Medan-medan Gravitoelektromagnetik pada Ruang Waktu Vaidya Diperumum dalam Teori Gravitasi Teleparalel

In[1]:=
```
Get["https://raw.githubusercontent.com/bshoshany/OGRe/master/OGRe.m"]
```

OGRe: An Object-Oriented General Relativity Package for Mathematica

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GitHub repository: https://github.com/bshoshany/OGRe

OGRe:
- To view the full documentation for the package, type TDocs[].
- To list all available modules, type ?OGRe`*.
- To get help on a particular module, type ? followed by the module name.
- To enable parallelization, type TSetParallelization[True].
- UpdateMessage
  To disable automatic checks for updates at startup, type TSetAutoUpdates[False].

Mendefinisikan Sistem Koordinat

In[2]:=
```
TNewCoordinates["Eddington", {v, r, θ, ϕ}]
```

Out[2]= Eddington

Mendefinisikan tensor metrik ruang singgung (Minkowski)

In[3]:=
```
TShow@TNewMetric["Minkowski", "Eddington", DiagonalMatrix[{1, -1, -1, -1}], "η"]
```

OGRe: Minkowski: $\eta_{\mu\nu}(v, r, \theta, \phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

Inverse bagi tensor metrik ruang singgung

In[4]:=
```
TShow@TNewTensor["InvMin", "Minkowski",
  "Eddington", {1, 1}, DiagonalMatrix[{1, -1, -1, -1}], "η"]
```

OGRe: InvMin: $\eta^{\mu\nu}(v, r, \theta, \phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

Mendefinisikan tetrad ansatz dan inversenya

In[5]:=
```
TShow@TNewTensor["Tetrad", "Minkowski",
 "Eddington", {1, -1}, {{Sqrt[2]*Δ[v, r], -(Sqrt[2] + 1)/Δ[v, r], 0, 0},
  {Δ[v, r], -(Sqrt[2] + 1)/Δ[v, r], 0, 0}, {0, 0, r, 0}, {0, 0, 0, r*Sin[θ]}}, "h"]
```

OGRe: Tetrad: $h^{\mu}{}_{\nu}(v, r, \theta, \phi) = \begin{pmatrix} \sqrt{2}\,\Delta[v, r] & -\frac{1+\sqrt{2}}{\Delta[v,r]} & 0 & 0 \\ \Delta[v, r] & -\frac{1+\sqrt{2}}{\Delta[v,r]} & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & r\,\mathrm{Sin}[\theta] \end{pmatrix}$

In[6]:=
```
TShow@TNewTensor["InvTetrad", "Minkowski",
 "Eddington", {-1, 1},
 {{(1 + Sqrt[2])/Δ[v, r], Δ[v, r], 0, 0}, {-((1 + Sqrt[2])/Δ[v, r]), -Sqrt[2] Δ[v, r], 0, 0},
  {0, 0, 1/r, 0}, {0, 0, 0, Csc[θ]/r}}, "h"]
```

OGRe: InvTetrad: $h_{\mu}{}^{\nu}(v, r, \theta, \phi) = \begin{pmatrix} \frac{1+\sqrt{2}}{\Delta[v,r]} & \Delta[v, r] & 0 & 0 \\ -\frac{1+\sqrt{2}}{\Delta[v,r]} & -\sqrt{2}\,\Delta[v, r] & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & \frac{\mathrm{Csc}[\theta]}{r} \end{pmatrix}$

Melakukan pengecekkan, apakah tetrad dan inversenya saling ortogonal

In[7]:=
```
TShow@TCalc["Hasil Kali Tetrad", "Tetrad"["aμ"].“InvTetrad"["aν"]]
```

OGRe: Hasil Kali Tetrad: $\square_{\mu}{}^{\nu}(v, r, \theta, \phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Mengonstruksi metrik Vaidya diperumum melalui kontraksi metrik Minkowski dengan tetrad

In[8]:=
```
TShow@TCalc["SpaceTimeMetric", "Minkowski"["ab"].“Tetrad"["aμ"].“Tetrad"["bν"], "g"]
```

