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Coupled normal-mode sensitivity to inner-core shear velocity and attenuation

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SUMMARY

We have studied the response of normal modes to perturbations in inner-core shear velocity and attenuation, using fully coupled mode synthetics. Our results indicate that (i) mode pairs ${}_nS_l$–${}_{n\pm1}S_l$ are strongly coupled by anelasticity, (ii) this coupling causes shear velocity perturbations to strongly affect the Q values of modes through exchange of inner-core characteristics, (iii) there is no evidence for a weakly attenuating inner core in shear, and (iv) the discrepancy between attenuation models returned from normal modes and body waves is small. These results suggest that inversions for inner-core attenuation and shear velocity should be performed simultaneously and should take account of the strong cross-coupling due to attenuation.

Key words: attenuation, inner core, mode coupling, normal modes.

1 INTRODUCTION

Since the first radial models of earth structure were produced in the 1930s, seismologists have continually sought refinements to the structure and the values of elastic and anelastic parameters. Most models of the inner core have focused on the compressional velocity, whilst the shear wave velocity structure and jump at the inner-core boundary are much less well known. The first estimate of inner-core shear velocity was based on thermodynamic calculations (Birch 1952). The study of Dziewonski & Gilbert (1971) on inner-core rigidity supported a shear velocity at normal-mode frequencies of 3.5 km s^{-1} and the Preliminary Reference Earth Model (PREM, Dziewonski & Anderson 1981) gives a value of 3.49 km s^{-1} at normal-mode frequencies. Forward modelling by Shearer & Masters (1990) of around 50 eigenfrequencies sensitive to inner-core v_S best fit the data with an average value at normal-mode frequencies of 3.45 km s^{-1}. They argued that better modelling using normal modes is hindered by some of the low harmonic order modes being non-linear functionals of inner-core v_S, causing dramatic changes to the mode characteristics for only small velocity changes.

Inner-core shear phases (*PKJKP*) have been reported at delays corresponding to PREM inner-core v_S (Deuss *et al.* 2000), or a slightly faster value than PREM (Cao *et al.* 2005). However, the phase is very hard to detect and is searched for at the delay and slowness predicted by the earth model. This creates a somewhat circular argument that excludes the possibility of detecting the phase at a different time and thus these observations cannot be taken as robust confirmation that the PREM value of inner-core v_S is correct.

Whilst it has been possible to constrain velocities with increasing accuracy, the anelastic attenuation has inherently greater uncertainty due to elastic scattering and lateral variations. In a recent review of the attenuation structure of the Earth (Romanowicz & Durek 2000), the attenuation of the inner core is highlighted as having remaining research problems, one of which is the discrepancy between body wave and normal-mode studies. Inner-core sensitive modes have been used by some authors to indicate that for long-period waves the Q_μ value of the core is high, or weakly attenuating. This conflicts with the body wave results, which have the inner core as the most attenuating part of the Earth.

Masters & Gilbert (1981) first identified the mode ${}_{10}S_2$ as evidence of a weakly attenuating inner core in shear at mode frequencies. They consequently inferred a frequency dependence of inner-core attenuation, since body wave studies showed strong inner-core attenuation (Cormier 1981). In a further paper by Masters & Gilbert (1983) on low-frequency attenuation, they constrained inner-core shear attenuation to be $Q_\mu = 3500$ based on normal-mode data, a value that is significantly less attenuating than PREM ($Q_\mu = 84.6$, Dziewonski & Anderson 1981). The idea was supported by Fukao & Suda (1989) and Suda & Fukao (1990), who used the Sompi method to analyse a number of relevant modes (${}_2S_2$, ${}_6S_2$ and ${}_7S_3$) with their results giving a depth-dependent, weakly attenuating inner-core model ($Q_\mu = 1500$ in the upper 200 km and $Q_\mu = 3800$ below).

More recent attenuation models based on inversions of normal-mode data find much lower inner-core Q_μ values. Widmer *et al.* (1991) found $Q_\mu = 110$, Durek & Ekstrom (1996) found $Q_\mu = 104$ and Resovsky *et al.* (2005) suggested that Q_μ is in the range 90–100, though none of these studies included measurements of the low-Q modes (${}_2S_2$, ${}_6S_2$ and ${}_7S_3$) in their data sets. Many authors (Masters & Shearer 1990; Widmer *et al.* 1991) questioned the validity of observations of some of the inner-core low-Q modes, suggesting that the peaks have been confused with quasi-toroidal energy that is present due to spheroidal–toroidal coupling. These data, plus observations of ${}_{10}S_2$, are therefore excluded from many modal inversions, the latter due to its sensitivity to inner-core shear structure. However, the early models of a weakly attenuating inner core have been mentioned in a number of papers highlighting the discrepancy with body wave results (Souriau & Roudil 1995; Souriau *et al.* 2003) or concerning interpretations of attenuation mechanisms (Yoshida *et al.* 1996; Singh *et al.* 2000). Nobody has conclusively shown the high-Q_μ inner-core model to be implausible.

It is generally accepted that the reconciliation of body wave and normal-mode results requires frequency dependence or finite bulk attenuation. Various models have been proposed to achieve this including a viscoelastic absorption-band model (Li & Cormier 2002), a scattering fabric model due to crystal or thermal heterogeneity (Vidale & Earle 2000; Cormier & Li 2002) and the presence of fluid inclusions (Singh *et al.* 2000). Body wave studies also find depth dependence and anisotropy of inner-core attenuation (Souriau & Roudil 1995; Romanowicz & Durek 2000; Cormier & Li 2002) that could affect normal modes and the magnitude and cause of discrepancy between the data sets.

In this study, we have used perturbation theory to create fully coupled synthetic normal-mode data for different inner-core values of v_S and attenuation for comparison with data. We have found several pairs of inner-core modes that are strongly coupled by anelasticity, despite being often studied with the self-coupling approximation. This work also indicates the importance of considering the shear wave velocity in the inner core jointly with attenuation due to this strong mode coupling, which results in a very non-linear response for the modes in our study. We also attempt to discriminate between the models with low and high inner-core shear attenuation using observations of core-sensitive modes.

