```python
import os
import numpy as np
import matplotlib.pyplot as plt
from PIL import Image
import imageio
from sklearn.model_selection import train_test_split
from sklearn.utils import shuffle
```

Q1. Explain the Problem

1) What is your binary outcome variable?
The binary outcome variable in this problem is the **type of waste**:

- `0` for **metal**
- `1` for **paper**

2) What's the size of the images? Provide five image examples.
All images have been resized to **64x64 pixels** and converted to **grayscale** to standardize the input format and reduce computational load.

Below are 5 sample images from each category:

```python
# Display size of each image under its picture
fig, axes = plt.subplots(1, 5, figsize=(15, 5))
for i, metal_file in enumerate(os.listdir("Dataset/metal")[:5]):
    metal_path = os.path.join("Dataset/metal", metal_file)
    try:
        metal_image = Image.open(metal_path)
        axes[i].imshow(metal_image, cmap='gray')
        axes[i].set_title(f"Metal\n{metal_image.size}")
        axes[i].axis('off')
    except Exception as e:
        print(f"Error loading metal image {i+1}: {e}")

plt.show()

fig, axes = plt.subplots(1, 5, figsize=(15, 5))
for i, paper_file in enumerate(os.listdir("Dataset/paper")[:5]):
    paper_path = os.path.join("Dataset/paper", paper_file)
    try:
        paper_image = Image.open(paper_path)
        axes[i].imshow(paper_image, cmap='gray')
        axes[i].set_title(f"Paper\n{paper_image.size}")
        axes[i].axis('off')
    except Exception as e:
        print(f"Error loading paper image {i+1}: {e}")

plt.show()
```


```python
def load_images_from_folder(folder_path, label, image_size=(64, 64)):
    X = []
    Y = []
    for filename in os.listdir(folder_path):
        img_path = os.path.join(folder_path, filename)
        try:
            img = imageio.imread(img_path, pilmode='L')  # Grayscale
            img = np.resize(img, image_size)
            X.append(img.flatten())
            Y.append(label)
        except:
            continue
    return np.array(X), np.array(Y)

# Define folder paths
metal_folder = 'Dataset/metal'
paper_folder = 'Dataset/paper'

# Load data
X_metal, y_metal = load_images_from_folder(metal_folder, label=0)
X_paper, y_paper = load_images_from_folder(paper_folder, label=1)

# Combine and shuffle
X = np.concatenate((X_metal, X_paper), axis=0)
Y = np.concatenate((y_metal, y_paper), axis=0)
X, Y = shuffle(X, Y, random_state=42)
```

```
C:\Users\yuhes\AppData\Local\Temp\ipykernel_33412\2026548941.py:7:
DeprecationWarning: Starting with ImageIO v3 the behavior of this
function will switch to that of iio.v3.imread. To keep the current
behavior (and make this warning disappear) use `import imageio.v2 as
```

```
imageio` or call `imageio.v2.imread` directly.
  img = imageio.imread(img_path, pilmode='L')  # Grayscale
```

Q2. Describe the Dataset

1) How many observations do you have in your dataset?
The dataset consists of images from the "metal" and "paper" folders. After loading and filtering valid images:

- **Total Observations:** *1819*

2) How many observations have an outcome of zero, and how many have an outcome of one?

- **Metal (label = 0):** *769*

- **Paper (label = 1):** *1050*

The dataset has a fairly balanced distribution of the two classes, ensuring effective learning for a binary classification model.

```
print("Metal directory size:", len(os.listdir(metal_folder)))
print("Paper directory size:", len(os.listdir(paper_folder)))
print("Total Observations loaded:", len(X))

Metal directory size: 769
Paper directory size: 1050
Total Observations loaded: 1819
```

Q3. Split the Dataset

We split the dataset into three parts:

- **Training set (~60%)**
- **Cross-validation set (~20%)**
- **Test set (~20%)**

The splitting was done using `train_test_split()` from Scikit-learn with stratification to maintain class balance.

```
X1, X_test, Y1, Y_test = train_test_split(X, Y, test_size=0.2,
random_state=42, stratify=Y)
X_train, X_cv, Y_train, Y_cv = train_test_split(X1, Y1,
test_size=0.25, random_state=42, stratify=Y1)

print("Training set size:", X_train.shape, Y_train.shape)
print("Cross-validation set size:", X_cv.shape, Y_cv.shape)
print("Test set size:", X_test.shape, Y_test.shape)
```

```
Training set size: (1091, 4096) (1091,)
Cross-validation set size: (364, 4096) (364,)
Test set size: (364, 4096) (364,)
```

