Minimal Background on Hidden Markov Models

Robert W Rankin and Krista E Nicholson

Georgetown University,Murdoch University

October 30, 2017

Hidden Markov Models (HMM)

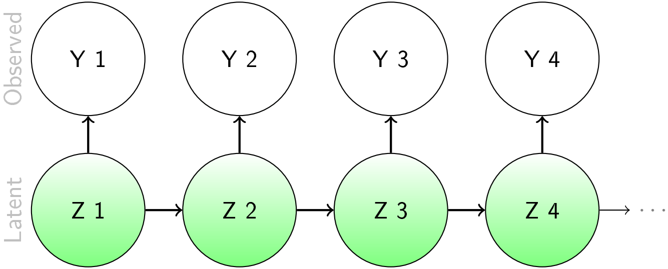
- provides a unified approach to capture-mark-recapture models
- essential for JAGS CMR models
- widely used framework outside CMR

H M M

- Hidden: the process model is *latent*, i.e., unobservable
- Markov: the dependence between states Z_t at t and Z_{t+1} at $t+1$ has order 1.

Latent States

separate the observed error (Y) from the process model
a.k.a, separate the observed event from the latent state



Why do HMM?

- decomposes the complex product-multinomial distribution of capture histories into a series of simple distributions.

e.g.,

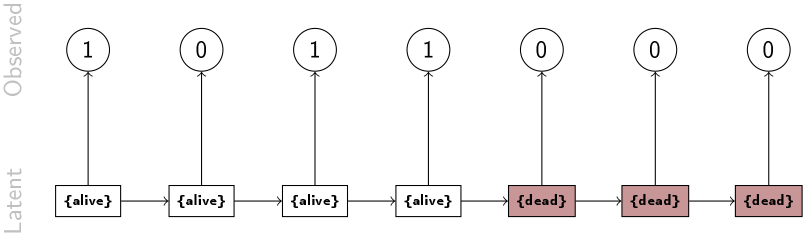
$$p(y_t = \text{capture}|z_t) = \begin{cases} p_t, & \text{if } z_t = \{\text{alive}\}, \\ 0, & \text{if } z_t = \{\text{dead}\} \end{cases} \tag{1}$$

- calculate biological (derived) parameters (N_t, births$_t$, super-population) as simple summaries of latent states

$$\begin{aligned} N_t &= \sum_{i=1}^{M} \mathbb{I}[z_t = \{\text{alive}\}] \\ B_t &= \sum_{i=1}^{M} (\mathbb{I}[z_t = \{\text{alive}\}] \cdot [z_{t-1} = \{\text{unrecruited}\}]) \end{aligned} \tag{2}$$

Latent States: Example (CJS)

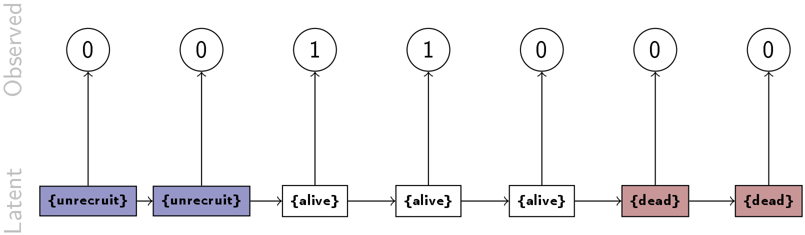
- example capture history ($\mathbf{y} = \{1011000\}$) and plausible latent state sequence (Z)



(This is the Cormack-Jolly-Seber model)

Latent States: Example (POPAN)

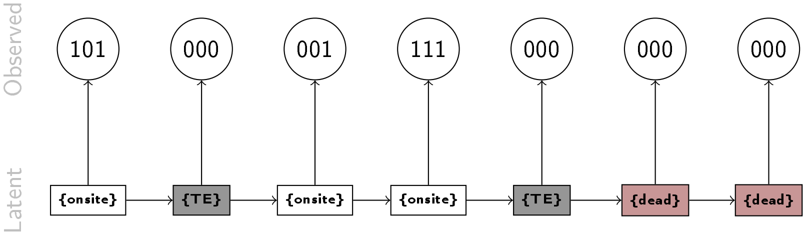
- example capture history ($\mathbf{y} = \{0011000\}$) and plausible latent state sequence (Z)



(This could be the POPAN model)

Latent States: Example (PCRD)

