

23/04/25

PC - Lab Guidelines' Questions/Answers

- (1) Plotting and fitting of Binomial distribution and graphical representation of probabilities.

Sol⁴ A Binomial distribution is used to describe the probability of obtaining k successes in n obtaining binomial experiment.
formula :

$$P(X = k) = {}^n C_k * p^k * (1-p)^{n-k}$$

where :

n = no. of trials

k = no. of successes

p = Probability of success on a given trial

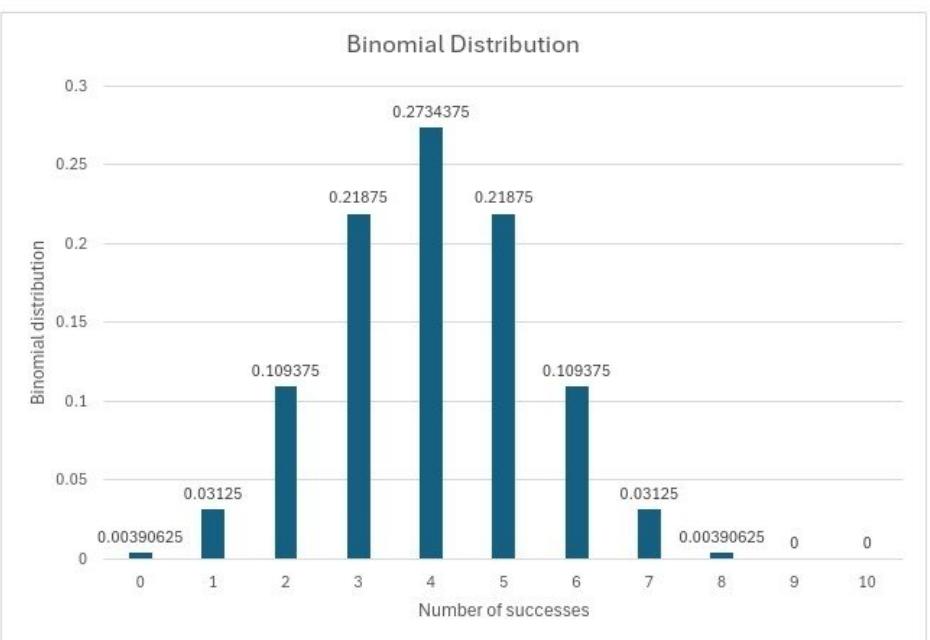
${}^n C_k$ = the number of ways to obtain k successes in n trials.

Example (in excel):

	A	B
1	n (no. of trials)	
2	p (Prob. of success on given trial)	0.5
3		
4	k (no. of successes)	Binomial Prob.
5		0.00390625
6		(BINOM.DIST(15,\$B\$1,
7		\$B\$2, False))
8		
9		
10		
11		
12		
13		

To create a binomial distribution graph we need to first decide on a value for n(number of trials) and p(probability of success in a given trial).

value of n(number of trial)	8
p(probability of success in given trial)	0.5
k number of successes	binomial distribution
0	0.00390625
1	0.03125
2	0.109375
3	0.21875
4	0.2734375
5	0.21875
6	0.109375
7	0.03125
8	0.00390625
9	#NUM!
10	#NUM!



(2) Plotting and fitting of Multinomial distribution and graphical representation of probabilities.

Sol^u The multinomial distribution describes the probability of obtaining a specific number of counts for k different outcomes, when each outcome has a fixed probability of occurring.

If a random variable X follows a multinomial distribution, then the probability that outcome 1 occurs exactly x_1 times, outcome 2 occurs exactly x_2 times, etc. can be found by the formula

$$\text{probability} = n! * (p_1^{x_1} * p_2^{x_2} * \dots * p_k^{x_k}) / (x_1! * x_2! * \dots * x_k!)$$

Here n : Total no. of events

x_i : no. of times outcome i occurs

P : Probability that outcome i occurs in one trial

Example 1

In a three way election for mayor, candidate A receives 10% of the votes, candidate B receives 40% of the votes and candidate C receives 50% of the votes.

If we select a random sample of 10 voters, what is the probability that 2 voted for A, 4 voted for B and 4 voted for C?

In Excel:

A

B

C

D

E

F

G

H

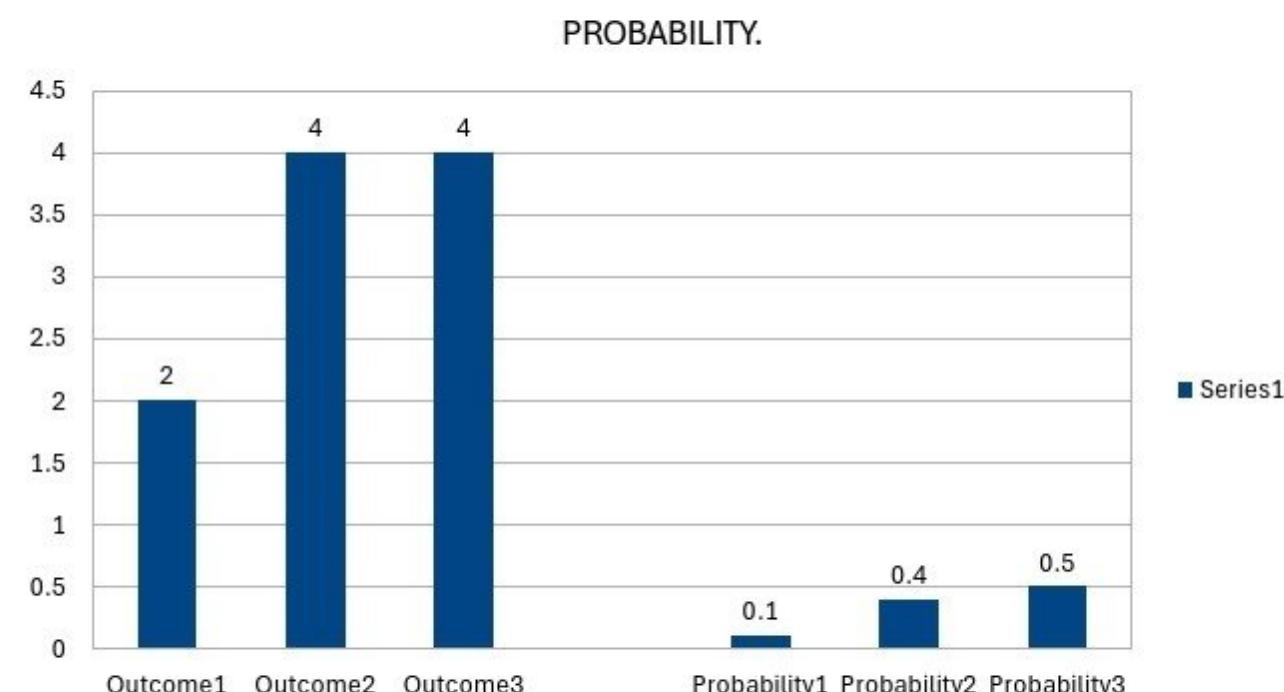
I

Plotting and Fitting of Multinomial distribution and graphical representation of probabilities.

Outcome1	2
Outcome2	4
Outcome3	4

Probability1	0.1
Probability2	0.4
Probability3	0.5

Multinomial	3150
Prob1-Outcome1	0.01
Prob2-Outcome2	0.026
Prob3-Outcome3	0.063
PROBABILITY	0.05



	A	B	C
1	Outcome 1		
2	Outcome 2	2	
3	Outcome 3	4	
4		4	
5	Probability 1		0.1
6	Probability 2		0.4
7	Probability 3		0.5
8			
9	Multinomial.	3150	= Multinomial (B1:B3)
10	Prob1^ Outcome 1	0.01	= B5^ B1
11	Prob2^ Outcome 2	0.0256	= B6^ B2
12	Prob3^ Outcome 3	0.0625	= B7^ B3
13	Probability	0.0504	= B9 * PRODUCT(B10:B12)

The probability that exactly 2 people voted for A, 4 people voted for B & 4 people voted for C is 0.0504.

(3) Plotting & fitting of poisson distribution & graphical representation of probabilities.

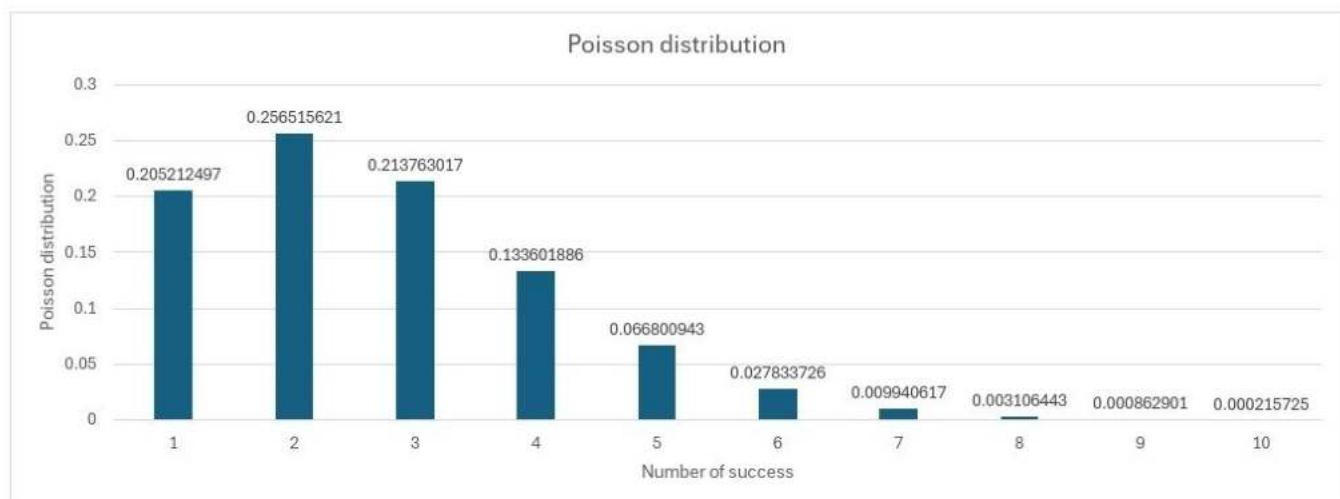
Solⁿ The Poisson distribution describes the probability of obtaining K successes during a given time interval.