2 METHODS

2.1 Theory

Synthetic seismograms, plus singlet frequencies and amplitudes, were created for comparison with the data. Synthetic seismograms are created by solving the momentum equation

$$\mathbf{H}\mathbf{s} + \mathbf{W}\partial_t \mathbf{s} + \mathbf{P}\partial_t^2 \mathbf{s} = \mathbf{S} \tag{1}$$

for the elastic displacement field $\mathbf{s}$, where $\mathbf{H}$ is the potential energy operator, $\mathbf{W}$ is the Coriolis operator, $\mathbf{P}$ is the kinetic energy operator and $\mathbf{S}$ is the excitation by an earthquake source. Since we need to solve this differential equation for large-frequency windows, we cannot approximate around a fiducial frequency. A more general and complete solution is therefore used, employing kinetic and Coriolis approximations to reduce computation time (see Deuss & Woodhouse 2001).

For every pair of coupled modes, the corresponding matrix

$$\mathbf{M} = [\mathbf{H} + i\tilde{\omega}\mathbf{W} - \tilde{\omega}^2\mathbf{P}] \tag{2}$$

is diagonalized using the approximation $\tilde{\omega} = \sqrt{\tilde{\omega}_1\tilde{\omega}_2}$. This gives $\mathbf{M} = \mathbf{U}\Lambda\mathbf{U}^{-1}$, where $\mathbf{\Lambda}$ is the matrix with the eigenvalues and $\mathbf{U}$ the matrix with the corresponding eigenvectors. The seismogram is then given by

$$u(t) = (\mathbf{r} \cdot \mathbf{U})e^{i\sqrt{\Lambda}t}(\mathbf{U}^{-1} \cdot \mathbf{S}), \tag{3}$$

where $\mathbf{S}$ is the source vector and depends upon the source moment tensor and the vector $\mathbf{r}$ is the receiver vector incorporating instrument response and orientation.

The earth model used as the basis of the synthetics was that of PREM (Dziewonski & Anderson 1981). A mantle heterogeneity model was added using the *S*-wave velocity model S20RTS (Ritsema *et al.* 1999) with scaling factors used to obtain v_P and density heterogeneity (as used in Deuss & Woodhouse 2001). The inner-core anisotropy model used in some of the calculations was from Woodhouse *et al.* (1986), in which the anisotropic parameters have a depth squared dependence. The synthetics were fully coupled for rotation, ellipticity, 3-D mantle structure, attenuation and also inner-core anisotropy where included. In this study, the coupling calculations were carried out on reasonably large numbers of modes so the term full coupling is used, though modes over a large, but finite, frequency band have been coupled. The greatest cross-coupling effects will occur for modes close in the frequency domain, so this is considered sufficient. Coupled synthetics have been computed for groups of 40–80 modes in the frequency band below 6 mHz. This is different to the frequently used self-coupling approximation, where only one mode is computed at a time, and cross-coupling with other modes is ignored.

To investigate the effect of spherical perturbations of inner-core v_S and attenuation on inner-core sensitive modes we have used perturbation theory. This was found to give the same results as recalculating the mode catalogue as long as cross-coupling for attenuation was included, despite the highly non-linear response to changes in shear velocity.

2.2 Data and *Q* measurements from the literature

The data used are from a large-magnitude event in Bolivia (1994 June 9), and selected spectral segments were Hann tapered and fast Fourier transformed for analysis. This event has been widely used for normal-mode studies as it provided very high quality data, and due to its depth of 647 km it strongly excited the inner-core overtones. These data were therefore chosen as the most likely to show the low-*Q* inner-core oscillations if they are observable. Data from events in the Kuril Islands (1994 October 4) and Chile (1995 July 30) were also analysed, but discarded due to the weak excitation of the overtones and the poor quality of the data at late-time windows.

Data from the 2004 December 26 earthquake in Sumatra have also not been used in this paper, though it seems that these data will be very useful in future inner-core mode studies. In the past, only deep earthquakes such as that in Bolivia have been thought to excite the inner-core modes sufficiently for analysis. However, due to the greater magnitude of the Sumatra event, it also seems to have produced good records for these modes (Rosat *et al.* 2005). As this paper is mostly focused on synthetic data, we have not extended our work to include the Sumatra event, but the data will be used in future, observation-based research.

Table 1 shows the literature sources of the modal *Q* observations used in this paper. The radial modes and most *PKIKP*-equivalent modes have been measured by a number of authors as they have high *Q* values and are fairly isolated. The inner-core oscillations have only been measured by Masters & Gilbert (1983) and Fukao & Suda (1989) and the measurements in the former study had very high variance. When plotted, the mean and entire range are shown to indicate the consistency of the measurements for each mode.

3 SHEAR VELOCITY

We first studied the effect of spherical perturbations of inner-core shear velocity on the inner-core modes. There is a small discrepancy between the results of the studies by Dziewonski & Gilbert (1971) and Shearer & Masters (1990) (3.5 and 3.45 km s^{-1},

Table 1. The table gives the sources of the modal Q measurements used in this study. The frequency quoted is that calculated for PREM. The source numbers correspond to the following papers: (1) Dziewonski & Anderson (1981), (2) Masters & Gilbert (1983), (3) Giardini *et al.* (1988), (4) Fukao & Suda (1989), (5) Li (1990), (6) Widmer *et al.* (1991), (7) Durek & Ekstrom (1996), (8) He & Tromp (1996), (9) Resovsky & Ritzwoller (1998), (10) Durek & Romanowicz (1999).