Functions

```
def sigmoid(z):
    return 1 / (1 + np.exp(-z))

def compute_cost(Y, Y_hat):
    m = Y.shape[0]
    cost = -np.sum(Y * np.log(Y_hat + 1e-9) + (1 - Y) * np.log(1 -
Y_hat + 1e-9)) / m
    return cost

def relu(z):
    return np.maximum(0, z)

def relu_derivative(z):
    return (z > 0).astype(float)

# Reshape Y to column vectors
Y_train = Y_train.reshape(-1, 1)
Y_cv = Y_cv.reshape(-1, 1)
```

Q4. Model Implementation and Results

We implemented and trained three models:

1) Logistic Regression (No Hidden Layers)

- **Training Accuracy:** *0.5829514207149404*
- **Validation Accuracy:** *0.5769230769230769*
- Cost was plotted every 100 iterations to visualize convergence.
- This model forms the baseline for binary classification performance.

```
def logistic_regression(X, Y, X_val, Y_val, alpha=0.01,
num_iter=2000):
    m, n = X.shape
    w = np.zeros((n, 1))
    b = 0
    train_costs = []
    val_costs = []

    for i in range(num_iter):
        Z = np.dot(X, w) + b
        A = sigmoid(Z)
```

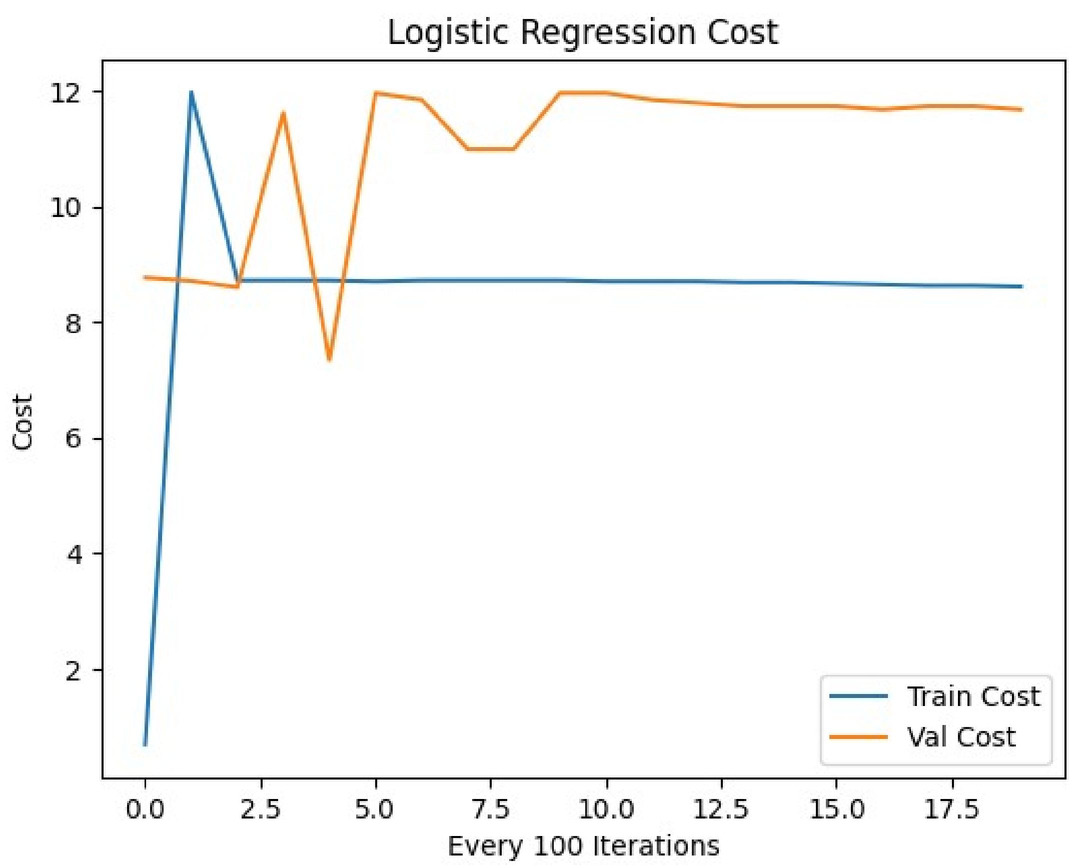
```
        cost = compute_cost(Y, A)

        dw = np.dot(X.T, (A - Y)) / m
        db = np.sum(A - Y) / m

        w -= alpha * dw
        b -= alpha * db

        if i % 100 == 0:
            A_val = sigmoid(np.dot(X_val, w) + b)
            val_cost = compute_cost(Y_val, A_val)
            train_costs.append(cost)
            val_costs.append(val_cost)
            print(f"Iteration {i} - Train Cost: {cost:.4f}, Val Cost:
{val_cost:.4f}")

    return w, b, train_costs, val_costs

# Load images from the dataset
w_lr, b_lr, train_costs_lr, val_costs_lr =
logistic_regression(X_train, Y_train, X_cv, Y_cv)

# Accuracy
def predict_lr(X, w, b):
    A = sigmoid(np.dot(X, w) + b)
    return (A > 0.5).astype(int)

