作业 08

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题 1

$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, find eigenvalue λ, eigenstates $|\lambda\rangle$ of A. With $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, show

$$\sigma_z \otimes A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 \end{pmatrix},$$

find eigenvalue and eigenvectors of $\sigma_z \otimes A$, show the relation with $\pm\lambda$, and $|0\rangle|\lambda\rangle$, $|1\rangle|\lambda\rangle$.

解. 已知 $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$，则本征值和本征态为

```
In[•]:= A = {{1, 1}, {1, 1}};

In[•]:= Grid[Insert[Transpose[{Eigenvalues[A], Normalize /@
        格子  插入  转置  特征值  正规化
            Eigenvectors[A]}], {"Eigenvalue", "Eigenvector"}, 1], Frame → All]
            特征向量  边框  全部
```

Out[•]=

Eigenvalue	Eigenvector
2	$\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}$
0	$\left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}$

又 $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$，二者的直积 $\sigma_z \otimes A$ 为

```
In[•]:= KroneckerProduct[PauliMatrix[3], A] // TraditionalForm
        克罗内克积  泡利自旋矩阵  传统格式
```

Out[•]//TraditionalForm=

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 \end{pmatrix}$$

同理，可以求得 $\sigma_z \otimes A$ 的本征值和本征态，

```
In[•]:= ZA = KroneckerProduct[PauliMatrix[3], A];
        |克罗内克积          |泡利自旋矩阵
        Grid[Insert[Transpose[{Eigenvalues[ZA], Normalize /@
        |格子  |插入    |转置          |特征值              |正规化
            Eigenvectors[ZA]}], {"Eigenvalue", "Eigenvector"}, 1], Frame → All]
            |特征向量                                          |边框   |全部
```

Out[•]=

Eigenvalue	Eigenvector
-2	$\left\{0, 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}$
2	$\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0\right\}$
0	$\left\{0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}$
0	$\left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0\right\}$

易知，$\sigma_z \otimes A$ 的本征值和本征态有如下关系：

本征值	本征态
λ	$\|0\rangle\|\lambda\rangle$
$-\lambda$	$\|1\rangle\|\lambda\rangle$

□

题 2

"w-state", for N-spin-$\frac{1}{2}$ particles, one can construct

$$|w_N\rangle \equiv \frac{1}{\sqrt{N}}(|10...0\rangle + |010...0\rangle + ... + |0...01\rangle)$$

with all the permutation of one of the particles at state $|1\rangle$ and the other particles at $|0\rangle$ state. What is the probability of measuring $|1\rangle$ state for the first particle? If we measure $|0\rangle$ for the first particle, find the relation of the remaining state and $|w_{N-1}\rangle$.

题 3

Evolution of coupled spin-$\frac{1}{2}$ system.

$$H = \Omega(\sigma_z \otimes I + I \otimes \sigma_z),\ |\psi(t=0)\rangle = \frac{1}{2}(|01\rangle + |10\rangle),$$

find $|\psi(t)\rangle$.

Hint: find the eigenstates and eigenvalues of H first