Mode	Frequency (mHz)	Mean Q (observed)	Number of observations	Source
${}_1S_0$	1.631	1922	4	1, 6, 7, 8
${}_2S_0$	2.510	1564	3	1, 6, 8
${}_3S_0$	3.27	1215	4	1, 6, 7, 8
${}_4S_0$	4.106	1167	4	1, 6, 7, 8
${}_5S_0$	4.884	1087	4	1, 6, 7, 8
${}_6S_0$	5.740	1042	4	1, 6, 7, 8
${}_8S_1$	2.873	945	5	2, 6, 8, 9, 10
${}_{13}S_1$	4.496	767	4	2, 6, 8, 10
${}_2S_2$	0.9458	2439	1	4
${}_3S_2$	1.106	344	7	2, 3, 5, 6, 8, 9, 10
${}_6S_2$	2.454	2207	2	2, 4
${}_{10}S_2$	4.032	937	3	2, 3, 6
${}_{13}S_2$	4.845	958	6	2, 3, 5, 6, 8, 10
${}_6S_3$	2.822	471	5	3, 5, 6, 8, 9
${}_7S_3$	3.142	2344	2	2, 4
${}_9S_3$	3.555	727	4	5, 6, 8, 10
${}_{13}S_3$	5.194	947	4	3, 5, 8, 10
${}_9S_4$	3.878	591	2	6, 10
${}_8S_5$	4.166	670	3	6, 8, 10

respectively) and no recent studies have been carried out to refine the value of inner-core v_S. We can thus assume a current uncertainty in v_S of at least 1 to 2 per cent. It is therefore important to assess the influence on the inner-core sensitive modes of small changes in inner-core shear velocity, before attempting an inversion to better constrain its value and uncertainty.

The largest observed effect of inner-core shear velocity perturbation is on the mode pair ${}_{10}S_2$–${}_{11}S_2$. In PREM, the volumetrically averaged shear velocity at 3 mHz is 3.49 km s^{-1}. Increasing this by 2 per cent, to 3.56 km s^{-1}, causes a large change to the eigenfunctions and sensitivity kernels of the modes ${}_{10}S_2$ and ${}_{11}S_2$ (Fig. 1). In PREM, both modes are a combination of a *PKIKP*-equivalent mode and an inner-core oscillation, but a change of the inner-core shear structure decouples the modes. As expected (Giardini *et al.* 1988), ${}_{10}S_2$ becomes a *PKIKP*-equivalent mode, whilst ${}_{11}S_2$ becomes almost entirely confined to the inner core. Fig. 2 shows that the change in shear velocity raises the Q of mode ${}_{10}S_2$ so that it becomes visible on the synthetic seismogram (though not quite as high as measured) without any alteration to inner-core attenuation values. Consequently, it is proposed that small alterations to the inner-core shear wave velocity would satisfy observations of this mode and no change to attenuation is necessary.

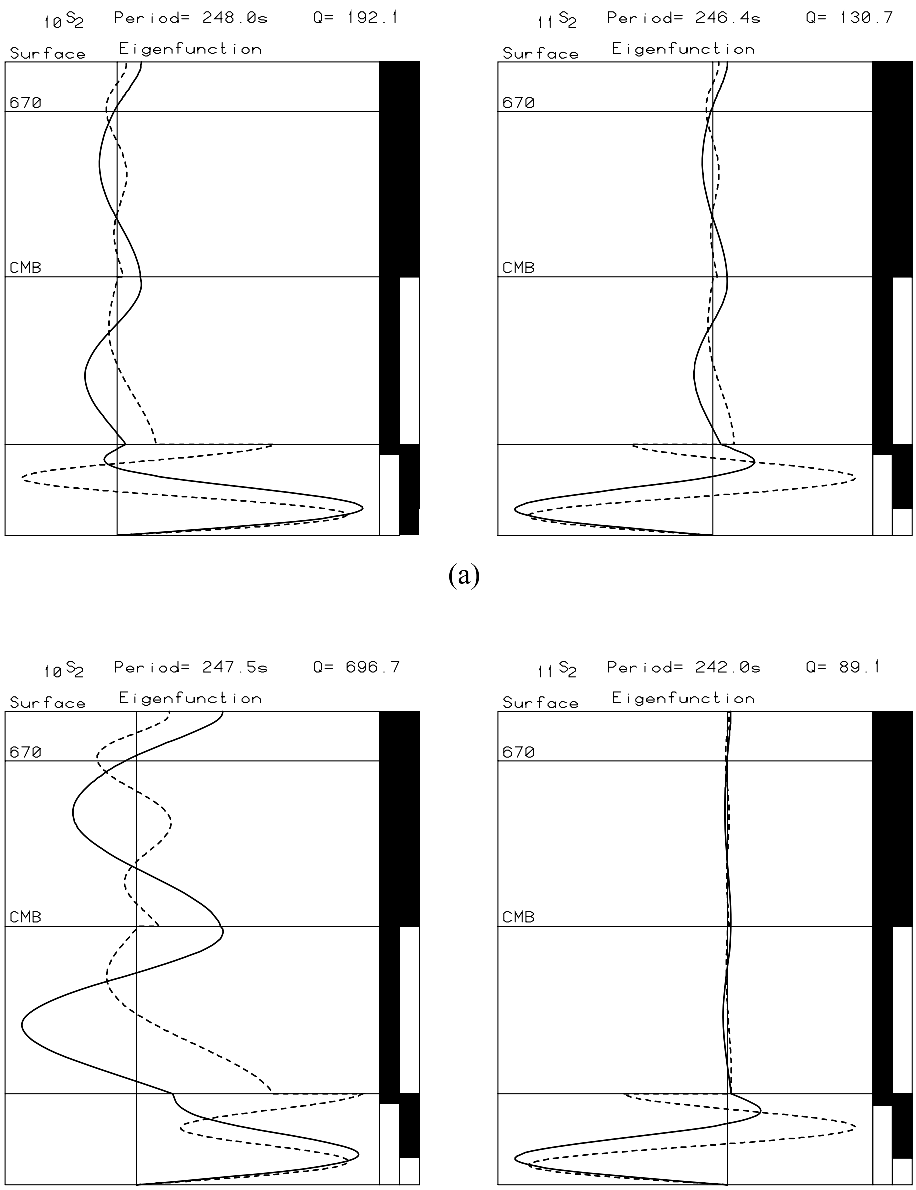
Due to the method of organizing the spheroidal modes, a number of them can exchange identities if shear velocity is altered by only a few per cent. With a 2 per cent change, ${}_5S_{10}$–${}_6S_{10}$ exchange their names (one is a mantle mode and the other an inner-core oscillation) due to the change in frequency of the inner-core oscillation (Dahlen & Tromp 1998). In addition, there are also a large number of *PKIKP*-equivalent inner-core oscillation pairs that couple due to anelasticity in a spherically symmetric earth (Tromp & Dahlen 1990). Such pairs can be written ${}_nS_l$–${}_{n\pm1}S_l$; they have the same angular order and radial orders usually differing by 1 or 2. Tromp & Dahlen (1990) first studied this anelastic coupling and assessed its importance in the earth model 1066A (Gilbert 1975) with the Q model of Masters & Gilbert (1983) (this contains a high inner-core Q_μ value of 3666). The resultant changes to both eigenfrequencies and Q values were found to be small, though they found the effects to be greater in general when a lower inner-core Q_μ value is used ($Q_\mu = 200$ after Widmer *et al.* 1991). However, the effect of this coupling becomes dramatic if additional small perturbations to inner-core v_S are applied. The resultant coupling allows large changes to eigenfunctions, frequency and Q values for small alterations to inner-core shear velocity as it allows exchange of inner-core characteristics.