OGRe: SpaceTimeMetric: $g_{\mu\nu}(v, r, \theta, \phi) = \begin{pmatrix} \Delta[v, r]^2 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \operatorname{Sin}[\theta]^2 \end{pmatrix}$

Mendefinisikan tetrad referensi dan inversenya

In[12]:=
```
TShow@TNewTensor["TetradRef", "Minkowski",
  "Eddington", {1, -1}, {{Sqrt[2], -(Sqrt[2] + 1), 0, 0},
   {1, -(Sqrt[2] + 1), 0, 0}, {0, 0, r, 0}, {0, 0, 0, r*Sin[θ]}}, "e"]
```

OGRe: TetradRef: $e^{\mu}{}_{\nu}(v, r, \theta, \phi) = \begin{pmatrix} \sqrt{2} & -1-\sqrt{2} & 0 & 0 \\ 1 & -1-\sqrt{2} & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & r \operatorname{Sin}[\theta] \end{pmatrix}$

In[13]:=
```
TShow@TNewTensor["InvTetradRef", "Minkowski", "Eddington", {-1, 1}, {{(1 + Sqrt[2])/1,
  1, 0, 0}, {-((1 + Sqrt[2])/1), -Sqrt[2] 1, 0, 0}, {0, 0, 1/r, 0}, {0, 0, 0, Csc[θ]/r}}, "e"]
```

OGRe: InvTetradRef: $e_{\mu}{}^{\nu}(v, r, \theta, \phi) = \begin{pmatrix} 1+\sqrt{2} & 1 & 0 & 0 \\ -1-\sqrt{2} & -\sqrt{2} & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & \frac{\operatorname{Csc}[\theta]}{r} \end{pmatrix}$

Menghitung koefisien anholonomi terkait tetrad referensi

In[14]:=
```
TShow@TCalc[
 "Anholonomi", -(TPartialD["ν"] . "TetradRef"["λμ"] - TPartialD["μ"] . "TetradRef"["λν"]) .
   "InvTetradRef"["αμ"] . "InvTetradRef"["βν"], "f"]
```

OGRe: Anholonomi: $f^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) = \begin{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ \frac{1}{r} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ -\frac{\sqrt{2}}{r} \\ 0 \end{pmatrix} & \begin{pmatrix} -\frac{1}{r} \\ \frac{\sqrt{2}}{r} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{r} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{\sqrt{2}}{r} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{\text{Cot}[\theta]}{r} \end{pmatrix} & \begin{pmatrix} -\frac{1}{r} \\ \frac{\sqrt{2}}{r} \\ -\frac{\text{Cot}[\theta]}{r} \\ 0 \end{pmatrix} \end{pmatrix}$

Menghitung koneksi spin terkait tetrad referensi

In[15]:=
```
TShow@TCalc[
 "KoneksiSpin", (1/2)*
   (-"Anholonomi"["μνρ"] + "Anholonomi"["νμρ"] + "Anholonomi"["ρμν"]).“TetradRef"["ρσ"], "ω"]
```

OGRe: KoneksiSpin: $\omega^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) = \begin{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \mathrm{Sin}[\theta] \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ \sqrt{2} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \sqrt{2}\,\mathrm{Sin}[\theta] \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ -\sqrt{2} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\mathrm{Cos}[\theta] \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ \mathrm{Sin}[\theta] \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\sqrt{2}\,\mathrm{Sin}[\theta] \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \mathrm{Cos}[\theta] \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \end{pmatrix}$

In[16]:=
```
TList["KoneksiSpin"]
```

OGRe:

KoneksiSpin:

$\omega^{v}{}_{\theta\theta} = \omega^{\theta}{}_{v\theta} = 1$

$\omega^{v}{}_{\phi\phi} = \omega^{\phi}{}_{v\phi} = \mathrm{Sin}[\theta]$

$\omega^{r}{}_{\theta\theta} = -\omega^{\theta}{}_{r\theta} = \sqrt{2}$

$\omega^{r}{}_{\phi\phi} = -\omega^{\phi}{}_{r\phi} = \sqrt{2}\,\mathrm{Sin}[\theta]$