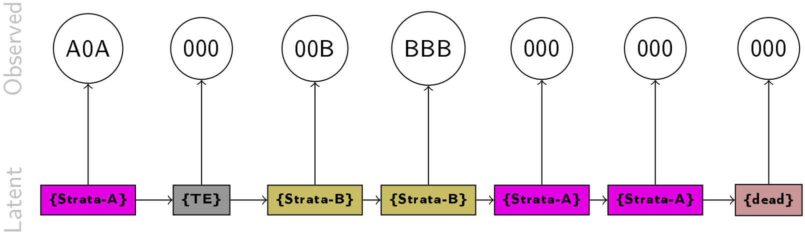
- example capture history ($\mathbf{y} = \{101000001111000000000\}$) and plausible latent state sequence



(This is the Pollock's Closed Robust Design model with 3 secondary periods)

Latent States: Example (MSCRD)

- example capture history ($\mathbf{y} = \{A0A00000BBBB000000000\}$) and latent state sequence



(This is a Multistate Robust Design Model with 2 strata (A,B) and Temporary Emigration (TE) and 3 secondary periods per primary period

HMM: A Unifying Framework

Many capture-mark-recapture models can be formulated as a type of HMM, e.g.,

- Cormack-Jolly-Seber
- POPAN
- Pollock's Closed Robust Design
- Multistate models

Mastering the HMM framework opens-up a lot of possibilities for standard and customized models

Stochastic Latent States

Quick Note:

- Latent states are *stochastic*.
- They are random variables, like ϕ,p,γ,ψ,etc.

Each capture history has a *distribution* of plausible latent states sequences
For example:

$$\mathbf{y} = \{1011000\}$$

could have latent state sequences...

$$\mathbf{z}^* \in \begin{cases} \{1111111\} \\ \{1111112\} \\ \{1111122\} \\ \{1111222\} \\ \dots \\ \{\cancel{1112222}\} \\ \{\cancel{2211111}\} \end{cases} \tag{3}$$

... where $1 \equiv \text{alive}, 2 \equiv \text{dead}$

Gibbs Sampling Latent States

- if we knew the latent states, we could easily estimate survival, births, etc.
- instead we marginalize over the latent states via Gibbs Sampling

i.e., sample $\mathbf{Z}$, impute their values and to update $\boldsymbol{\theta}$
a.k.a Monte-Carlo integration

Gibbs Sampling HMM

within each j MCMC iteration, we ...

- 1) Sample parameters conditional on latent states $\mathbf{Z}$ and data $\mathbf{Y}$

$$\boldsymbol{\theta}^{(j+1)} \sim \pi(\boldsymbol{\theta}|\mathbf{Z}^{(j)}, \mathbf{Y}) \tag{4}$$

- 2) Sample latent states conditional on parameters $\boldsymbol{\theta}$ and data $\mathbf{Y}$

$$\mathbf{Z}^{(j+1)} \sim \pi(\mathbf{Z}|\boldsymbol{\theta}^{(j+1)}, \mathbf{Y}) \tag{5}$$

- 3) repeat for all MCMC iterations

Gibbs Sampling Latent States in JAGS

for JAGS scripts, we must:

- specify Priors (like before)

```
# priors
p ~ dbeta(a,b)
```

- specify the Conditional Data Likelihood (like before, but conditional on the latent states)

```
# likelihood
y[t] ~ dbern(p * equals(z[t],1))
```

- AND specify the Markov latent state process (new)

```
# latent state process
z[t] ~ dcat(transition.matrix[,z[t-1]])
```

Enter the Matrix

To understand HMM models, you must master the matrix

Transition Matrix

- governs the probabilistic transition: $z_t \rightarrow z_{t+1}$

$$\boldsymbol{\Phi} = \begin{array}{l} \\ \textbf{Unborn} \\ \textbf{Alive} \\ \textbf{Dead} \end{array} \begin{array}{c} \begin{array}{ccc} \textbf{Unborn} & \textbf{Alive} & \textbf{Dead} \end{array} \\ \begin{pmatrix} .7 & 0 & 0 \\ .3 & .9 & 0 \\ 0 & .1 & 1 \end{pmatrix} \end{array}$$

- read: from *columns* (t) to *rows* ($t+1$)
- columns sum to 1

Emission Matrix

- governs capture process $p(y_t|z_t)$ (i.e., probability of a capture, given each latent state)

