THE INTERPRETATION OF WELL SHOT DATA—II*

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ABSTRACT

Five methods of shooting a well for the determination of vertical velocity, and the methods of reduction of the data, are discussed. Means are described for determining the existence of interference due to transmission of vibration down the cable which suspends the well geophone.

The primary purpose of shooting a well is to obtain information which one can use in conjunction with seismic reflection data to determine depths, dips, and horizontal offsets of subsurface reflectors. In some cases, a time-depth relationship can be used directly. For other purposes it is essential that a distribution of velocity with depth be obtained.

The present discussion covers the various procedures currently in use, together with a few new procedures. Additionally, we discuss the various difficulties encountered, including errors of interpretation and errors due to extraneous noise.

We may suppose at the outset that there are no actual mistakes in computations. Then the errors are normal observational errors which may in some cases be minimized by designing the observational procedure properly. We shall therefore be concerned with various methods of setting up in the field to shoot a well and with the various methods of interpreting the resultant data.

One result that should be emphasized is that the error is only partially dependent upon the error in measuring the travel time. A part of the error is due to the failure of our simplifying assumptions to correspond with the facts. Consequently, any information that we can obtain regarding the validity of our simplifying assumptions is information regarding a probable reliability of the final interpretation.

METHOD A

The usual method of shooting a well and of interpreting the data we shall call Method A. This consists of first correcting the directly read travel time to a datum at the shot point. These corrected directly read times are then reduced to vertical times by multiplication by the cosine of the angle between the shot point and the well mouth as seen from the geophone position in the well. Referring to

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Fig. 1, this angle is shown as angle A. This method is generally sufficiently accurate when the geophone depth exceeds the distance from the well mouth to the shot point. Otherwise, it may become very inaccurate indeed. The method is exact except for the error in reading the time on the record when the following conditions are satisfied: (a) the well is straight and vertical; and (b) the velocity is constant to the depth measured.

Naturally in case the velocity is constant the ray will be a straight line from the shot to the well geophone. In this case, Method A gives

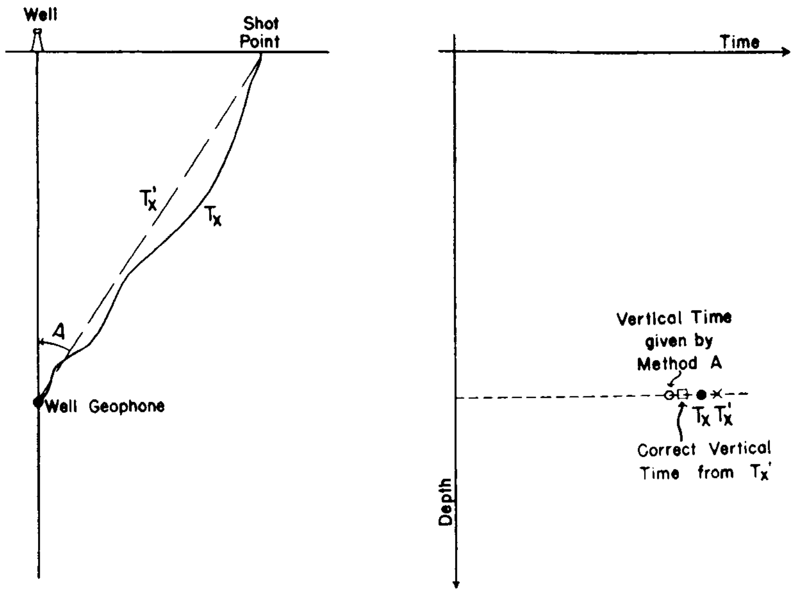

FIG. 1. Illustrating Method A for determining vertical velocity.

an accurate result. In case the velocity is not constant but is dependent upon the depth only, the actual ray will be different from the straight ray. Since the actual ray corresponds to a path of *minimum travel time*, the straight ray will correspond to a path of *longer travel time* and consequently the blind application of Method A will yield a vertical travel time which is too small. This is illustrated on the right hand side of Fig. 1 in which the cross is the time over the straight path and the square is the vertical travel time determined by using Method A and the time T_x' over the straight path. The observed time

T_x shows as a solid circle and the corrected vertical time based on this observed time is shown as the open circle. This open circle would be the value ordinarily given by Method *A*. Unfortunately, Method *A* itself gives no measure of the error involved by this procedure. Ordinarily this error is small enough so that it may be neglected.

