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Free Oscillations of a Laterally Heterogeneous and Anelastic Earth

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Abstract—We calculate normal modes of the laterally heterogeneous and anelastic earth model by using the variational method to include the coupling of the modes due to the asphericity of the earth. If the aspherical anelasticity correlates with the heterogeneity of elastic velocity structure, the quality factor Q of the split singlets has a correlation with the eigenfrequency. This can cause a center frequency shift of the spectral peak with time. We perform a synthetic experiment to examine whether the magnitude of the shift can become an observable for the realistic lateral heterogeneity model of anelasticity. The result of the experiment reveals that the shift of the center frequency is consistent with the initial estimate for the fundamental spheroidal modes used in the experiment. We then examine the actual seismograms of the June 9, 1994, Bolivian earthquake to determine if this shift of center frequency can be observed. Although the amount of the center frequency shift of each multiplet is large, there is no consistent shift of the center frequency that is predicted in the synthetic experiment.

Key words: Free oscillation, lateral heterogeneity, anelasticity.

Introduction

Free oscillations of the earth have been used as an useful tool for studies of both elastic wave velocity structure within the earth and earthquake source mechanics. If the earth is spherically symmetric and nonrotating, the free oscillation can be represented as two types of normal modes: spheroidal modes (${}_nS_l^m$) and toroidal modes (${}_nT_l^m$), where n is the overtone order, l is the angular order and m is the azimuthal order. Both modes have $2l + 1$ degeneracy with respect to the azimuthal order m. However, this degeneracy is removed in the actual earth, since the earth is rotating and slightly deviates from spherically symmetric velocity structure. Whereupon the splitting of the modes occurs, which means $2l + 1$ modes have different frequencies respectively. It has been shown that the asphericity of the earth also causes coupling among the modes. The analysis of seismograms from the recently developed global digital seismograph network reveals that these effects of the earth's aspherical structure on the modes of free oscillation are significant enough to be observed by these high quality data. For example, there are many observations of

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amplitude anomalies of long-period Rayleigh and Love waves, which are the traveling wave representation of the fundamental normal modes that indicate focusing and defocusing caused by deviation of the propagation path from the great circle due to the laterally heterogeneous velocity structure of the earth (e.g., LAY and KANAMORI, 1985).

In this paper, we present a technique to calculate wholly coupled modes for realastic aspherical earth models by employing the variational method. In the variational method, the normal modes of the aspherical earth are represented as the sum of the modes of the spherically symmetric earth. The eigenfrequencies and eigenfunctions are obtained by solving the general matrix eigenvalue problem. To take into account the effect of aspherical structure rigorously, it is necessary to use as many modes as is computationally feasible. It has been shown that the observed amplitude anomalies of surface waves are reproduced successfully by the synthetic seismograms calculated from these coupled modes eigenfunctions (e.g., TSUBOI, 1992). Recent research reveals that the amplitude anomalies of normal modes observed at broadband seismograph stations situated close to the epicenter of the shallow strike slip fault source are basically explained by the splitting and coupling of the normal modes caused by the laterally heterogeneous velocity structure (e.g., TSUBOI and UM, 1993). These examples illustrate that the coupled mode synthetics are quite useful to interpret the earth's aspherical structure from modern high quality seismograms. Because the elastic 3D velocity structure has become accurate enough to explain those observed anomalies, we examine the effect of lateral heterogeneity of anelasticity on the coupled modes. There are several published models of aspherical anelasticity obtained by using the surface waves (e.g., ROMANOWICZ, 1990). It is considered that the origin of the lateral heterogeneity of elastic velocity structure is related to the lateral variation of the temperature. Therefore, it is believed that the laterally heterogeneous anelastic structure is anticorrelated with the lateral heterogeneity of elastic velocity structure, that is, the region in which the velocity is faster than the average is characterized by the low attenuation. If the aspherical anelasticity correlates with the heterogeneity of elastic velocity structure, the quality factor Q of split singlets for the laterally heterogeneous anelastic earth model has a correlation with eigenfrequency. This will cause a center frequency shift of spectral peak with time. We will discuss whether this shift of center frequency can be observed in the actual seismograms by comparing the synthetic experiments with the analysis of the actual seismograms.

