LEARNING GUIDE

Week No. __1__

TOPIC/S

INTRODUCTION TO ENGINEERING ECONOMICS & ACCOUNTING

I. EXPECTED COMPETENCIES

At the end of the topic, the student should be able to:

:

1. Determine the role of engineering economy in the decision-making process
2. Identify what is needed to successfully perform an engineering economy study.

II. CONTENT / TECHNICAL INFORMATION

Engineering Economy:

- Discipline concerned with the economic aspects of engineering.
- Involves the scientific evaluation of the costs and benefits of proposed technical projects.
- Analyze alternative uses of financial resources, particularly in relation to the physical assets and the operation of an organization.

Alternatives

An alternative is a stand-alone solution for a given situation. We are faced with alternatives in virtually everything we do, from selecting the method of transportation we use to get to work every day to deciding between buying a house or renting one and the likes. Every situation has at least two alternatives. In addition to the one or more formulated alternatives, there is always the alternative of inaction, called the do-nothing (DN) alternative. This is the as-is or status quo condition.

Principle of Engineering Economy:

1. Develop the alternatives
- Identification of alternatives and defining for subsequent analysis.

2. Focus on the differences
- Only the differences in expected future outcomes among the alternatives are relevant to their comparison and should be considered in the decision.

3. Use a consistent viewpoint
- Prospective outcomes of the alternatives, economic and others, should be consistently developed from a defined viewpoint or perspective.
- (ex. Viewpoint of the customer should be adopted in decision)

4. Use a common unit of measure
- Using a common unit of measurement to enumerate as many of the prospective outcomes as possible will make easier the analysis and comparison of the alternatives.

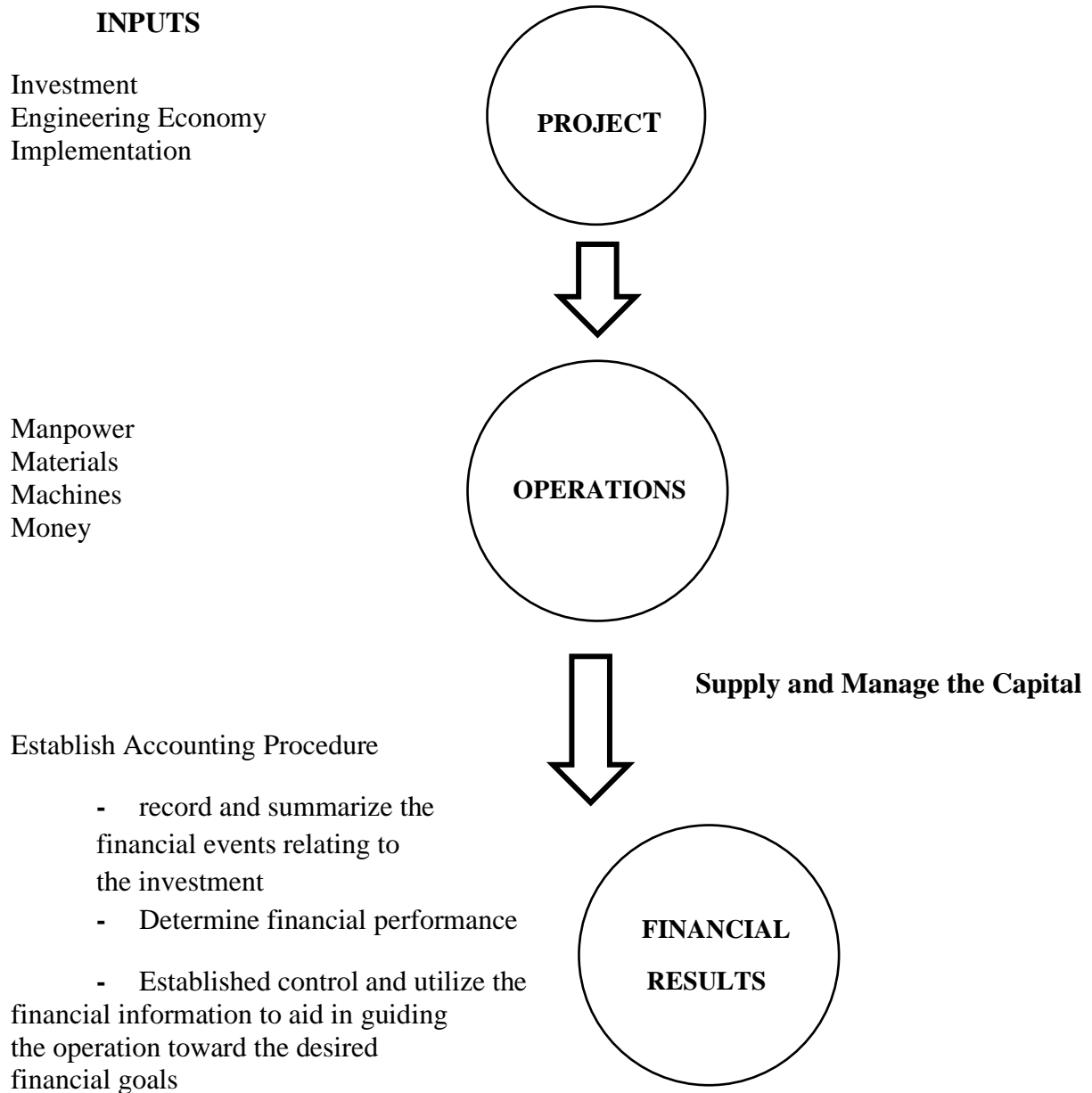5. Consider all relevant criteria

6. Make uncertainty explicit
- Uncertainty is inherent in projecting or estimating the future outcomes of the alternatives and should be recognized in their analysis and comparison.

