Comparison between Constrained Energy Minimization based Approaches for Hyperspectral Imagery

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***Abstract* - Constrained Energy Minimization (CEM) has been widely used for target detection in hyperspectral remote sensing imagery. It detects the desired target signal source using a unity constraint while suppressing noise and unknown signal sources by minimizing the average output power. Base on the design, CEM can only detect one target source at a time. In order to simultaneously detect multiple targets in single image, several approaches are developed, including Multiple-Target CEM (MTCEM), Sum CEM (SCEM) and Winner-Take-All CEM (WTACEM). Interestingly, the sensitivity of noise and interference seems to play a role in the detection performance. Unfortunately, this issue has not been investigated. In this paper, we take up this problem and conduct a quantitative study of the noise and interference suppression abilities of LCMV, SCEM, WTACEM for multiple-target detection.**

I. INTRODUCTION

Target detection in remotely sensed images has been a major research area for years. Many techniques have been studied in the past based on linear mixture model, such as singular value decomposition [1], subspace projection [2-5] and maximum likelihood [6-7]. Unfortunately, such linear unmixing methods require the complete knowledge of the image endmembers. On many practical occasions obtaining this prior knowledge may not be realistic. In order to resolve this issue, a method, referred to as Constrained Energy Minimization (CEM) was developed [7-8] where the only required knowledge is the desired image endmember rather than the entire set of image endmembers.

Unfortunately, this advantage may also turn to a disadvantage if the knowledge of the desired target signature used in CEM is not accurate. This is because the performance of CEM is completely determined by the information used to describe the desired target signature. It has evidenced that CEM is very sensitive to noise and is not a robust classifier. It cannot classify the same type of targets with similar signatures. In order to solve this problem, we consider general approaches, Multiple-Target CEM (MTCEM), Sum CEM (SCEM) and Winner-Take-All CEM (WTACEM), which can mitigate this problem and includes CEM as a special case. Rather than using a single target signature constraint in CEM which results in a constraint scalar, they consider multiple target signatures in its filter design so as to detect and classify similar targets. This enables those approaches to extract target pixels missed by detection of CEM. On the other hand, it also allows MTCEM to pick up some background pixels whose signatures similar to constrained signatures. In this paper, we take up this problem and conduct a quantitative study of the noise and interference suppression abilities of MTCEM, SCEM, WTACEM for multiple-target detection.

II. COSTRAINED ENERGY MINIMIZATION

Constrained Energy Minimization (CEM) [7-8] was developed for the case that the only required knowledge is the signature to be classified. It used a finite impulse response (FIR) filter to constrain the desired signature by a specific gain while minimizing the filter output power. The idea of CEM arises in Minimum Variance Distortionless Response (MVDR) in array processing [9-10] with the desired signature interpreted as the direction of arrival from a desired signal. It can be derived as follows.

Assume that we are given a finite set of observations $S = \{\mathbf{r}_1 \mathbf{r}_2 \cdots \mathbf{r}_N\}$ where $\mathbf{r}_i = (r_{i1} r_{i2} \cdots r_{iL})^T$ for $1 \le i \le N$ is a sample pixel vector. Suppose that the desired signature $\mathbf{d}$ is also known *a priori*. The objective of CEM is to design a

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finite impulse response (FIR) linear filter with L filter coefficients $\left\{w_1 w_2 \cdots w_L\right\}$, denoted by an L-dimensional vector $\mathbf{w} = \left(w_1 w_2 \cdots w_L\right)^T$ that minimizes the filter output power subject to the following constraint

$$\mathbf{d}^T \mathbf{w} = \sum_{l=1}^{L} d_l w_l = 1 . \tag{1}$$

Let y_i denote the output of the designed FIR filter resulting from the input $\mathbf{r}_i$. Then y_i can be written as

$$y_i = \sum_{l=1}^{L} w_l r_{il} = \mathbf{w}^T \mathbf{r}_i = \mathbf{r}_i^T \mathbf{w} . \tag{2}$$

