# Homework 06

## Problem 1

Prove $e^{-i\omega t\hat{\sigma}_x} = \cos\omega t I - i\sin\omega t\hat{\sigma}_x$

## Problem 2

Detuned Rabi flopping for a spin-1/2 particle with energy spacing $\omega_0$ apply an oscillating magnetic with frequency $\omega_0 + \delta$, and Rabi rate $\Omega$, so we have $H(t) = \hbar\frac{\omega_0}{2}\hat{\sigma}_z + \hbar\Omega\hat{\sigma}_x cos((\omega+\delta)t)$

1. Choose a proper transformation and apply rotating wave approximation to make $H(t)$ time-independent, so that $H_{int} = -\hbar\frac{\delta}{2}\sigma_z + \hbar\frac{\Omega}{2}\sigma_x$
2. Solve for eigenvalue $\lambda_+, \lambda_-$ and eigenstate $|\psi_+\rangle, |\psi_-\rangle$ for $H_{int}$ in the basis of $\sigma_z$ $\{|0\rangle, |1\rangle\}$
3. With $|\psi(t=0)\rangle = |\psi_0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ solve for the overlap between $|\psi_0\rangle$ and $|\psi(t)\rangle$, defined as $|\langle\psi_0|\psi(t)\rangle|^2$. Hint: use $|\psi(t)\rangle = e^{-\frac{i}{\hbar}Ht}|\psi(0)\rangle$, and $H = \lambda_+|\psi_+\rangle\langle\psi_+| + \lambda_-|\psi_-\rangle\langle\psi_-|$. We can assume $\Omega$ is real for simplicity.

## Problem 3

Proof $Tr(\rho^2) = 1$ correspond to $\rho = |\psi\rangle\langle\psi|$, a pure state.