Method

We calculate synthetic seismograms for a laterally heterogeneous and anelastic earth model employing the method of TSUBOI and UM (1993). We use the variational method which treats coupling of the multiplets without approximation to obtain eigensolutions of the laterally heterogeneous earth. In the variational method we expand the eigenfunction of the laterally heterogeneous earth model in terms of the eigenfunctions of the degenerate singlets of the laterally homogeneous part of the earth model (e.g., PARK, 1986; MORRIS *et al.*, 1987; TSUBOI and GELLER, 1989). The eigenfunction $|v\rangle$ is given by

$$|v\rangle = \sum_{k=1}^{N} C_k |k\rangle \tag{1}$$

where $|k\rangle$ is a fully normalized degenerate singlet of the spherically symmetric earth model. We solve a matrix eigenvalue problem to determine the expansion coefficients C_i. The matrix elements are calculated by the coupling between or within the multiplets through the asphericity of the earth such as rotation, ellipticity and laterally heterogeneous structure.

We choose multiplets of angular order $l-5$ to $l+5$ along the same dispersion branch in our basis set to obtain the singlet whose equivalent multiplet angular order is l. The eigenfrequencies and eigenfunctions of the spherically symmetric earth model used in the basis sets are those calculated for 1066A (GILBERT and DZIEWONSKI, 1975). We include the effect of rotation and ellipticity in our calculation but we do not include toroidal-spheroidal coupling in the present paper. As we will discuss later, we will choose the frequency range in which the toroidal-spheroidal coupling due to the rotation and the ellipticity is not significant. For the elastic part of lateral heterogeneity, we use laterally heterogeneous upper mantle model M84A of WOODHOUSE and DZIEWONSKI (1984), where the S-wave velocity β is expanded by the spherical harmonics Y_l^m

$$\beta(r, \theta, \phi) = \sum_s \sum_t \beta_s^t(r) Y_s^t(\theta, \phi) \tag{2}$$

where the maximum angular order s of the expansion is 8 for M84A. In calculating the matrix elements, we assume the scaling relation $\delta\kappa/\kappa = 1.7\,\delta\beta/\beta$, $\delta\mu/\mu = 2.4\,\delta\beta/\beta$ and $\delta\rho/\rho = 0.4\,\delta\beta/\beta$ to derive a material perturbation from model M84A.

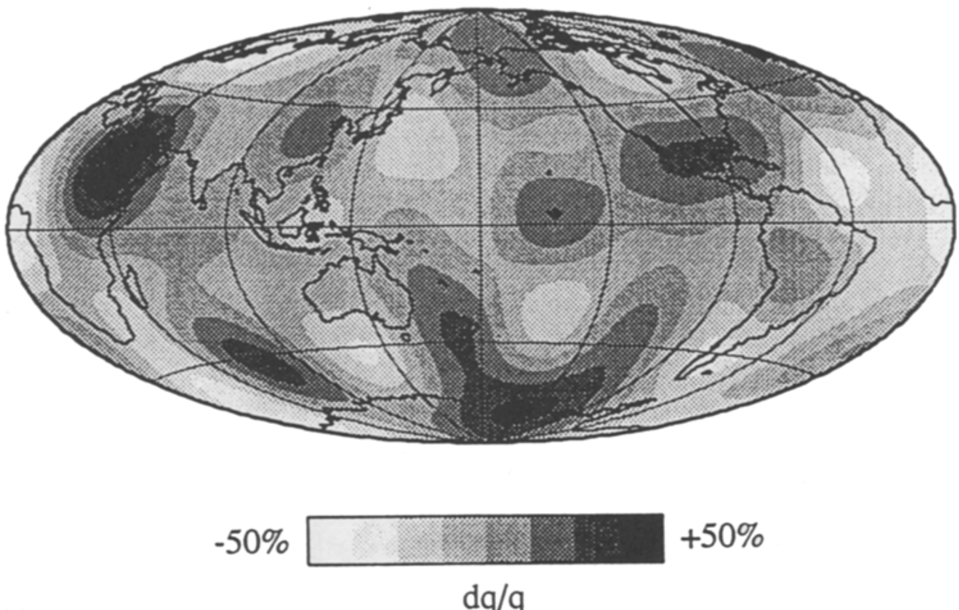
We include an anelastic part of lateral heterogeneity in the same manner as used by DUREK *et al.* (1993). Since it is considered that the origin of the lateral heterogeneity of elastic velocity structure is related to the lateral variation of the temperature, it is believed that the laterally heterogeneous anelastic structure is anticorrelated with the lateral heterogeneity of elastic velocity structure. This means that the region in which the velocity is faster than the average is characterized by low attenuation. We model the aspherical anelasticity which is anticorrelated with velocity in the following manner: (1) We add the aspherical anelasticity to the imaginary part of the shear modulus μ_s^t by assuming the scaling relation