7. Revisit your decisions.
- Improved decision-making results from an adoptive process; projected outcomes of the selected alternative should be compared with actual results achieved.
-

Engineering Economy Studies

- Provide information on which current decisions pertaining to the future operation of an organization can be based.

Structure:

INPUTS

Investment
Engineering Economy
Implementation

PROJECT

Manpower
Materials
Machines
Money

OPERATIONS

Supply and Manage the Capital

Establish Accounting Procedure

- record and summarize the
financial events relating to
the investment

- Determine financial performance

- Established control and utilize the
financial information to aid in guiding
the operation toward the desired
financial goals

FINANCIAL

RESULTS

Terms & Definitions:

General Accounting

Cost Accounting

Procedures necessary services in Business organization

Accounting Data — primarily concerned with past and current financial events; data are often used to make projects about the future

General Accounting — is a source of much of the past financial data needed for estimating future financial conditions.

Cost Accounting — also known as management accounting is subset of accounting that is of particular importance because it is concerned principally with decision making and control in action.

Accounting — also a source of data for analyses of how well the results of a capital investment turned out compared to the results that were predicted in the engineering economic analysis.

Objectives of Modern Cost Accounting:

1. Determine the cost of products or services.
2. Provide a rational basis for pricing goods or services
3. Provide a means of controlling expenditures.
4. Provide information on which operating decisions may be based and the results evaluated.

Time Value of Money

It is often said that money makes money. The statement is indeed true, for if we elect to invest money today, we inherently expect to have more money in the future. The change in the amount of money over a given time period is called time value of money; it is the most important concept in engineering economy.

The price of any commodity or product will depend largely on the market situation. The following is a tabulation of the different market situations:

Market situation	Sellers	Buyers
Perfect Competition	many	Many
Monopoly	One	Many
Monopsony	Many	One
Bilateral monopoly	One	One
Duopoly	Two	Many
Duopsony	Many	Two
Oligopoly	Few	many
Oligopsony	Many	Few
Bilateral oligopoly	few	Few

Perfect Competition (also known as atomistic competition) refers to the market situation in which any given product is supplied by a very large number of vendors and there is no restriction against additional vendors from entering the market.

Perfect competition is a type of market situation characterized by the following:

A. Many sellers and many buyers

B. Homogenous products

C. Free market-entry and exit

D. Perfect Information

E. Absence of all economic friction

This market situation provides an assurance of complete freedom on the part of both the vendors and the buyers though the latter benefits more from the reduced prices brought about by the perfect competition while more and better services are afforded by the vendors or players in the industry.

Monopoly is the opposite of perfect competition. This market situation is characterized by the following:

A. One seller and many buyers

B. Lack of substitute products

C. Blockaded entry

There exists a perfect monopoly if the single vendor can prevent the entry of all other vendors into the market. The monopolist is in the position to set the market price.

Demand is the need, want or desire for a product backed by the money to purchase it. In economic analysis, demand is always based on "willingness and ability to pay" for a product, not merely want or need for the product. The demand for a product is inversely proportional to its selling price, i.e. as the selling price is increased, there will be less demand for the product; and as the selling price is decreased, the demand will increase.

Supply is the amount of a product made available for sale. If the selling price for a product is high, more producers will be willing to work harder and risk more capital in order to reap more profit. However, if the selling price for a product declines, capitalist will not produce as much because of the smaller profit they can obtain for their labor and risk. The **law of supply and demand** is a theory that explains the interaction between the **supply** of a resource and the **demand** for that resource. Generally, low **supply** and high **demand** increase price. In contrast, the greater the **supply** and the lower the **demand**, the price tends to fall.

Therefore, the relationship between price and supply is that they are directly proportional, i.e. the bigger the selling price, the more the supply; and the smaller the selling price, the less is the supply.

Types of Cost

Costs of engineering products can broadly be grouped under **DIRECT COST or INDIRECT COST.**

Direct Costs - are the costs of those factors which can be directly attributed to the manufacture of a specific product. These are the costs of **material** and **labor**.

Material Cost is the cost of that material which goes into the finished product and includes all the waste which has been cut away from the original bar, casting, etc.

Labor Cost will be the product of the number of pieces produced and the piecework rate (in the case of a simple incentive scheme) or the product of the time spent in manufacturing the product.

Indirect Costs - are the costs of those factors which can only be indirectly attributed to the manufacture of a specific product. They are sometimes called **OVERHEADS** or **ONCOSTS**. They can be subdivided for convenience under three headings:

a. **Works Overheads**. These consist of the cost of the wages of works of superintendents, foremen, inspectors, storekeepers, etc., cost of cutting oil, depreciation of machines, lighting, rents, rates, etc.
b. **Office overheads**. These consist of the cost of the wages of all office staff, postage, legal expenses, depreciation of office equipment, etc.

c. **Sales Overheads**. These consist of the cost of the wages of all sales staff, advertising, sales commissions, etc.