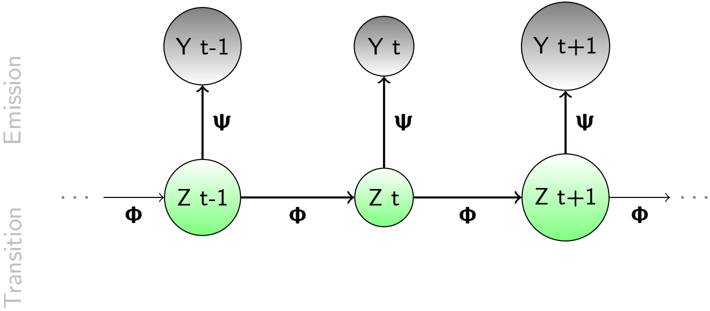
$$\boldsymbol{\Psi} = \begin{array}{l} \\ \textbf{Capture} \\ \textbf{No Capture} \end{array} \begin{array}{c} \begin{array}{ccc} \textbf{Unborn} & \textbf{Alive} & \textbf{Dead} \end{array} \\ \begin{pmatrix} 0 & .5 & .0 \\ 1 & .5 & 1 \end{pmatrix} \end{array}$$

- read: from *columns* (z at t) to *row* (y at t)
- columns sum to 1

HMM is a Bunch of Matrices

$$\boldsymbol{\Psi} = \begin{array}{l} \\ \textbf{Capture} \\ \textbf{No Capture} \end{array} \begin{array}{c} \begin{array}{ccc} \textbf{Unborn} & \textbf{Alive} & \textbf{Dead} \end{array} \\ \left(\begin{array}{ccc} 0 & .5 & .0 \\ 1 & .5 & 1 \end{array} \right) \end{array}, \quad \boldsymbol{\Phi} = \begin{array}{l} \\ \textbf{Unborn} \\ \textbf{Alive} \\ \textbf{Dead} \end{array} \begin{array}{c} \begin{array}{ccc} \textbf{Unborn} & \textbf{Alive} & \textbf{Dead} \end{array} \\ \left(\begin{array}{ccc} .7 & 0 & 0 \\ .3 & .9 & 0 \\ 0 & .1 & 1 \end{array} \right) \end{array}$$

\ \



- Read: FROM columns TO rows
- example: Let's say $z_{t-1} = \{\text{Unborn}\}$ at $t-1$

What is $p(z_t = \{\text{Alive}\} | z_{t-1} = \{\text{Unborn}\}, \mathbf{\Phi})$?

$$\mathbf{\Phi} = \begin{array}{l} \\ \textit{Unborn} \\ \textit{Alive} \\ \textit{Dead} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Unborn} & \textit{Alive} & \textit{Dead} \end{array} \\ \left(\begin{array}{ccc} .7 & 0 & 0 \\ \mathbf{.3} & .9 & 0 \\ 0 & .1 & 1 \end{array} \right) \end{array}$$

What is $p(y_t = \{\text{Capture}\} | z_t = \{\text{Alive}\}, \mathbf{\Psi})$?

$$\mathbf{\Psi} = \begin{array}{l} \\ \textit{Capture} \\ \textit{No Capt} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Unborn} & \textit{Alive} & \textit{Dead} \end{array} \\ \left(\begin{array}{ccc} 0 & \mathbf{.5} & .0 \\ 1 & .5 & 1 \end{array} \right) \end{array}$$

The matrices merely codify the conditional probabilities of the HMM processes

Understanding Transition and Emission Matrices (2)

- Likewise, let's say $z_t = \{\text{Alive}\}$ at t

$$\mathbf{\Phi} = \begin{array}{c} \\ \textit{Unborn} \\ \textit{Alive} \\ \textit{Dead} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Unborn} & \textit{Alive} & \textit{Dead} \end{array} \\ \begin{pmatrix} .7 & 0 & 0 \\ .3 & .9 & 0 \\ 0 & .1 & 1 \end{pmatrix} \end{array}$$

Translation: IF z_t is in state $\{\text{Alive}\}$ at time t, then there is:
· 0 chance of returning to $\{\text{Unborn}\}$,
· 0.9 chance of staying $\{\text{Alive}\}$,
· 0.1 chance of going to $\{\text{Dead}\}$

$$\mathbf{\Psi} = \begin{array}{c} \\ \textit{Capture} \\ \textit{No Capt} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Unborn} & \textit{Alive} & \textit{Dead} \end{array} \\ \begin{pmatrix} 0 & .5 & .0 \\ 1 & .5 & 1 \end{pmatrix} \end{array}$$

Translation: IF z_t is in state $\{\text{Alive}\}$ at time t, then there is:
· 50% chance getting a $\{\text{Capture}\}$,
· 50% chance of getting $\{\text{No Capture}\}$

Let's do a worked example!