$$\mu_s'^{\,t} = \mu_s^t + i q_s^t \tag{3}$$

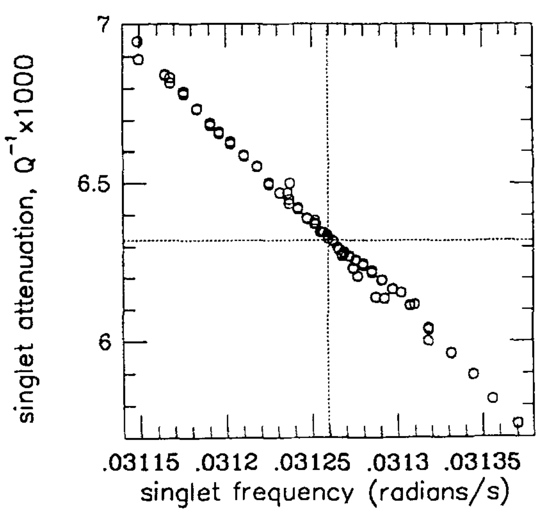
where

$$q_s^t = \alpha(r) \frac{\delta\beta}{\beta} \mu_0(r) q_0(r)$$

μ_0 is the spherically symmetric shear modulus and $q_0 = 1/Q_\mu$. (2) We take Q_μ from those of PREM. For the scaling factor $\alpha(r)$, we take -8 for $25\text{ km} \le r \le 80\text{ km}$, -15 for $80\text{ km} \le r \le 220\text{ km}$ and -25 for $220\text{ km} \le r \le 670\text{ km}$. A variation in Q of $\pm 50\%$ is imposed by this model. Figure 1(a) illustrates a variation in Q^{-1} that is anticorrelated with velocity for ${}_0S_{43}$ (DUREK *et al.*, 1993). We also display a plot



(a)



(b)

Figure 1

(a) The variation of Q^{-1} for ${}_0S_{43}$ and (b) the plot of singlet eigenfrequencies of ${}_0S_{43}$ with the attenuation.

of singlet eigenfrequencies and attenuation in Figure 1(b). The singlet eigenfrequencies are anticorrelated with the singlet attenuation. The variation of singlet attenuation is roughly $\pm 10\%$, which is comparable with the observation. One Q cycle, which is the time required for the signal amplitude to decay by $\exp(-\pi)$ and defined by $2Q_k\pi/\omega_k$, of these modes is about 9.7 hours for the highest eigenfrequency singlet whereas it is 8.1 hours for the lowest. Due to the introduction of aspherical anelasticity in the model, the matrix becomes non-Hermitian. Consequently, the orthogonality condition of eigenfunctions should be modified to

$$\mathbf{v}_i^T \cdot \mathbf{u}_j = \delta_{ij} \tag{4}$$

where $\mathbf{v}_i$ is the left eigenvector and $\mathbf{u}_i$ is the right eigenvector. When we solve the matrix eigenvalue problem, we obtain both the left and the right eigenvectors and calculate the orthogonality relation to review whether the eigenvectors are sufficiently accurate.