Regarding Method *A*, we may state the following proposition: *If* the velocity is stratified horizontally; that is, is dependent only upon the depth *and* is not constant, *then* Method A gives vertical travel times that are too small. A somewhat more general proposition might also be considered by leaving out the assumption that the velocity is horizontally stratified. Such a proposition would usually be true, but it would be possible to devise a velocity distribution for which the proposition would be false. Such an instance would occur if a fault separated the well and the shot point and if the shot point were located on the side of the fault having much higher velocity than the well side. Then if the vertical travel time referred to vertical travel time at the well, the time would be too short, but if the vertical travel time referred to travel time under the shot point, the time would be too long.

The great virtue of Method *A* is its simplicity which will probably be sufficient to keep it in use for some time to come. The greatest fault of Method *A* is that it gives no information regarding observational errors not due to errors in timing.

METHOD B

A second method, Method *B*, may be employed when shot points at several distances from the well mouth are available. Method *B* is illustrated in Fig. 2. The profile view of three shot points and the well is shown on the left hand side of the Fig. The ray paths between the shot points and the well geophone are shown as straight lines. It is of course understood that these ray paths may not actually be straight lines. However, *if* the ray paths are straight lines we may plot the squares of the directly observed times (for simplicity assuming all shots fired at the datum) against the squares of the horizontal distances from the well to the shot points. This plot is shown on the right hand side of Fig. 2. *If* the velocity is constant and the rays are straight lines, *then* the points thus obtained will fall on a straight line. This straight line will intersect the t^2 axis at the point $t_o{}^2$ as shown. The positive square root of $t_o{}^2$ is the vertical travel time t_o.

Actually the ray paths will not be straight lines as indicated on the plate but will deviate somewhat from these straight lines. This deviation will show in the failure of the x^2, t^2 graph to be an exact straight line. Ordinarily, the graph as shown on the plate will be slightly concave upwards in case the velocity is stratified horizontally. In fact, we may state the following proposition: *If* the velocity is stratified horizontally and is not constant, *then* Method B gives a vertical travel time that is a little too large. Here I am assuming that Method B will be applied by fitting a straight line to the points of the x^2, t^2

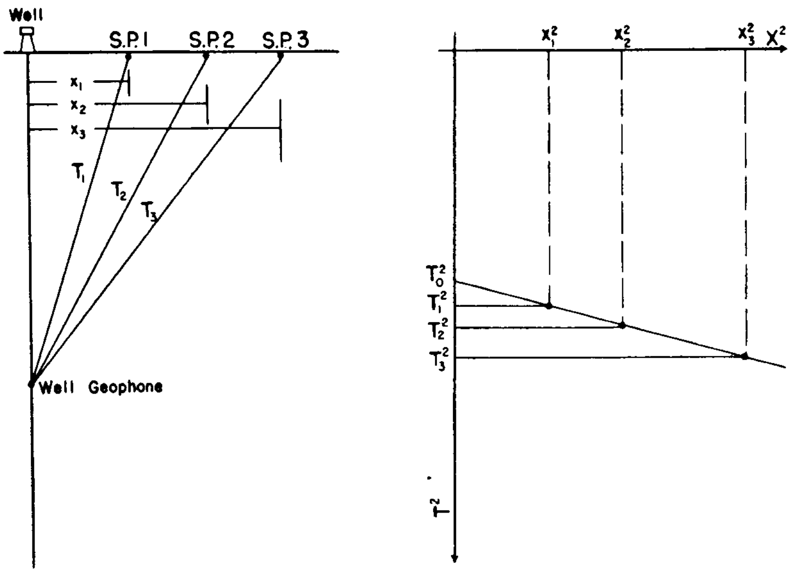

FIG. 2. Illustrating Method B for determining vertical velocity.

graph and that since the actual curve should be somewhat concave upwards, the straight line will cut the t^2-axis somewhat below where the true curved line would cut this axis.

This method is of some interest because it may furnish some information regarding the validity of our usual assumptions regarding velocity stratification. For example, if the x^2, t^2 curve is concave downwards, we can conclude that the velocity is not constant, and that it is not stratified horizontally. Here we are assuming, of course, that our errors in timing and in reading the times are small enough so that they will not invalidate our conclusion.

METHOD C

A third method, Method C, consists of applying Method A to each shot point distance where several shot point distances are available, and obtaining a reduced travel time which may be plotted for each depth against the shot point distance, x. A straight line fit to this set of points intersects the T axis at the point which approximately corresponds to the correct vertical travel time. This method will ordinarily correct the results given by Method A; it may in some cases tend to over-correct these results.

