Notes on algorithms

Informally, an ***algorithm*** is any well-defined computational procedure that takes some value, or set of values, as ***input*** and produces some value, or set of values, as ***output***. An algorithm is thus a sequence of computational steps that transform the input into the output.

We can also view an algorithm as a tool for solving a well-specified ***computational problem***. The statement of the problem specifies in general terms the desired input/output relationship. The algorithm describes a specific computational procedure for achieving that input/output relationship.

An algorithm is said to be ***correct*** if, for every input instance, it halts with the correct output. We say that a correct algorithm ***solves*** the given computational problem. An incorrect algorithm might not halt at all on some input instances, or it might halt with an answer other than the desired one.

A ***data structure*** is a way to store and organize data in order to facilitate access and modifications. No single data structure works well for all purposes, and so it is important to know the strengths and limitations of several of them.

Stacks and queues are dynamic sets in which the element removed from the set by the DELETE operation is prespecified. In a ***stack***, the element deleted from the set is the one most recently inserted: the stack implements a ***last-in, first-out***, or ***LIFO***, policy. Similarly, in a ***queue***, the element deleted is always the one that has been in the set for the longest time: the queue implements a ***first-in, first out***, or ***FIFO***, policy. There are several efficient ways to implement stacks and queues on a computer. In this section we show how to use a simple array to implement each.

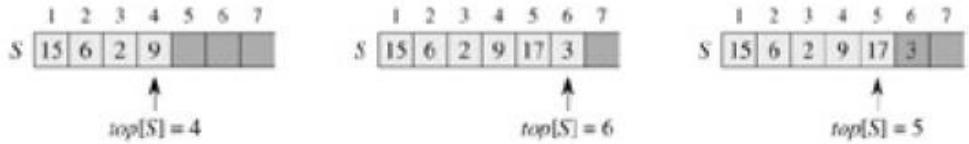
The stack operations can each be implemented with a few lines of code.

```
STACK-EMPTY(S)
1  if top[S] = 0
2       then return TRUE
3       else return FALSE

PUSH(S, x)
1  top[S] ← top[S] + 1
2  S[top[S]] ← x

POP(S)
1  if STACK-EMPTY(S)
2     then error "underflow"
```

```
3     else top[S] ← top[S] - 1
4          return S[top[S] + 1]
```



Example on use of Stack:

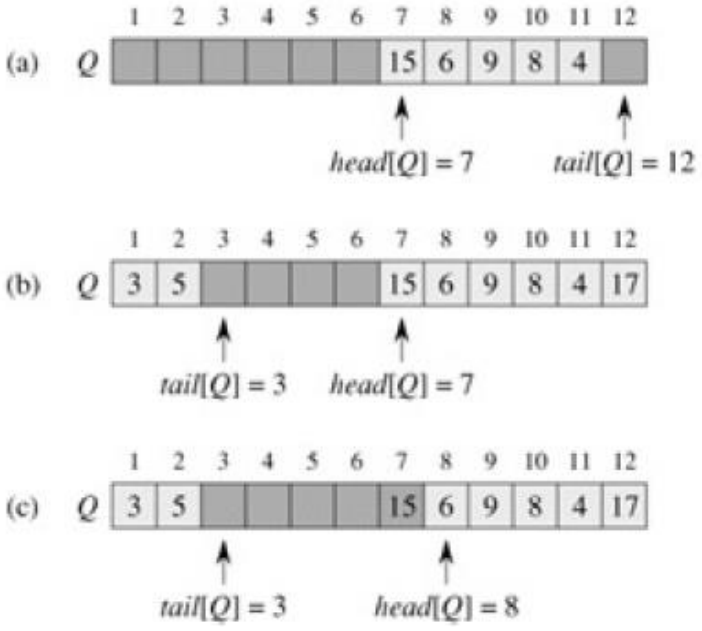
Queues

We call the INSERT operation on a queue ENQUEUE, and we call the DELETE operation DEQUEUE; like the stack operation POP, DEQUEUE takes no element argument. The FIFO property of a queue causes it to operate like a line of people in the registrar's office. The queue has a ***head*** and a ***tail***. When an element is enqueued, it takes its place at the tail of the queue, just as a newly arriving student takes a place at the end of the line. The element dequeued is always the one at the head of the queue, like the student at the head of the line who has waited the longest. (Fortunately, we don't have to worry about computational elements cutting into line.)

```
ENQUEUE(Q, x)
1   Q[tail[Q]] ← x
```

```
2   if tail[Q] = length[Q]
3      then tail[Q] ← 1
4      else tail[Q] ← tail[Q] + 1

DEQUEUE(Q)
1   x ← Q[head[Q]]
2   if head[Q] = length[Q]
3      then head[Q] ← 1
4      else head[Q] ← head[Q] + 1
5   return x
```

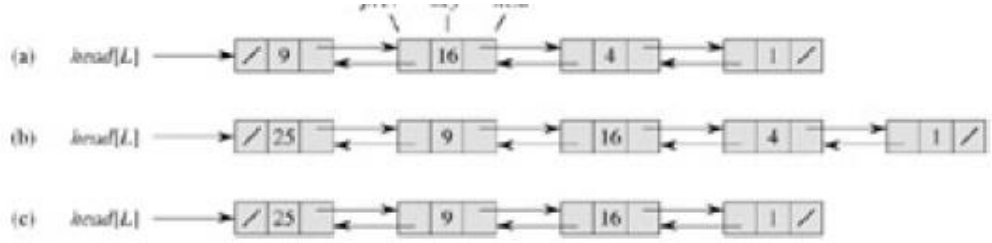


Whereas a stack allows insertion and deletion of elements at only one end, and a queue allows insertion at one end and deletion at the other end, a ***deque*** (double-ended queue) allows insertion and deletion at both ends. Write four $O(1)$-time procedures to insert elements into and delete elements from both ends of a deque constructed from an array.

Linked Lists:

A ***linked list*** is a data structure in which the objects are arranged in a linear order. Unlike an array, though, in which the linear order is determined by the array indices, the order in a linked list is determined by a pointer in each object.

Each element of a ***doubly linked list*** L is an object with a *key* field and two other pointer fields: *next* and *prev*. The object may also contain other satellite data. Given an element x in the list, *next*[x] points to its successor in the linked list, and *prev*[x] points to its predecessor. If *prev*[x] = NIL, the element x has no predecessor and is therefore the first element, or ***head***, of the list. If *next*[x] = NIL, the element x has no successor and is therefore the last element, or ***tail***, of the list. An attribute *head*[L] points to the first element of the list. If *head*[L] = NIL, the list is empty



A list may have one of several forms. It may be either singly linked or doubly linked, it may be sorted or not, and it may be circular or not. If a list is ***singly linked***, we omit the *prev*

pointer in each element. If a list is ***sorted***, the linear order of the list corresponds to the linear order of keys stored in elements of the list; the minimum element is the head of the list, and the maximum element is the tail. If the list is ***unsorted***, the elements can appear in any order. In a ***circular list***, the *prev* pointer of the head of the list points to the tail, and the *next* pointer of the tail of the list points to the head. The list may thus be viewed as a ring of elements. In the remainder of this section, we assume that the lists with which we are working are unsorted and doubly linked.

Searching a linked list

```
LIST-SEARCH(L, k)
1  x ← head[L]
2  while x ≠ NIL and key[x] ≠ k
3      do x ← next[x]
4  return x
```

To search a list of n objects, the LIST-SEARCH procedure takes $\Theta(n)$ time in the worst case, since it may have to search the entire list.

Inserting into a linked list

```
LIST-INSERT(L, x)
1  next[x] ← head[L]
2  if head[L] ≠ NIL
3     then prev[head[L]] ← x
4  head[L] ← x
5  prev[x] ← NIL
```

The running time for LIST-INSERT on a list of n elements is $O(1)$.

Deleting from a linked list

```
LIST-DELETE(L, x)
1  if prev[x] ≠ NIL
2       then next[prev[x]] ← next[x]
3       else head[L] ← next[x]
4  if next[x] ≠ NIL
5       then prev[next[x]] ← prev[x]
```