Therefore, it can be seen that indirect costs are the total costs running the organization less the direct material costs and the direct labor costs.

$$\textbf{Sales Price = Direct Cost + Overheads + Profit}$$

Past experience in this case shows that Overheads are 300% of Direct Labor Cost.

The Total Cost of a product is the Direct Cost of manufacturing the product plus any Indirect Costs attributed to the manufacture of the product.

Illustrative example:

1. A batch of 500 components is produced on a capstan lathe. The piece work rate is P2.^{50}, and the direct material cost per piece is P4.^{00}. Overheads are 450% of direct labor cost. What is the total cost of the batch of components?

   Solution:

   Direct material cost $= 500 \times P4.^{00}$ $= P2\ 000.^{00}$

   Direct labor cost $= 500 \times P2.^{50}$ $= P1\ 250.^{00}$

   Indirect cost $= P1250.^{00} \times 450/100$ $= P5\ 625.^{00}$

   Total cost $= \textbf{P8 875.}^{\textbf{00}}$

Cost may be more conveniently grouped under **fixed costs** and **variable costs.**

Fixed Costs are those costs which are independent of the quantity of the product manufactured. These include preparation costs such as the cost of tooling, setting up, etc. and also interest cost and depreciation cost. Obviously, the more products that are made, the less will be the fixed cost per piece.

Variable Costs are those costs which vary as the quantity of products made varies. Usually variable costs increase proportionally as the number of products made increases and includes the direct labor and material costs.

$$\textbf{Cost of Product = Fixed Cost + Variable Cost}$$

Segregate the fixed cost and the variable cost of running a motor car for one year.

Fixed Cost (independent of distance covered in the year)

Variable Cost (varies proportionally to distance covered in the year)

1. Depreciation per year
2. Interest costs per year
3. Road tax and Insurance
4. Garage rent
5. Maintenance cost

6. Cost of Petrol
7. Cost of Oil

   The greater the distance covered in the year, the less will be the fixed cost per kilometer, and the less will be the total cost per kilometer.

III. PROGRESS CHECK

1. The basic accounting equation is
   A. Assets = Liability + owner's equity
   B. Liability = Assets + owner's equity
   C. Owner's equity = Assets + Liability
   D. Owner's equity = Liability – Assets

2. Cash money credit necessary to establish and operate an enterprise.
   A. Funds   B. Capital   C. Liabilities   D. Assets

3. Which of the following statements is incorrect?
   A. Economics decision are time invariant
   B. Time and risk are the most important factors in any investment evaluation.
   C. For a large-scale engineering project, engineers must consider the impact of the project on the company's financial statements.
   D. One of the primary roles of engineers is to make capital expenditure decisions.

4. When evaluating a large-scale engineering project, which of the following items is important?
   A. Expected profitability
   B. Timing of cash flows
   C. Degree of financial risk
   D. All of the above

5. Which of the following statements defines the discipline of engineering economics most closely?
   A. Economic decisions made by engineers.
   B. Economic decisions related to financial assets.
   C. Economic Decisions primarily for real assets and services from engineering projects
   D. Any economic decision related to the time value of money.

IV. REFERENCES

Leland Blank et. Al (2012), Engineering Economy, 7th Edition Graw-Hill, Wiley Publishing Inc. New York.

Park, Chan (2013) et. Al (2013), Fundamentals of engineering Economics, 3rd Edition, Pearson Education South Asia PTE. LTD., Singapore.

Horngren, Charles T. et. Al (2009), Accounting, 8yh Edition, Pearson Education South Asia, Philippines.

<center>**LEARNING GUIDE**</center>

<center>Week No. ___2___</center>

<center>**TOPIC/S**</center>

<center>**CASH FLOW DIAGRAMS**</center>

I. EXPECTED COMPETENCIES

At the end of the topic, the student should be able to:

1. Explain cash flows, their estimation, and how to graphically represent them.
2. Identify and use engineering economy terminology and symbols.
3. Construct a simplified cash flow diagram from the statement given.

II. CONTENT / TECHNICAL INFORMATION

Cash flow diagrams are, in general, used in the teaching and practice of engineering economic analysis to evaluate present and future worth receipts and disbursements of investment alternatives. Periodic receipts and payments usually occur in five different series: uniform (or equal) amount series, single present or future receipt, arithmetic gradient series, geometric series, and irregular series

DEFINITION:

1. Interest - is called to the benefits of the lender by receiving a fee for letting the Borrower use the money, the borrower benefits, too, by availing of money he could use for investment, production of goods or even consumption. Designated as "I or r" as the current simple interest rate or referred to as "money's worth"

2. Origin Date – is called to the date on which borrowed money is received by the borrower.

3. Maturity Date – is called to the date on which the loan is to be completely repaid; also called "Repayments"

4. Term of the Loan – is called to the length of time between the origin and the maturity dates.

5. Simple Interest – is called the interest that is computed only on the amount received by the borrower on origin date and that is added to this amount on maturity date.

