

## EE2211 Tutorial 4

(Systems of Linear Equations)

### Question 1:

Given  $\mathbf{X}\mathbf{w} = \mathbf{y}$  where  $\mathbf{X} = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$ ,  $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ .

even  
 $\det \mathbf{X} = 1 \neq 0$ , invertible

- (a) What kind of system is this? (even-, over- or under-determined?)
- (b) Is  $\mathbf{X}$  invertible? Why?
- (c) Solve for  $\mathbf{w}$  if it is solvable.

$\mathbf{w} = (-1, 1)$

(Systems of Linear Equations)

### Question 2:

Given  $\mathbf{X}\mathbf{w} = \mathbf{y}$  where  $\mathbf{X} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ ,  $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ .

even  
 $\det \mathbf{X} = 0$ , non invertible

- (a) What kind of system is this? (even-, over- or under-determined?)
- (b) Is  $\mathbf{X}$  invertible? Why?
- (c) Solve for  $\mathbf{w}$  if it is solvable.

not solvable? can use least square?

(Systems of Linear Equations)

### Question 3:

Given  $\mathbf{X}\mathbf{w} = \mathbf{y}$  where  $\mathbf{X} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 1 & -1 \end{bmatrix}$ ,  $\mathbf{y} = \begin{bmatrix} 0 \\ 0.1 \\ 1 \end{bmatrix}$ .

over

- (a) What kind of system is this? (even-, over- or under-determined?)
- (b) Is  $\mathbf{X}$  invertible? Why?
- (c) Solve for  $\mathbf{w}$  if it is solvable.

$\mathbf{X}$  is full rank, rank = 2, left inverse exist

$\mathbf{w} = [0.68 \ -0.32]$

(Systems of Linear Equations)

### Question 4:

Given  $\mathbf{X}\mathbf{w} = \mathbf{y}$  where  $\mathbf{X} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$ ,  $\mathbf{y} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ .

under  
rank  $\mathbf{X} = 3$ , full rank, right inverse exist  
 $\mathbf{w} = [0.5 \ 0.5 \ 0.5 \ 0.5]$

- (a) What kind of system is this? (even-, over- or under-determined?)
- (b) Is  $\mathbf{X}$  invertible? Why?
- (c) Solve for  $\mathbf{w}$  if it is solvable.

(Systems of Linear Equations)

### Question 5:

Given  $\mathbf{w}^T \mathbf{X} = \mathbf{y}^T$  where  $\mathbf{X} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ ,  $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ .

$$\mathbf{X}^T \mathbf{w} = \mathbf{y}$$

- (a) What kind of system is this? (even-, over- or under-determined?)
- (b) Is  $\mathbf{X}$  invertible? Why?
- (c) Solve for  $\mathbf{w}$  if it is solvable.

even

det = 0, non invertible

no solution

(Systems of Linear Equations)

**Question 6:**

Given  $\mathbf{w}^T \mathbf{X} = \mathbf{y}^T$  where

$$\mathbf{X} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 1 & -1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

$$\mathbf{X}^T \mathbf{w} = \mathbf{y}$$

over

after transpose its under determined

- (a) What kind of system is this? (even-, over- or under-determined?)
- (b) Is  $\mathbf{X}$  invertible? Why?
- (c) Solve for  $\mathbf{w}$  if it is solvable.

rank 2 = full rank, left inverse exist

$$\mathbf{w} = [0.06666667 \quad 0.13333333 \quad -0.33333333]$$

$$= [1/15, 2/15, -1/3]$$

(Systems of Linear Equations)

**Question 7:**

This question is related to determination of types of system where an appropriate solution can be found subsequently. The following matrix has a left inverse.

$$\mathbf{X} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

full rank, wide, has right inverse, false

- a) True
- b) False

(Systems of Linear Equations)

**Question 8:**

MCQ: Which of the following is/are true about matrix  $\mathbf{A}$  below? **There could be more than one answer.**

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

- a)  $\mathbf{A}$  is invertible
- b)  $\mathbf{A}$  is left invertible
- c)  $\mathbf{A}$  is right invertible
- d)  $\mathbf{A}$  has no determinant
- e) None of the above

$\mathbf{A}$  is full rank, wide right inverse,

c, d