A demonstration of the data-generating process for a POPAN-like recruitment model

3 latent states:

- $1 \equiv$ unborn (or pre-immigrant),
- $2 \equiv$ alive (and marked),
- $3 \equiv$ dead (or permanent-emigrant)

1 capture history to generate

7 primary periods

time-invariant recruitment[1], survival and capture-probabilities

[1] In practise, the recruitment parameters are ALWAYS time-varying

Generative Model for POPAN (2)

Initialize: $z_0 = 1$ at $t = 0$ ({unborn}).
No captures in t=0
done!

t_0 dummy state

Generative Model for POPAN (3)

Sample: $z_1|z_0$ at $t=1$.
Use column 1 from $\mathbf{\Phi}[,1]$ (because $z_0 = 1$)
Use the Multinomial distribution (a.k.a categorical)

$$\mathbf{\Phi} = \begin{array}{cc} & \begin{array}{ccc} \textit{Unborn} & \textit{Alive} & \textit{Dead} \end{array} \\ \begin{array}{c} \textit{Unborn} \\ \textit{Alive} \\ \textit{Dead} \end{array} & \begin{pmatrix} .7 & 0 & 0 \\ .3 & .9 & 0 \\ 0 & .1 & 1 \end{pmatrix} \end{array}$$

$\boxed{\textbf{unborn}} \rightarrow z_1 \sim \textbf{Mult}([.7,.3,0])$

t_0 dummy state

t_1

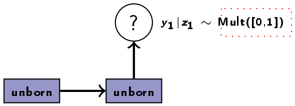
Generative Model for POPAN (4)

We got $z_1 = 1$! (unborn)!

Next: Sample a capture $y_1|z_1$
Use column 1 from $\mathbf{\Psi}[,1]$ (because $z_1 = 1$)
Use the Multinomial Distribution

$$\mathbf{\Phi} = \begin{array}{c} \\ \textit{Unborn} \\ \textit{Alive} \\ \textit{Dead} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Unborn} & \textit{Alive} & \textit{Dead} \end{array} \\ \begin{pmatrix} .7 & 0 & 0 \\ .3 & .9 & 0 \\ 0 & .1 & 1 \end{pmatrix} \end{array}$$

$$\Psi = \begin{array}{c} \\ \textit{Capture} \\ \textit{No Capt} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Unborn} & \textit{Alive} & \textit{Dead} \end{array} \\ \begin{pmatrix} 0 & .5 & .0 \\ 1 & .5 & 1 \end{pmatrix} \end{array}$$



t_0 dummy state

t_1

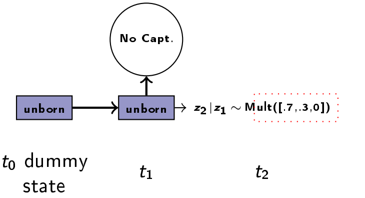
Generative Model for POPAN (5)

We got $y_1 = 2$! (No-capture)

Next: sample $z_2|z_1$ at $t = 2$.
Use column 1 from $\mathbf{\Phi}[,1]$ (because $z_1 = 1$)

$$\mathbf{\Phi} = \begin{array}{c} \\ \textit{Unborn} \\ \textit{Alive} \\ \textit{Dead} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Unborn} & \textit{Alive} & \textit{Dead} \end{array} \\ \begin{pmatrix} .7 & 0 & 0 \\ .3 & .9 & 0 \\ 0 & .1 & 1 \end{pmatrix} \end{array}$$

$$\mathbf{\Psi} = \begin{array}{c} \\ \textit{Capture} \\ \textit{No Capt} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Unborn} & \textit{Alive} & \textit{Dead} \end{array} \\ \begin{pmatrix} 0 & .5 & .0 \\ 1 & .5 & 1 \end{pmatrix} \end{array}$$



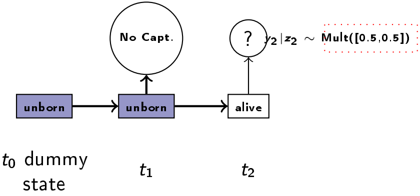
Generative Model for POPAN (6)