The case of ${}_{10}S_2$ and ${}_{11}S_2$ was identified due to their proximity, but it is also found that wide band cross-coupling can occur for such pairs. These include ${}_6S_2$–${}_7S_2$, ${}_8S_4$–${}_9S_4$, ${}_8S_5$–${}_9S_5$ and even ${}_7S_3$–${}_9S_3$, with effects still seen at mode separations of 0.4 mHz. Fig. 3 shows examples of ${}_nS_l$–${}_{n\pm1}S_l$ pairs that are appreciably coupled by anelasticity and illustrates the range of frequency separations over which this effect can be important. These pairs can exchange *PKIKP*-equivalent and inner-core oscillation characteristics (eigenfunctions become more or less confined to the inner core) causing significant changes to attenuation values.

Fig. 4 shows the comparison of self- and cross-coupling when using perturbation theory. Panel (a) is computed for self-coupling of the modes only and shows that whilst the modes are shifted slightly in frequency by the v_S perturbation, the variation of modal Q values is practically negligible. Most notably, no energy is predicted for the modes ${}_{10}S_2$ and ${}_{11}S_2$. In contrast, the cross-coupled synthetic in Fig. 4(b) shows the strong effect of v_S perturbations on modal Q values. The spectrum shows that mode ${}_9S_4$ becomes almost unobservable for a small increase in v_S, whilst mode ${}_8S_5$ shows the opposite response, decreasing in amplitude for a small decrease in v_S. Many of these modes are currently included in inversions for both inner-core attenuation and velocity anisotropy but no current procedures simultaneously invert for velocity and attenuation or include cross-coupling.

Fig. 5 shows the alteration to the Q values of the *PKIKP*-equivalent modes when the shear velocity is perturbed by ±2 per cent of PREM. The modes identified as being coupled to a J_{SV} mode (inner-core oscillation) show greater variation of their quality factors than those that do not couple, the most extreme being the previously identified ${}_{10}S_2$. The magnitude of the effect depends upon the proximity of the coupled modes and also the relative change in their separation due to the v_S perturbation. For some modes therefore, the effect is quite small, but the change to Q is still greater than for self-coupled synthetics. For mode ${}_3S_2$, the Q change predicted using self-coupling perturbation theory is 0.3 per cent for ±2 per cent v_S changes. However, both recomputation of the mode catalogue and perturbation theory including cross-coupling produce Q changes of between 2 and 6 per cent.

The Q values of the *PKIKP*-equivalent modes respond differently to the increase or decrease in shear velocity. The anelastic coupling moves the relevant mode pairs slightly closer together in frequency (Tromp & Dahlen 1990) acting to hybridize the modes. However, the dominant effect is the alteration of the frequencies of the different mode types due to the shear velocity change, which affects the frequency separation, and thus extent of hybridization, of the modes in the pair. J_{SV} modes have a larger proportion of their energy in shear, so the increase in shear velocity will increase the frequency of their vibration much more than it will increase the frequency of the *PKIKP*-equivalent mode. For the pair ${}_8S_4$–${}_9S_4$ where ${}_9S_4$ is the *PKIKP*-equivalent mode, this means an increase in v_S will decrease their frequency separation leading to greater hybridization and a



(b)

Figure 1. The eigenfunctions for $_{10}S_2$ and $_{11}S_2$ for (a) PREM and (b) recalculated for PREM with a 2 per cent increase in inner-core shear velocity.

decrease in the Q value of $_9S_4$, whilst a decrease in v_S will increase their frequency separation and lead to an increased Q value for mode $_9S_4$. The opposite is true of the pair $_8S_5$–$_9S_5$ where $_8S_5$ is the *PKIKP*-equivalent mode and lower in frequency than the J_{SV} mode. This effect places additional strong constraints on future inversions for inner-core v_S and Q.

Comparison of measured Q values with synthetics (Fig. 5) tentatively suggests a small increase in v_S is needed for most modes, but this will increase the misfit to mode $_9S_4$. This suggests that a parameter, additional to spherically symmetric v_S and attenuation, affects these modes. Further factors which may explain, and be constrained by, the observations include bulk attenuation, anisotropy, inner-core radius and lateral variations in inner-core structure.

Inner-core anisotropy has been invoked to explain the anomalous splitting of certain normal modes, which cannot be explained by ellipticity, rotation or lateral heterogeneity (Woodhouse *et al.* 1986). The first models of Woodhouse *et al.* (1986) were simple, with either constant or depth squared dependent anisotropic parameters, and further inversions for more complex anisotropy models have since been performed (Durek & Romanowicz 1999; Ishii *et al.* 2002; Beghein & Trampert 2003). As well as causing anomalous splitting, inner-core anisotropy affects the modal Q values, so it may be instructive to repeat our calculations in the presence of anisotropy.