If a random variable X follows a Poisson distribution, then the probability that $X = K$ successes can be found by the following formula:

Plotting and fitting of poisson distribution and graphical representation of probabilities

λ (mean number of success)	2.5
------------------------------------	-----

k(number of success)	
1	0.205212497
2	0.256515621
3	0.213763017
4	0.133601886
5	0.066800943
6	0.027833726
7	0.009940617
8	0.003106443
9	0.000862901
10	0.000215725



$$P(X=k) = \lambda^k * e^{-\lambda} / k!$$

where :

λ : mean of successes that occur during a specific interval

k : number of successes

e : a constant equal to approx 2.7183

Example :

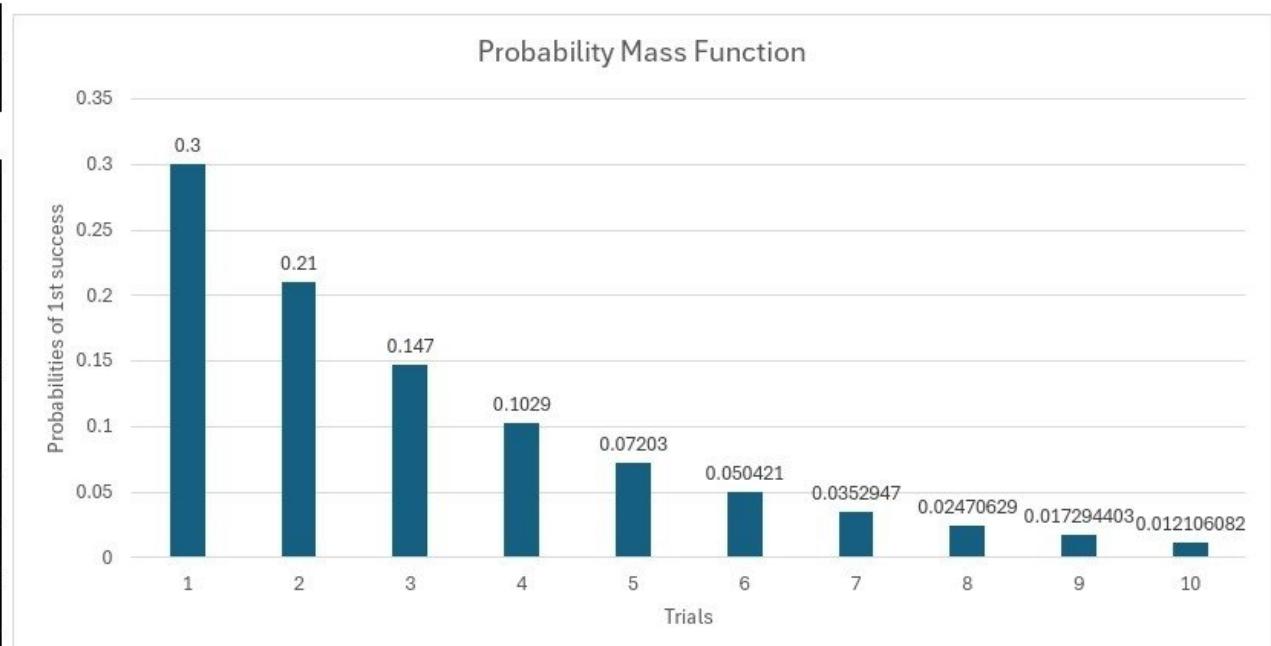
A	B	C
1 λ (mean no. of successes)	4	
2		
3 k (no. of successes)		= POISSON.DIST
4	0.018316	(A4, \$B\$1, false)
5	0.073263	
6	0.146525	
7	0.195367	
8	0.195367	
9	0.150093	
10	0.104196	
11	0.05954	
12	0.02977	
13	0.013231	
14	0.005292	

(4) Plotting and fitting of Geometric distribution and graphical representation of probabilities.

Ans The Geometric distribution describes the probability of experiencing a certain amount of failures before experiencing the first success in a series of Bernoulli trials.

Plotting and fitting of geometric distribution and graphical representation of probabilities.

P(Probability of success)	0.3
1-P(Probability of failure)	0.7
Trials(x)	PMF(X=x)
1	0.3
2	0.21
3	0.147
4	0.1029
5	0.07203
6	0.050421
7	0.0352947
8	0.02470629
9	0.017294403
10	0.012106082



If a random variable X follows a geometric distribution, then the probability of experiencing k failures before experiencing the first success can be found by the formula:

$$P(X=k) = (1-p)^k p.$$

where:

K : number of failures before first success

p : Probability of success on each trial.

Example : Flipping a coin

Suppose we are flipping a coin and we want to know the probability that it will take exactly three "failures" until a coin finally lands on heads.

	A	B
1	Prob. of success of given trial (p)	0.5
2	No. of failures before first success (k)	3
3		
4	Geometric Distribution formula	$= (1 - B1)^k B2 * B1$
5	Probability	0.0625

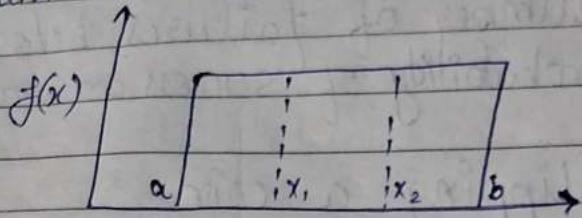
5 Plotting & fitting of Uniform distribution and graphical representation of probabilities.

Sol^m A uniform distribution is a probability distribution in which every value between an interval from a to b is equally

likely to be chosen.