The excitation of the normal modes for the laterally heterogeneous earth model is calculated essentially correspondent to the spherically symmetric case. Assuming epicenter coordinates and moment tensor of the source, the synthetic seismogram at a receiver for a spherically symmetric earth is (e.g., DAHLEN, 1981)

$$\mathbf{s}(\mathbf{r}, t) = \mathrm{Re}\left[\sum_k [e_{ij}^{(k)}(\mathbf{r}_s)M_{ij}] \cdot \mathbf{u}^{(k)}(\mathbf{r})(1 - e^{-\omega_k t/2Q_k} \cos \omega_k t)\omega_k^2\right] \tag{5}$$

where ω_k and $\mathbf{u}^{(k)}(\mathbf{r})$ are the eigenfrequency and the right eigenfunction of the k-th mode respectively. $e_{ij}^{(k)}$ is the strain tensor calculated from $\mathbf{v}^{(k)}$ at the source position, and M_{ij} is the moment tensor. Q_k is the temporal attenuation factor of the k-th mode. For the laterally heterogeneous earth, we use the eigenfunction that is expressed as the sum of the trial functions as shown in (1). The eigenfrequency ω_k and the attenuation factor Q_k corresponding to that eigenfunction are obtained by solving the matrix eigenvalue problem.

Synthetic Experiment

Figure 1(b) shows that the singlet attenuation is anticorrelated with the singlet eigenfrequency, if the aspherical anelasticity is anticorrelated with velocity. Since the singlets with lower eigenfrequencies decay more rapidly than the singlets with higher eigenfrequencies, this may result in a shift of center frequency of a spectral peak of each multiplet with time. We attempt to examine whether this shift in the center frequency of the spectral peak is observable in the actual seismograms. If it is observable, then it should be possible to model the aspherical anelasticity from this observation. First we calculate synthetic seismograms by using the method described in the previous section and measure the center frequency of each multiplet for different time intervals. Since we do not include toroidal-spheroidal coupling in our calculation, we choose to calculate eigensolutions in the period range between 300–200 sec., which correpsonds to the spheroidal multiplets, ${}_0S_{24} - {}_0S_{43}$. The effect of the rotation and the ellipticity of the earth is not significant for these modes, except ${}_0S_{32}$.

One Q cycle of these modes is about 10 hours. Therefore, it is necessary to choose significantly large earthquakes to measure the center frequency of each spectral peak for different time windows. Otherwise the signal-to-noise ratio becomes worse for later time windows and it is not easy to identify each spectral peak. We use the June 9, 1994 Bolivian earthquake (13.7°S 67.3°W, 0:33:13, depth 627 km, $M_W = 8.3$ (HRV)). This is the largest deep focus earthquake ever recorded by modern digital seismograph networks. We calculate vertical component synthetic seismograms for six IDA stations (CMO, ESK, HAL, PFO, SUR and TWO) and two broadband seismograph stations (SHK and TSK) operated by the Earthquake Research Institute of the University of Tokyo (TAKANO *et al.*, 1990). Figure 2 displays the synthetic seismogram calculated for station TSK. It should be noted that we do not add a noise to the synthetics. Since R1 arrival is saturated in IDA