Fig. 6(a) compares synthetics calculated with and without inner-core anisotropy. The anisotropy clearly causes splitting of some of the inner-core modes and alters the modal Q values, notably causing energy to be visible for mode $_{10}S_2$. For two example inner-core modes $_9S_4$ and $_8S_5$, the anisotropy causes changes of less than 10 per cent to the Q values, whereas ±2 per cent v_S perturbations cause changes of 10–30 per cent. Thus, permissible alterations to the spherical velocity structure have a much greater effect

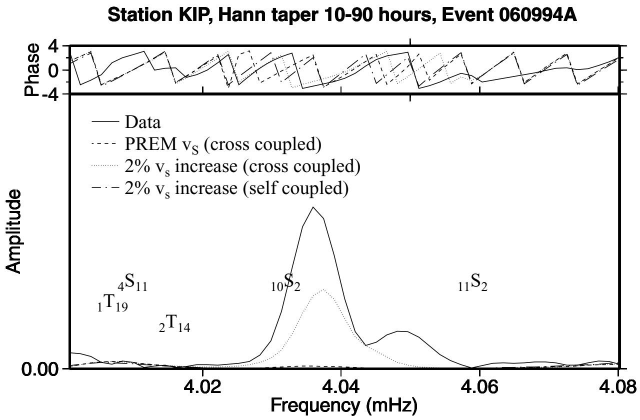


Figure 2. Modes $_{10}S_2$ and $_{11}S_2$ on the observed and three synthetic spectra. The line marked PREM v_S has been calculated for the model based on PREM plus S20RTS and fully coupled for rotation, ellipticity, mantle structure and attenuation. The line marked 2 per cent v_S increase (cross-coupled) was calculated for the same model except for a 2 per cent increase of inner-core shear velocity. The line marked 2 per cent v_S increase (self-coupled) was calculated for the same model (with the 2 per cent increase of inner-core shear velocity), but computed using self-coupling only. This last line can just be seen near the edges of the plot but has zero amplitude for modes $_{10}S_2$ and $_{11}S_2$; thus, it cannot be seen in the central part of the plot.

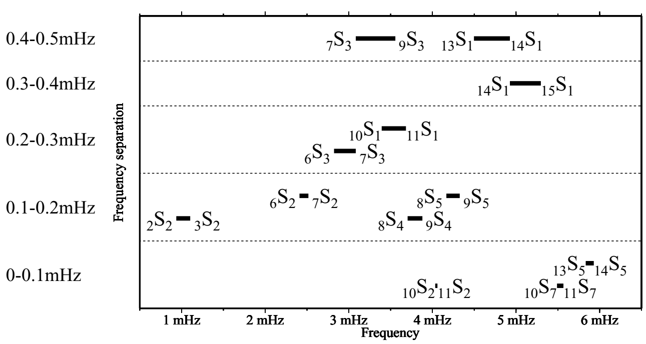


Figure 3. The frequency separation of mode pairs $_nS_l$–$_{n\pm1}S_l$ that are appreciably coupled by anelasticity.

on modal Q values than the anisotropic structure recovered by inversion. Fig. 6(b) demonstrates that the effects of ±2 per cent v_S changes for a model that includes inner-core anisotropy are similar to those for an isotropic model, though the anisotropy does cause additional splitting and small adjustments to Q values. This suggests that anisotropy may be important in resolving the issue raised previously, in satisfying the opposing responses of $_9S_4$ and $_8S_5$. However, this is an added complexity and a joint inversion for the background v_S and Q values should be performed initially.

4 ATTENUATION

4.1 Inner-core (J_{SV}) modes

Modes with energy mainly confined in the inner core are termed inner-core oscillations (J_{SV}). In models with low inner-core Q_μ, these modes are highly attenuated and are not expected to be observed at the surface as most of their energy is stored in shear. Additionally, none of these modes can be well excited by earthquakes in the crust or upper mantle unless there is a certain amount of energy along the rest of the earth's radius. As a result, these modes are not usually studied and the only measurements made of their Q values have been those of Masters & Gilbert (1983) and Fukao & Suda (1989).

Here, J_{SV} modes $_2S_2$, $_6S_2$ and $_7S_3$ are reanalysed as they formed the evidence for a weakly attenuating inner core. Synthetics were calculated for two models: one with the inner-core values of PREM and labelled 'PREM Q_μ', and the other with the inner-core Q_μ model of Suda & Fukao (1990) labelled 'High Q_μ'. The latter has $Q_\mu = 1500$ in the upper 200 km, $Q_\mu = 3800$ below and Q_κ set to infinity throughout the inner core. If the core modes are weakly attenuated, they should still be visible in late-time windows, as shown by the High Q_μ synthetic spectra (Fig. 7). While the data and PREM synthetic do not show any significant signal, large peaks for $_6S_2$ and $_7S_3$ are present in the high inner-core Q_μ spectra. The signal around $_2S_2$ is complicated by the presence of high-Q modes, $_3S_1$ and $_1S_3$.

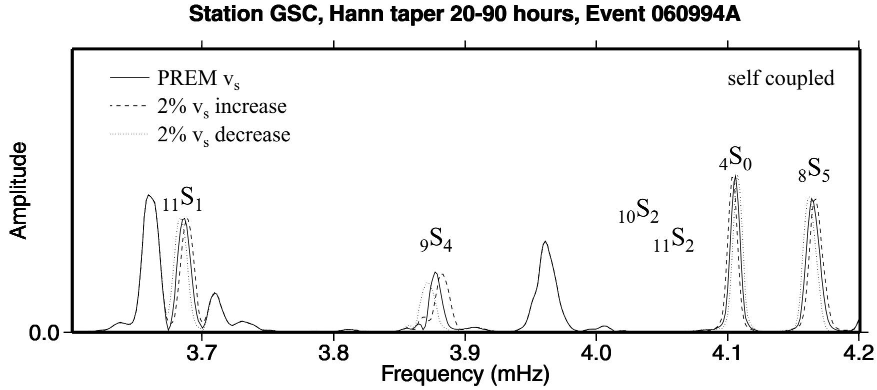
The observations of $_6S_2$ and $_7S_3$ reported by Fukao & Suda (1989) find the modes at slightly different frequencies from those predicted by PREM: $_6S_2$ at 2.436 mHz and $_7S_3$ at 3.122 mHz. This would require a perturbation to the inner-core shear velocity of PREM in addition to the change in attenuation structure. Fig. 7, however, shows no significant energy at the frequencies reported by Fukao & Suda (1989) either. The data also do not show any quasi-toroidal energy from the modes $_0T_{17}$ and $_0T_{23}$, suggested as an explanation of Suda & Fukao's observations by Masters & Shearer (1990).