The probability that we will obtain a value between x_1 and x_2 on an interval from a to b can be found using the formula:

$$P(\text{obtain the value btw } x_1 \text{ & } x_2) = (x_2 - x_1)/(b - a)$$



A uniform distribution has the following properties:

- The mean of the distribution is $\mu = (a+b)/2$
- The variance of distribution is $\sigma^2 = (b-a)^2/12$.
- The standard derivation of the distribution is $\sigma = \sqrt{\sigma^2}$

Example :

A bus shows up at a bus stop every 20 min. If you arrive at the bus stop, what is the probability that the bus will show up in 8 minutes or less?

In Excel. $a = 0 \text{ min}$, $b = 20 \text{ min}$, $x_1 = 0 \text{ min}$, $x_2 = 8 \text{ min}$

	A	B	C
1	a (min value of distribution)	0	
2	b (max value of distribution)	20	
3	x_1 (min value you're interested in)	0	
4	x_2 (max value you're interested in)	8	
5			
6	Probability	0.4	$= (B4-B3)/(B2-B1)$

A

B

C

D

E

F

G

H

I

J

K

L

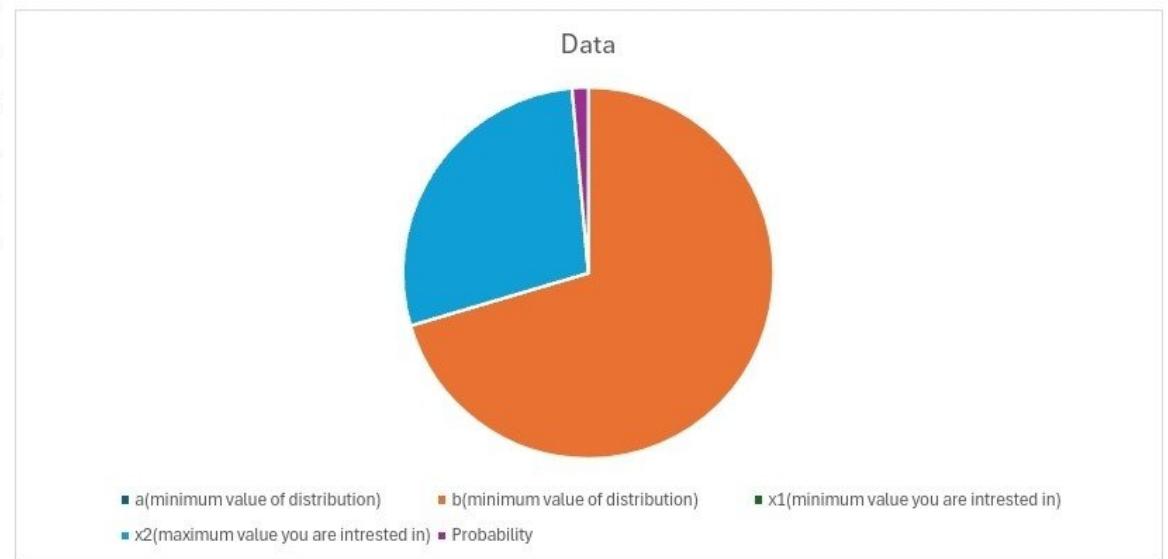
M

N

Plotting and fitting of uniform distribution and graphical representation

A bus shows up at the bus stop for every 20 minutes. If you arrive at the bus stop, What is the probability that the bus will show up in 8 minutes or less?

a(minimum value of distribution)	0
b(minimum value of distribution)	20
x1(minimum value you are interested in)	0
x2(maximum value you are interested in)	8
Probability	0.4



The probability that the bus shows up in 8 min or less is 0.4.

(6) Plotting & fitting of exponential distribution and graphical representation of Probabilities.

Solⁿ The exponential distribution is a probability distribution that is used to model the time we must wait until a certain event occurs.

If a random variable X follows an exponential distribution then the cumulative density function of X can be written as:

$$F(x; \lambda) = 1 - e^{-\lambda x}$$

where

λ : the rate parameter (calculated as $\lambda = 1/\mu$)

e : A constant roughly equal to 2.718.

Example: A new customer enters a shop in every two minutes, on average. After a customer arrives, find the probability that a new customer arrives in less than one min.

Solⁿ The average time between customers is 2 minutes. Thus, the rate can be calculated as : $\lambda = 1/\mu$

$$\lambda = 1/2$$

$$\lambda = 0.5.$$

A

B

C

D

E

F

G

H

I

J

K

L

M

N

Plotting and fitting of Exponential distribution and graphical representation of probabilities