We got $z_2 = 2!$ (alive)

Next: Sample a capture $y_2|z_2$
Use column 2 from $\mathbf{\Psi}[,2]$ (because $z_1 = 2$)

$$\mathbf{\Phi} = \begin{array}{l} \\ \textit{Unborn} \\ \textit{Alive} \\ \textit{Dead} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Unborn} & \textit{Alive} & \textit{Dead} \end{array} \\ \begin{pmatrix} .7 & 0 & 0 \\ .3 & .9 & 0 \\ 0 & .1 & 1 \end{pmatrix} \end{array}$$

$$\Psi = \begin{array}{l} \\ \textit{Capture} \\ \textit{No Capt} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Unborn} & \textit{Alive} & \textit{Dead} \end{array} \\ \begin{pmatrix} 0 & .5 & .0 \\ 1 & .5 & 1 \end{pmatrix} \end{array}$$



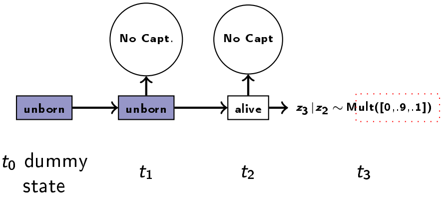
Generative Model for POPAN (7)

We got $y_2 = 2$! (No-capture)

Next: sample $z_3 | z_2$ at $t = 3$.
Use column 2 from $\mathbf{\Phi}[, 2]$ (because $z_2 = 2$)

$$\mathbf{\Phi} = \begin{array}{c c} & \begin{array}{ccc} \textit{Unborn} & \textit{Alive} & \textit{Dead} \end{array} \\ \begin{array}{c} \textit{Unborn} \\ \textit{Alive} \\ \textit{Dead} \end{array} & \begin{pmatrix} .7 & 0 & 0 \\ .3 & .9 & 0 \\ 0 & .1 & 1 \end{pmatrix} \end{array}$$

$$\Psi = \begin{array}{c c} & \begin{array}{ccc} \textit{Unborn} & \textit{Alive} & \textit{Dead} \end{array} \\ \begin{array}{c} \textit{Capture} \\ \textit{No Capt} \end{array} & \begin{pmatrix} 0 & .5 & .0 \\ 1 & .5 & 1 \end{pmatrix} \end{array}$$

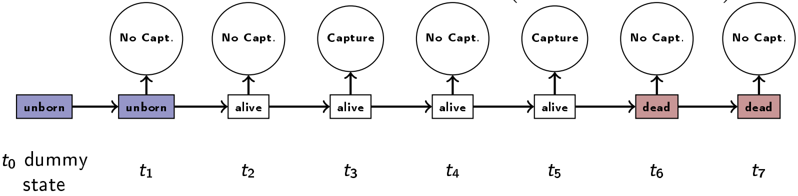


Generative Model for POPAN (n)

REPEAT UNTIL t=7

$$\Phi = \begin{array}{c} \\ \textit{Unborn} \\ \textit{Alive} \\ \textit{Dead} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Unborn} & \textit{Alive} & \textit{Dead} \end{array} \\ \begin{pmatrix} .7 & 0 & 0 \\ .3 & .9 & 0 \\ 0 & .1 & 1 \end{pmatrix} \end{array}$$

$$\Psi = \begin{array}{c} \\ \textit{Capture} \\ \textit{No Capt} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Unborn} & \textit{Alive} & \textit{Dead} \end{array} \\ \begin{pmatrix} 0 & .5 & .0 \\ 1 & .5 & 1 \end{pmatrix} \end{array}$$



- Generated capture history: $\{0, 0, 1, 0, 1, 0, 0\}$

conceptually simple generative HMM
in R, it would look like:

```
z<-numeric(7)
y<-numeric(7)
z[1]<-rmultinom(n=1,size=1,prob=PHI[,1]) # first primary
y[1]<-rmultinom(n=1,size=1,prob=PSI[,z[1]]) # first capture
for(t in 2:7){
  z[t]<-rmultinom(n=1,size=1,prob=PHI[,z[t-1]])
  y[t]<-rmultinom(n=1,size=1,prob=PSI[,z[t]])
}
```

Generate Model for POPAN

conceptually simple generative HMM

- generate: $\mathbf{y}^*$ given $\mathbf{\Phi}$ (*transition matrix*) and $\mathbf{\Psi}$ (*emission matrix*) (via z)

BUT What about inference about $\mathbf{\Phi}$ and $\mathbf{\Psi}$ for a *given* ($\mathbf{y}$)?