Misfit values were calculated for the three modes, $_2S_2$, $_6S_2$ and $_7S_3$, between the data and two synthetics (Table 2): the two fully coupled models PREM Q_μ and High Q_μ mentioned above. At early-time windows, the results for the two models are close, as both predict energy to be present at this time. However, for late-time windows a strongly attenuating core model would predict all energy to have decayed away, whilst a weakly attenuating core model would predict a peak still to be visible. For modes $_6S_2$ and $_7S_3$, this predicted peak was not present on the data, resulting in misfit values for the High Q_μ model four times greater than that given for the model with strong inner-core attenuation.

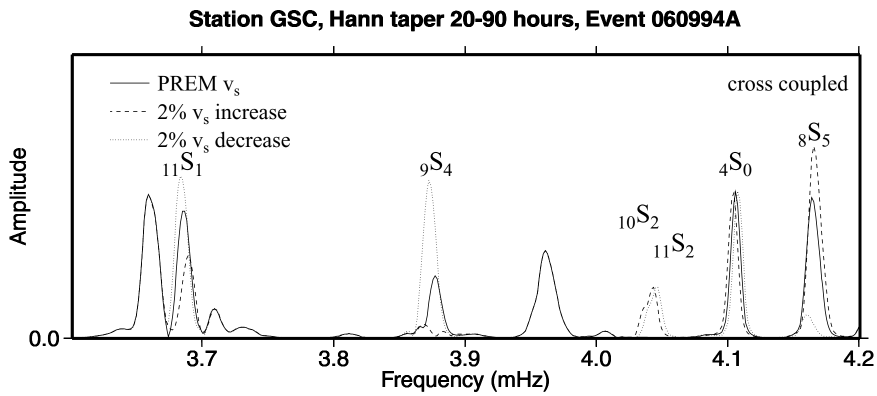
4.2 *PKIKP*-equivalent modes

The *PKIKP* equivalent modes have energy throughout most of the Earth including the inner core. They usually have only 20–30 per cent of their energy in shear and are thus useful for constraining both bulk and shear attenuation, though few modes have significant shear energy sensitivity in the inner core. For these modes, the synthetic data created for different inner-core attenuation models were compared against data from the Bolivia earthquake and measured modal Q values taken from the literature. Results showed that the Q_μ value could not be as high as required by Fukao & Suda (1989), since the few modes with high shear energy in the inner core are predicted to have higher amplitudes than those observed in the spectra (Fig. 8) and greater modal Q values than those measured (Fig. 9). The amplitude of mode $_3S_2$ is shown in Fig. 8 to be overpredicted by PREM, with the discrepancy becoming worse as the inner core is made even less attenuating as in the High Q_μ model. In Fig. 9, theoretical and observed Q measurements are shown for a number of inner-core sensitive modes. For the *PKIKP*-equivalent modes PREM predicts Q values that fall within the range of observations, whilst the predictions for a weakly attenuating core generally fall outside of the observed range, most clearly for modes $_3S_2$, $_9S_3$ and $_9S_4$.

Due to the sensitivity of the *PKIKP*-equivalent modes to bulk as well as shear attenuation, it should also be investigated whether



(a)



(b)

Figure 4. In panel (a), the three synthetics shown are calculated for the model based on PREM plus S20RTS and self-coupled for rotation, ellipticity, mantle structure and attenuation. Two of the synthetics are calculated for models with inner-core v_S perturbations of ±2 per cent. Small shifts of frequency are observed, but the effect of the v_S perturbation on modal Q values is very slight. Panel (b) shows the same synthetics calculated using cross-coupling. Modes with sensitivity to shear velocity in the inner core, such as ${}_9S_4$, can often be strongly affected by small spherical perturbations. For clarity, only pertinent modes are labelled.

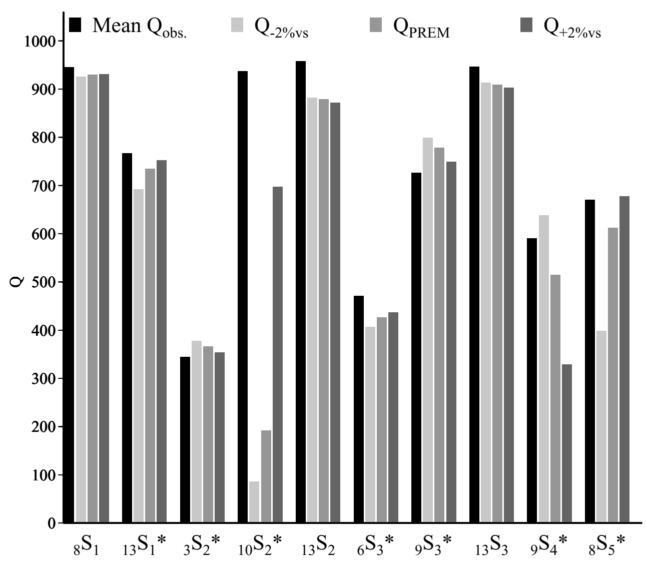


Figure 5. The histogram shows the alteration of the Q values of the *PKIKP*-equivalent modes due to changes in inner-core v_S. The Q values have been found by computation of mode catalogues for three earth models: PREM, and earth models identical to PREM except for 2 per cent changes to inner-core shear velocity. Only those modes marked with an * have been identified as coupling to a J_{SV} mode.

the modes that are poorly predicted by weak shear attenuation could be reconciled by introducing finite bulk attenuation in the inner core. The zone of bulk attenuation in the Earth, as required to fit the radial mode observations, has been located in the inner core, outer core and upper mantle by different authors. Radial Q value observations included in the inversion for PREM are slightly higher than those recorded more recently, and the inversion of Durek & Ekstrom (1995) gave much lower misfits when no bulk attenuation was present in the inner core, suggesting $Q_\kappa = \infty$ there. The single-layer inversion of Durek & Ekstrom (1995) can also be used to place a minimum constraint of $Q_\kappa = 2350$ if all bulk attenuation in the Earth occurs throughout the inner core. Synthetics and predicted Q values were thus also calculated using this latter value. The significantly misfit modes (${}_3S_2$, ${}_9S_3$ and ${}_9S_4$) are noted to have very low sensitivity to bulk attenuation and show no change with the variations in Q_κ.