λ	0.5
-----------	-----

X	P(X)
---	------

1	0.30326533
---	------------

2	0.183939721
---	-------------

3	0.11156508
---	------------

4	0.067667642
---	-------------

5	0.041042499
---	-------------

6	0.024893534
---	-------------

7	0.015098692
---	-------------

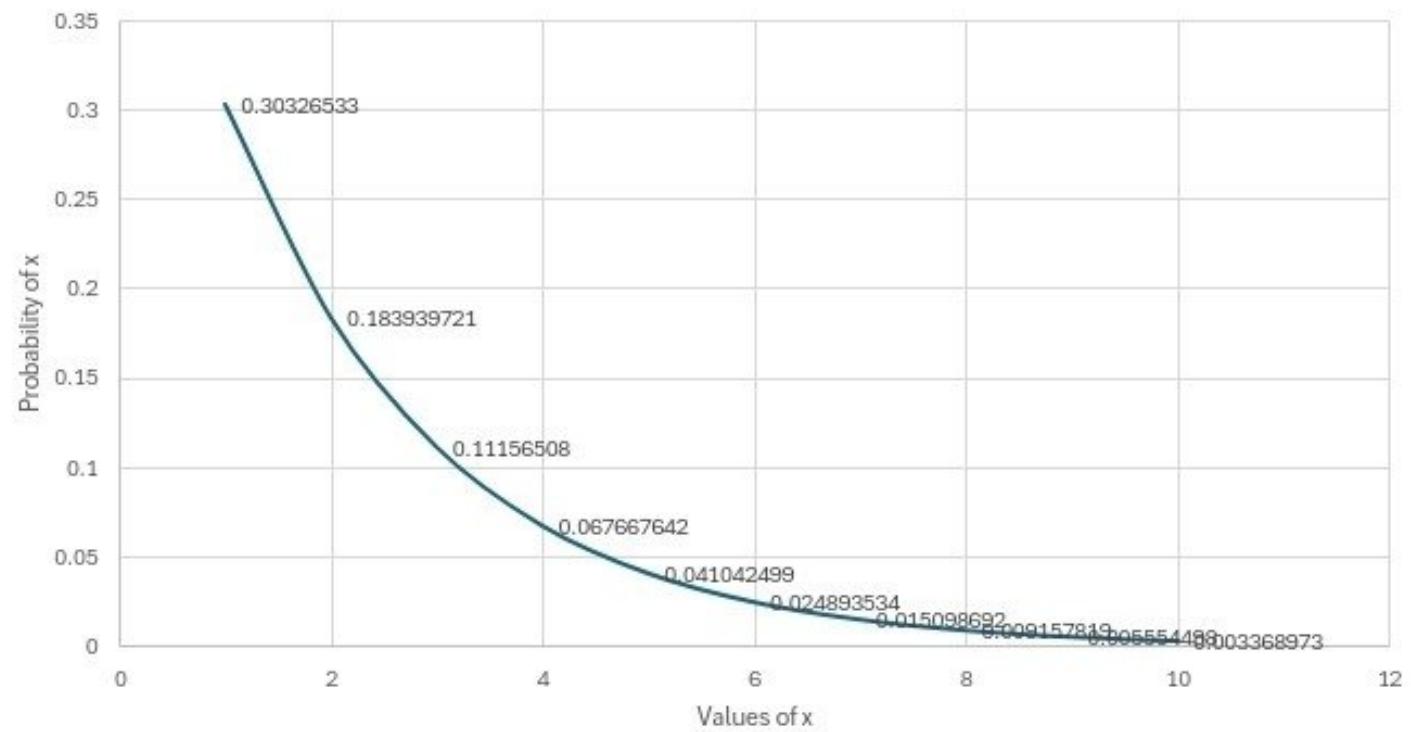
8	0.009157819
---	-------------

9	0.005554498
---	-------------

10	0.003368973
----	-------------

0.30326533

Exponential Distribution



formula for excel:
 $\text{EXPON.DIST}(x, \lambda, \text{cumulative})$
 where : x = value of exponentially distributed random variable.
 λ = the rate parameter.
 cumulative : True or False.

In Excel:

	A	B
1	λ	$\rightarrow \text{EXPON.DIST}(0.5$
2	x	2
3	= EXPON.DIST(B2, B1, TRUE)	0.393469

Q7. Plotting & fitting of Normal Distribution and graphical representation of probabilities.

Sol⁷ - A normal distribution is the most commonly used distribution in all of statistics.

To calculate probabilities related to the normal distribution in Excel, you can use the NORMDIST function, which uses the following basic syntax :

= NORMDIST(x , mean, standard-dev, cumulative)