METHOD D

Method D is the method described in the author's previous paper

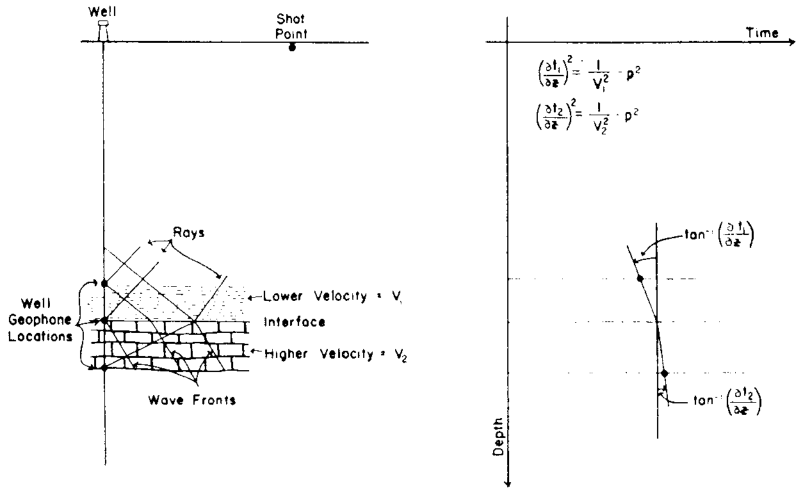

FIG. 3. Illustrating Method D for determining vertical velocity.

on the same subject.[1] Method D is somewhat more difficult to apply than the other methods but it has the virtue that it yields a velocity distribution which would yield the observed data if this velocity distribution were the actual velocity distribution. Method D is generally more accurate than the other methods even though it is dependent upon the determination of interval velocities. It can in fact

[1] GEOPHYSICS, Vol. IV, pp. 24–32 (1939).

be made as accurate as we please, excepting that the accuracy is always limited by the accuracy of reading the travel times.

Method *D* may be described by reference to Fig. 3. On the left side is shown the well, an interface between media of velocities V_1 and V_2, a set of rays, wave fronts, and three geophone locations in the well. Because of the bending of the rays at the interface, there is a sharp bend in the time-depth graph as shown on the right of the Fig. It should be emphasized that the times plotted on this time-depth graph are the directly read times assuming each shot at the datum. The slopes of the two parts of the time-depth graph are the two derivatives of directly read time, t, with respect to depth, z, one taken just above the interface and the other taken just below the interface. The first two equations on the right hand side of Fig. 3 show the relationships between the squares of these slopes and the corresponding velocities and the quantity p^2. This quantity, p^2, is a derivative of t with respect to horizontal distance. This derivative, p, may be eliminated by taking the difference between the first two equations to give the following equation:

$$\left(\frac{\partial t_1}{\partial z}\right)^2 - \left(\frac{\partial t_2}{\partial z}\right)^2 = \frac{1}{V_1^2} - \frac{1}{V_2^2} = \Delta' \text{ (measured)}$$

But

$$\Delta' = (V_2^2 - V_1^2)/V_1^2V_2^2 = \Delta V^2/\{V_1^2(V_1^2 + \Delta V^2)\}$$

where

$$\Delta V^2 = V_2^2 - V_1^2.$$

Hence

$$\Delta V^2 = V_1^4\Delta'/(1 - V_1^2\Delta').$$

Thus if V_1 is known and Δ' is measured, ΔV^2 can be found and hence V_2 can be found.

Method *D* is of greatest value for long offset shot points and for the part of the section shallower in depth than the shot hole offset distance. The advantage is especially noticeable when the variation of velocity in the upper section is large.

Method *D* will be in error if the velocity is not stratified horizontally or if the well is not sufficiently straight and vertical. In fact if z' is the well direction, z is the true vertical, and n is the ray direction, then if ψ is the angle between n and z and ϕ is the angle between z and z' then

$$\frac{\partial t}{\partial z} = \frac{1}{V} \cos \psi, \quad p = \frac{1}{V} \sin \psi, \quad \text{and} \quad \frac{\partial t}{\partial z'} = \frac{1}{V} \cos (\psi - \phi)$$

or

$$\frac{\partial t}{\partial z'} = \frac{\partial t}{\partial z} \cos \phi + p \sin \phi$$

or approximately

$$\partial t/\partial z' = \partial t/\partial z + p\phi^\circ/57.$$

Hence the error made in taking $\partial t/\partial z'$ for $\partial t/\partial z$ is approximately $p\phi^\circ/57$. p can be estimated from $p^2 = 1/V^2 - (\partial t/\partial z)^2$ for the section in the well just above where the deviation from vertical becomes appreciable. Since p decreases continuously and slowly with increasing depth, we can estimate our error if we know the deviation from vertical ϕ° in degrees. Thus the fact that the well is not vertical can be taken into account in Method *D* if the deviations from vertical are known.