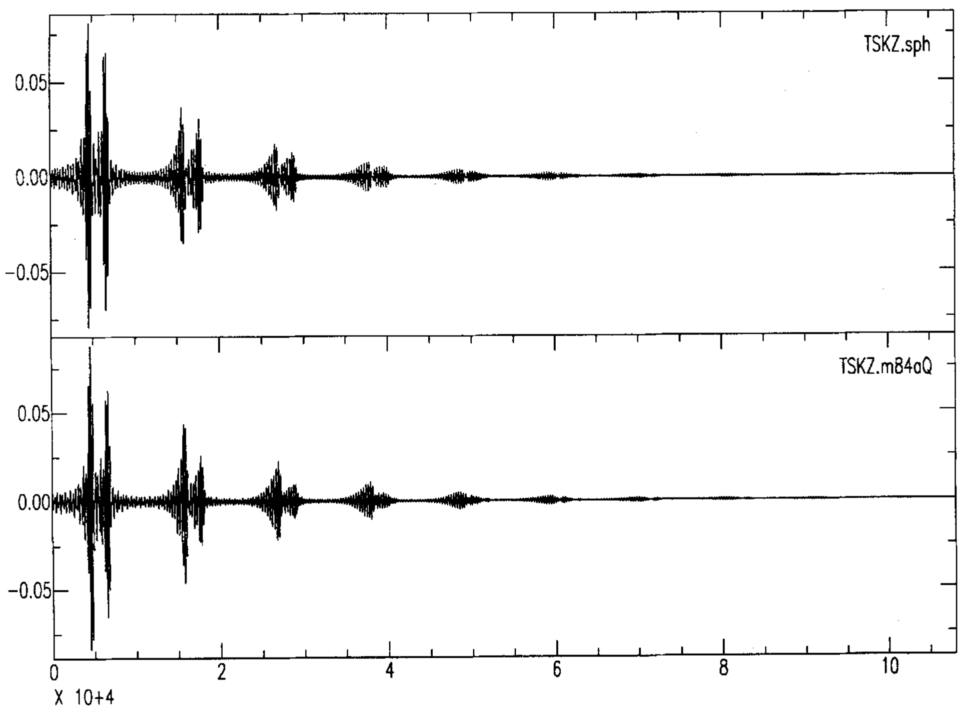


Figure 2
Synthetic seismograms calculated for station TSK. For comparison, synthetics for spherically symmetric earth model 1066A are shown in the above. Synthetic for aspherical model (lower trace) shows amplitude anomalies, which are that the amplitude of R7 is greater than that of R6, for example.

traces, we do not use first five hours after the origin time of each trace. We select two time windows to measure the center frequency of spectra peaks. One starts at 5 hours after the origin time and the duration is 10 hours. The other starts at 15 hours after the origin time and the duration is again 10 hours. We edit original synthetic seismograms to obtain two data sets according to the time windows defined above and Hann-tapered the traces to apply FFT. Figure 3 shows examples of the amplitude spectrum of two different time windows. We have found that even for this size of earthquake the modes with eigenfrequencies above 4 mHz decay rapidly enough and the spectral peaks are not significant in the amplitude spectrum of the second windowed data set. It is required that the record length should be 1.1 Q cycle to measure the eigenfrequency (DAHLEN, 1982). Therefore we cannot shorten the record length to maintain the significant amplitude of the modes with the eigenfrequencies above 4 mHz. Therefore we decided to measure the center frequencies of the modes from ${}_0S_{24}$ to ${}_0S_{32}$. We fit resonance peaks to data by nonlinear least squares and estimate the center frequencies. This procedure is first applied to the data by MASTERS and GILBERT (1983). We use the Levenberg-Marquardt method to apply a nonlinear least squares fitting. We take the program *mrqmin* from PRESS *et al.* (1992). If the amplitudes of the modes at some stations are not large enough to obtain reliable measures of the center frequency, we discard those data. We take an average of center frequency obtained from available stations and calculate a standard deviation. Table 1 displays the results of the synthetic experiments. Although the standard deviation is comparable with the amount of shift of the center frequency, the center frequency shift with time is consistently positive, except ${}_0S_{24}$. This is consistent with what we expect from the results of normal mode calculation (Fig. 1) and with the results of DUREK *et al.* (1993). Since the singlets with lower eigenfrequencies decay more rapidly than the singlets with higher eigenfrequencies, this can result in a positive shift of a center frequency of a

Table 1

The shift of center frequency with time for synthetics. Δf is the difference of the center frequency measured with the two time windows

Mode	Mean of Δf (μHz)	Standard deviation (μHz)	No. of observation
${}_0S_{24}$	−0.704	0.944	8
${}_0S_{25}$	0.775	2.048	6
${}_0S_{26}$	1.327	1.654	8
${}_0S_{27}$	0.003	2.401	5
${}_0S_{28}$	0.936	1.476	7
${}_0S_{29}$	0.378	1.080	6
${}_0S_{30}$	0.545	1.620	7
${}_0S_{31}$	0.430	1.244	4
${}_0S_{32}$	−0.050	0.822	7