So, the average output power produced by the observation set S and the FIR filter with coefficient vector $\mathbf{w} = \left(w_1 w_2 \cdots w_L\right)^T$ specified by Eq. (2) is given by

$$\frac{1}{N}\left[\sum_{i=1}^{N} y_i^2\right] = \frac{1}{N}\left[\sum_{l=1}^{L} \left(\mathbf{r}_i^T \mathbf{w}\right)^T \mathbf{r}_i^T \mathbf{w}\right] = \mathbf{w}^T \left(\frac{1}{N}\left[\sum_{i=1}^{N} \mathbf{r}_i \mathbf{r}_i^T\right]\right)\mathbf{w} = \mathbf{w}^T R_{L\times L} \mathbf{w} \tag{3}$$

where $R_{L\times L} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{r}_i \mathbf{r}_i^T$ turns out to be the $L \times L$ sample autocorrelation matrix of S.

Minimizing Eq. (3) with the filter response constraint $\mathbf{d}^T \mathbf{w} = \sum_{l=1}^{L} d_l w_l = 1$ yields

$$\min_{\mathbf{w}} \frac{1}{N} \sum_{i=1}^{N} y_i^2 = \min_{\mathbf{w}} \left\{\mathbf{w}^T R_{L\times L}\, \mathbf{w}\right\} \text{ subject to } \mathbf{d}^T \mathbf{w} = 1. \tag{4}$$

The solution to Eq. (4) was shown in [9-10] and called Constrained Energy Minimization (CEM) classifier with the weight vector $\mathbf{w}^*$ given by

$$\mathbf{w}^* = \frac{R_{L\times L}^{-1} \mathbf{d}}{\mathbf{d}^T R_{L\times L}^{-1} \mathbf{d}} . \tag{5}$$

III. Extension Of CEM Filter

While CEM has shown some success in hyperspectral image classification, it has been also shown to be very sensitive to the knowledge of **d**. This is primarily due to the constraint given by Eq. (1). With this specific constraint CEM can only detect the exact signature **d** with very little flexibility. So, it cannot detect signatures similar to **d**. As a result, it is not robust to noise. This may create a problem since in many applications acquiring the precise knowledge of **d** may not be ralistic. More importantly, even the knowledge of **d** can be obtained from real data, it may not be accurate. If it is used in Eq. (1), CEM may not perform satisfactorily. To remedy this drawback, three approaches are proposed to include all desired similar signatures as multiple constraints, including Multiple-Target CEM (MTCEM), Sum CEM (SCEM) and Winner-Take-All CEM (WTACEM) [11], which are described as following.

A. Multiple-Target CEM (MTCEM)

Assume that $D = \left\{\mathbf{d}_1, \mathbf{d}_2, \cdots, \mathbf{d}_q\right\}$ is a desired signature matrix consisting of q signatures of interest, $\mathbf{d}_1, \mathbf{d}_2, \cdots, \mathbf{d}_q$. The constraint scalar in Eq. (1) can be expanded to constraint vector by including multiple constraints and expressed by

$$D^T \mathbf{w} = \mathbf{1} . \tag{6}$$

where **1** is a $p\times 1$ column constraint vector with ones in all components.

Substituting Eq. (6) for Eq. (1) Eq. (4) becomes

$$\min_{\mathbf{w}} \left\{\mathbf{w}^T R_{L\times L}\, \mathbf{w}\right\} \text{ subject to } D^T \mathbf{w} = \mathbf{1} . \tag{7}$$

The solution to Eq. (7) is called Linearly Constrained Minimum Variance (LCMV) beamformer with the weight vector $\mathbf{w}^*$ given by

$$\mathbf{w}^* = R_{L\times L}^{-1} D \left(D^T R_{L\times L}^{-1} D\right)^{-1} \mathbf{1} . \tag{8}$$

