

1. How will you measure the running time of an algorithm?

$T(n)$ = Running time of an algorithm

Identify basic operation:

$C(n)$ = basic operation count

Operation = Time taken by basic operation for its execution

$T(n) \cong \text{Operation} \cdot C(n)$.

2. What are the steps involved in the analysis framework?

Steps:

i) Finding time and space efficiency (complexity)

ii) Measuring an input's size

iii) Measuring running time and Units for Measuring Running time

iv) Orders of growth

v) worst, best, average case efficiencies

3. What is a basic operation?

The most time-consuming operation in the algorithm is called as basic operation. For example, most sorting algorithms work by comparing elements (keys) of a list being sorted with each other; for such algorithms, the basic operation is a key comparison.

4. Arrange the following functions based on its order of decay $n!$, $n \log n$, $15 \log n$, n^3 , $100n^2 + n$, 2^n

$n!$, 2^n , n^3 , $100n^2 + n$, $n \log n$, $15 \log n$

5. Arrange the following functions based on its order of growth $n!$, $n \log n$, $15 \log n$, n^3 , $100n^2 + n$, 2^n

$15 \log n$, $n \log n$, $100n^2 + n$, n^3 , 2^n , $n!$

6. Give formal and informal definition of Big Theta.

Formal:

* $t(n) \in \theta(g(n))$ iff $t(n)$ is bounded above and below by some constant multiple of $g(n)$.

$c_2 \cdot g(n) \leq t(n) \leq c_1 \cdot g(n); n_0 = \max(n_1, n_2)$

Informal:

*Set of all functions whose order of growth is same as $g(n)$

(eg): $10n^2 - 5 \in \theta(n^2)$

7. Give formal and informal definition of Big Oh.

Formal:

$t(n) \in O(g(n))$ iff order of growth of $t(n)$ is less than or equal to some constant multiple of $g(n)$.

(i.e): $t(n) \leq c \cdot g(n)$ for some position constant c , $n \geq n_0$

Informal:

Set of all functions whose order of growth is less than or same as order of growth of $g(n)$.

Eg: $10n^2 + n \in O(n^2)$

8. Give formal and informal definition of Big Omega.

Formal: $t(n) \in \Omega(g(n))$ iff order of $t(n)$ is greater than or equal to and bounded below some constant multiple of $g(n)$.

(i.e): $t(n) \geq c \cdot g(n)$ for some positive constant c and $n \geq n_0$

Informal: set of all function whose order of growth is greater or same as $g(n)$.

Eg: $10n^2 + n \in \Omega(n^2)$

9. Define best, average and worst case efficiency.

Best case: The best-case efficiency of an algorithm is its efficiency for the best-case input of size n , which is an input (or inputs) of size n for which the algorithm runs the fastest among all possible inputs of that size

Eg: linear search

Average case: "average" over inputs of size n Number of times the basic operation will be executed on typical input. In average case, some assumptions about possible inputs of size n .

Eg: Linear search

Worst case: maximum over inputs of size n . In worst case, the case that causes a maximum number of operations to be executed.

Eg: Linear search with time complexity $O(n)$.

9. Identify true or false $5n^2 - 6n \in O(n^2)$

True

10. Identify true or false $n(n^2 + 1) \in \Omega(n^2)$

True

11. Find the time complexity of sum of n given numbers.

//////DON'T KNOW

1. Algorithm for linear search and its best, avg, worst cases:

//Algorithm linear search(a[0..n-1], key)

//Input: list of n numbers and key to be search

//Output: Returns position of key in a list if successful search, otherwise return -1.

for i=0 to n-1 do

 if a[i] == key

 return i

return -1

Best case:

key is present in 1st position

Number of comparison = 1 (best case)

$T(n) \in \Omega(1)$

Average case:

Probability of successful search = p

Probability of unsuccessful search = 1-p

$$\begin{aligned} T_{avg}(n) &= \frac{(1.p + 2.p + \dots + n.p)}{n} + n(1-p) \\ &= \frac{p(1+2+3+\dots+n)}{n} + n(1-p) \\ &= p \frac{(n+1)}{2} + n(1-p) \end{aligned}$$

Case i): For successful search: p=1	case ii): for unsuccessful search: p=0
$T(n) = \frac{n+1}{2} + 0$ $= \frac{n+1}{2}$	$T(n) = 0 * \frac{n+1}{2} + n(1-0)$ $= n$

Worst case:

Algorithm runs slowest among all possible inputs of size n.

$T(n) \in O(n)$

2. Discuss about asymptotic notations in detail.

BIG OH(O):

Formal:

$t(n) \in O(g(n))$ iff order of growth of $t(n)$ is less than or equal to some constant multiple of $g(n)$.

(i.e): $t(n) \leq c \cdot g(n)$ for some positive constant c , $n \geq n_0$

Informal:

Set of all functions whose order of growth is less than or same as order of growth of $g(n)$.

Eg: $10n^2 + n \in O(n^2)$

BIG OMEGA(Ω):

Formal: $t(n) \in \Omega(g(n))$ iff order of $t(n)$ is greater than or equal to and bounded below some constant multiple of $g(n)$.

(i.e): $t(n) \geq c \cdot g(n)$ for some positive constant c and $n \geq n_0$

Informal: set of all function whose order of growth is greater or same as $g(n)$.

Eg: $10n^2 + n \in \Omega(n^2)$

BIG THETA(θ):

Formal:

* $t(n) \in \theta(g(n))$ iff $t(n)$ is bounded above and below by some constant multiple of $g(n)$.

$c_2 \cdot g(n) \leq t(n) \leq c_1 \cdot g(n)$; $n_0 = \max(n_1, n_2)$

Informal:

*Set of all functions whose order of growth is same as $g(n)$

(eg): $10n^2 - 5 \in \theta(n^2)$

3. Prove the following assertions:

i) $n^2 + 5n - 10 \in O(n^2)$

ii) $n^3 + n^2 \in \Theta(n^3)$

iii) $100n^2 - n - 10 \in \Omega(n^2)$

//SOLVE IT YOURSELF