Fast MIMO Blind Detection via Modified MMA Approach over the Stiefel Manifold

(Supplementary material)

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Abstract——In Sec. III-A of our paper "Fast MIMO Blind Detection via Modified MMA Approach over the Stiefel Manifold", a novel modified MMA (M^3A) algorithm is proposed to improve the performance of the blind detection of the multi-modulus (MM) signal. This document presents the proof for Eq. (11).

I. Introduction

By the derivation of Eqs. (5)-(11), we find that when the residual demixing error $\Delta\mathbf{V}$ (defined in Sec. 3-A) is reduced to a certain level (i.e., $\|\Delta\mathbf{V}\|_F < 2|a_{\pm 1}|/3$), the SIR of J_{MMA} will increase with the magnitude of $x_{\mathrm{R,ij}}$. This implies that the outer constellation points are more important. Therefore, we conclude that giving larger weight to the outer constellation points can improve the performance of the algorithm.

Specifically, since we have division of the desired and inferential parts of $\hat{x}_{R,ij}^2$ (the input of J_{MMA}) in Eq. (6), the effect of $\hat{x}_{R,ij}^2$ to J_{MMA} can be measured by an SIR defined in Eq. (10) with the variance of the desired and inferential parts (as shown in Eqs. (7)-(9)). The minimum value point of SIR (denoted as κ) is easy to obtain and is presented in Eq.(11). Once the input $\hat{x}_{R,ij}^2 > \kappa$, the SIR will increase with $\hat{x}_{R,ij}^2$. We will proof that with suitable deflation1, a rough upper bound on κ can be obtained, and it is depended on $\|\Delta\mathbf{V}\|_F$.

II. Derivation of a rough upper bound on κ

Recall the expression for SIR (Eq. (10)) as follows

$$\begin{aligned}\mathrm{SIR} &= \frac{\mathbb{E}[(z_A^2 + z_{B'} + z_{C'})^2]}{\mathbb{E}[\mathcal{N}^2]}, \\ &= \frac{z_4 x_{R,ij}^4 + 2z_1 x_{R,ij}^2 + 2z_2 + z_3 + z_5}{4z_1 x_{R,ij}^2 + 4z_2 + z_3},\end{aligned} \tag{1}$$

where

$$\begin{aligned}
z_1 &= (1+\Delta v_{R,ii})^2 \left(\sum_{j=1,j\neq i}^{N_t} \Delta v_{R,ij}^2 + \sum_{j=1}^{N_t} \Delta v_{I,ij}^2 \right) \mathbb{E}[x_{R,ij}^2], \\
z_2 &= 4\mathbb{E}[x_{R,ij}^2] \sum_{j=1,j\neq i}^{N_t} \sum_{k=1}^{N_t} \Delta v_{R,ij}^2 \Delta v_{I,ik}^2, \\
z_3 &= \mathbb{E}[x_{R,ij}^2]^2 \sum_{j=1,j\neq i}^{N_t} \sum_{k=1,k\neq i,k\neq j}^{N_t} \Delta v_{R_i j}^2 \Delta v_{R,ik}^2 + \mathbb{E}[x_{R,ij}^2]^2 \sum_{j=1}^{N_t} \sum_{k=1,k\neq j}^{N_t} \Delta v_{I_i j}^2 \Delta v_{I_i k}^2, \\
z_4 &= (1+\Delta v_{R,ii})^4, \\
z_5 &= \left(\sum_{j=1,j\neq i}^{N_t} \Delta v_{R,ij}^4 + \sum_{j=1}^{N_t} \Delta v_{I,ij}^4 \right) \mathbb{E}[x_{R,ij}^4].
\end{aligned} \tag{2}$$

By solving $\partial \mathrm{SIR} / \partial x_{R,ij} = 0$, the minimum value point κ can be obtained as

$$\kappa = \frac{1}{2} \sqrt{\frac{\sqrt{(z'^2 + 8z_4 z_1^2 (z_3 + 2z_5)} - z'}{z_1 z_4}}, \tag{3}$$

where $z' = z_4(4z_2 + z_3)$. For z_1~z_5, we perform a rough deflation as follows

$$
\begin{aligned}
z_1 &\leq \left(1+\Delta \mathbf{V}_{R,ii}\right)^2 \| \Delta \mathbf{V}_i \|_2^2 \, \mathbb{E}\left[s_i^2\right], \\
z_1 &\geq \left(1-|\Delta \mathbf{V}_{R,ii}|\right)^2 \| \Delta \mathbf{V}_i \|_2^2, \\
z_2 &\leq \| \Delta \mathbf{V}_i \|_2^4 \, \mathbb{E}\left[s_i^2\right]^2, \\
z_3 &\leq \| \Delta \mathbf{V}_i \|_2^4 \, \mathbb{E}\left[s_i^2\right]^2 \leq \| \Delta \mathbf{V}_i \|_2^2 \, \mathbb{E}\left[s_i^2\right]^2, \\
z_5 &\leq \| \Delta \mathbf{V}_i \|_2^4 \, \mathbb{E}\left[s_i^2\right]^2.
\end{aligned} \tag{4}
$$

The above deflation holds for $\mathbb{E}\left[s_i^2\right] \leq 1$. As a result, we have

$$
\begin{aligned}
\kappa &\leq \frac{1}{2}\sqrt{\frac{\sqrt{z_4\left(z_4\left(4z_2+z_3\right)^2+8z_1^2\left(z_3+2z_5\right)\right)}}{z_1 z_4}}, \\
&\leq \frac{1}{2}\sqrt{\frac{7\| \Delta v_i \|_2^4}{\left(1-|\Delta v_{R,ii}|\right)^2}}, \\
&\leq \frac{3\| \Delta v_i \|_2}{2\left(1-|\Delta v_{R,ii}|\right)} \triangleq \kappa_u .
\end{aligned} \tag{5}
$$

By letting $\kappa_u \leq | \min\{s_i\} | \triangleq P_{s,min}$, where $P_{s,min}$ is the power of the innermost constellation point, the following conclusions can be drawn

$$
\frac{3}{2}\| \Delta v_i \|_2 < \frac{3}{2}\| \Delta \mathbf{V} \|_F < P_{s,min}. \tag{6}
$$

Hence, we can conclude that when $\| \Delta \mathrm{V} \|_F < 2\,|a_{\pm 1}|/3$, κ will smaller than the innermost constellation point. Then, the SIR will increase with the magnitude of $\hat{x}_{R,ij}^2$, which means that the constellation points on the outside contain more information and thus more important than those in the inner loop.