B. Sum CEM (SCEM)

Let $\mathbf{r}$ denote an image pixel and $\left\{CEM_j(\mathbf{r})\right\}_{j=1}^{J}$ be J target abundance fraction generated by J CEM detectors, each of which is used to detect same target with similar spectrum. The SCEM classifier combines J CEM classifiers using a linear function "sum". It referred to as SCEM(**r**) defined by

$$SCEM(\mathbf{r}) = \sum_{j=1}^{J} CEM_j(\mathbf{r}) \tag{9}$$

which simply sums up the J abundance fractions of each output of CEM.

C. Winner-Take-All CEM (WTACEM)

The Winner-Take-All CEM referred to as WTACEM(**r**) uses a nonlinear function "maximum" to combine J CEM classifiers, which also known as winner-take-all rule to determine the gray scale value. It is defined by

$$WTACEM(\mathbf{r}) = \max\{CEM_j(\mathbf{r})\}_{j=1}^{J} \tag{10}$$

IV. COUPUTER SIMULATION

The data set used for simulation was the AVIRIS (Airborne Visible/Infrared Imaging Spectrometer) reflectance data library provided by United States Geological Survey website. AVIRIS data has 224 bands covers from 0.4 μm to 2.5 μm with 10 nm spectral resolution. Figure 1 shows five reflectance spectra, black brush, maple leaves and three alunite spectra, denoted as alunite 1, alunite 2 and alunite3.

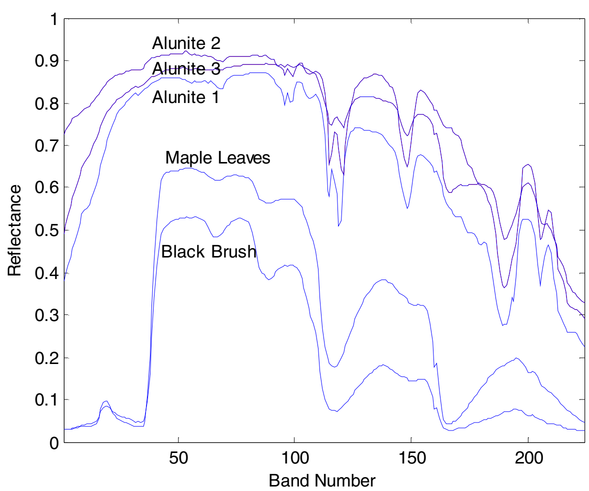


Figure 1. Spectra of five AVIRIS reflectance

In the following AVIRIS experiment, 401 mixed pixel vectors were simulated. It started the first pixel vector with 100% black brush and 0% maple leaves, then began to increase 0.25% maple leaves and decrease 0.25% black brush every pixel vector until the 401-st pixel vector which contained 100% maple leaves. We then added alunite 1 to pixel vector numbers 99-103 at abundance fractions 20% while reducing the abundance of black brush and maple leaves by multiplying their abundance fractions by 80%. For example, after addition of alunite 1, the resulting pixel vector 101 contained 20% alunite1, 60% black brush and 20% leaves. Similarly, the spectra of alunite 2 and alunite 3 were also added to pixel numbers from 199 to 203 and from 299 to 303 respectively with 20% abundance in the same way as done for the alunite 1, shown in figure 2. A Gaussian noise was added to each pixel vector to achieve a 30:1 signal-to-noise ratio which was defined in [1] as 50% reflectance divided by the standard deviation of the noise.