6. Principal – is called the sum received by the borrower on the origin date; also called as "Present Worth or Present Value."

7. Maturity Value – is called the sum of the principal and the simple interest that the lender receives from the borrower; also called the future worth.

8. Ordinary Interest – is called when the loan term is expressed in days (D) using 360 days for ordinary year, also called as approximate interest; assume 30 days per month.

9. Exact Interest – is called when the loan term is expressed in days (D) using 365 days per year and 366 days for leap year.

10. Actual Time – is obtained by counting every day, except the origin date, within the term of the loan.

11. Approximate Time – is obtained by assuming that every month contains 30 days.

Below are 4 different time factors which are possible whenever simple interest is involved.

1. Actual Time/360   3. Approximate time/360

2. Actual Time/365   4. Approximate time/365

When "Actual Time/360 is used, the interest obtained is referred to as" Ordinary Interest by Banker's Rule" This is to be used whenever the problem does not specify which time factor to use.

12. Discount Interest – is an amount paid for borrowing money; is charged at the time the loan that has been negotiated or executed. Whereas simple interest is paid on the maturity date when it is added to the amount of the loan applied for on the origin date, discount interest is charged in advance and is taken from the amount of the loan applied for on the origin date.

13. Promissory Notes – a written commitment by a person or business (called the drawer) to pay a certain sum to another person or business (called the drawee) within a specified time; it is also called simply a "note"

14. Discounting Notes – is called to the discount that a drawee will give to the buyer of the note in case he will sell it to a bank or an individual prior to the maturity date; this happens when the drawee needs money prior to its maturity date. This discount is equivalent to the advance interest. In such a case, the buyer is said to have rediscounted the note.

NOTATIONS AND CASH FLOWS:

The use of cash flow (time) diagrams and / or tables is strongly recommended for situations in which the analyst needs to clarify or visualize what is involved when flows of money occur at various times. In addition, viewpoint is an essential feature of cash flow diagrams. This is analogous to a free body diagram for mechanic problem.
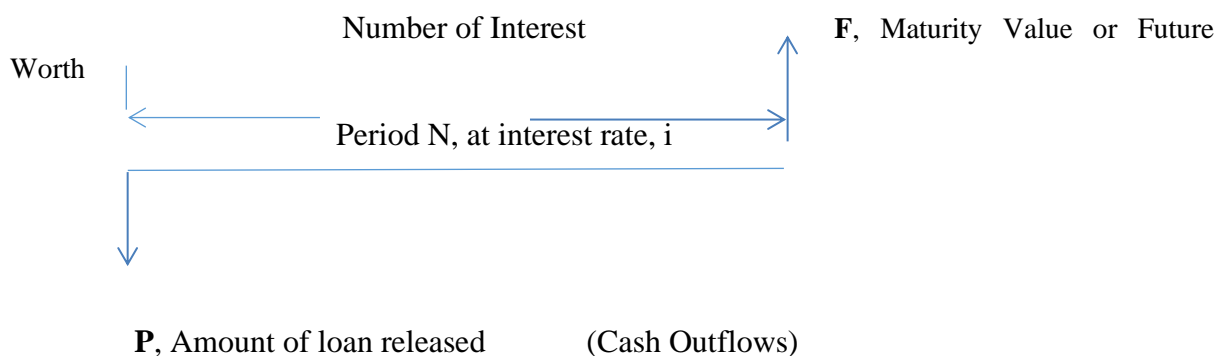
All cash flows are usually placed at the end of specified period of time (one year). Some of the terms related to cash flow diagram are:
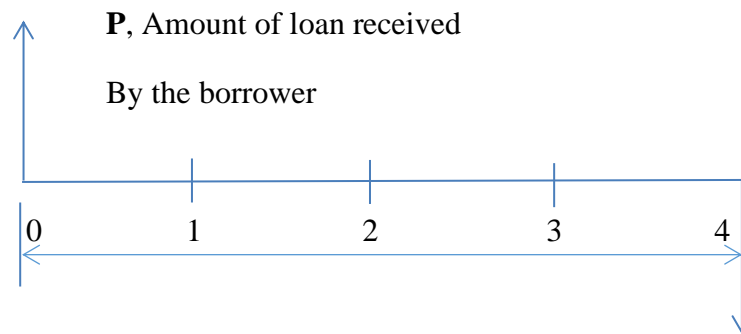
1. Time Scale - is called to the horizontal line, with progression of time moving left to right.
2. Arrows - signify cash flows and are placed at the end of the period.
3. Cash Outflows - are represented by an arrow downward to connote expenses; also called as negative cash flows.
4. Cash Inflows - are represented by an arrow upward to connote receipts; also called as positive cash flows.
5. Cash Flows - are usually placed at the end of the period
6. Net Cash Flow - is called to the difference between the total cash flows (receipts) and cash outflows.
7. Cash Flow Diagram - is a diagram that shows what is involved when flows of money occur at various times dependent on the point of view/ viewpoint. Help visualize and simplify problems having diverse receipts and disbursements.

