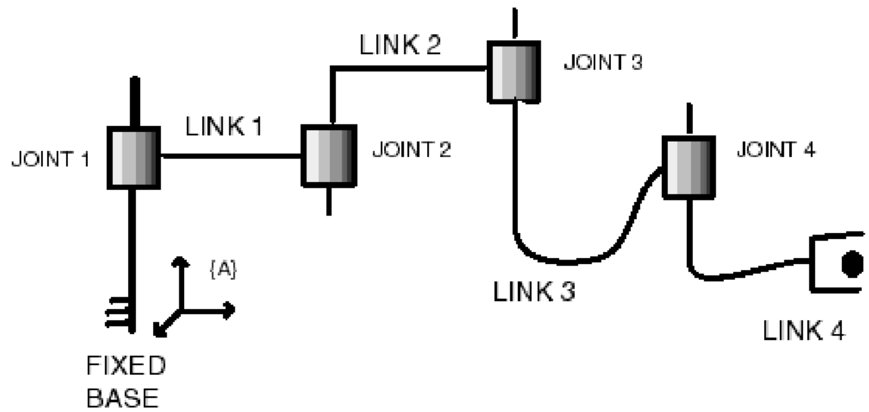
Introduction to Robotics (CS223A) — Homework #2

(Winter 2007/2008) — Due: **Wednesday, January 30**

1. The following sketch represents a generic open, serial, kinematic-chain.



位移

Here each kinematic joint connects two adjacent members. Assume that the relative displacement between adjacent members $i-1$ and i is described by an operator T_i that is a 4x4 matrix whose elements are computed in a coordinate frame $\{A\}$ fixed to the base of the chain. Now, if each member is displaced in sequence, *starting from the free end*, the displacement operator for the resultant total displacement of the free end will be given by $T_1T_2T_3T_4$. (Note: In this problem you are to use only displacements operators, not coordinate transformations) $T_1(T_2(T_3(T_4)))$

However, if the displacements are done in the reverse order, ie. *starting at the fixed end*, and moving in the sequence 1, 2, 3, 4, then the operators T_2, T_3 , and T_4 no longer represent the actual displacements.

Determine, in terms of the original T_i:

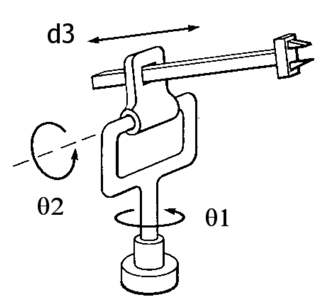
(a) The operator for joint 2, when its displacement is done *after* the displacement in joint 1. Let us call this operator T_2'

(b) The operator for joint 3 when its displacement follows the displacement in joints 1 and 2 (from part (a)). Let us call this operator T_3'

(c) The operator for joint 4 when its displacement follows the displacement in joints 1, 2 and 3 (from part (b)). Let us call this operator T_4'

(d) Using your results for parts (a), (b) and (c), show that the resulting displacement operator for the free end is still $T_1T_2T_3T_4$

2. Consider the following RRP manipulator (figure courtesy of J. J. Craig):



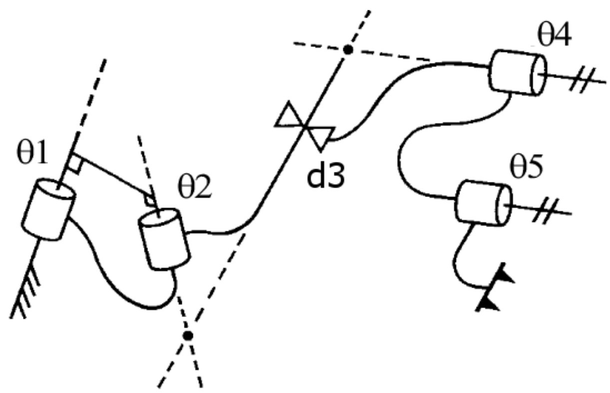
(a) Draw a schematic of this manipulator, with the axes of frames {0} through {3} labeled. Also, include the parameters θ_1, θ_2, a_2, and d_3 on your schematic. Assume that in this diagram, the slider bar is parallel to the ground and that this is the configuration where $\theta_1 = 0$, $\theta_2 = 90^o$.

(b) Write down the Denavit-Hartenberg parameters for this manipulator, in the form of a table:

i	a_{i-1}	α_{i-1}	d_i	θ_i
1				
2				
3				

(c) Derive the forward kinematics for this manipulator — that is, find ${}^0_3T$.

3. Consider the following 2RP2R manipulator (figure courtesy of J. J. Craig):



(a) Draw a schematic of this manipulator, with the axes of frames {0} through {5} labeled. Include all non-zero Denavit-Hartenberg parameters and the joint variables. Draw your schematic in the position where, as far as possible, the angles θ_i are in their zero positions.