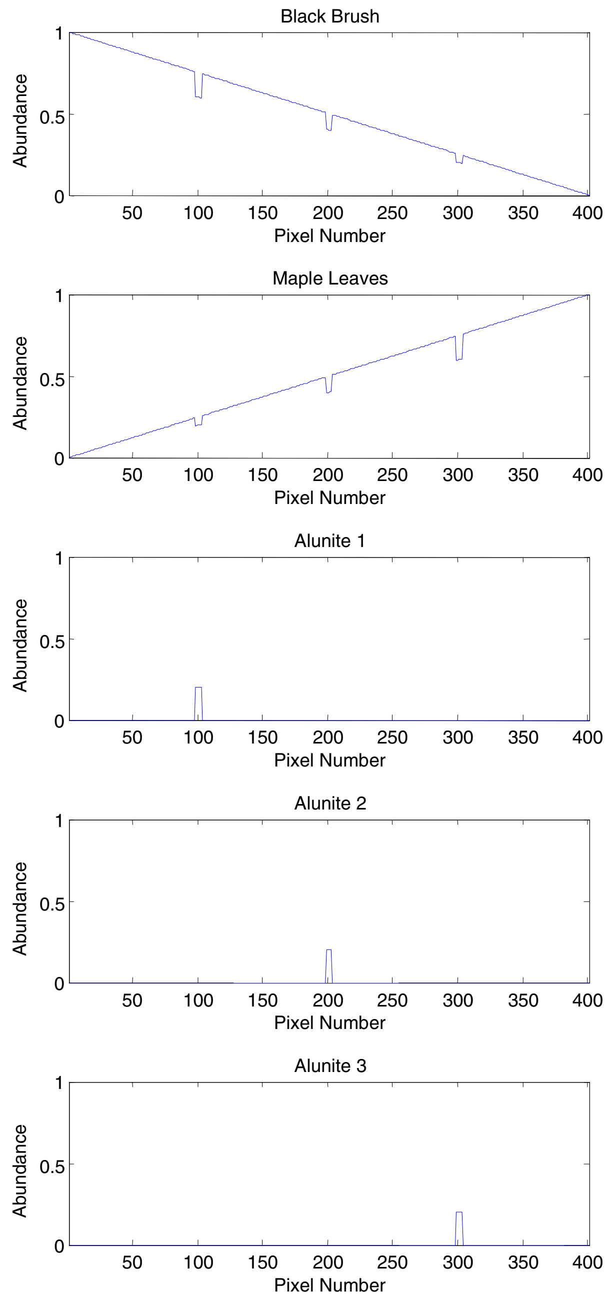


Figure 2: Assigned Abundance

Here we use the spectra of alunite as desired target, so three alunite spectra are available. Since CEM can only use one desired signature at a time, we applied CEM three times with different alunite signatures. Figure 3 shows the results of CEM for alunite 1, alunite 2 and alunite 3 respectively. Ideally, the abundance fraction of all 99-103, 199-203 and 299-303 pixels should indicate the presence of the alunite, but because CEM is too sensitive to target information, only those pixels with exactly alunite signature are detected.