$\omega^{\theta}{}_{\phi\phi} = -\omega^{\phi}{}_{\theta\phi} = -\mathrm{Cos}[\theta]$

Menghitung koneksi Weitzenböck (dengan indeks kontravariannya mewakili ruang singgung)

In[17]:=
```
TShow@TCalc["Weitzenbock", TPartialD["μ"].
 "Tetrad"["aν"] + "KoneksiSpin"["abμ"].  "Tetrad"["bν"], "W"]
```

OGRe: Weitzenbock: $W^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) =$

$$\begin{pmatrix} \begin{pmatrix} \sqrt{2}\,\partial_v \Delta[v,r] \\ \frac{(1+\sqrt{2})\,\partial_v \Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \sqrt{2}\,\partial_r \Delta[v,r] \\ \frac{(1+\sqrt{2})\,\partial_r \Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ r \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ r\,\mathrm{Sin}[\theta]^2 \end{pmatrix} \\ \begin{pmatrix} \partial_v \Delta[v,r] \\ \frac{(1+\sqrt{2})\,\partial_v \Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \partial_r \Delta[v,r] \\ \frac{(1+\sqrt{2})\,\partial_r \Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ \sqrt{2}\,r \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \sqrt{2}\,r\,\mathrm{Sin}[\theta]^2 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ \frac{1}{\Delta[v,r]} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ -r\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta] \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \mathrm{Sin}[\theta] \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ r\,\mathrm{Cos}[\theta] \end{pmatrix} & \begin{pmatrix} 0 \\ \frac{\mathrm{Sin}[\theta]}{\Delta[v,r]} \\ r\,\mathrm{Cos}[\theta] \\ 0 \end{pmatrix} \end{pmatrix}$$

Menghitung torsi

In[18]:=
```
TShow@TCalc["Torsi", "Weitzenbock"["aνμ"] - "Weitzenbock"["aμν"], "T"]
```

OGRe: Torsi: $T^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) = \begin{pmatrix} \begin{pmatrix} 0 \\ \sqrt{2}\, \partial_r \Delta[v, r] - \frac{(1+\sqrt{2})\partial_v \Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} -\sqrt{2}\, \partial_r \Delta[v, r] + \frac{(1+\sqrt{2})\partial_v \Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ \partial_r \Delta[v, r] - \frac{(1+\sqrt{2})\partial_v \Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} -\partial_r \Delta[v, r] + \frac{(1+\sqrt{2})\partial_v \Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ -1 + \frac{1}{\Delta[v,r]} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 1 - \frac{1}{\Delta[v,r]} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \mathrm{Sin}[\theta]\left(-1 + \frac{1}{\Delta[v,r]}\right) \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ \mathrm{Sin}[\theta] - \frac{\mathrm{Sin}[\theta]}{\Delta[v,r]} \\ 0 \\ 0 \end{pmatrix} \end{pmatrix}$

Menyesuaikan torsi dengan hitungan manual

In[19]:= `TShow@TCalc["TorsiBener", (-1)*"Torsi"["aμν"], "T"]`

OGRe: TorsiBener: $T^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) =$

$$\begin{pmatrix} \begin{pmatrix} 0 \\ -\sqrt{2}\,\partial_r\Delta[v,r] + \frac{(1+\sqrt{2})\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \sqrt{2}\,\partial_r\Delta[v,r] - \frac{(1+\sqrt{2})\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ -\partial_r\Delta[v,r] + \frac{(1+\sqrt{2})\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \partial_r\Delta[v,r] - \frac{(1+\sqrt{2})\partial_v\Delta[v,r]}{\Delta[v,r]^2} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 1 - \frac{1}{\Delta[v,r]} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ -1 + \frac{1}{\Delta[v,r]} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\mathrm{Sin}[\theta]\left(-1 + \frac{1}{\Delta[v,r]}\right) \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ \mathrm{Sin}[\theta]\left(-1 + \frac{1}{\Delta[v,r]}\right) \\ 0 \\ 0 \end{pmatrix} \end{pmatrix}$$

