EE2211 Tutorial 6

(Ridge Regression in Dual Form)
Question 1:

Derive the solution for linear ridge regression in dual form (see Lecture 6 notes page 16).

(Polynomial Regression, 1D data)
Question 2:

Given the following data pairs for training

$$\{x = -10\} \rightarrow \{y = 5\}$$
$$\{x = -8\} \rightarrow \{y = 5\}$$
$$\{x = -3\} \rightarrow \{y = 4\}$$
$$\{x = -1\} \rightarrow \{y = 3\}$$
$$\{x = 2\} \rightarrow \{y = 2\}$$
$$\{x = 8\} \rightarrow \{y = 2\}$$

(a) Perform a 3rd-order polynomial regression and sketch the result of line fitting.
(b) Given a test point $\{x = 9\}$ predict y using the polynomial model.
(c) Compare this prediction with that of a linear regression.

(Polynomial Regression, 3D data, Python)
Question 3:

(a) Write down the expression for a 3rd order polynomial model having a 3-dimensional input.
(b) Write down the **P** matrix for this polynomial given $\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix}$.
(c) Given $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, can a unique solution be obtained in dual form? If so, proceed to solve it.
(d) Given $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, can the primal ridge regression be applied to obtain a unique solution? If so, proceed to solve it.
(e)

(Binary Classification, Python)
Question 4:
Given the training data:

$$\{x = -1\} \rightarrow \{y = class1\}$$
$$\{x = 0\} \rightarrow \{y = class1\}$$
$$\{x = 0.5\} \rightarrow \{y = class2\}$$
$$\{x = 0.3\} \rightarrow \{y = class1\}$$
$$\{x = 0.8\} \rightarrow \{y = class2\}$$

Predict the class label for $\{x = -0.1\}$ and $\{x = 0.4\}$ using linear regression with signum discrimination.

(Multi-Category Classification, Python)

Question 5:

Given the training data:

$$\{x = -1\} \rightarrow \{y = class1\}$$
$$\{x = 0\} \rightarrow \{y = class1\}$$
$$\{x = 0.5\} \rightarrow \{y = class2\}$$
$$\{x = 0.3\} \rightarrow \{y = class3\}$$
$$\{x = 0.8\} \rightarrow \{y = class2\}$$

(a) Predict the class label for $\{x = -0.1\}$ and $\{x = 0.4\}$ based on linear regression towards a one-hot encoded target.

(b) Predict the class label for $\{x = -0.1\}$ and $\{x = 0.4\}$ using a polynomial model of 5th order and a one-hot encoded target.

(Multi-Category Classification, Python)

Question 6 (continued from Q3 of Tutorial 2):

Get the data set "`from sklearn.datasets import load_iris`". Use Python to perform the following tasks.

(a) Split the database into two sets: 74% of samples for training, and 26% of samples for testing. Hint: you might want to utilize `from sklearn.model_selection import train_test_split` for the splitting.

(b) Construct the target output using one-hot encoding.

(c) Perform a linear regression for classification (without inclusion of ridge, utilizing one-hot encoding for the learning target) and compute the number of test samples that are classified correctly.

(d) Using the same training and test sets as in above, perform a 2nd order polynomial regression for classification (again, without inclusion of ridge, utilizing one-hot encoding for the learning target) and compute the number of test samples that are classified correctly. Hint: you might want to use `from sklearn.preprocessing import PolynomialFeatures` for generation of the polynomial matrix.

Question 7

MCQ: there could be more than one answer. Given three samples of two-dimensional data points $\mathbf{X} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 3 & 3 \end{bmatrix}$ with corresponding target vector $\mathbf{y} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Suppose you want to use a full third-order polynomial model to fit these data. Which of the following is/are true?

a) The polynomials model has 10 parameters to learn
b) The polynomial learning system is an under-determined one
c) The learning of the polynomial model has infinite number of solutions
d) The input matrix **X** has linearly dependent samples
e) None of the above

Question 8

MCQ: there could be more than one answer. Which of the following is/are true?

a) The polynomial model can be used to solve problems with nonlinear decision boundary.

b) The ridge regression cannot be applied to multi-target regression.

c) The solution for learning feature $\mathbf{X}$ with target $\mathbf{y}$ based on linear ridge regression can be written as $\hat{\mathbf{w}} = (\mathbf{X}^T\mathbf{X}+\lambda\mathbf{I})^{-1}\mathbf{X}^T\mathbf{y}$ for $\lambda > 0$. As λ increases, $\hat{\mathbf{w}}^T\hat{\mathbf{w}}$ decreases.

d) If there are four data samples with two input features each, the full second-order polynomial model is an over-determined system.