CS223A - Introduction to Robotics - Homework #2

Yunsen Xing
Autonomous Systems and Intelligent Robots
yunsen@kth.se

KTH Royal Institute of Technology && Aalto University — August 25, 2025

Question 1

$$
\begin{aligned}
T_2' &= T_1 T_2 T_1^{-1} \\
T_3' &= T_1 T_2 T_3 (T_1 T_2)^{-1} \\
T_4' &= T_1 T_2 T_3 T_4 (T_1 T_2 T_3)^{-1}
\end{aligned}
$$

Total displacement =

$$
\begin{aligned}
T_4' T_3' T_2' T_1 &= T_1 T_2 T_3 T_4 (T_1 T_2 T_3)^{-1} T_1 T_2 T_3 (T_1 T_2)^{-1} T_1 T_2 T_1^{-1} T_1 \\
&= T_1 T_2 T_3 T_4
\end{aligned}
$$

Question 2

DH parameters:

$$
\begin{bmatrix}
\cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\
\sin\theta_i \cos\alpha_{i-1} & \cos\theta_i \cos\alpha_{i-1} & -\sin\alpha_{i-1} & -\sin\alpha_{i-1} d_i \\
\sin\theta_i \sin\alpha_{i-1} & \cos\theta_i \sin\alpha_{i-1} & \cos\alpha_{i-1} & \cos\alpha_{i-1} d_i \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

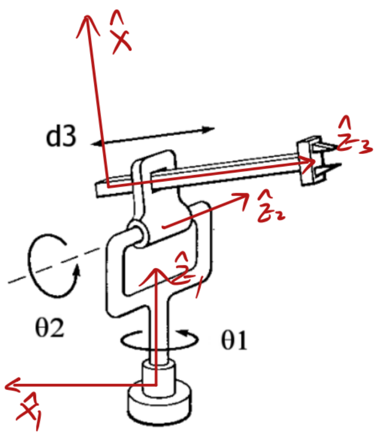

Figure 1: Question 2

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$90°$	0	0	θ_2
3	$90°$	a_2	d_3	0

Table 1: the answer to question 2

$$
{}^0_1T = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1_2T = \begin{bmatrix} \cos\theta_2 & -\sin_{\theta_2} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2_3T = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\begin{aligned}
\therefore {}^3_0T &= {}^1_0T\,{}^1_2T\,{}^2_3T \\
&= \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \cos\theta_1\cos\theta_2 & -\cos\theta_1\sin\theta_2 & \sin\theta_1 & 0 \\ \sin\theta_1\cos\theta_2 & -\sin\theta_1\sin\theta_2 & -\cos\theta_1 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \cos\theta_1\cos\theta_2 & \sin\theta_1 & \cos\theta_1\sin\theta_2 & a_2\cos\theta_1\cos\theta_2 + d_3\cos\theta_1\sin\theta_2 \\ \sin\theta_1\cos\theta_2 & -\cos\theta_1 & \sin\theta_1\sin\theta_2 & a_2\sin\theta_1\cos\theta_2 + d_3\sin\theta_1\sin\theta_2 \\ \sin\theta_2 & 0 & -\cos\theta_2 & a_2\sin\theta_[2] - d_3\cos\cos\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Another Example

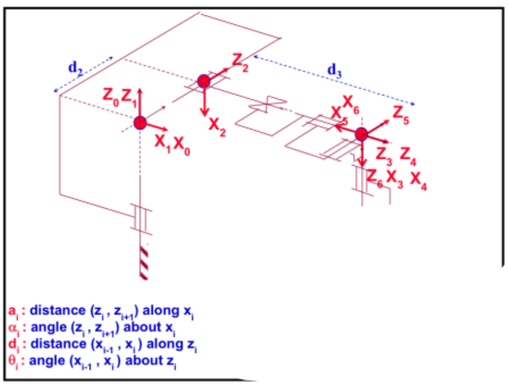

Figure 2: Question 3

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$-90°$	0	d_2	θ_2
3	$90°$	0	d_3	0
4	0	0	0	0
5	$-90°$	0	0	θ_5
6	$90°$	0	0	θ_6

Table 2: the answer to the example

Question 3

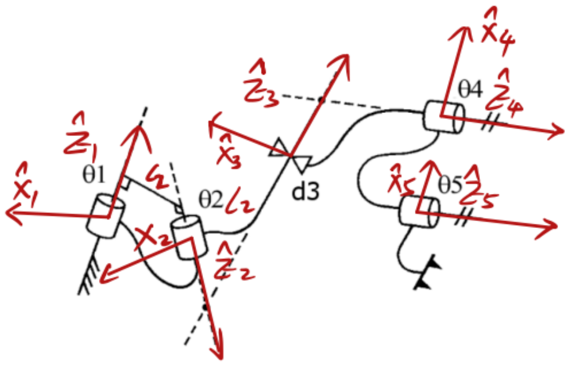

Figure 3: Question 3

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	α_1	a_1	d_2	θ_2
3	α_2	0	d_3	θ_3
4	α_3	0	d_4	θ_4
5	0	a_4	0	θ_5

Table 3: the answer to question 3