Menentukan torsi ruang waktu dengan melakukan kontraksi torsi yang telah diperoleh dengan tetrad

In[20]:=
```
TShow@TCalc["TorsiRW", "InvTetrad"["aρ"] . "TorsiBener"["aμν"], "T"]
```

OGRe: TorsiRW: $T^{\mu}{}_{\nu\rho}(v, r, \theta, \phi) = \begin{pmatrix} \begin{pmatrix} 0 \\ -\frac{\partial_r \Delta[v,r]}{\Delta[v,r]} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{\partial_r \Delta[v,r]}{\Delta[v,r]} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ -\frac{\partial_v \Delta[v,r]}{\Delta[v,r]} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{\partial_v \Delta[v,r]}{\Delta[v,r]} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ \frac{1-\frac{1}{\Delta[v,r]}}{r} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ \frac{-1+\frac{1}{\Delta[v,r]}}{r} \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{-1+\frac{1}{\Delta[v,r]}}{r} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ \frac{-1+\frac{1}{\Delta[v,r]}}{r} \\ 0 \\ 0 \end{pmatrix} \end{pmatrix}$

Mentransformasikan torsi dari sistem koordinat Eddington-Finkelstein ke Kartesius

Mula-mula, kita definisikan sistem koordinat Kartesius

In[21]:=
```
TNewCoordinates["Cartesian", {t, x, y, z}]
```

Out[21]=
```
Cartesian
```

Definisikan aturan transformasi dari sistem koordinat Eddington-Finkelstein ke Kartesius

In[22]:=
```
TAddCoordTransformation[
 "Eddington" → "Cartesian", {v → t + r + 2*M*Log[Abs[r/(2*M) - 1]],
  r → Sqrt[x^2 + y^2 + z^2], θ →
   ArcCos[z / Sqrt[x^2 + y^2 + z^2]], ϕ → ArcTan[y/x]}]
```

Out[22]=
```
Eddington
```

Tampilkan torsi ruang waktu ke sistem koordinat Kartesius

In[23]:= `TShow["TorsiRW", "Cartesian"]`

OGRe: TorsiRW: $T^{\mu}{}_{\nu\rho}(t, x, y, z) =$

$$\begin{pmatrix}
\begin{pmatrix} 0 \\ -\frac{x\,\partial_{\sqrt{x^2+y^2+z^2}}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{\sqrt{x^2+y^2+z^2}\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\ -\frac{y\,\partial_{\sqrt{x^2+y^2+z^2}}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{\sqrt{x^2+y^2+z^2}\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\ -\frac{z\,\partial_{\sqrt{x^2+y^2+z^2}}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{\sqrt{x^2+y^2+z^2}\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \end{pmatrix} &
\begin{pmatrix} \frac{x\,\partial_{\sqrt{x^2+y^2+z^2}}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{\sqrt{x^2+y^2+z^2}\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\ 0 \\ 0 \\ 0 \end{pmatrix} & \cdots \\
\begin{pmatrix} 0 \\ -\frac{x^2\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\ -\frac{x\,y\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\ -\frac{x\,z\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \end{pmatrix} &
\begin{pmatrix} \frac{x^2\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\ 0 \\ \frac{y\left(-1+\frac{1}{\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}\right)}{x^2+y^2+z^2} \\ \frac{z\left(-1+\frac{1}{\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}\right)}{x^2+y^2+z^2} \end{pmatrix} & \cdots \\
\begin{pmatrix} 0 \\ -\frac{x\,y\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\ -\frac{y^2\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\ -\frac{y\,z\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \end{pmatrix} &
\begin{pmatrix} \frac{x\,y\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\ 0 \\ \frac{x\left(1-\frac{1}{\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}\right)}{x^2+y^2+z^2} \\ 0 \end{pmatrix} & \cdots \\
\begin{pmatrix} 0 \\ -\frac{x\,z\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\ -\frac{y\,z\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\ -\frac{z^2\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \end{pmatrix} &
\begin{pmatrix} \frac{x\,z\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\ 0 \\ 0 \\ \frac{x\left(1-\frac{1}{\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}\right)}{x^2+y^2+z^2} \end{pmatrix} & \cdots
\end{pmatrix}$$