METHOD E

Another method of interpretation, applicable when velocity is stratified horizontally, utilizes the gradient of the travel time in the form

$$\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2 = \frac{1}{V^2(z)}$$

where $T = T(x, z)$ is the travel time from a shot point at a distance x from the well mouth to a geophone at a depth z in the well. Fig. 4 illustrates the field setup for Method *E*. In this Fig. the well is located at W and the shot points used are s_1, s_2 and s_3. Since the velocity is assumed to be stratified horizontally, we may imagine that we have a shot point S in place of the well and three wells W_1, W_2 and W_3 in place of the three shot points. This exchange of shot point and well does not change the travel time when the velocity is stratified horizontally.

Method E was suggested in the first paper[2] but it was implied that the derivatives could be replaced by their corresponding difference qoutients. Such a replacement is not sufficiently accurate

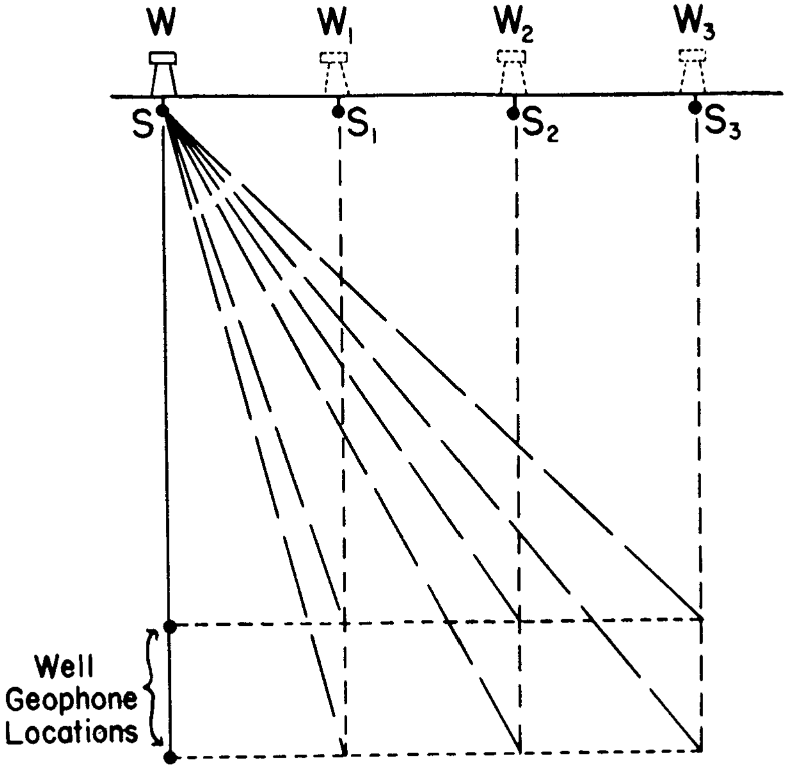

FIG. 4. Illustrating Method E for determining vertical velocity.

however. This is most evident in the calculation of $\partial T/\partial x$. Approximately, $T^2(x, z) = T_o^2(z) + k(z)x^2$ so that

$$\begin{aligned}\partial T/\partial x &= kx/T = (\Delta T^2/\Delta x^2)(x/T)\\ &= ((T_1 + T_2)/(x_1 + x_2))(x/T)(\Delta T/\Delta x)\end{aligned}$$

where $\Delta T/\Delta x = (T_2 - T_1)/(x_2 - x_1)$. Hence unless x and t are carefully

[2] *Loc. cit.*

chosen $\partial T/\partial x \neq \Delta T/\Delta x$. However we avoid this difficulty if we use the formula $\partial T/\partial x = kx/T$ where $k = \Delta T^2/\Delta x^2$.

The best method of computing $\partial T/\partial x$ and $\partial T/\partial z$ appears to be to use the interpolation formulae for these derivatives in terms of divided differences (See Whittaker and Robinson's "Calculus of Observations" (1940) Page 66, Equation (4)).

The interpolation formula could be used to determine an algebraic formula for $\partial T/\partial x$ and $\partial T/\partial z$ and consequently for $1/V^2(z)$ from first to last observation point in the well. However, this is not desirable as the higher powers of z will become too important in the formula while remaining without real significance. The preferred process is therefore to use first, second, and possibly third divided differences only for a limited range of depth within which no major sudden changes of velocity are indicated. Where major changes of velocity occur our interpolation stops so that we get different first, second or possibly third degree curves on different sides of the sudden change of velocity.