Below is an example illustrating the above terms with notations (Simple Interest)

Cash Flow Diagram

Lender's Viewpoint:

Number of Interest   **F**, Maturity Value or Future

Worth

Period N, at interest rate, i

P, Amount of loan released   (Cash Outflows)

Borrower's View Point:

P, Amount of loan received

By the borrower

No. of Interest Period, N

At interest rate/interest period, i   **F**, Maturity Value or Future Worth

Consider the following example:

An equipment costs P30, 000. Maintenance costs P3, 000 each year. It will generate revenues of P15, 000 each year for 5 years after which the salvage value is expected to be P12, 000. Draw and simplify the cash flow diagram.
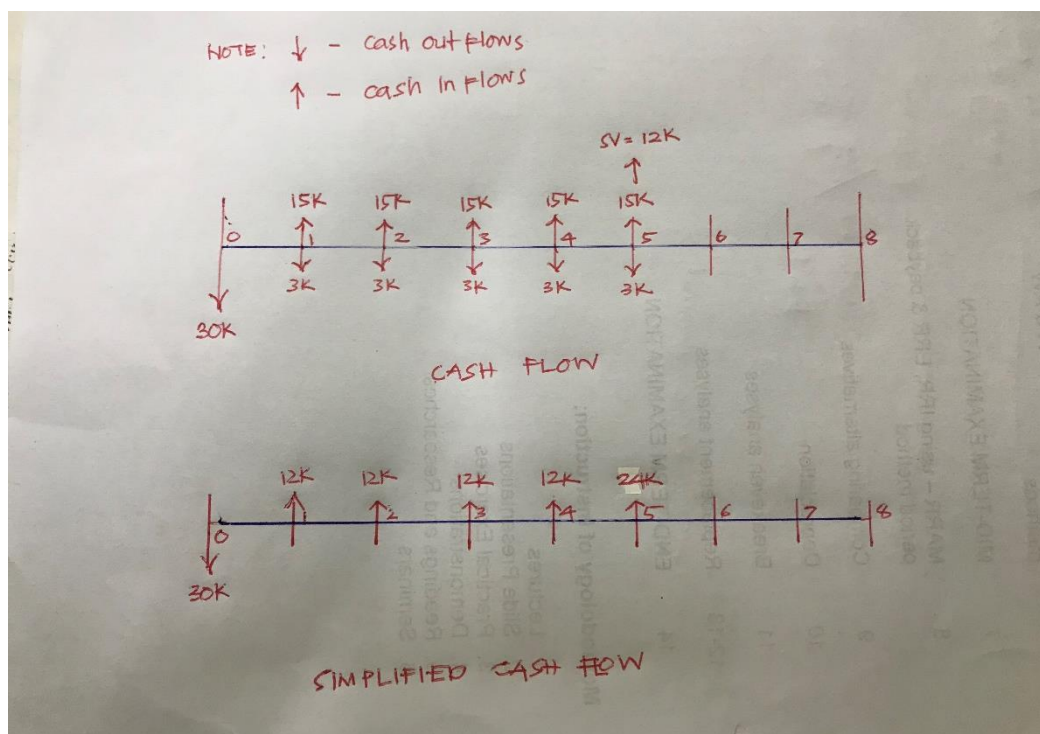

Figure 1 Simplified Cash Flow.

Sort of a Review:

1. **Percent** – A percent means "so many per hundred."
   Thirty percent, 30% is 30 out of 100

   Fifty percent is 50 out of 100

$$50\% = 50/100 = ½$$

Note: You need to be careful when you see expressions like 0.04 percent. This is not the same as 4 percent, which equals 0.04. You must divide by another hundred:

$$.04\% = .04 / 100 = .0004$$

2. **Discounts**
• A jacket priced at P695.00 is marked "40% off". How much does it cost?

Solution:

Multiply P695 by $(1 − 0.4) = 0.6$

$$0.6 × 695 = 417$$

Forty percent off means you must pay 60 percent of the original price.

3. **Sales Tax**
• If the sales tax is 6.5%, what is the final price of a DVD player that is marked P1179.98?

Solution:

Multiply 1179.98 by $(1+.065) = 1.065$:

$$1.065 × 1179.98 = 1256.6787$$

III. PROGRESS CHECK

Read and analyze and construct the cashflow needed.

1. Suppose that you apply for an education loan in the amount of P300,000 from a bank at a 9% annual interest rate. In addition, you pay a P30, 000 loan origination fee when the loan commences. The bank offers two repayment plans, one with equal payments made at the end of every year for the next five years (installment plan) and the other with a single payment made after the loan period of five years (deferment plan). Construct a cash flow diagram for each payment plan.

2. A businessman obtained a commercial loan from a financing institution amounting to P 1,000,000 bearing an interest of 11% compounded annually on a condition that he will pay five equal end-of-year payments after the fifth year. Assuming no single amount will be paid, and the loan is restructured for another ten years, draw the diagram on businessman's point of view.

IV. REFERENCES

Leland Blank et. Al (2012), Engineering Economy, 7th Edition Graw-Hill, Wiley Publishing Inc. New York.

Park, Chan (2013) et. Al (2013), Fundamentals of engineering Economics, 3rd Edition, Pearson Education South Asia PTE. LTD., Singapore.

