ORDINARY ANNUITIES

A man paid 10% down payment of P200,000 for a house and lot and agreed to pay the 90% balance on the monthly installments for 60 months at an interest rate of 15% compounded monthly. Compute the amount of monthly payment

SOLUTION:

Cost of house = 200,000/0.10

= P2,000,000

Balanced = 2,000,000-200,000

= P1,800,000

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i}$$

$$i = \frac{0.15}{12} = 0.0125$$

A=P42, 821.87

$$1{,}800{,}000 = \frac{A[(1.0125)^{60} - 1]}{(1.0125)^{60}(0.0125)}$$

What is the present worth of a 3 years annuity paying P3,000.00 at the end of each year, with interest at 8% compounded annually?

SOLUTION

$$P = \frac{A[(1+i)^n - 1}{(1+i)^n i} \qquad P = \frac{3000[(1.08)^3 - 1]}{(1.08)^3(0.08)}$$

<u>P = P7,731.29</u>

How much must be deposited at 6% each year beginning on January1, year 1, in order to accumulate P5,000.00on the date of the last deposit, January 1, year 6?

SOLUTION

$$F = A\left[\frac{(1+i)^n - 1}{i}\right] \quad 5{,}000 = A\left[\frac{(1.06)^6 - 1}{0.06}\right]$$

A = P717.00

An instructor plans to retire in one year and want an account that will pay him P25,000 a year for the next 15 years. Assuming a 6% annual effective interest rate, what is the amount he would need to deposit now? (the fund will be depleted after 15 years)

SOLUTION

$$P = A\left[\frac{(1+i)^n - 1}{(1+i)^n i}\right] \qquad P = 25{,}000\left[\frac{(1.08)^3 - 1}{(1.06)^{15}(0.06)}\right]$$

P = P242,806.00

A manufacturing firm wishes to give each 80 employees a holiday bonus. How much is needed to invest monthly for a year at 12% nominal interest rate, compounded monthly, so that each employee will receive a P2,000 bonus?

$$P = A\left[\frac{(1+i)^n - 1}{(1+i)^n i}\right] \quad 80(2{,}000) = A\left[\frac{(1.01)^{12} - 1}{0.01}\right]$$

A = P12,615.80

How much money must you invest today in order to withdraw P2,000 annually for 10 years if the interest rate is 9%?

SOLUTION

$$P = \frac{A[\,(1+i)^n - 1\,]}{(1+i)^n i}$$

$$P = \frac{2{,}000[\,(1.09)^{10} - 1}{(1.09)^{10}(0.09)}$$

P = P12,835.32

Money borrowed today is to be paid in 6 equal payments at the end of 6 quarters. If the interest is 12% compounded quarterly, how much was initially borrowed if quarterly, payments is P2,000.00?

$$P = \frac{A[\,(1+i)^n - 1\,]}{(1+i)^n i}$$

$$P = \frac{2{,}000[\,(1.03)^6 - 1}{(1.03)^6(0.03)}$$

$$i = \frac{0.12}{4} = 0.03$$

<u>P = P10,834.38</u>

What is the accumulated amount of the five year annuity paying P6,000 at the of each year, with interest at 15% compounded annually?

$$F = \frac{A[(1+i)^n - 1]}{i}$$

$$F = \frac{6{,}000[(1.15)^5 - 1]}{0.06}$$

<u>F = P40,454.29</u>

Mr. Robles plans a deposit of P500 at the end of each month for 10 years at 12% annual interest, compounded monthly. The amount that will be available in two years is.

SOLUTION

$$F = \frac{A[(1+i)^n - 1]}{i}$$

$$i = \frac{0.12}{12} = 0.01$$

$$F = \frac{500[(1.01)^{24} - 1]}{0.01}$$

n = 2(12) = 24

<u>F = P13,486.73</u>

A debt of P10,000 with 10% interest compounded semi-annually is to be amortized by semi-annual payments over the next 5 years. The first due is 6 months. Determine the semi-annual payments.

SOLUTION

$$P = \frac{A[\,(1+i)^n - 1\,]}{(1+i)^n i}$$

$$10{,}000 = \frac{A[\,(1.05)^{10} - 1\,]}{(1.05)^{10}(0.05)}$$

<u>**A = P1,295.05**</u>

ANNUITY DUE

PROBLEMS

Engr. Sison borrows P100,000.00 at 10% effective annual interest. He must pay back the loan over 30 years with the uniform monthly payments due on the first day of each month. What does Engr. Sison pay each month?