print("Training Accuracy:", np.mean(predict_lr(X_train, w_lr, b_lr) ==
Y_train))
print("Validation Accuracy:", np.mean(predict_lr(X_cv, w_lr, b_lr) ==
Y_cv))

# Plot cost
plt.plot(train_costs_lr, label='Train Cost')
plt.plot(val_costs_lr, label='Val Cost')
plt.title("Logistic Regression Cost")
plt.xlabel("Every 100 Iterations")
plt.ylabel("Cost")
plt.legend()
plt.show()
```

```
Iteration 0 - Train Cost: 0.6931, Val Cost: 8.7675
```

```
C:\Users\yuhes\AppData\Local\Temp\ipykernel_33412\1874142768.py:2:
RuntimeWarning: overflow encountered in exp
  return 1 / (1 + np.exp(-z))
```

```
Iteration 100 - Train Cost: 11.9675, Val Cost: 8.7106
Iteration 200 - Train Cost: 8.7199, Val Cost: 8.5977
Iteration 300 - Train Cost: 8.7201, Val Cost: 11.6141
Iteration 400 - Train Cost: 8.7203, Val Cost: 7.3442
```

```
Iteration 500 - Train Cost: 8.7011, Val Cost: 11.9557
Iteration 600 - Train Cost: 8.7204, Val Cost: 11.8419
Iteration 700 - Train Cost: 8.7206, Val Cost: 10.9879
Iteration 800 - Train Cost: 8.7207, Val Cost: 10.9879
Iteration 900 - Train Cost: 8.7209, Val Cost: 11.9557
Iteration 1000 - Train Cost: 8.7020, Val Cost: 11.9557
Iteration 1100 - Train Cost: 8.7022, Val Cost: 11.8419
Iteration 1200 - Train Cost: 8.7025, Val Cost: 11.7849
Iteration 1300 - Train Cost: 8.6837, Val Cost: 11.7280
Iteration 1400 - Train Cost: 8.6839, Val Cost: 11.7280
Iteration 1500 - Train Cost: 8.6651, Val Cost: 11.7280
Iteration 1600 - Train Cost: 8.6463, Val Cost: 11.6711
Iteration 1700 - Train Cost: 8.6275, Val Cost: 11.7280
Iteration 1800 - Train Cost: 8.6277, Val Cost: 11.7280
Iteration 1900 - Train Cost: 8.6089, Val Cost: 11.6711
Training Accuracy: 0.5829514207149404
Validation Accuracy: 0.5769230769230769
```



2) Neural Network with 1 Hidden Layer (4 Units)