Menentukan torsi (dengan indeks kontravariannya mewakili ruang singgung) dalam sistem koordinat Kartesius

Mula-mula, definisikan tetrad ansatz yang telah ditransformasi ke sistem

koordinat Kartesius

In[24]:=
```
TShow@TNewTensor["TetradCart", "Minkowski",
 "Cartesian", {1, -1}, {{Sqrt[2]*Δ[v, r], x*P, y*P, z*P},
   {Δ, x*R, y*R, z*R}, {0, x*z/(r*U), y*z/(r*U), U/r}, {0, -y/U, x/U, 0}}, "h"]
```

OGRe: TetradCart: $h^{\mu}{}_{\nu}(t, x, y, z) = \begin{pmatrix} \sqrt{2}\,\Delta[v, r] & P\,x & P\,y & P\,z \\ \Delta & R\,x & R\,y & R\,z \\ 0 & \frac{x\,z}{r\,U} & \frac{y\,z}{r\,U} & \frac{U}{r} \\ 0 & -\frac{y}{U} & \frac{x}{U} & 0 \end{pmatrix}$

Kontraksikan tetrad tersebut dengan torsi ruang waktu dalam sistem koordinat Kartesius

In[25]:= `TShow@TCalc["TetradCart"["aρ"]."TorsiRW"["ρμν"], "T"]`

OGRe: Result: $T^{\mu}{}_{\nu\rho}(t,x,y,z) =$

$$\left(\begin{matrix}
\begin{pmatrix}
0 \\
-\frac{x\left(\sqrt{2}\,\Delta[v,r]\,\partial_{\sqrt{x^2+y^2+z^2}}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]+P\sqrt{x^2+y^2+z^2}\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right.\right.}{\sqrt{x^2+y^2+z^2}\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\
-\frac{y\left(\sqrt{2}\,\Delta[v,r]\,\partial_{\sqrt{x^2+y^2+z^2}}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]+P\sqrt{x^2+y^2+z^2}\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right.\right.}{\sqrt{x^2+y^2+z^2}\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\
-\frac{z\left(\sqrt{2}\,\Delta[v,r]\,\partial_{\sqrt{x^2+y^2+z^2}}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]+P\sqrt{x^2+y^2+z^2}\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right.\right.}{\sqrt{x^2+y^2+z^2}\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}
\end{pmatrix} \\
\begin{pmatrix}
0 \\
-\frac{x\left(\Delta\,\partial_{\sqrt{x^2+y^2+z^2}}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]+R\sqrt{x^2+y^2+z^2}\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]\right)}{\sqrt{x^2+y^2+z^2}\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\
-\frac{y\left(\Delta\,\partial_{\sqrt{x^2+y^2+z^2}}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]+R\sqrt{x^2+y^2+z^2}\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]\right)}{\sqrt{x^2+y^2+z^2}\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\
-\frac{z\left(\Delta\,\partial_{\sqrt{x^2+y^2+z^2}}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]+R\sqrt{x^2+y^2+z^2}\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]\right)}{\sqrt{x^2+y^2+z^2}\,\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}
\end{pmatrix} \\
\begin{pmatrix}
0 \\
-\frac{x\left(U^2+x^2+y^2\right)z\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{r\,U\left(x^2+y^2+z^2\right)\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\
-\frac{y\left(U^2+x^2+y^2\right)z\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{r\,U\left(x^2+y^2+z^2\right)\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]} \\
-\frac{\left(U^2+x^2+y^2\right)z^2\,\partial_{r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right]}\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}{r\,U\left(x^2+y^2+z^2\right)\Delta\left[r+t+2M\,\mathrm{Log}\left[1-\frac{r}{2M}\right],\sqrt{x^2+y^2+z^2}\right]}
\end{pmatrix} \\
\begin{pmatrix}
0 \\ 0 \\ 0 \\ 0
\end{pmatrix}
\end{matrix}\right)$$