SOLUTION

$$P = \frac{A[\,(1+i)^{n-1} - 1\,]}{i\,(1+i)^{n-1}} + A$$

$$i = \frac{0.10}{12} = 0.00833$$

n = 30(12) = 360

$$100{,}000 = \frac{A[\,(1.00833)^{359} - 1\,]}{(1.00833)\,(0.00833)^{359}} + A$$

<u>**A = 879**</u>

Instead of paying a P100,000 in annual rent for office space at the beginning of each year for the next 10 years, an engineering firm has decided to take a 10-years, P1,000,000.00 loan for anew building at 6% interest. The firm will invest P100,000.00 of the rent saved and earn 18% annual interest on that amount. What will be the difference between the firm's annual revenue and expenses?

SOLUTION

$$P = \frac{A[\,(1+i)^{n-1} - 1\,]}{i\,(1+i)^{n-1}} + A$$

$$1{,}000{,}000 = \frac{A[\,(1.06)^{9} - 1\,]}{(0.6)\,(1.06)^{9}} + A$$

A = P128,177.32 (annual amortization)

Annual income= (100,000)(1.18)

= P118,000

Difference = 128,177.32-118,000

<u>= P10,177.32</u>

A man wishes to have P35,000 when he retires 15 years from now. If he can expect to receive 4% annual interest, how much he set aside in each of 15 equal annual beginning of year deposits?

SOLUTION

$$F = \frac{A[\,(1+i)^{n+1} - 1\,]}{i} + A$$

$$35{,}000 = \frac{A[\,(1.04)^{16} - 1\,]}{0.04} + A$$

<u>**A = P1680.71**</u>

A man owes P10,000.00 with interest at 6% payable semi annually. What equal payments at the beginning of each 6 months for 8 years will discharge his debt?

SOLUTION

$$10{,}000 = \frac{A[\,(1+i)^{n-1} - 1\,]}{i\,(1+i)^{n-1}} + A$$

$$i = \frac{0.06}{2} = 0.03$$

n = (8)(2) = 16

$$10{,}000 = \frac{A[\,(1.03)^{15} - 1\,]}{(0.03)\,(1.03)^{15}} + A$$

A = P772.92 equal payments at each 6 months

On retirement, a workman finds that this company pension calls for payment of P300 to him or to his estate if he dies at the beginning of each month for 20 years. Find the present value of this pension at 5% compounded monthly.

SOLUTION

$$P = \frac{A[\,(1+i)^{n-1} - 1\,]}{i\,(1+i)^{n-1}} + A$$

$$i = \frac{0.05}{12} = 0.00417$$

$$P = \frac{300[\,(1.00417)^{239} - 1\,]}{(0.00417)\,(1.00417)^{239}} + 300$$

n = 20(12) = 240

<u>P = P45,631.87</u>

Under a factory savings plan, a workman deposits P25 at the beginning of each month for 4 years, and the management guarantees accumulation at 6% compounded monthly. How much stands to the work man’s credit at the end of 4 years.

SOLUTION

$$i = \frac{0.06}{12} = 0.005$$

$$F = \frac{A[\,(1+i)^{n+1} - 1\,]}{i} - A$$

n = (4)12 = 48

$$F = \frac{25[\,(1.005)^{49} - 1\,]}{0.005} - 25$$

F = P1,359.21

A man owes P12,000 today and agrees to discharge the debt by equal payments at the beginning of each three months for 8 years, where this payment include all interest at 8% payable quarterly. Find the quarterly payment

SOLUTION

$$P = \frac{A[\,(1+i)^{n-1} - 1\,]}{i\,(1+i)^{n-1}} + A$$

$$i = \frac{0.08}{4} = 0.002$$

$$12{,}000 = \frac{A[\,(1.02)^{31} - 1\,]}{(0.02)\,(1.02)^{31}} + A$$

n = 8(4) = 32

<u>A = P501.30</u>

A man will deposit P200 with a savings and loan association at the beginnings of each 3 months for 9 years. If the association pays interest at the rate of 5.5% quarterly, find the sum to his credit just after the last deposit.

SOLUTION

$$F = \frac{A[(1+i)^{n+1} - 1]}{i} - A$$

$$i = \frac{0.055}{4} = 0.01357$$

$$F = \frac{200[(1.01375)^{37} - 1]}{0.01375} - 200$$

n = 9(4) = 36

<u>F = P9363</u>

Engr. Peter loan an amount of P100,000 at local commercial bank qt 10% compounded annually. How much is his monthly payment if he is required to pay at the beginning of the first day of the month for a period of 30 years?