- Real Answer: Forwards-Backwards algorithm[2]
- Quick Answer JAGS will take care of it for us. . .

so long as we specify:

- priors
- conditional data likelihood
- latent state process

[2] Commonly referenced tutorial: Rabiner et al.

HMM Matrices: Connection with CMR (CJS)

How to specify the emission and transition matrices for a CMR model?
Each cell in the matrices are parameterized by products of CMR parameters
Example in Cormack Jolly Seber:

Transition Matrix

from $t-1$ to t

$$\mathbf{\Phi}_t = \begin{array}{c c} & \begin{array}{cc} \textbf{Alive} & \textbf{Dead} \end{array} \\ \begin{array}{c} \textbf{Alive} \\ \textbf{Dead} \end{array} & \begin{pmatrix} \phi_{t-1} & 0 \\ 1-\phi_{t-1} & 1 \end{pmatrix} \end{array}$$

where:

- ϕ_{t-1} is the apparent survival between $t-1$ and t

Emission Matrix

capture at t

$$\mathbf{\Psi}_t = \begin{array}{c c} & \begin{array}{cc} \textbf{Alive} & \textbf{Dead} \end{array} \\ \begin{array}{c} \textbf{Capture} \\ \textbf{No Capture} \end{array} & \begin{pmatrix} p_t & 0 \\ 1-p_t & 1 \end{pmatrix} \end{array}$$

where

- p_t is the capture-probability at t

HMM Matrices: Connection with CMR (POPAN)

How to specify the emission and transition matrices for a CMR model?

Each cell in the matrices are parameterized by products of CMR parameters
Example in POPAN:

Transition Matrix

from $t-1$ to t

$$\mathbf{\Phi}_t = \begin{array}{c|c} & \begin{array}{ccc} \textbf{Unborn} & \textbf{Alive} & \textbf{Dead} \end{array} \\ \begin{array}{c} \textbf{Unborn} \\ \textbf{Alive} \\ \textbf{Dead} \end{array} & \begin{pmatrix} 1-\psi_t & 0 & 0 \\ \psi_t & \phi_{t-1} & 0 \\ 0 & 1-\phi_{t-1} & 1 \end{pmatrix} \end{array}$$

where:

- ϕ_t is the apparent survival at t
- ψ_t are *related* to the POPAN pent

Emission Matrix

capture at t

$$\mathbf{\Psi}_t = \begin{array}{c|c} & \begin{array}{ccc} \textbf{Unborn} & \textbf{Alive} & \textbf{Dead} \end{array} \\ \begin{array}{c} \textbf{Capture} \\ \textbf{No Capture} \end{array} & \begin{pmatrix} 0 & p_t & 0 \\ 1 & 1-p_t & 1 \end{pmatrix} \end{array}$$

- p_t is the capture-probability at t

HMM Matrices: Connection with CMR (PCRD)

Example in the Pollock's Closed Robust Design (*conditioning on first capture*):

Transition Matrix

from $t-1$ to t

$$\boldsymbol{\Phi}_t = \begin{array}{c} \\ \textbf{Onsite} \\ \textbf{TE} \\ \textbf{Dead} \end{array} \begin{array}{c} \begin{array}{ccc} \textbf{Onsite} & \textbf{TE} & \textbf{Dead} \end{array} \\ \begin{pmatrix} \phi_{t-1}(1-\gamma''_t) & \phi_{t-1}(1-\gamma'_t) & 0 \\ \phi_{t-1}\gamma''_t & \phi_{t-1}\gamma'_t & 0 \\ 1-\phi_{t-1} & 1-\phi_{t-1} & 1 \end{pmatrix} \end{array}$$

- ϕ_{t-1} is the apparent survival between primary periods $t-1$ and t
- γ''_t is the probability of <u>becoming</u> a temporary emigrant between $t-1$ and t
- γ'_t is the probability of <u>staying</u> as a temporary emigrant between $t-1$ and t

Emission Matrix

capture at s_t

$$\boldsymbol{\Psi}_{s,t} = \begin{array}{c} \\ \textbf{Capture} \\ \textbf{No Capture} \end{array} \begin{array}{c} \begin{array}{ccc} \textbf{Onsite} & \textbf{TE} & \textbf{Dead} \end{array} \\ \begin{pmatrix} p_{s,t} & 0 & 0 \\ 1-p_{s,t} & 1 & 1 \end{pmatrix} \end{array}$$

