

1. Gini impurity

$$Q_m = 1 - \sum_{i=1}^k p_i^2$$

Entropy

$$Q_m = - \sum_{i=1}^k p \log_2 p$$

Misclassification rate

$$Q_m = 1 - \max_i p_i$$

Gini impurity

$$A: 1 - \left(\frac{5}{18}\right)^2 - \left(\frac{5}{18}\right)^2 - \left(\frac{4}{9}\right)^2 = 0.648$$

$$B: 1 - \left(\frac{2}{5}\right)^2 - 0^2 - \left(\frac{3}{5}\right)^2 = 0.48$$

$$C: 1 - \left(\frac{1}{8}\right)^2 - \left(\frac{5}{8}\right)^2 - \left(\frac{1}{4}\right)^2 = 0.5312$$

$$\text{Overall: } \frac{10}{18} \times 0.48 + \frac{8}{18} \times 0.5312 = 0.5028$$

Entropy :

$$A: -\left(\frac{5}{18}\right) \log_2\left(\frac{5}{18}\right) - \left(\frac{5}{18}\right) \log_2\left(\frac{5}{18}\right) - \left(\frac{4}{9}\right) \log_2\left(\frac{4}{9}\right) = 1.5466$$

$$B: -\left(\frac{2}{5}\right) \log_2\left(\frac{2}{5}\right) - 0 - \left(\frac{3}{5}\right) \log_2\left(\frac{3}{5}\right) = 0.9710$$

$$C: -\left(\frac{1}{8}\right) \log_2\left(\frac{1}{8}\right) - \left(\frac{5}{8}\right) \log_2\left(\frac{5}{8}\right) - \left(\frac{1}{4}\right) \log_2\left(\frac{1}{4}\right) = 1.2988$$

$$\text{Overall: } \frac{10}{18} \times 0.9710 + \frac{8}{18} \times 1.2988 = 1.1167$$

Misclassification rate :

$$A: 1 - \frac{4}{9} = 0.5556$$

$$B: 1 - \frac{3}{5} = \frac{2}{5}$$

$$C: 1 - \frac{5}{8} = \frac{3}{8}$$

$$\text{overall: } \frac{10}{18} \times \frac{2}{5} + \frac{8}{18} \times \frac{3}{8} = 0.3889$$

$$2. \quad x \leq 5$$

$$\bar{y} = \frac{1}{7} (2 + 3 + 2.5 + 1 + 2.3 + 2.8 + 1.5) = 2.1571$$

$$MSE = \frac{1}{7} [(2 - \bar{y})^2 + (3 - \bar{y})^2 + \dots + (1.5 - \bar{y})^2] = 0.4367$$

$$x > 5$$

$$\bar{y} = \frac{1}{6} (2.6 + 3.5 + 4 + 3.5 + 5 + 4.5) = 3.85$$

$$MSE = \frac{1}{6} [(2.6 - \bar{y})^2 + (3.5 - \bar{y})^2 + \dots + (4.5 - \bar{y})^2] = 0.5958$$

Overall

$$MSE = \frac{7}{13} \times 0.4367 + \frac{6}{13} \times 0.5958 = 0.5102$$

Root

$$\bar{y} = \frac{1}{13} (2 + 3 + 2.5 + \dots + 4.5) = 2.9385$$

$$MSE = \frac{1}{13} [(2 - \bar{y})^2 + (3 - \bar{y})^2 + \dots + (4.5 - \bar{y})^2] = 1.2224$$