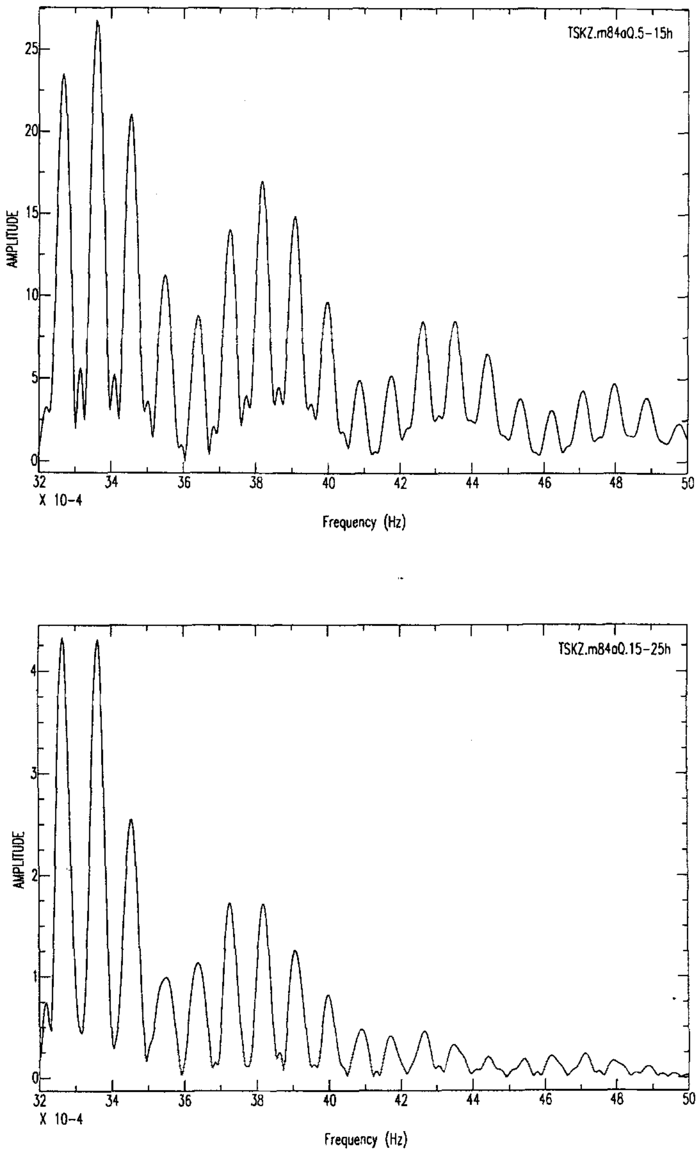


Figure 3
The amplitude spectrum of synthetic seismograms calculated for aspherical anelastic model at TSK. The upper figure is for the time window 5–15 hours after the origin time and the lower figure is for the time window 15–25 hours.

spectral peak of each multiplet with time. Although we do not add a noise to the synthetics, this consistent shift of center frequencies should be observed in the actual data, if the aspherical anelasticity is anticorrelated to the velocity in the actual earth.

Data Analysis

We then analyze the actual data to examine if the shift of center frequencies of spectral peaks can be observed. We edit the data from 6 IDA stations and our 2 broadband seismograph stations in the same manner as the synthetic traces. Figure 4 shows examples of the amplitude spectrum of two data sets which applied two time windows that are equivalent to the synthetic cases. We fit a resonance peak to the observed spectrum by nonlinear least squares and measure the center frequencies. When the peak is apparently split or the amplitude is comparable with the noise level, we do not use those peaks to measure the center frequencies. Since all of the modes used in this study in the spectrum of HAL for 15–25 hour time windows were split or decayed rapidly, we do not use HAL in this analysis. Therefore we have at most seven observations for each mode. We take an average of center frequency obtained from available stations and calculate a standard deviation. Table 2 shows the results of the measurement of center frequencies for the actual seismograms. We did not have reliable measurements for ${}_0S_{32}$. The amount of shift of center frequencies as well as the standard deviation is large compared with that of the synthetic case. It is because we do not include a noise in the synthetic seismogram. The direction of the shift of the center frequencies varies from modes to modes. There is no consistent positive shift which exists in the synthetic results.