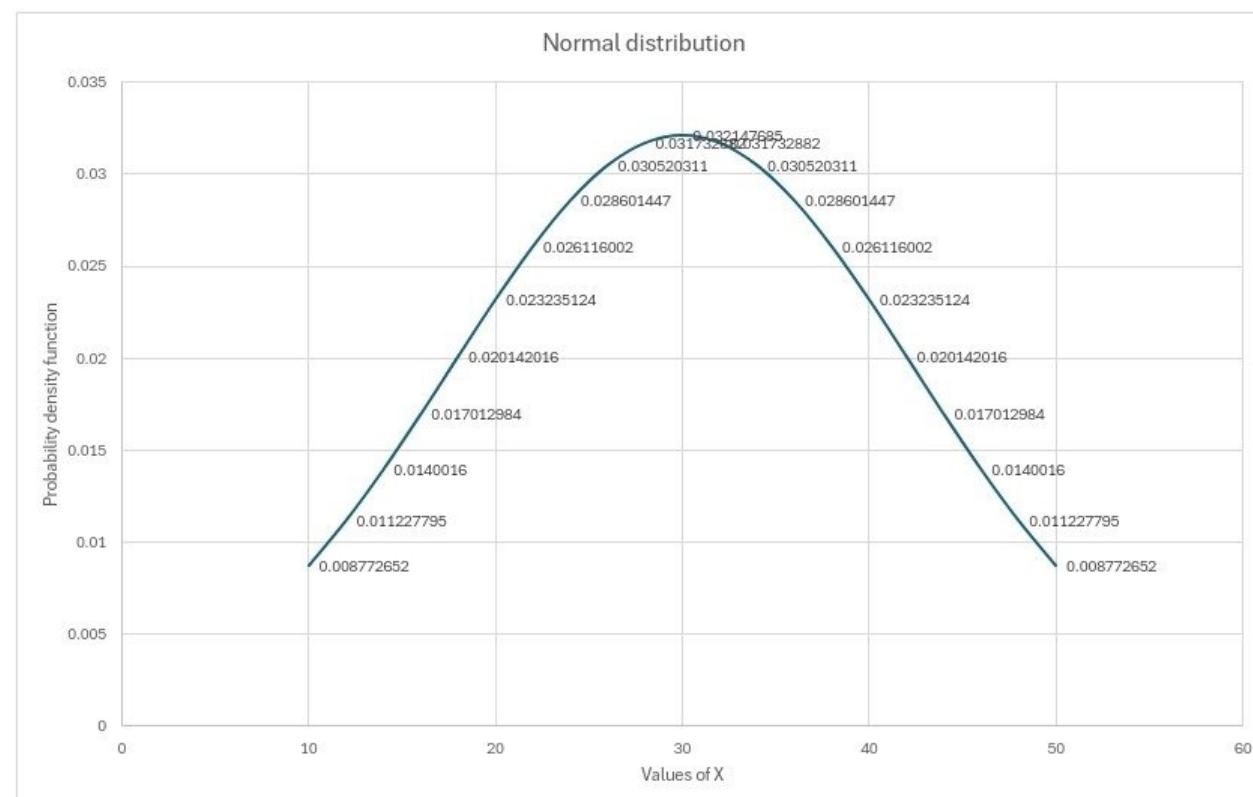
where

x → The value of interest in the normal distribution
 mean : The mean of the normal distribution
 Standard-dev : The standard deviation of the normal distribution

cumulative : whether to calculate cumulative probabilities (this is usually true).

1
2 Plotting and fitting of Normal distribution and graphical representation of probabilities.
3

X	PDF
10	0.008772652
12	0.011227795
14	0.0140016
16	0.017012984
18	0.020142016
20	0.023235124
22	0.026116002
24	0.028601447
26	0.030520311
28	0.031732882
30	0.032147685
32	0.031732882
34	0.030520311
36	0.028601447
38	0.026116002
40	0.023235124
42	0.020142016
44	0.017012984
46	0.0140016
48	0.011227795
50	0.008772652
Mean	30
Standard deviation	12.40967365



Example 1 :

Suppose the scores for an exam are normally distributed with a mean of 90 & a standard deviation of 10.

Find probability that a randomly selected student receives a score less than 80.

In Excel:

	A	B	C
1	Mean	90	
2	Standard Deviation	10	
3			
4	Prob $x < 80$	0.1587	$\leftarrow =NORMDIST(80, B1, B2, TRUE)$

The probability that a randomly selected student receives a score less than 80 is 0.1587.

(8) Calculation of cumulative distribution function for exponential & normal distribution.

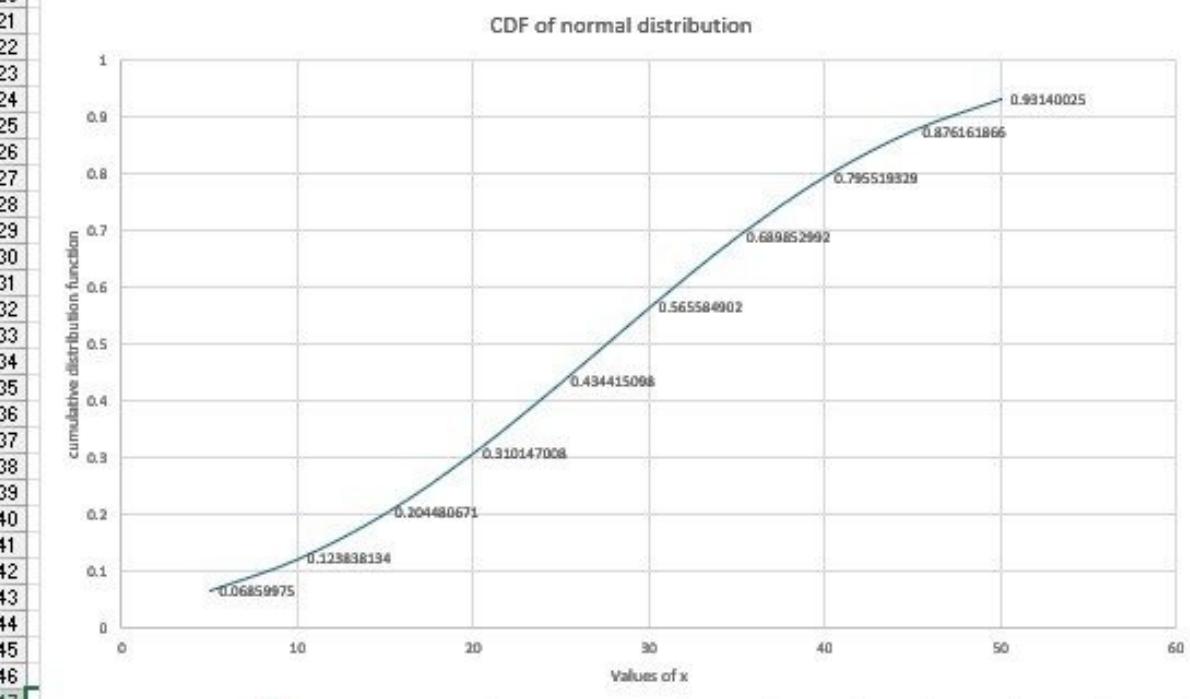
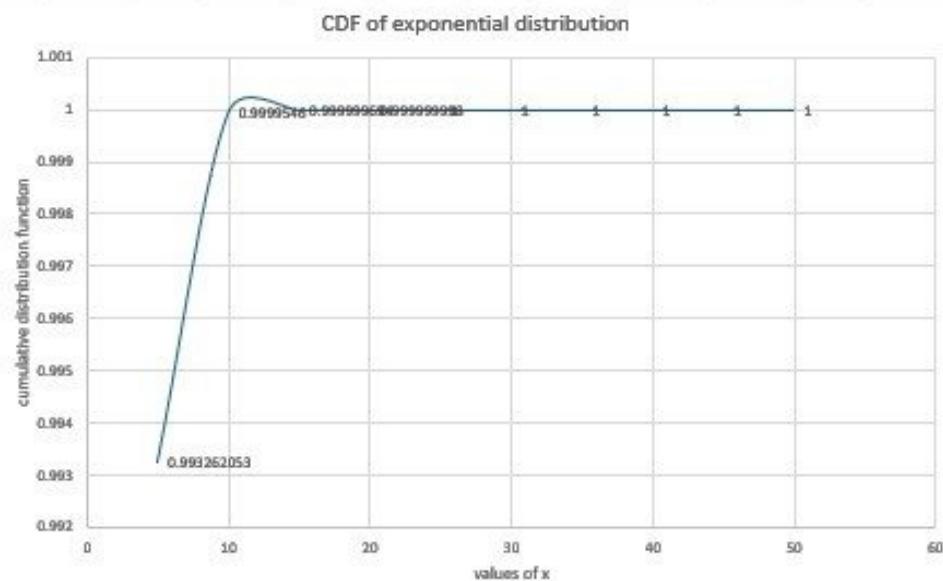
Sol¹⁴
Exponential Distribution CDF
formula of Exponential CDF is:

$$F(x; \lambda) = 1 - e^{-\lambda x}$$

In Excel :

	A	B
1	X	
2	λ	0.5
3	Exponential Distribution (CDF Result)	0.6321205588 $\rightarrow =EXPON.DIST(B1, B2, True)$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Calculation of cumulative distribution functions for Exponential and Normal distribution															
4	λ	1													
5															
6	X	Exponential CDF	Normal CDF												
7	5	0.993262053	0.06859975												
8	10	0.9999546	0.123838134												
9	15	0.999999694	0.204480671												
10	20	0.999999998	0.310147008												
11	25	1	0.434415098												
12	30	1	0.565584902												
13	35	1	0.689852992												
14	40	1	0.795519329												
15	45	1	0.876161866												
16	50	1	0.93140025												
17															
18	Mean	27.5													
19	Standard deviation	15.13825177													
20															



Normal Distribution CDF

The formula for the normal CDF is more complex but Excel makes it simple:

formula : $\text{NORM.DIST}(x, \text{mean}, \text{standard_dev}, \text{True})$

<u>In Excel</u>	A	B
1	x	70
2	μ	65
3	σ	10
4		
5	CDF formula =NORM.DIST(B1, B2, B3, TRUE)	$\rightarrow 0.6914$

- (9) Given data from two distributions, find the distance between the distributions.

SOLY (1) Consider let X -Binomial ($n=20, p=0.2$) & Y -Poisson ($m=4$). Compute the probability that $X=4$ & $Y=4$. Find the absolute diff. between these two probabilities. Compute the contribution to the total variation Distance at $X=4$.

<u>In Excel</u>	A	B
1	$P(X=4)$	0.2182
2	$P(Y=4)$	0.1952
3		
4	$ P(X=4) - P(Y=4) =$	0.023

\leftarrow Absolute Difference

6	Contribution to Total Variation Distance at $X=4$:
7	$ P(X=4) - P(Y=4) * 0.5 = 0.0115$
8	

(10) Application problems based on the Binomial Distribution.

Sol^y Example : Defective Products.

Problem: A factory produces 1000 light bulb, and the probability of a light bulb being defective is 2%. What's the probability that at least 25 of the light bulbs are defective?

Sol^y No. of trials (n) = 1000
probability of success (defective bulb) (p) = 0.02

Let X be the no. of defective bulbs:
 $X \sim \text{Binomial}(n=1000, p=0.02)$

We want probability that at least 25 bulbs are defective, i.e.

$P(X \geq 25)$, By the complement rule:
 $P(X \geq 25) = 1 - P(X \leq 24) = 1 - P(X \leq 24)$

By CDF formula of Binomial distribution.

$$P(X \leq 24) \approx 0.8455$$

$$P(X \geq 25) \approx 1 - 0.8455 = 0.1545$$

The probability that at least 25 bulbs are defective is approx. 15.45%.

Suppose you play a game that you can only either win or lose. The probability that you win a game is 55% and the probability that you lose the game is 45%. Each game you play is independent. If you play the game 20 times. Find the function that describe the probability that you win 15 of 20 times.