For these *PKIKP*-equivalent modes, misfit values calculated between the data and synthetics are more than doubled by the introduction of very weak shear attenuation values (Table 3). The high misfit values for all synthetics for these modes are in part explained by their anomalous splitting most commonly attributed to inner-core anisotropy. However, we have discovered that, as yet, models of inner-core anisotropy do not perform consistently well in predicting these modes once full coupling is included and this will be the subject of a further publication.

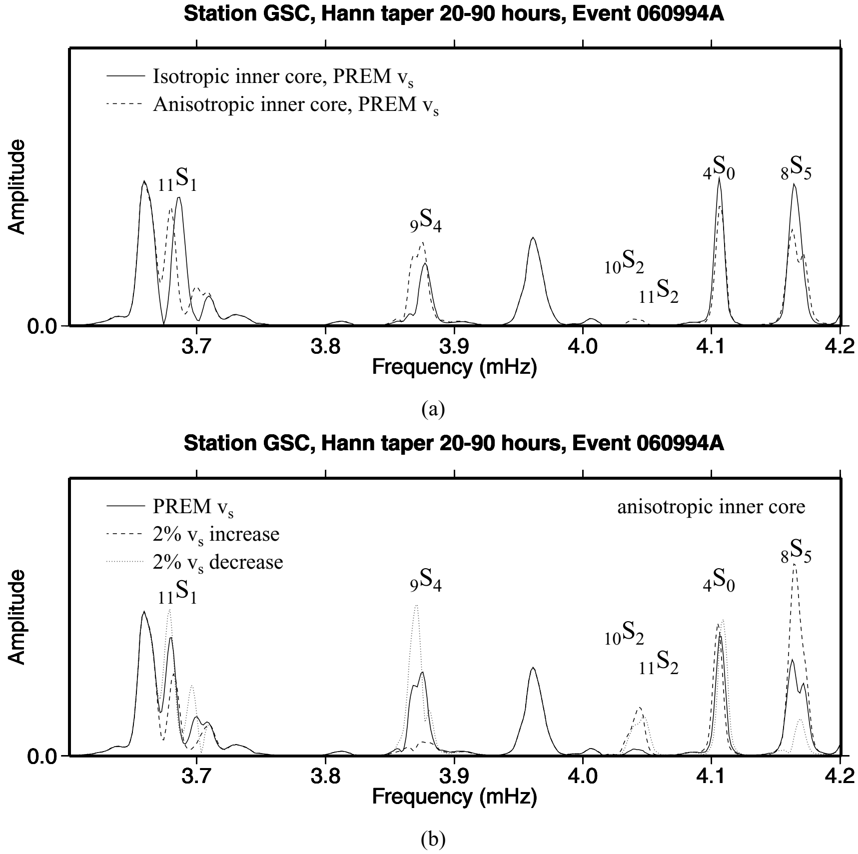


Figure 6. In panel (a), one of the synthetics shown is calculated for the model based on PREM plus S20RTS and fully coupled for rotation, ellipticity, mantle structure and attenuation. The second synthetic additionally includes an inner-core anisotropy model. The anisotropy causes splitting of some of the inner-core modes. In panel (b), the three synthetics shown are calculated for the model based on PREM plus S20RTS and an inner-core anisotropy model. The synthetics are fully coupled for rotation, ellipticity, mantle structure, inner-core anisotropy and attenuation. Two of the synthetics are calculated for models with inner-core v_S perturbations of ± 2 per cent. For clarity, only pertinent modes are labelled.

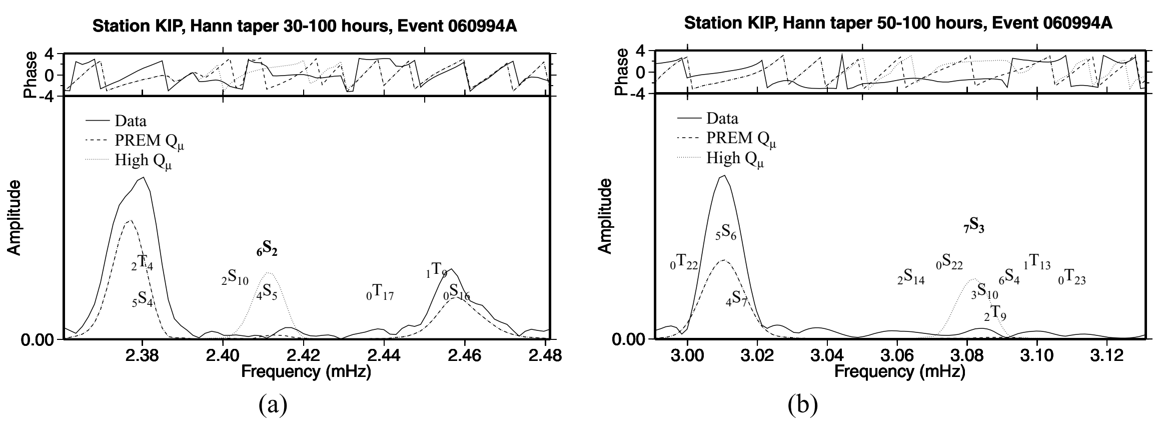


Figure 7. Inner-core oscillations ${}_6S_2$ and ${}_7S_3$ shown on the data and two synthetics. The synthetic marked PREM Q_μ is for an earth model based on PREM plus mantle model S20RTS and fully coupled for rotation, ellipticity, mantle structure and attenuation. The synthetic marked High Q_μ is the same except the inner-core attenuation model used was that of Suda & Fukao (1990). The weakly attenuating core of the latter model causes a peak to appear in this late-time window that is not present in the data. Fukao & Suda (1989) reported their observations to be at slightly different frequencies from PREM; they found ${}_6S_2$ at 2.436 mHz and ${}_7S_3$ at 3.122 mHz. The figure, however, shows no significant energy at these frequencies.

