Chapter 4: Least Squares
Due date: 11/09/2022

A correct answer without proper explanation will not receive full credit.

4.1 Least Squares and the Normal Equations

- Exercises 2, 8, 9.
- Computer problems 1, 2.

4.2 A Survey of Models

- Exercises 6.
- Computer problems 3.

4.3 QR Factorization

- Exercises 2, 7 (Use the QR factorization from Exercise 2).
- Computer problems 1, 5.

Exercises Not from the Textbook

1. Let's generate our data in the following way: Start by sampling the cubic polynomial

$$q(t) = -11 + \frac{55}{3}t - \frac{17}{2}t^2 + \frac{7}{6}t^3$$

at 33 equidistant points `x` between 0.9 and 4.1. Then add to these values 30% noise using Matlab's random number generator `rand`, e.g. `x + 0.3*rand(size(x))`. After that, we "forget" that the data came from q, and we work only with the data points.

Program the following 3 approximations. In each case, plot the data and the obtained approximations. Which approximation makes more sense? Discuss.

   (a) An interpolating polynomial of degree 32. Use e.g. Matlab's function `polyfit`. (You don't need to know how interpolation works).

   (b) An interpolating cubic spline. Use e.g. Matlab's function `spline`. (Again, you don't need to know how splines work for this exercise).

   (c) A cubic polynomial which best fits the data in the l_2 sense, obtained by your own least squares function.