Horngren, Charles T. et. Al (2009), Accounting, 8yh Edition, Pearson Education South Asia, Philippines.

LEARNING GUIDE

TOPIC/S

SIMPLE INTEREST

I. EXPECTED COMPETENCIES

At the end of the topic, the student should be able to:

:

1. Explain what simple interest is and how it is calculated.
2. Calculate simple interest for one or more interest periods.

II. CONTENT / TECHNICAL INFORMATION

INTEREST

- It is the manifestation of the time value of money and it essentially represents "rent" pain for use of the money.

SIMPLE INTEREST

- Is called when the interest earned at a cut of date is automatically reinvested to earn more interest.

FORMULA: Interest = end amount – original amount

Simple Interest:

$I = P\,N\,i$

$F = P\,(1 + N\,I)$

where: F = Future Worth or Maturity Value
P = Present Worth or Principal
N = No. of Interest Period or term of the loan
I = Interest Rate per interest period

For Exact Interest: N = D/365 days for ordinary year & D/366 for leap year

For Approximate Interest, N = D/ 360 days; assume 30 days per month

Discount Interest:

$$F = P + I_d$$ where I_d = Discount Interest = F (dt) or F(iN)

$$P = F - I_d$$ d = discount Interest rate; equivalent to I expressed in %

t = term of the loan; equivalent to N

F = Maturity Value of the Loan; the amount of the loan applied for on the origin date

P = Amount received by the borrower on the origin date or proceeds of the loan; equivalent to principal

$$P = F (1 - dt)$$

Rate of discount, d   = Interest / Original Principal

Rate of Interest, i   = Interest / Actual Principal

i   = d / (1-d)

Promissory Notes:

A type of promissory note is the simple interest note in which:

1. the note is drawn on the origin date.

2. the note is redeemed on the maturity date.

3. the stated value of the note – the face value- corresponds to the principal; and

4. the face value plus the interest, is the maturity value.

Below is an illustration of a promissory note

P 5,000	Manila City, Philippines	January 11, 2006

Seventy – five days after the above date, the undersigned promises to pay to the order of Budjawi K. Budjanga, Five Thousand and no/100 pesos with interest at 16.2% per annum payable at Metro Bank of Manila City, Philippines

Signed by:

Bogok T. Taburnok

From the note, the following information is obtained:

Drawer	:	Bogok T. Taburnok
Drawee	:	Budjawi K. Budjanga
Face Value	:	P 5, 000
Interest	:	16.2 %
Term	:	75 days
Maturity Date	:	March 27 (75 days after January 11 2006)
Interest	:	P 5, 000 x 162 x 75 / 360 = P P 168.75
Maturity Value	:	P 5, 000 + P 168.75 = P P 5,168.75

To apply your knowledge and analytical skills in using the above formula in various situations, let us consider the following problems.

Exercise Problems.

1.1 Find the remaining quantities for each of item.

P	r/year	t	Is	F
1. P 2000	14.4 %	8 months	---------	-----
2. ---------	17.4 %	2 years	P 5000	-------
3. P 20,200	---------	210 days	P 3000	-------

1.2 The conditions of a credit purchase specify an interest charge of 18 % per annum on any unpaid balance. If the purchaser was charged P 270 on an invoice for P 4,500, how long was the bill overdue.

1.3 If the money is currently worth 14.8 %, how much must be invested now in order to have P 26,500   in 2 years?

1.4 Find the proceeds of P 40,000 due at the end of 8 months if the discount interest rate is 10.5 %

1.5 If P 12,400 is the present value of P 16,800 due at the end of 16 months, find a.) discount interest rate b.) The simple interest rate

1.6 Billy signed a P 28,000 bank discount note on January 6, 2006. If the note was for 10 months at 18% per annum. Find a.) the maturity date, b.) the interest deducted in advance and c.) the proceeds

1.7 Jam holds a P 30,000, 120-day simple interest note from Bil Tayson. The interest rate is 21 %. The note was made on December 12, 2005. On January 7, 2006, Jam wanted to encash the note. Hefound a bank which would buy the note that day at 25 % interest, collectible in advance. How much will Jam receive from the bank on January 7, 2006?

1.8 Billy John deposits P 50,000 in a bank. It earns 15 % simple interest. Using banker's rule, find the amount due in 180 days.

1.9 A man borrowed P 2000.00 from a bank & promises to pay the amount for one year. He received only the amount of P 1,920 after the bank collected an advance interest of P 80.00. What were the rate of discount and the rate of interest that the bank collected in advance?

2.0 P20, 000 was invested @ simple interest, part @ 11% for 2 years, and the rest @ 8% for 3 years. The total interest earned was P4, 690. How much was invested @ 11%.

2.1. An employee at BISCOM borrows P10, 000 on May 1 and must repay a total of P10, 700 exactly 1 year later. Determine the interest amount and the interest rate paid.

2.2. Calculate the amount deposited 1 year ago to have 10000 now at an interest rate of 5% per year. Calculate the amount of interest earned during this time period.

If the investment term is long, the computation of the compound amount would be very lengthy using the interest computed for a certain period of time and adding this to the principal to be the new principal for the next computation of interest. For this reason, a formula will be derived to simplify the computation.