- $p_{s,t}$ is the capture-probability at *secondary period* s_t

HMM Matrices: Connection with CMR (PCRD)

Example in the Pollock's Closed Robust Design (*full capture* model):

Transition Matrix

from $t-1$ to t

$$\mathbf{\Phi}_t = \begin{array}{c|cccc} & \textbf{Unborn} & \textbf{Onsite} & \textbf{TE} & \textbf{Dead} \\ \hline \textbf{Unborn} & 1-\psi_t & 0 & 0 & 0 \\ \textbf{Onsite} & \psi_t & \phi_{t-1}(1-\gamma''_t) & \phi_{t-1}(1-\gamma'_t) & 0 \\ \textbf{TE} & 0 & \phi_{t-1}\gamma''_t & \phi_{t-1}\gamma'_t & 0 \\ \textbf{Dead} & 0 & 1-\phi_{t-1} & 1-\phi_{t-1} & 1 \end{array}$$

- ϕ_{t-1} is the apparent survival between primary periods $t-1$ and t
- γ''_t is the probability of becoming a temporary emigrant between $t-1$ and t
- γ'_t is the probability of staying as a temporary emigrant between $t-1$ and t
- ψ_t are "recruitment" parameters between $t-1$ and t

Emission Matrix

$$\mathbf{\Psi}_{s,t} = \begin{array}{c|cccc} & \textbf{Unborn} & \textbf{Onsite} & \textbf{TE} & \textbf{Dead} \\ \hline \textbf{Capture} & 0 & p_{s,t} & 0 & 0 \\ \textbf{No Capture} & 1 & 1-p_{s,t} & 1 & 1 \end{array}$$

HMM Matrices: Connection with CMR (MSCRD)

Example in the Multi-State Closed Robust Design with 3 strata and TE (*first-capture* conditioning):

Transition Matrix

from $t-1$ to t

$$\mathbf{\Phi}_t = \begin{array}{l} \\ \textbf{Strata A} \\ \textbf{Strata B} \\ \textbf{Strata C} \\ \textbf{TE} \\ \textbf{Dead} \end{array} \begin{array}{c} \begin{array}{ccccc} \textbf{Strata A} & \textbf{Strata B} & \textbf{Strata C} & \textbf{TE} & \textbf{Dead} \end{array} \\ \begin{pmatrix} \phi_{t-1}\gamma_t^{aa} & \phi_{t-1}\gamma_t^{ba} & \phi_{t-1}\gamma_t^{ca} & \phi_{t-1}\gamma_t^{ta} & 0 \\ \phi_{t-1}\gamma_t^{ab} & \phi_{t-1}\gamma_t^{bb} & \phi_{t-1}\gamma_t^{cb} & \phi_{t-1}\gamma_t^{tb} & 0 \\ \phi_{t-1}\gamma_t^{ac} & \phi_{t-1}\gamma_t^{bc} & \phi_{t-1}\gamma_t^{cc} & \phi_{t-1}\gamma_t^{tc} & 0 \\ \phi_{t-1}\gamma_t^{at} & \phi_{t-1}\gamma_t^{bt} & \phi_{t-1}\gamma_t^{ct} & \phi_{t-1}\gamma_t^{tt} & 0 \\ 1-\phi_{t-1} & 1-\phi_{t-1} & 1-\phi_{t-1} & 1-\phi_{t-1} & 1 \end{pmatrix} \end{array}$$

- ϕ_{t-1} is the apparent survival between primary periods $t-1$ and t
- γ_t^{xy} are the probability of moving from strata X to Y between $t-1$ and t, and $\gamma_t^{xa} + \gamma_t^{xb} + \gamma_t^{xc} + \gamma_t^{xt} = 1$

Example in the Multi-State Closed Robust Design with 3 strata and TE (*first-capture* conditioning):