5 CONCLUSIONS

We have shown that a number of inner-core modes are highly sensitive to perturbations in the shear velocity of the inner core, including some modes currently included in inversion data sets. Through the strong anelastic coupling of the J_{SV} *PKIKP*-equivalent pairs, perturbations to shear velocity have a large impact on the Q values measured for the modes. We have found that for changes to inner-core shear velocity, perturbation theory only gives the same result as recomputation of the mode catalogue when cross-coupling due

Table 2. The table shows the misfit values calculated for each model for three J_{SV} modes. The models, marked PREM Q_μ and High Q_μ, are fully coupled for rotation, ellipticity, mantle structure S20RTS and attenuation. Both use PREM plus S20RTS except for attenuation models in the inner core. The model denoted PREM Q_μ uses the inner-core attenuation values in PREM and the model marked High Q_μ uses the model of Suda & Fukao (1990). The data used were from the Bolivian event (1994 June 6), taking either early (1–40 h or 1–50 h) or late (50–100 h) time windows.

Mode	Early-time window		Late-time window	
	PREM Q_μ	High Q_μ	PREM Q_μ	High Q_μ
${}_2S_2$	0.1599	0.1658	0.4591	0.4743
${}_6S_2$	0.2795	0.3351	0.7523	3.104
${}_7S_3$	0.3916	0.3949	1.048	4.880

Table 3. The table shows the misfit values calculated for each model for three *PKIKP*-equivalent modes. As before, the models marked PREM Q_μ and High Q_μ are fully coupled for rotation, ellipticity, mantle structure S20RTS and attenuation. Both use PREM plus S20RTS except for attenuation models in the inner core, where the model marked High Q_μ uses the model of Suda & Fukao (1990). The data used were from the Bolivian event (1994 June 6) and the time windows are late: 20–80 h for ${}_3S_2$ and 30–100 h for ${}_9S_3$ and ${}_8S_5$.

Mode	PREM Q_μ	High Q_μ
${}_3S_2$	2.748	6.213
${}_9S_3$	2.732	6.625
${}_8S_5$	1.455	3.182

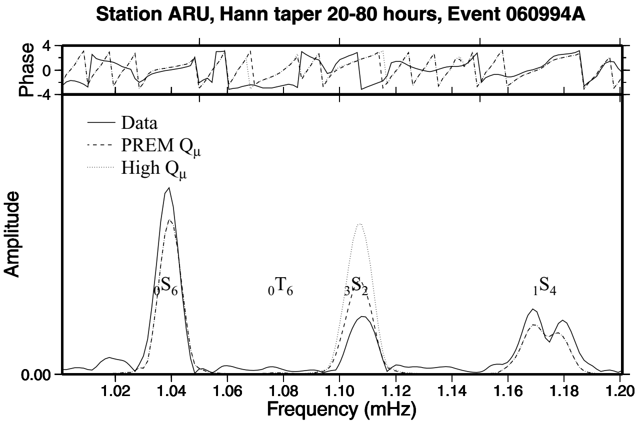


Figure 8. Mode ${}_3S_2$ is shown on the observed spectrum plus two synthetic spectra. The synthetic marked PREM Q_μ is for an earth model based on PREM plus mantle model S20RTS and fully coupled for rotation, ellipticity, mantle structure and attenuation. The synthetic marked High Q_μ is the same except the inner-core attenuation model used was that of Suda & Fukao (1990).

to attenuation is included. Inversions that use perturbation theory assuming modes to be isolated will therefore return incorrect shear velocity values, and ideally a joint inversion for both attenuation and shear velocity should be performed. Additionally, if the shear velocity is poorly known, to the extent of a few per cent, then inversions for inner-core attenuation involving data from the highly sensitive coupled modes will also be incorrect.

We have also demonstrated the complexity of working with J_{SV} modes that have low amplitude and are difficult to observe. Our data show no evidence of inner-core oscillations with high Q values and no correlation with synthetics based on high inner-core Q_μ models. This finally provides conclusive evidence that the inner core is strongly attenuating in the normal-mode frequency band. This implies a small discrepancy between normal-mode and body wave results that should be explained by finite bulk attenuation or a frequency-dependent attenuation mechanism.

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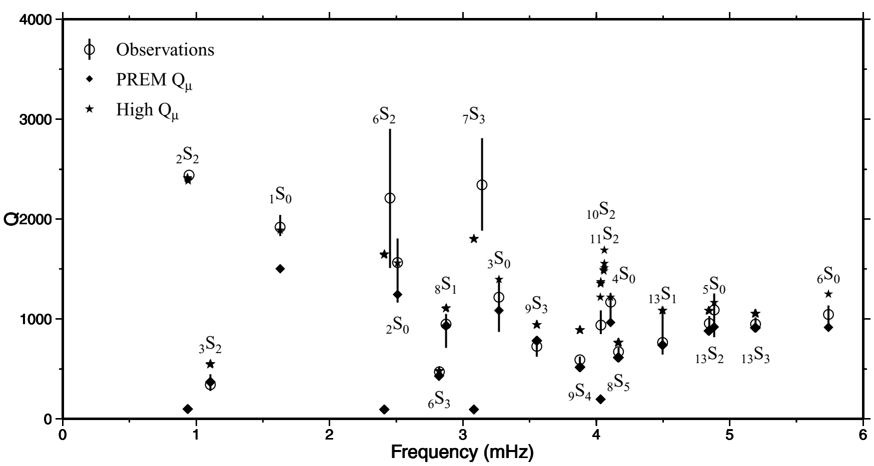

Figure 9. Observed and synthetic (singlet) modal Q values for the radial modes (${}_1S_0$ to ${}_6S_0$), inner-core oscillations (${}_2S_2$, ${}_6S_2$ and ${}_7S_3$) measured by Masters & Gilbert (1983) and Fukao & Suda (1989) and the *PKIKP*-equivalents (note ${}_3S_2$ at 1.1 mHz, ${}_9S_3$ at 3.55 mHz and ${}_9S_4$ at 3.88 mHz). The mean and range of the observations are plotted, indicating the limits of current measured values. The synthetics are marked PREM Q_μ for an earth model based on PREM plus mantle model S20RTS and fully coupled for rotation, ellipticity, mantle structure and attenuation. The synthetics marked High Q_μ are the same except the inner-core attenuation model used was that of Suda & Fukao (1990). The points near 4 mHz are the spread of singlets for modes ${}_{10}S_2$ and ${}_{11}S_2$. The poor match to ${}_1S_0$ using the PREM core attenuation model is due to the finite bulk attenuation in the model, and this also contributes to the discrepancy between synthetics for the other radial modes.

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