SOLUTION

$$(1+i)^1 = (\,1 + \frac{i}{12})^{12}$$

$$(1.10)^1 = (\,1 + \frac{i}{12})^{12}$$

$$1.007974 = \ 1 + \frac{i}{12}$$

$$\frac{i}{12} = 0.007974\ (monthly\ interest)$$

SOLUTION

n = 30(12) = 360

$$P = \frac{A[\,(1+i)^{n-1} - 1\,]}{i\,(1+i)^{n-1}} + A$$

$$100{,}000 = \frac{A[\,(1.007974)^{359} - 1\,]}{(0.007974)\,(1.007974)^{359}} + A$$

<u>A = P839.19 monthly payment</u>

DEFFERED ANNUITY

A man loan s P187,400 from a bank with interest at 5% compounded annually. He agrees to pay his obligation by paying 8 equal annual payments, the first being due at the end of 10 years. Find the annual payments.

SOLUTION

$$P_1 = \frac{A[\,(1+i)^n - 1\,]}{i\,(1+i)^n}$$

$$P_1 = \frac{A[\,(1.05)^8 - 1\,]}{(0.05)\,(1.05)^8}$$

= 6.463A

$$P_1 = (1+i)^n P_2$$

$$6.463A = (1.05)^9\,(187{,}400)$$

<u>A = P44,982.04</u>

A house and lot can be acquired a down payment of P500,000 and a yearly payment of P100,000at the end of each year for a period of 10 years, starting at the end of 5 years from the date of purchase. If money worth is 14%compounded annually what is the cash price of the property?

SOLUTION

$$P_1 = 100{,}000\left[\frac{(1.14)^{10} - 1}{.14\,(1.14)^{10}}\right]$$

= 521,611.56

$$P_2 = \frac{521611.56}{(1{,}14)^4}$$

= P308,835.92

Cash price = 500,000 + 308,835.92

<u>= P808,835</u>

A parent on the day the child is born wishes to determine what lump sun would have to be paid into an account annually, in order to withdraw P20,000 each on the child's 18^{th} ,19^{th} ,20^{th} and 21^{st} birthdays

SOLUTION

$$P_1 = A[\frac{(1+i)^n - 1}{i\,(1+i)^n}]$$

$$P_2 = \frac{70{,}919}{(1.05)^{17}}$$

<u>= P30,941.73</u>

$$P_1 = 20{,}000[\frac{(1.05)^4 - 1}{.05\,(1.05)^4}]$$

=70,919.00

A man borrowed P300,000 from a lending institution which will be paid after 10 yrs. At an interest rate of 12% compounded annually. If money is 8% per annum how much should be deposit to a bank monthly in order to discharge his debt 10 yrs. Hence?

$$F_1 = A(1+i)^n$$

$$F_2 = \frac{A[(1+i)^n - 1]}{i}$$

= P931754.46

$$931{,}754.46 = \frac{A[(1.00667)^{120} - 1]}{.00667}$$

$$F_1 = 300{,}000(1.12)^{10}$$

<u>**A = P5,091.92**</u>

In five years P18,000 will be needed to pay for a building renovation. In order to generate this sum, a sinking fund consisting of three annual payments is established now. For tax purposes, no further payments will be made after three years. What payments are necessary if money is worth 15%per annum?

SOLUTION

$$F_2 = \frac{A[\,(1+i)^n - 1\,]}{i}$$

= 3.4725A

$$F = \frac{A[\,(1.15)^3 - 1\,]}{.15}$$

$$F_2 = F(1+i)^n$$

$$18{,}000 = 3.4725A(1.15)^2$$

A = P3919.54

A father wishes to provide P4,000 for his son on his 21^{st} birthday. How much should he deposit every 6 months in a savings bank. Which pays 3% converted semi-annually if the first deposit is made when the son is 3 ½ years old?

SOLUTION

$$F = \frac{A[(1+i)^n - 1]}{i} \qquad 4{,}000 = \frac{A[(1.03)^{36} - 1]}{.03}$$

<u>**A = P63.22 amount he should deposit every 6 months**</u>

An annual deposit of P1270 is placed on the fund at the end of each year for 6 years if the fund invested has a rate of interest of 5% compounded annually, how much is the worth of this fund at the end of 9 years?