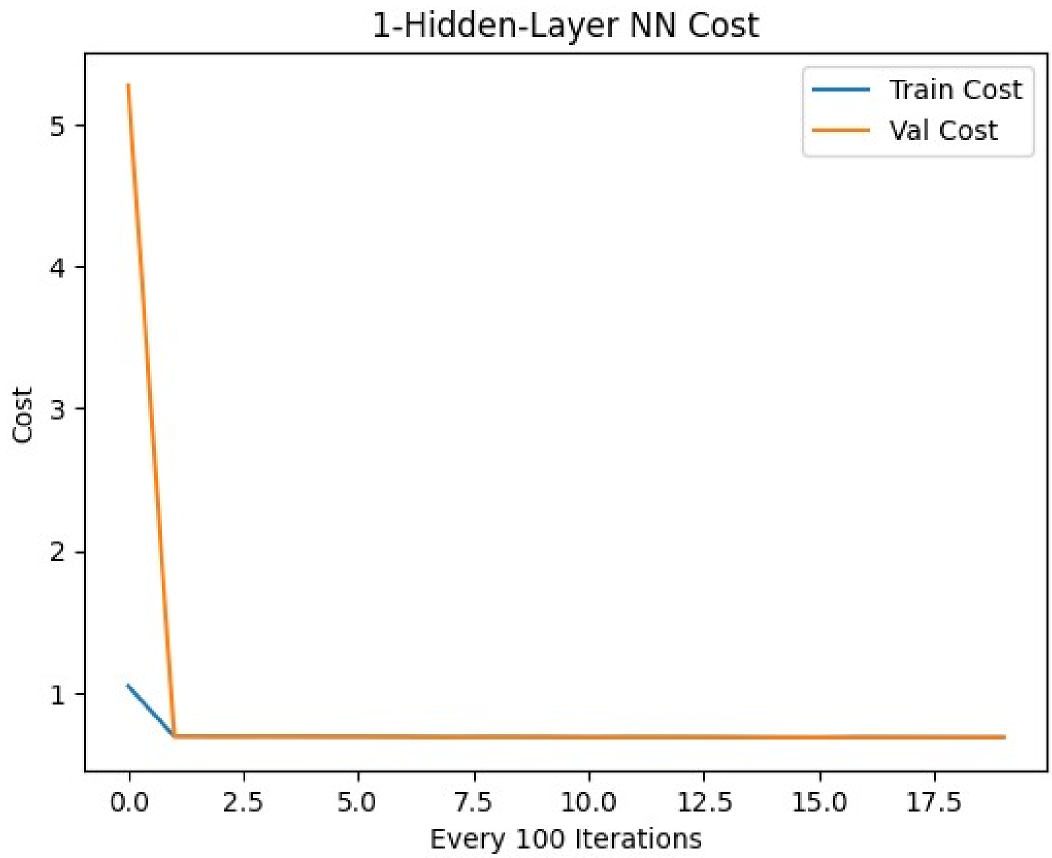
- **Architecture:**
  - Input layer → 4 Hidden units (ReLU) → Output layer (Sigmoid)
- **Training Accuracy:** *0.5774518790100825*