Two Types of Interest

Simple interest is computed on the original principal only. At an annual rate of 4.5%, the accrued simple interest on a principal of P1000 after three years will be $3 \times 45 = 135$. With *Compound interest*, the bank pays you interest on the interest you made in the first and second years.

Simple Interest:

The formula for simple interest is very simple.

Variables:

r = annual interest rate

t = number of years

P = original principal

F = amount accumulated after t year

$$F = P (1 + r t)$$

When money is borrowed, interest is charged for the use of that money for a certain period of time. When the money is paid back, the principal (amount of money that was borrowed) and the interest is paid back. The amount to interest depends on the interest rate, the amount of money borrowed (principal) and the length of time that the money is borrowed.

Simple interest is generally charged for borrowing money for short periods of time. Compound interest is similar but the total amount due at the end of each period is calculated and further interest is charged against both the original principal but also the interest that was earned during that period.

III. PROGRESS CHECK

Instruction: Read, Analyze and solve what is being ask.

1. A time deposit of P110,000 for 31 days earns P890.39 maturity date after deducting the 20 % withholding tax of interest income. Find the rate of interest per annum.

2. The tag prize of a T-shirt is for 100 days. If paid in 31 days, there is a 3 % discount. What is the simple interest paid?

3. How long must a P40,000 investment bearing 4 % simple interest run to amount to P41,350.00.

4. P20, 000 was invested @ simple interest, part @ 11% for 2 years, and the rest @ 8% for 3 years. The total interest earned was P4, 690. How much was invested @ 11%.

5.A student deposits P1, 500 in a 9% account today. He intends to deposit another P3, 000 at the end of two years. He plans to purchase in five years his favorite shoes worth P5, 000. Calculate the money that will be left in his account one year after the purchase.

IV. REFERENCES

Leland Blank et. Al (2012), Engineering Economy, 7th Edition Graw-Hill, Wiley Publishing Inc. New York.

Park, Chan (2013) et. Al (2013), Fundamentals of engineering Economics, 3rd Edition, Pearson Education South Asia PTE. LTD., Singapore.

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LEARNING GUIDE

Week No. __4__

TOPIC/S

NOMINAL & EFFECTIVE RATES / COMPOUND INTEREST

I. EXPECTED COMPETENCIES

At the end of the topic, the student should be able to:

:

1. Explain what nominal and effective rates is and how it is calculated.
2. Calculate compound interest one or more interest periods.

II. CONTENT / TECHNICAL INFORMATION

NOMINAL RATE AND EFFECTIVE RATE OF INTEREST

Nominal Rate – is called when the interest is compounded more than once a year.

Effective Rate – is the rate that, when compounded annually, produces the same amount each year as the nominal rate i compounded m times a year.

Example:

Two annual rates of interest with different conversion periods are said to be equivalent if they earn the same compound interest for the same time. For instance, in one year, the compound interest of P10.00 invested:

a. At 12% compounded semi-annually:

$$F = 10 \ (1 + 0.06)^2 = P11.236$$

b. At 12.36% compounded annually:

$$F = 10 \ (1 + 0.1236) = P11.236$$

Then: 12% is a nominal rate

12.36% is an effective rate

Formula:

$F_1 = P \ (1 + i)$ – effective rate at the end of 1 year

$F_2 = P \ (1 + j \ / \ m)^m$ – nominal rate i compounded m times a year

Since the two compounds are equal: $\mathbf{F_1 = F_2}$, then

$\mathbf{P\ (1 + i) = P\ (1 + j\ /\ m)^m}$

$\mathbf{(1 + i) = (1 + j\ /\ m)^m}$

$\qquad \mathbf{i = (1 + j\ /\ m)^m - 1}$

Note: If the nominal rate i is compounded annually, then $\mathbf{i = j}$

Various i and n Values for single-amount equations using r = 12% per year, compounded monthly

Effective Interest Rate, i	Units for n
1% per month	Months
3.03 % per quarter	Quarters
6.15% per 6 months	Semiannual periods
12.68% per year	Years
26.97% per 2 years	2-year periods

Interest rate is used for i, that is $(1.01)^3 - 1 = 3.03\%$, then the n time unit I 4 quarters, Alternatively, it is always correct to determine the effective i per payment period.

COMPOUND INTEREST

- Is called when the interest earned at a cut of date is automatically reinvested to earn more interest.
- Interest may be compounded annually, quarterly (4 times a year), monthly (12 times a year), or even daily.

Compounded annually – means that the interest earned in 1 year is added to the principal to earn additional interest for the next year.

FORMULA:

If the investment term is long, the computation of the compound amount would be very lengthy using the interest computed for a certain period of time and adding this to the principal to be the new principal for the next computation of interest. For this reason, a formula will be derived to simplify the computation.

BORROWER'S VIEWPOINT

$$I = P \times i$$

$$P' = P + Pi = P(1 + i)$$

$$P = (P \times (1+i)) \times i$$

The new principal at the end of second period. $P_2 = P(1+i) + i(P \times (1+i))$

Simplifying: $P_2 = P(1+i)^2$

If the conversion period is n, then the compound amount is:

$$F = P(1+i)^n$$

Where:

P = original principal

F = compound amount

j = nominal rate of interest

m = frequency of conversion

i = interest rate per period = j / m

i = j if m = 1

t = term of investment in years

n = total no. of conversion periods in the investment term

"To accumulate a Principal P for n period means to find the compound amount F at the end of n periods, if P is invested at a nominal rate."