By using the interpolation formulae we derive explicit formulae for the velocity as a function of depth for each of the several shot points. Such results will give a measure of the reliability of the well survey since they show, by their consistency or lack of consistency, how well the results and the assumptions hang together.

Whenever there is a major break in velocity using Method E, we can test the internal consistency of our work by applying Method D to each such major break.

CABLE NOISE

We occasionally encounter well seismic records with highly disturbing anomalous breaks When instrumental troubles are excluded there still remain the casing and the cable to give trouble. By pounding on the casing one can verify the already suspected very high velocity of elastic disturbances in the casing. However, if one clamps the cable with a suitable smooth and padded wooden clamp and strikes this clamp with a hammer while holding a geophone firmly against the clamp to record the instant of striking, one can record the arrival of the cable disturbance on the well geophone trace. Consequently the travel time down the cable is measured and the average velocity down the cable is calculated. This velocity in the cable is usually found to be between 8500 and 9500 feet per second.

The disturbance we usually observe travels with the cable velocity in those cases where we have measured the cable velocity directly. This practically assigns the trouble to the cable. However further evidence can be obtained. Sometimes further evidence is really needed because the seismic velocities may be near the cable velocity and such that the cable kicks fall in with the seismic kicks so that one cannot tell which points on the time depth graph are seismic and which are due to the cable.

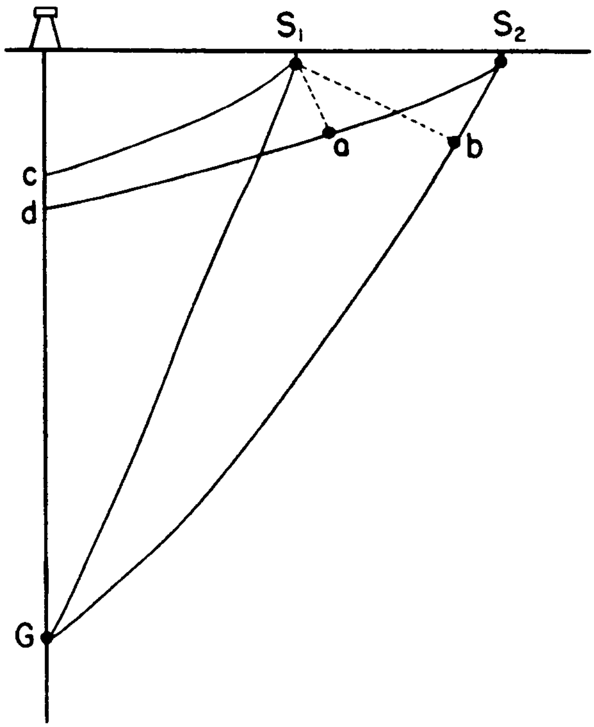

FIG. 5. Illustrating a field set up by means of which the existence of interference due to transmission of vibration down the cable may be established.

The field setup illustrated in Fig. 5 will make the detection of cable noise quite definite. In Fig. 5, s_1 and s_2 are two shot points and G is the geophone in the well. In case the disturbance travels down the cable it travels from s_1 to c to G and from s_2 to d to G. These paths may be reversed (for purposes of comparing travel times) in which case the difference of the two travel times is readily seen to be the travel time from a to s_2. This time difference is clearly independent of the depth of the geophone G—as long as G is below d.

However if the paths are the desired geological paths $s_1 G$ and s_2bG then the difference of the two times is the travel time from b to s_2. Note that as G gets deeper bS_2 gets shorter. Also the time difference over bS_2 is shorter than the time difference over aS_2.

The process of detecting cable noise involves two precautions: (1) the direct measurement of cable velocity; and (2) the shooting of the well from two different distances. Then there are three possibilities:

(a) Time (directly read), depth curves parallel and with velocity same as velocity in cable implies energy travels down cable;

(b) Time, depth curves converging with increasing depth, neither showing cable velocity, implies data from both shot points is the desired data undisturbed by cable; and

(c) One time-depth curve showing cable velocity but the other curve not parallel implies we are safer in using the time-depth curve not showing cable velocity although both curves may correspond to travel times not disturbed by the cable.

We are on a firm basis with either cases (b) or (c) above.

Cable signal trouble can possibly be eliminated by placing a coil spring between the well geophone and the bottom of the cable. The major part of the energy traveling down the cable will then be reflected back up the cable at the base of the cable.