- **Validation Accuracy:** *0.5741758241758241*
- Cost curves were plotted for both training and validation sets.

```
def nn_one_hidden(X, Y, X_val, Y_val, alpha=0.001, num_iter=2000):
    m, n = X.shape
    np.random.seed(1)
    W1 = np.random.randn(n, 4) * 0.01
    b1 = np.zeros((1, 4))
    W2 = np.random.randn(4, 1) * 0.01
    b2 = np.zeros((1, 1))

    train_costs = []
    val_costs = []

    for i in range(num_iter):
        Z1 = np.dot(X, W1) + b1
        A1 = relu(Z1)
        Z2 = np.dot(A1, W2) + b2
        A2 = sigmoid(Z2)

        cost = compute_cost(Y, A2)

        dZ2 = A2 - Y
        dW2 = np.dot(A1.T, dZ2) / m
        db2 = np.sum(dZ2) / m

        dA1 = np.dot(dZ2, W2.T)
        dZ1 = dA1 * relu_derivative(Z1)
        dW1 = np.dot(X.T, dZ1) / m
        db1 = np.sum(dZ1, axis=0, keepdims=True) / m

        W2 -= alpha * dW2
        b2 -= alpha * db2
        W1 -= alpha * dW1
        b1 -= alpha * db1

        if i % 100 == 0:
            A2_val = sigmoid(np.dot(relu(np.dot(X_val, W1) + b1), W2)
+ b2)
            val_cost = compute_cost(Y_val, A2_val)
            train_costs.append(cost)
            val_costs.append(val_cost)
            print(f"Iteration {i} - Train Cost: {cost:.4f}, Val Cost:
{val_cost:.4f}")

    return W1, b1, W2, b2, train_costs, val_costs

W1, b1, W2, b2, train_costs_nn1, val_costs_nn1 =
nn_one_hidden(X_train, Y_train, X_cv, Y_cv)

# Accuracy
```

```python
def predict_nn1(X):
    A1 = relu(np.dot(X, W1) + b1)
    A2 = sigmoid(np.dot(A1, W2) + b2)
    return (A2 > 0.5).astype(int)

print("Training Accuracy:", np.mean(predict_nn1(X_train) == Y_train))
print("Validation Accuracy:", np.mean(predict_nn1(X_cv) == Y_cv))

plt.plot(train_costs_nn1, label='Train Cost')
plt.plot(val_costs_nn1, label='Val Cost')
plt.title("1-Hidden-Layer NN Cost")
plt.xlabel("Every 100 Iterations")
plt.ylabel("Cost")
plt.legend()
plt.show()
```

```
Iteration 0 - Train Cost: 1.0472, Val Cost: 5.2755
Iteration 100 - Train Cost: 0.6922, Val Cost: 0.6929
Iteration 200 - Train Cost: 0.6911, Val Cost: 0.6913
Iteration 300 - Train Cost: 0.6914, Val Cost: 0.6922
Iteration 400 - Train Cost: 0.6909, Val Cost: 0.6917
Iteration 500 - Train Cost: 0.6903, Val Cost: 0.6912
Iteration 600 - Train Cost: 0.6893, Val Cost: 0.6906
Iteration 700 - Train Cost: 0.6878, Val Cost: 0.6876
Iteration 800 - Train Cost: 0.6886, Val Cost: 0.6899
Iteration 900 - Train Cost: 0.6875, Val Cost: 0.6895
Iteration 1000 - Train Cost: 0.6860, Val Cost: 0.6870
Iteration 1100 - Train Cost: 0.6872, Val Cost: 0.6888
Iteration 1200 - Train Cost: 0.6868, Val Cost: 0.6885
Iteration 1300 - Train Cost: 0.6854, Val Cost: 0.6882
Iteration 1400 - Train Cost: 0.6845, Val Cost: 0.6857
Iteration 1500 - Train Cost: 0.6841, Val Cost: 0.6846
Iteration 1600 - Train Cost: 0.6864, Val Cost: 0.6873
Iteration 1700 - Train Cost: 0.6856, Val Cost: 0.6870
Iteration 1800 - Train Cost: 0.6852, Val Cost: 0.6868
Iteration 1900 - Train Cost: 0.6848, Val Cost: 0.6866
Training Accuracy: 0.5774518790100825
Validation Accuracy: 0.5741758241758241
```



3) Neural Network with 2 Hidden Layers (7 and 4 Units)