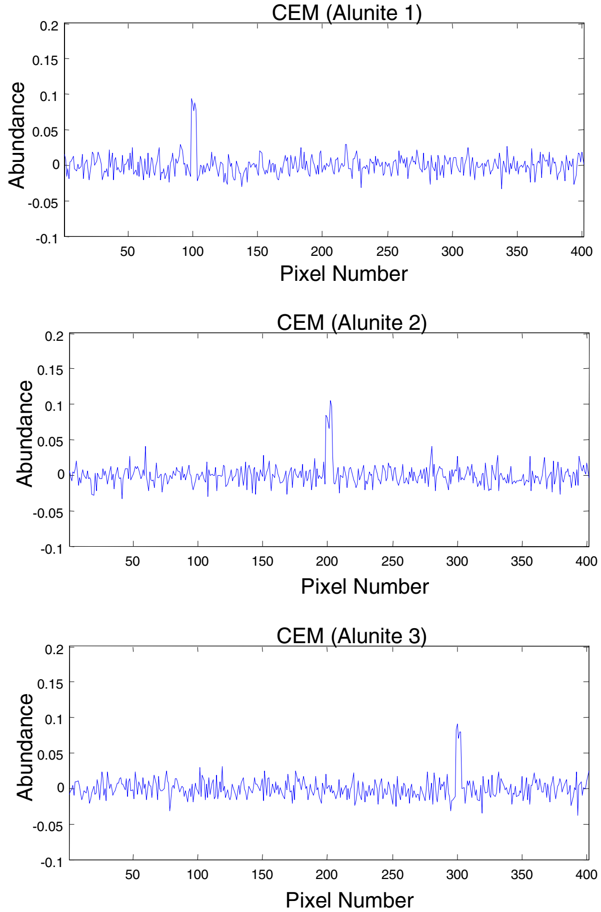
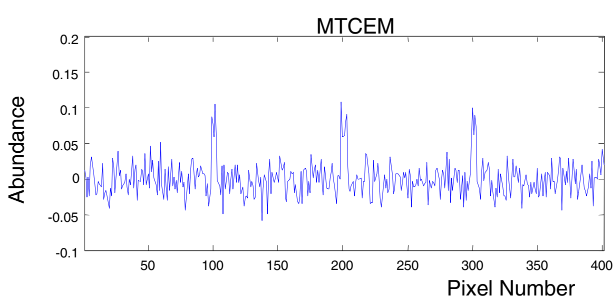


Figure 3: CEM results for Alunite

If we want to detect all the samples contained alunite, we need to use those extended CEM techniques. The result of MTCEM, SCEM and WTACEM are shown in Figure 4, MTCEM and SCEM give compatible results. From the derivations, MTCEM is designed to detect multiple target with all target signatures constrained to 1 and minimize the output power. On the other hand, SCEM sums all the CEM results. In this case, if one of the CEM classifier partially detect the similar target, for example, if CEM with alunite 1 as desired target detected 5% of alunite 2, the SCEM will over estimate the abundance. The WTACEM uses the function "maximum" to combine CEM results. When WTACEM is applied, this nonlinear function will combine the detected abundance without add up the noise.



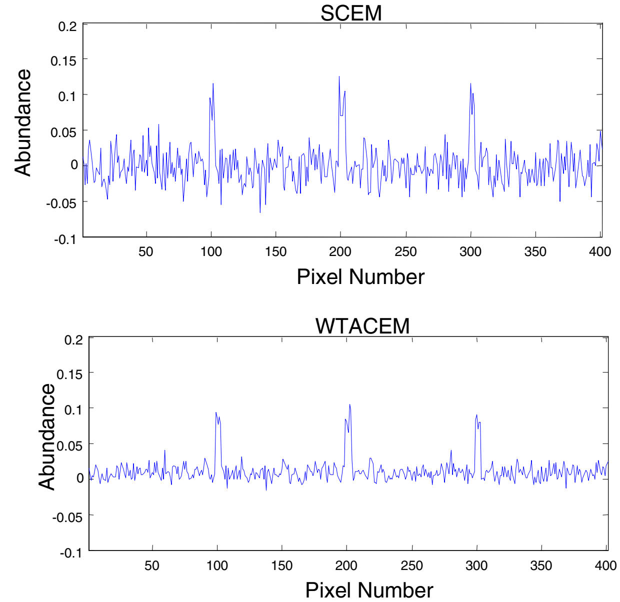


Figure 4: MTCEM, SCEM and WTACEM results

Table 1: sum of absolutely error

	MTCEM	SCEM	WTACEM
Error	7.73	8.37	5.59

Table 1 shows the sum of absolutely error of these six classifiers. The result shows that WTACEM gives best performance in this experiment.

V. Conclution

The CEM classifier has been proved that it is too sensitive to the target information. In order to detect target with multiple signatures, extended CEM are designed. In this paper, we demonstrate the performance of CEM, MTCEM, SCEM and WTACEM. We also compare the detection error and it shows WTACEM usually gives the best result among these three extended versions.

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