MSCRD Emission Matrix

$$\boldsymbol{\Psi}_{s,t} = \begin{array}{c} \\ \textbf{Capture A} \\ \textbf{Capture B} \\ \textbf{Capture C} \\ \textbf{No Capture} \end{array} \begin{array}{c} \begin{array}{ccccc} \textbf{Strata A} & \textbf{Strata B} & \textbf{Strata C} & \textbf{TE} & \textbf{Dead} \end{array} \\ \begin{pmatrix} p^a_{s,t} & 0 & 0 & 0 & 0 \\ 0 & p^b_{s,t} & 0 & 0 & 0 \\ 0 & 0 & p^c_{s,t} & 0 & 0 \\ 1-p^a_{s,t} & 1-p^b_{s,t} & 1-p^c_{s,t} & 1 & 1 \end{pmatrix} \end{array}$$

- $p^x_{s,t}$ is the capture probability per secondary period conditional on being in strata X at t

A POPAN (unrealistic[3]) Example: making a matrix in JAGS

```
# FROM unborn (col1) to...
tr[1,1] <- 1-psi # unborn to unborn
tr[2,1] <- psi   # unborn to alive
tr[3,1] <- 0     # (illegal)
# FROM alive (col2) to...
tr[1,2] <- 0     # (illegal)
tr[2,2] <- phi   # alive to alive
tr[3,2] <- 1-phi # alive to dead
# FROM dead (col3) to...
tr[1,3] <- 0     # (illegal)
tr[2,3] <- 0     # (illegal)
tr[3,3] <- 1     # dead to dead
```

Note: `matrix[r, c]` :

- 1^{st} integer is the `row`,
- 2^{nd} is the `column` (like `R`)

[3] in practise, the recruitment parameters are always time-varying

Let's do a simple HMM in JAGS

Parameters:

- ϕ time-constant survival (`phi`)
- ψ time-constant "recruitment" (`psi`)
- p time-constant capture probability (`p`)

Goal:

- *inferencce* about `phi`, `psi`
- demonstrate the latent states posterior distribution `z`

Inputs

- One capture history: `y<-c(0,0,0,0,1,0,1,1,0,0,0)`
- `T=11` capture periods
- prior parameters: `pr.phi`, `pr.psi`, `pr.p`

JAGS HMM Demo: STEP 1: Priors

- In JAGS:

```
# specify the priors
p ~ dbeta(pr.p[1], pr.p[2]) # prior on capture history
phi ~ dbeta(pr.phi[1], pr.phi[2]) # prior on survival
psi ~ dbeta(pr.psi[1], pr.psi[2]) # prior on recruitment
```

- On the R side:

```
jags.data<-list(
   pr.p=c(1,1),  # flat prior on capture probability
   pr.phi=c(1,1),# flat prior on survival
   pr.psi=c(1,1) # flat prior on recruitment
)
```

- Qu: why beta?[4]

[4] reasonable distribution for probability parameters

```
# specify the conditional data likelihod
for(t in 1:length(y)){ # loop through capture periods
   y[t] ~ dcat(em[,z[t]])
}
```

... where `em` is the emission matrix

1. Qu: Why `dcat` and not `dbern` ? [5]
2. QU: Why is it called the “conditional" likelihood? [6]

[5] We could use dbern . The Multinoulli distribution (dcat) generalizes the Bernoulli for more than 2 outcomes.

[6] Conditional on the value of z[t]

```
# specify the latent state process (for t=1)
z[1] ~ dcat(tr[,1])
# specify the latent state process (for t>1)
for(t in 2:length(y)){
   z[t] ~ dcat(tr[,z[t]])
}
```

...where `tr` is the transition matrix

- Qu: why is $t = 1$ handled differently from $t > 1$ [7]

[7] The latent state process is Markovian, depends on values one-step-back. For t=1, there is no one-step-back with observation information.

JAGS HMM Demo: STEP 3b: Matrices

Transition Matrix

```
# Build the Transition Mat
# state 1
tr[1,1]<-1-psi
tr[2,1]<-psi
tr[3,1]<-0
# state 2
tr[1,2]<-0
tr[2,2]<-phi
tr[3,2]<-1-phi
# state 3
tr[1,3]<-0
tr[2,3]<-0
tr[3,3]<-1
```

Emission Matrix

```
# Build the Emission Mat
# state 1: unborn (100% no capture)
em[1,1]<-1
em[2,1]<-0
# state 2: alive
em[1,2]<-1-p
em[2,2]<-p
# state 3: dead (100% no capture)
em[1,3]<-1
em[2,3]<-0
```

---

where y=1 $\equiv$ no-capture and y=2 $\equiv$ capture

JAGS HMM Demo

Time to open up JAGS!

open the file `PART4_introHMM/R_hmm_intro.R` and see exercise 1