- **Architecture:**
  - Input layer → 7 Hidden units (ReLU) → 4 Hidden units (ReLU) → Output layer (Sigmoid)
- **Training Accuracy:** *0.5783684692942255*
- **Validation Accuracy:** *0.5741758241758241*
- This deeper model showed improved performance and generalization capability.

```
def nn_two_hidden(X, Y, X_val, Y_val, alpha=0.02, num_iter=2000):
    m, n = X.shape
    np.random.seed(1)

    # Initialize parameters
    W1 = np.random.randn(n, 7) * 0.01
    b1 = np.zeros((1, 7))

    W2 = np.random.randn(7, 4) * 0.01
    b2 = np.zeros((1, 4))

    W3 = np.random.randn(4, 1) * 0.01
    b3 = np.zeros((1, 1))
```

```python
    train_costs = []
    val_costs = []

    for i in range(num_iter):
        # Forward Propagation
        Z1 = np.dot(X, W1) + b1
        A1 = relu(Z1)

        Z2 = np.dot(A1, W2) + b2
        A2 = relu(Z2)

        Z3 = np.dot(A2, W3) + b3
        A3 = sigmoid(Z3)

        # Cost
        cost = compute_cost(Y, A3)

        # Backward Propagation
        dZ3 = A3 - Y
        dW3 = np.dot(A2.T, dZ3) / m
        db3 = np.sum(dZ3, axis=0, keepdims=True) / m

        dA2 = np.dot(dZ3, W3.T)
        dZ2 = dA2 * relu_derivative(Z2)
        dW2 = np.dot(A1.T, dZ2) / m
        db2 = np.sum(dZ2, axis=0, keepdims=True) / m

        dA1 = np.dot(dZ2, W2.T)
        dZ1 = dA1 * relu_derivative(Z1)
        dW1 = np.dot(X.T, dZ1) / m
        db1 = np.sum(dZ1, axis=0, keepdims=True) / m

        # Parameter updates
        W3 -= alpha * dW3
        b3 -= alpha * db3
        W2 -= alpha * dW2
        b2 -= alpha * db2
        W1 -= alpha * dW1
        b1 -= alpha * db1

        if i % 100 == 0:
            # Validation Cost
            Z1_val = np.dot(X_val, W1) + b1
            A1_val = relu(Z1_val)
            Z2_val = np.dot(A1_val, W2) + b2
            A2_val = relu(Z2_val)
            Z3_val = np.dot(A2_val, W3) + b3
            A3_val = sigmoid(Z3_val)
            val_cost = compute_cost(Y_val, A3_val)
```

```python
            train_costs.append(cost)
            val_costs.append(val_cost)
            print(f"Iteration {i} - Train Cost: {cost:.4f}, Val Cost:
{val_cost:.4f}")

    return W1, b1, W2, b2, W3, b3, train_costs, val_costs

def predict_nn2(X, W1, b1, W2, b2, W3, b3):
    A1 = relu(np.dot(X, W1) + b1)
    A2 = relu(np.dot(A1, W2) + b2)
    A3 = sigmoid(np.dot(A2, W3) + b3)
    return (A3 > 0.5).astype(int)

W1_2, b1_2, W2_2, b2_2, W3_2, b3_2, train_costs_2, val_costs_2 =
nn_two_hidden(
    X_train, Y_train, X_cv, Y_cv, alpha=0.02, num_iter=2000
)

# Accuracy
print("Training Accuracy:", np.mean(predict_nn2(X_train, W1_2, b1_2,
W2_2, b2_2, W3_2, b3_2) == Y_train))
print("Validation Accuracy:", np.mean(predict_nn2(X_cv, W1_2, b1_2,
W2_2, b2_2, W3_2, b3_2) == Y_cv))

# Plot Cost
plt.plot(train_costs_2, label='Train Cost')
plt.plot(val_costs_2, label='Val Cost')
plt.title("2-Hidden-Layer NN Cost")
plt.xlabel("Every 100 Iterations")
plt.ylabel("Cost")
plt.legend()
plt.show()
```

```
Iteration 0 - Train Cost: 0.6936, Val Cost: 0.6922
Iteration 100 - Train Cost: 0.6877, Val Cost: 0.6859
Iteration 200 - Train Cost: 0.6841, Val Cost: 0.6842
Iteration 300 - Train Cost: 0.6822, Val Cost: 0.6823
Iteration 400 - Train Cost: 0.6814, Val Cost: 0.6817
Iteration 500 - Train Cost: 0.6811, Val Cost: 0.6814
Iteration 600 - Train Cost: 0.6808, Val Cost: 0.6814
Iteration 700 - Train Cost: 0.6806, Val Cost: 0.6815
Iteration 800 - Train Cost: 0.6805, Val Cost: 0.6817
Iteration 900 - Train Cost: 0.6804, Val Cost: 0.6818
Iteration 1000 - Train Cost: 0.6804, Val Cost: 0.6819
Iteration 1100 - Train Cost: 0.6804, Val Cost: 0.6821
Iteration 1200 - Train Cost: 0.6803, Val Cost: 0.6821
Iteration 1300 - Train Cost: 0.6803, Val Cost: 0.6822
Iteration 1400 - Train Cost: 0.6803, Val Cost: 0.6822
Iteration 1500 - Train Cost: 0.6803, Val Cost: 0.6823
Iteration 1600 - Train Cost: 0.6803, Val Cost: 0.6823
```

```
Iteration 1700 - Train Cost: 0.6803, Val Cost: 0.6824
Iteration 1800 - Train Cost: 0.6803, Val Cost: 0.6824
Iteration 1900 - Train Cost: 0.6803, Val Cost: 0.6823
Training Accuracy: 0.5783684692942255
Validation Accuracy: 0.5741758241758241
```