: find the eigenstates and eigenvalues of H first