Monthly Compound Interest

A bank pays you 6% annual interest, compounded monthly. What does this mean?

Answer: Break up the year into 12 monthly periods and pay you 6/12 = ½ percent interest for each period. Your account after 1 year is worth

$$(1 + 0.06/12)^{12}$$

$$(1.005)^{12} \times P = 1.0617 \times P$$

Where: P is the beginning principal.

The formula for compound interest with periodic interest conversions per year is:

Variables:

j = annual interest rate

m = number of interest periods per year

t = number of years

P = original principal

F = amount accumulated after t years

$$F = P\ (1 + r\ /\ m)^{\,mt}$$

Monthly versus Yearly:

At 4.5% annual interest, the amount accrued in your account after 30 years with P10, 000.00 is?

Yearly?   **P37, 453.[20]**

Monthly?   **P38, 476.[98]**

Sample Problems:

1. Find the accumulated value of P2, 000.00 in 5 years if it is invested at 11% compounded quarterly. (P3440.86)

2. Find the present value of P5, 000.00 due in 8 years if money is worth 12% compounded semi-annually. (P1968.23)

3. Find the compound amount if P5, 000.00 is invested for 3 years and 9 months at 12% compounded semi-annually, assuming simple interest over the fractional part. (P7,743.70)

4. At what nominal rate compounded semi-annually will P1, 000.00 accumulate to P2, 500.00 in 10 years?

5. How long will it take the P2, 500.00 to amount to P5, 800.00 if invested at 12% compounded quarterly?

6. Find the effective rate equivalent to 12% compounded quarterly? (12.55%)

7. What nominal rate compounded semi-annually is equivalent to 8% effective rate. (7.85%)

8. What rate compounded quarterly is equivalent to 14% compounded semi-annually? (13.76%)

9. Using the compound amount formula, find the accumulated value of P2, 000.00 in 5 years if it is invested at 11% compounded quarterly. (P3440.86)

10. Find the present value of P5, 000.00 due in 8 years if money is worth 12% compounded semi-annually. (P1968.23)

11. Find the compound amount if P5, 000.00 is invested for 3 years and 9 months at 12% compounded semi-annually, assuming simple interest over the fractional part. (P7,743.70)

12. At what nominal rate compounded semi-annually will P1, 000.00 accumulate to P2, 500.00 in 10 years?

13. How long will it take the P2, 500.00 to amount to P5, 800.00 if invested at 12% compounded quarterly?

14. Find the effective rate equivalent to 12% compounded quarterly? (12.55%)

15. What nominal rate compounded semi-annually is equivalent to 8% effective rate. (7.85%)

16. What rate compounded quarterly is equivalent to 14% compounded semi-annually? (13.76%)

17. Find the accumulated value of P2, 000.00 in 5 years if it is invested at 11% compounded quarterly. (P3440.86)

18. Find the present value of P5, 000.00 due in 8 years if money is worth 12% compounded semi-annually. (P1968.23)

19. Find the compound amount if P5, 000.00 is invested for 3 years and 9 months at 12% compounded semi-annually, assuming simple interest over the fractional part. (P7,743.70)

20. At what nominal rate compounded semi-annually will P1, 000.00 accumulate to P2, 500.00 in 10 years?

21. How long will it take the P2, 500.00 to amount to P5, 800.00 if invested at 12% compounded quarterly?

22. Find the effective rate equivalent to 12% compounded quarterly? (12.55%)

23. What nominal rate compounded semi-annually is equivalent to 8% effective rate. (7.85%)

24. What rate compounded quarterly is equivalent to 14% compounded semi-annually? (13.76%)

25. A student deposits P1, 500 in a 9% account today. He intends to deposit another P3, 000 at the end of two years. He plans to purchase in five years his favorite shoes worth P5, 000. Calculate the money that will be left in his account one year after the purchase.

26. A sum of P10, 000 is invested now and left for eight years, at which time the principal is withdrawn. The interest that has accrued is left for another eight years. If the effective annual interest rate is 5%, what will be the withdrawal amount at the end of the 16th year?

III. PROGRESS CHECK

Instruction: Read, Analyze and solve what is being ask.

1. A bank is advertising 9.5 % account that yield 9.84 % annually. How often is the interest compounded?
2. Convert 12 % compounded semi-annually to x % compounded quarterly.
3. What is the effective rate corresponding to 16% compounded daily? Take 1 year = 360 days
4. A sum of P100,000.00is invested now and left for eight years, at which time the principal is withdrawn. The interest that has accrued is left for another eight years. If the effective annual interest rate is 5 %, what will be the withdrawal amount at the end of the 16th year?
5. By the conditions of a will, the sum of P 25,000 is left to a girl to be held in a trust fund by her guardian until it amounts to P 45,000. When will the girl receive the money if the fund is invested at 8 % compounded quarterly?

IV. REFERENCES

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