1.

(a)

X_b , y

$$\hat{w} = (X^T X)^{-1} X^T y$$

$$= \begin{bmatrix} 6 & -12 \\ -12 & 242 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ -10 & -8 & -3 & -1 & 2 & 8 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \\ 4 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

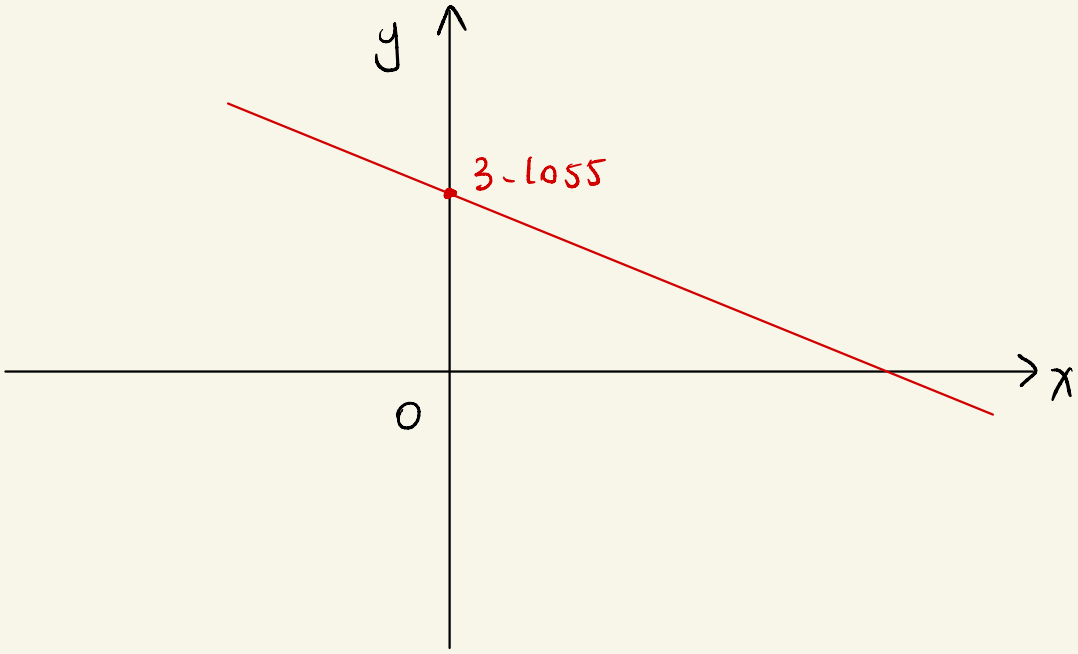
2×2 $\quad 2 \times 6$ $\quad 6 \times 1$

$$= \begin{bmatrix} 3.1055 \\ -0.1972 \end{bmatrix}$$ 2×1

3.1055 → constant

-0.1972 → coeff. of x

$$y = X\hat{w} = -0.1972x + 3.1055$$



(b) X, y

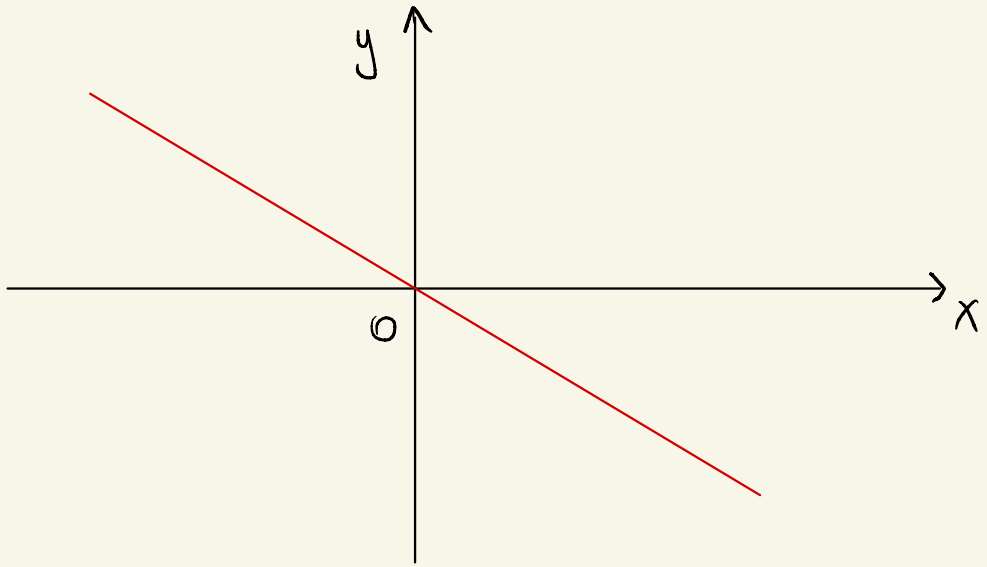
$$\hat{w} = (X^T X)^{-1} X^T y$$

$$= [242]^{-1} [-10, -8, -3, -1, 2, 8] \begin{bmatrix} 5 \\ 5 \\ 4 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

1×1 $\quad$ 1×6 $\quad$ 6×1

$$= -0.3512 \quad 1 \times 1$$

$$y = -0.3512 X$$



2.

(a) $\hat{w} = X^{-1} y$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3333 \\ -0.6667 \\ 0.6667 \end{bmatrix}$$

$$y = X\hat{w} = X \begin{bmatrix} 0.3333 \\ -0.6667 \\ 0.6667 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 8 \\ 1 & 5 & -1 \end{bmatrix} \begin{bmatrix} 0.3333 \\ -0.6667 \\ 0.6667 \end{bmatrix}$$

$$= \begin{bmatrix} 3.6667 \\ -3.6667 \end{bmatrix} \begin{matrix} \rightarrow y_1 \\ \rightarrow y_2 \end{matrix}$$

(b)

$$\hat{w} = X^T (XX^T)^{-1} y$$

$$= \begin{bmatrix} -0.1429 \\ 0.5238 \\ -0.4762 \\ 0.6190 \end{bmatrix}$$

$$\hat{y} = X\hat{w} = \begin{bmatrix} 1 & -1 & 2 & 8 \\ 1 & 1 & 5 & -1 \end{bmatrix} \begin{bmatrix} -0.1429 \\ 0.5238 \\ -0.4762 \\ 0.6190 \end{bmatrix}$$

$$= \begin{bmatrix} 3.3333 \\ -2.6190 \end{bmatrix} \begin{matrix} \rightarrow y_1 \\ \rightarrow y_2 \end{matrix}$$

3.

(a) $\hat{w} = (X^T X)^{-1} X^T y$

$$= \begin{bmatrix} 9.30 \\ 0.6727 \end{bmatrix}$$

→ constant

→ coeff. of X

$$y = Xw = X \begin{bmatrix} 9.30 \\ 0.6727 \end{bmatrix}$$

(b)

$$y = \begin{bmatrix} 1 & 30 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 9.30 \\ 0.6727 \end{bmatrix}$$

$$= \begin{bmatrix} 29.4818 \\ 12.6636 \end{bmatrix}$$

→ (b) 30 ~1

→ (c) 5 ~2

4.

$$\hat{w} = (X^T X)^{-1} X^T y$$

$$= \begin{bmatrix} -10.4126 \\ 1.2143 \end{bmatrix}$$

$$y = [1 \ 30]\, \hat{w} = 26.017$$

(c)

$$\hat{w} = (X^T X)^{-1} X^T y$$

$$= \begin{bmatrix} -3.5584 \\ 1.0260 \end{bmatrix}$$

$$y = [1 \ 30]\, \hat{w} = 27.2208$$