1. Given $(X^T X + \lambda I) w = X^T y$

derive $\hat{w} = X^T (XX^T + \lambda I)^{-1} y$

Hint: $w = X^T a$ where

$a = \lambda^{-1} (y - Xw)$, $\lambda > 0$

$$: X^T X w + \lambda \cancel{I} w = X^T y$$

$$\lambda w = X^T y - X^T X w$$

$$= X^T (y - Xw)$$

$$w = \lambda^{-1} X^T (y - Xw)$$

$$= X^T a \quad \text{—— ①}$$

$$: \quad a = \lambda^{-1} (y - Xw)$$

$$\lambda a = y - Xw$$

$$= y - XX^T a$$

$$XX^T a + \lambda a = y$$

$$(XX^T + \lambda) a = y$$

$$a = (XX^T + \lambda I)^{-1} y \quad \text{—— ②}$$

combining ① with ②

$$w = X^T a$$

$$= X^T (XX^T + \lambda I)^{-1} y$$

dual form

2.

(a)

$$P = \begin{array}{cccc} \text{cons} & \text{1st} & \text{2nd} & \text{3rd} \\ \end{array}$$
$$\begin{bmatrix} 1 & -10 & 100 & -1000 \\ 1 & -8 & 64 & -512 \\ 1 & -3 & 9 & -27 \\ 1 & -1 & 1 & -1 \\ 1 & 2 & 4 & 8 \\ 1 & 8 & 64 & 512 \end{bmatrix}$$

$$\hat{w} = (P^T P)^{-1} P^T y$$

$$\hat{w} = \begin{bmatrix} 2.6894 \\ -0.3772 \\ 0.0134 \\ 0.0029 \end{bmatrix} \begin{array}{l} w_0 \\ w_1 \\ w_2 \\ w_3 \end{array}$$

(b)

$$X = \begin{bmatrix} 1 & 9 & 9^2 & 9^3 \end{bmatrix}$$

$$y = Xw = 2.4661$$

(c)

$$\hat{w} = (X^T X)^{-1} X^T y$$

$$= \begin{array}{c} \begin{bmatrix} 3.1055 \\ -0.1972 \end{bmatrix} \end{array} \begin{array}{l} w_0 \\ w_1 \end{array}$$

3.

(a)

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$$f(x) = w_0 \qquad \text{constant}$$

$$+ w_1 x_1 + w_2 x_2 + w_3 x_3 \qquad \text{1st}$$

$$+ w_{12} x_1 x_2 + w_{23} x_2 x_3 + w_{13} x_1 x_3$$

$$+ w_{11} x_1^2 + w_{22} x_2^2 + w_{33} x_3^2 \qquad \text{2nd}$$

$$\left.\begin{aligned} &+ w_{211} x_2 x_1^2 + w_{311} x_3 x_1^2 + w_{122} x_1 x_2^2 \\ &+ w_{322} x_3 x_2^2 + w_{133} x_1 x_3^2 \\ &+ w_{123} x_1 x_2 x_3 \\ &+ w_{111} x_1^3 + w_{222} x_2^3 + w_{333} + x_3^3 \end{aligned}\right\} \text{3rd}$$

(b)

$$P = \begin{array}{c}
\begin{array}{cccc}
\text{cons} & \text{1st} & \text{2nd} & \text{3rd}
\end{array} \\
\left[\begin{array}{c|ccc|cccccc|cccccccccc}
1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1
\end{array}\right]
\end{array}$$

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