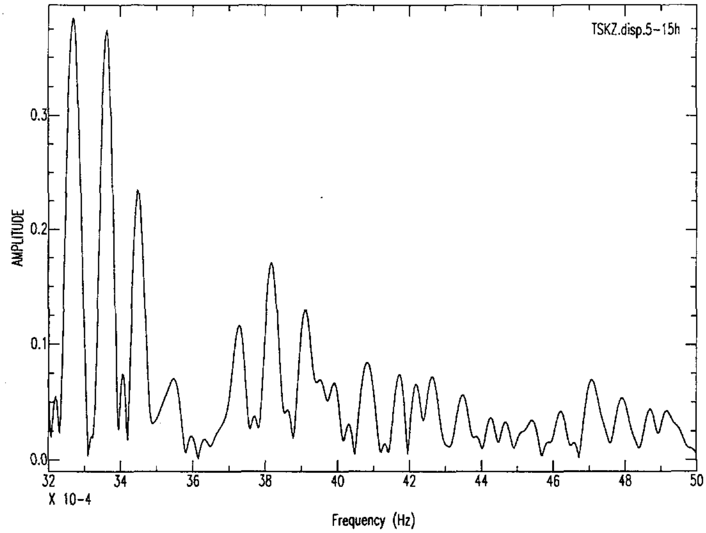
Discussion

As is shown in the previous section, the observed center frequencies of normal modes do not show a consistent positive shift with time, which is predicted in the synthetic calculation. Although the number of stations used in the analysis might

Table 2

The shift of center frequency with time for data. Δf is the difference of the center frequency measured with the two time windows

Mode	Mean of Δf (μHz)	Standard deviation (μHz)	No. of observation
${}_0S_{24}$	3.804	4.043	7
${}_0S_{25}$	−3.748	10.043	6
${}_0S_{26}$	2.016	4.994	7
${}_0S_{27}$	−6.907	4.043	5
${}_0S_{28}$	0.834	5.397	6
${}_0S_{29}$	−3.220	6.328	5
${}_0S_{30}$	0.351	9.045	4
${}_0S_{31}$	−2.169	—	1
${}_0S_{32}$	—	—	0



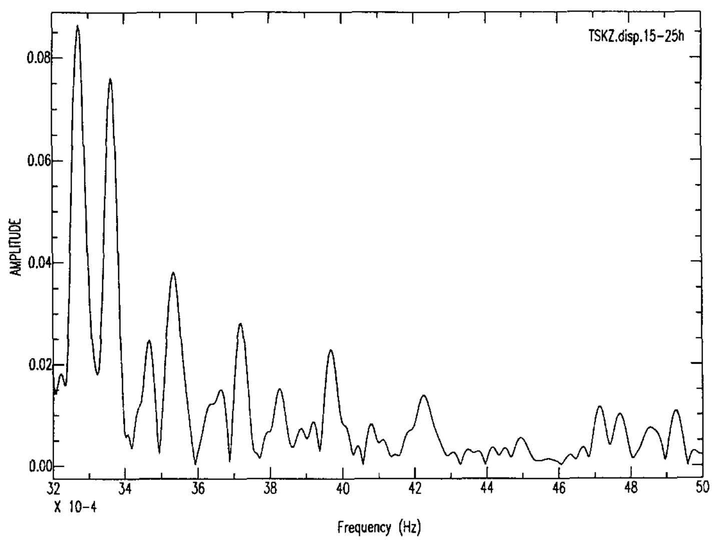


Figure 4
The amplitude spectrum of actual data for TSK. The upper figure is for the time window 5–15 hours after the origin time and the lower figure is for the time window 15–25 hours.