Value for n(number of times)	20	p	q
probability of success in a given trial	0.55	0.55	0.45

Number of success	Binomial distribution
0	1.15945E-07
1	2.8342E-06
2	3.29082E-05
3	0.000241327
4	0.001253559
5	0.004902808
6	0.014980803
7	0.036619741
8	0.072730875
9	0.118524388
10	0.159349455
11	0.177054951
12	0.162300371
13	0.122072074
14	0.074599601
15	0.036470916

In Excel

1	-n
2	no. of trials (n)
3	probability of success
4	No. of trial.
5	1
6	2
7	3
8	4
9	5
10	6
11	7
12	8
13	9
14	10
15	

B

1.000

0.02

Binomial Distribution
BINOM.DIST (A6, B2, B3, 0)

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Solv.

(11) Application problem based on Poisson Distribution

Solv: A hardware store sells 3 hammers per day on average. What is the probability that they will sell 5 hammers on a given day?

$$\eta = 5$$

$$\text{mean} = 3$$

cumulative = False

A

formula. = POISSON.DIST (5,3,FALSE)

Answers = 0.100819

The probability that the store sells 5 hammer is 0.100819

Calculate the probability that there will be 220 infected people by covid-19 in 1 day if a total of 2800 new cases are recorded in last 14 days

	infected per day	days	average	poisson distribution
total cases				
2800	220	14	200	0.0102102

(12)

Application problems based on Normal distribution.

Soln.

Suppose the scores for an exam are normally distributed with a mean of 90 & a standard deviation of 10.

Find the probability that a randomly selected student receives a score greater than 80.

In Excel.

	A	B	C
1	Mean	90	
2	Standard Deviation	10	
3	Prob < 80	0.8413	$\Rightarrow \text{NORM.DIST}(80, B1, B2, \text{True})$

So the probability of less than 80 is 0.8413.

(13)

Presentation of bivariate data through scatter plot diagrams and calculations of covariance.

14

In Excel.

	A	B	C	D	E
1	X (hours studied)	Y (Test score)	$x_i - \bar{x}(x'')$	$y_i - \bar{y}(y'')$	$x'' * y''$
2	2	50	-4	50	200
3	4	65	-2	65	-130
4	6	70	0	70	0
5	8	85	2	85	170
6	10	95	4	95	380
7					
8	Mean of $x(x'')$	6	Total Sum	220	
9	Mean of $y(y'')$	73	Covariance	55	

A B C D E F G H I J K L M N O P Q

1 Application problems based on the Normal distribution.

2

3

4 Q.The heights of adult males in a certain city are normally distributed with a mean of 175 cm and a standard deviation of 8 cm.

5 What is the probability that a randomly selected man is taller than 185 cm?

6

7

8

9 Mean 175

10 Standard deviation 8

11 Value of interest(X) 185

12

13 P(X>185) 0.105649774

14

15

(14) Calculation of Karl Pearson's correlation coefficient

~~In Excel~~ The Pearson correlation coefficient is the most widely used correlation coefficient and is known as Bivariate correlation.

In Excel

	A	B		= PERSON (A1:B9, A1:B9)
1	11			
2	12			
3	13			
4	14			
5	15			
6	16			
7	17			
8	18			
9	19			

(15)

To find the correlation coefficient for bivariate frequency distribution.

SOL⁴

To calculate the correlation coefficient for a bivariate frequency distribution in excel we use the CORREL function. For example if your X-values are in column A & Y-values on column B, the formula CORREL (A1:A10, B1:B10) will calculate the correlation coefficient. This formula can be placed in any blank cell to display the result.

(16)

SOL⁴

	A	B	C	D	E	F	G
1	Calculate the Karl Pearson's correlation coefficient						
2							
3							
4	A	B					
5	1	11	1				
6	2	12	1				
7	3	13	1				
8	4	14	1				
9	5	15	1				
10	6	16	1				
11	7	17	1				
12	8	18	1				
13	9	19	1				
14	10	20					
15							
16							
17							
18							

(16) Generating random numbers from discrete (Bernoulli, Binomial, Poisson) distributions.

Sol 1) A Bernoulli distribution returns 1 with probability p and 0 with probability 1-p.

formula :

= If (RAND() <= p, 1, 0)

for example

For p = 0.3:

If (RAND() <= 0.3, 1, 0)

(2) A Binomial distribution ~~returns~~ models the no. of successes in n-trials with success probability p.

Formula :

= BINOM.INV(n, p, RAND())

Ex. for n=10, p=0.5:

BINOM.INV(10, 0.5, RAND())

(3) A Poisson distribution models the number of events occurring in a fixed interval with average rate λ .

Formula :

= POISSON.INV(RAND(), lambda)

Example : for $\lambda = 4$

= POISSON.INV(RAND(), 4)

A

B

C

D

E

F

G

H

I

J

K

1 Generating Random numbers from discrete (Bernoulli, Binomial, Poisson) distributions.

4 Binomial Distribution

5 P(Probability of success)	0.2
6 1-P(Probability of failure)	0.8

8 Random Numbers

9 0

10 0

11 1

12 1

13 0

14 0

15 0

16 1

17 0

18 0

19 1

21 Binomial Distribution

22 n(Trials)	10
23 P(Probability of success)	0.5
24 1-P(Probability of failure)	0.5

26 Random Number

27 4

28 6

29 4

30 4

31 6

32 5

33 5

34 9

35 4

36 5

37 7

38

39 Poisson Distribution

40

 λ

4

41

42

Random numbers

43

1

44

3

45

7

46

4

47

5

48

3

49

4

50

4

51

4

52

5

53