SOLUTION

$$F = \frac{A[(1+i)^n - 1]}{i}$$

$$S = F(1+i)^n$$

= P8,638.43

<u>S = P10,000</u>

$$S = 8638.43(1.05)^3$$

$$F = \frac{1270[(1.05)^6 - 1]}{.05}$$

An old boiler cost P2,400 a year to maintain. What expenditures for anew boiler is justified if no maintenance will be required for the first three years, P600 per year for the next 7 years and P2400 a year thereafter ? Assume money to cost 4% compounded annually and no other costs to be considered.

SOLUTION

$$P = \frac{A[\,(1+i)^n - 1\,]}{i\,(1+i)^n}$$

$$P_1 = \frac{600[\,(1.04)^{10} - 1\,]}{0.04\,(1.04)^{10}}$$

= 1665.05

$$P_2 = \frac{1800[\,(1.04)^{10} - 1\,]}{0.04\,(1.04)^{10}}$$

= 14599.61

P = P1 + P2

<u>= P16,264.66</u>

A shirt factory has just installed a boiler. It is expected that there will be no maintenance expenses until the end of 11th year when P400 will be spent at the end of each successive year until the boiler is scrapped at the age of 35 years what some of money set aside at his time at 6% interest will take care of all maintenance expenses for the boiler?

$$F = \frac{A[(1+i)^n - 1]}{i}$$

$$F = \frac{400[(1.06)^{25} - 1]}{.06}$$

= P21945.80

$$P_1 = \frac{21945.80}{(1.06)^{35}}$$

= P2855.26

amount set aside

A new generator has just been installed it is expected that there will be no maintenance charges until the end of the 6th year when P300 will be spent at the end of its 14th year of service. What sum of money set aside at the time of installation of the generator at 6% will take care of all maintenance expenses for generator?

SOLUTION

$$F = \frac{A[(1+i)^n - 1]}{i}$$

$$P = \frac{3447.39}{(1.06)^{14}}$$

$$F = \frac{300[(1.06)^9 - 1]}{.06}$$

P = P1524.79

= P3447.39

PERPETUITY

What present sum would be needed for annual end of year payments of P15,000 each forever if money is worth 8%.

SOLUTION

$$P = \frac{A}{i}$$

$$P = \frac{15000}{.08}$$

<u>= P187,500.00</u>

P45,000 is deposited in a savings account that pays 5% interest compounded semi-annually. Equal annual withdrawals are to be made from the account, beginning one year from now and continuing forever. Compute the maximum amount of the equal annual withdrawal.

SOLUTION

$$\left(1 + \frac{.05}{2}\right)^2 = (1 + i)^2$$

$$P = \frac{A}{i}$$

i = 0.050625

A = (45000)(0.050625)

<u>A = P2278.13</u>

What amount of money deposited 20 years ago at 4% interest would provided perpetual payment of P2,000 per year ?

SOLUTION

$$P = \frac{A}{i}$$

$$P = \frac{2000}{.04} = P50,000$$

$$F = P(1 + i)^n$$

$$50,000 = P(1.04)^{20}$$

<u>P = P22,819.35</u>

Find the present value of a perpetuity of P100 payable semi-annually if money is worth 4% compounded quarterly.

SOLUTION

$$(1+\frac{.04}{4})^4=(1+i)^2$$

1+i = 1.0201

i = 0.0201

$$A=\frac{P}{i}$$

<u>A = P4,975.12</u>

$$A=\frac{100}{.0201}$$

If money worth is 4% find the present value of perpetuity of P100 payable at the beginning of each year

SOLUTION

$$P = A + \frac{A}{i}$$

$$A = 100 + \frac{100}{.04}$$

<u>A = P2600</u>

If money worth is 8% obtain the present value of a perpetuity of P1000 payable annually when the first payment due at the end of 5 years

SOLUTION

$$A = \frac{P}{i} - \frac{P[(1+i)^n - 1}{(1+i)^n i}$$

$$A = \frac{1000}{.08} - \frac{1000[(1.08)^4 - 1}{(1.08)^4 .08}$$

<u>**A = P9187.87**</u>

Find the present value in pesos of a perpetuity of P15000 payable semi annually if money is worth 8% compound quarterly.

SOLUTION

$$P = \frac{A}{i}$$

$$(1 + \frac{.08}{4})^4 = (1 + i)^2$$

$$(1.02)^4 = (1 + i)^2$$

i = 0.0404

$$P = \frac{15000}{.0404}$$

P = 371287