not be sufficient to draw a conclusion, this result is consistent with the observation which appeared in MASTERS and GILBERT (1983). They demonstrated that the eigenfrequency of ${}_0S_{16}$ varies as record length increases, but there is no consistent positive or negative shift of eigenfrequency with time. It seems that the center frequencies of spectral peak of normal modes do not vary consistently with time in the actual earth. Synthetic calculation predicts that the eigenfrequencies should shift positively with time, if the anelastic asphericity is anticorrelated with the velocity and the variation of Q is $\pm 50\%$ in the upper mantle. If the results of the present study are valid and the eigenfrequency of the normal mode does not shift positively with time, it means that either the aspherical anelasticity is uncorrelated with velocity or the variation of Q in the upper mantle is substantially smaller than presently considered. We first consider the second possibility. It is well known that the variation of observed Q of the normal modes is quite large. This observation itself does not signify, however, that the variation of Q in the mantle is large, since the synthetic calculation reveals that the laterally heterogeneous velocity structure can cause the variation of Q of the normal modes, even though the spherically symmetric Q model is used in the calculation (DAVIS, 1985; DUREK *et al.*, 1993). This can be interpreted as caused by focusing and defocusing in terms of surface waves. Therefore it is thought that a significant part of the variation of observed Q value of normal modes originates from laterally heterogeneous elastic velocity structure. As a result of this, it is difficult to obtain a model of aspherical anelasticity for the actual earth. Recently however, several models were published for the aspherical anelasticity by using the surface wave amplitude measurement (e.g., ROMANOWICZ, 1990). It might be insufficient to discuss the order of aspherical anelasticity, since the maximum angular order of these models is just 2. However, the present aspherical anelasticity model gives $\pm 50\%$ variation of Q in the upper mantle. Therefore we believe that the aspherical anelasticity model used in the synthetic calculation in this paper has comparable magnitude with the actual earth. Then there is a possibility that the aspherical anelasticity is uncorrelated with the aspherical elastic velocity. This result is actually consistent with that of MASTERS and GILBERT (1983). They showed that the shift in Q as a function of the pole position of the great circle joining source and receiver does not manifest any large-scale pattern which was apparent in the spatial distribution of the shift of center frequency (MASTERS *et al.*, 1982). DUREK *et al.* (1993) calculated normal modes for an aspherical anelastic model which is uncorrelated with the velocity. Thus the eigenfrequency and the Q of the singlet does not show any apparent correlation. The eigenfrequency also does not shift either a positive or negative direction consistently with time in their synthetic experiments. The results of the present paper seem to support this uncorrelated model. For the moment, we have no interpretation as to how aspherical anelasticity which is uncorrelated with velocity, realizes in the actual earth.

The standard deviation of the measured center frequency is almost comparable with the shift of the center frequency in both synthetic experiments and data analysis. To consider this problem, we plot the excitation coefficients of each singlet of arbitrarily taken mode. Figure 5 shows the real part of the excitation coefficients of each singlet of ${}_0S_{25}$ computed for station SHK. This figure presents a typical example of how each singlet is excited at stations used in this study. The singlets

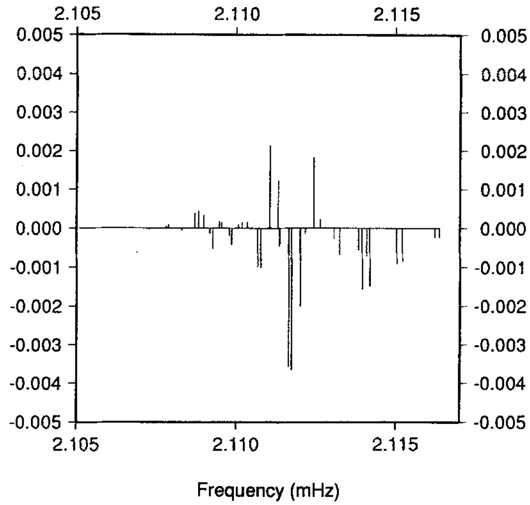


Figure 5
The real part of excitation coefficients of each singlet for ${}_0S_{25}$ calculated for station SHK. The amplitude of each singlet is plotted at its eigenfrequency position in horizontal axis.

with higher or lower eigenfrequencies are not excited well. The well excited singlets in Figure 5 exist in the vicinity of the degenerate eigenfrequency and separate about a few μHz, which is comparable with the standard deviation. We fit the spectral peak with a single resonance function in frequency domain. If multiple singlets exist, excited in a narrow frequency band, it may be likely that the results of the fitted center frequency will have an ambiguity of the order of its frequency differences. Also, there is only a moderate difference of attenuation factor Q among excited singlets. Therefore we may be unable to observe a significant shift of center frequency with time. We may have to investigate another source and receiver pairs that have excited singlets with enough separation in frequency domain. In that case, we may have to use the splitting functions (e.g., GIARDINI *et al.*, 1987) to resolve spectral peaks. We also need to increase the number of observations of center frequency shift with time. The results of the present paper must be reviewed in future studies which include these requirements.

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