(b) Write down the Denavit-Hartenberg parameters for this manipulator, in the form of a table:

i	a_{i-1}	α_{i-1}	d_i	θ_i
1				
2				
3				
4				
5				

1. $T_2' = T_1 T_2 T_1^{-1}$

$$T_3' = T_1 T_2 T_3 (T_1 T_2)^{-1}$$

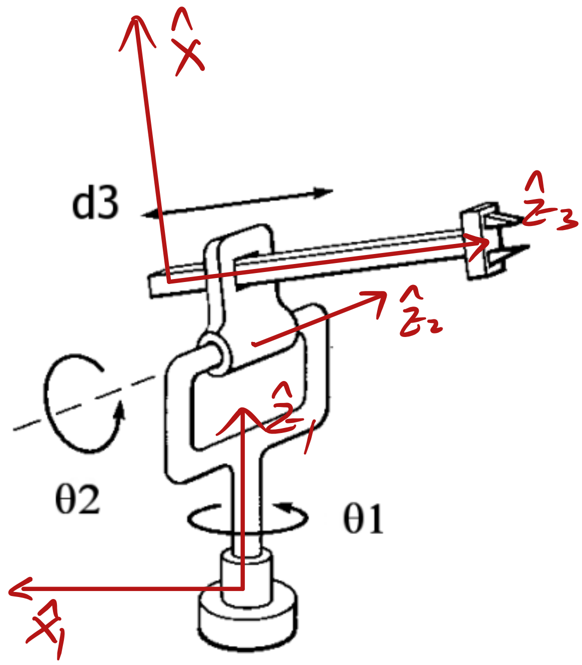
$$T_4' = T_1 T_2 T_3 T_4 (T_1 T_2 T_3)^{-1}$$

换了顺序得到的总位移：

$$T_4' T_3' T_2' T_1 = T_1 T_2 T_3 T_4 \cancel{(T_1 T_2 T_3)^{-1} T_1 T_2 T_3 (T_1 T_2)^{-1} T_1 T_2 T_1^{-1} T_1}$$

$$= T_1 T_2 T_3 T_4$$

2.

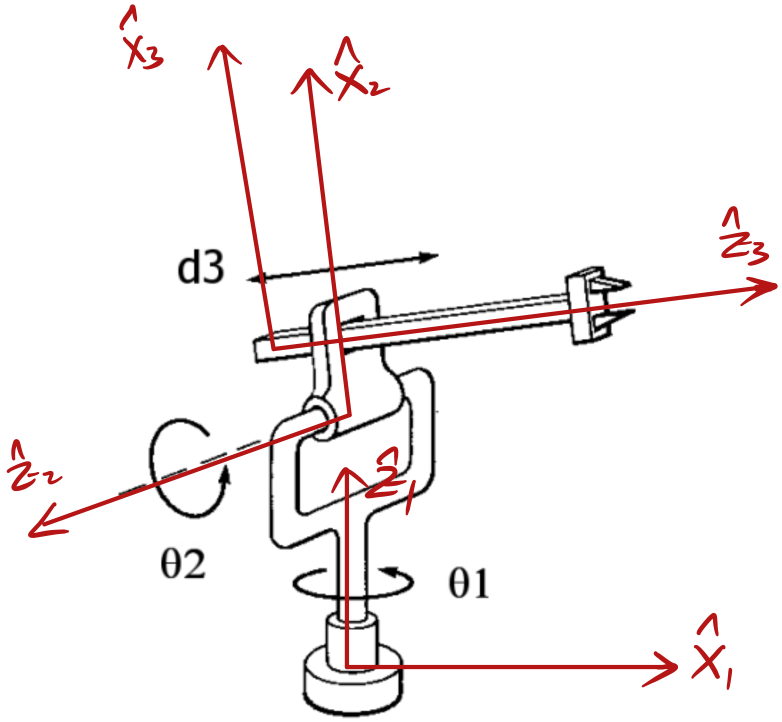


	R_x	T_x	T_z	R_z
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$90°$	0	0	θ_2
3	$90°$	a_2	d_3	0

DH公式：

标准DH： $A_i = R_z(\theta_i)\, T_z(d_i)\, T_x(a_{i-1})\, R_x(\alpha_{i-1})$

$$= \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i \cos\alpha_{i-1} & \cos\theta_i \cos\alpha_{i-1} & -\sin\alpha_{i-1} & -\sin\alpha_{i-1} d_i \\ \sin\theta_i \sin\alpha_{i-1} & \cos\theta_i \sin\alpha_{i-1} & \cos\alpha_{i-1} & \cos\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	0	θ_2
3	90°	a_2	d_3	0

$$
{}^0_1T = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1_2T = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

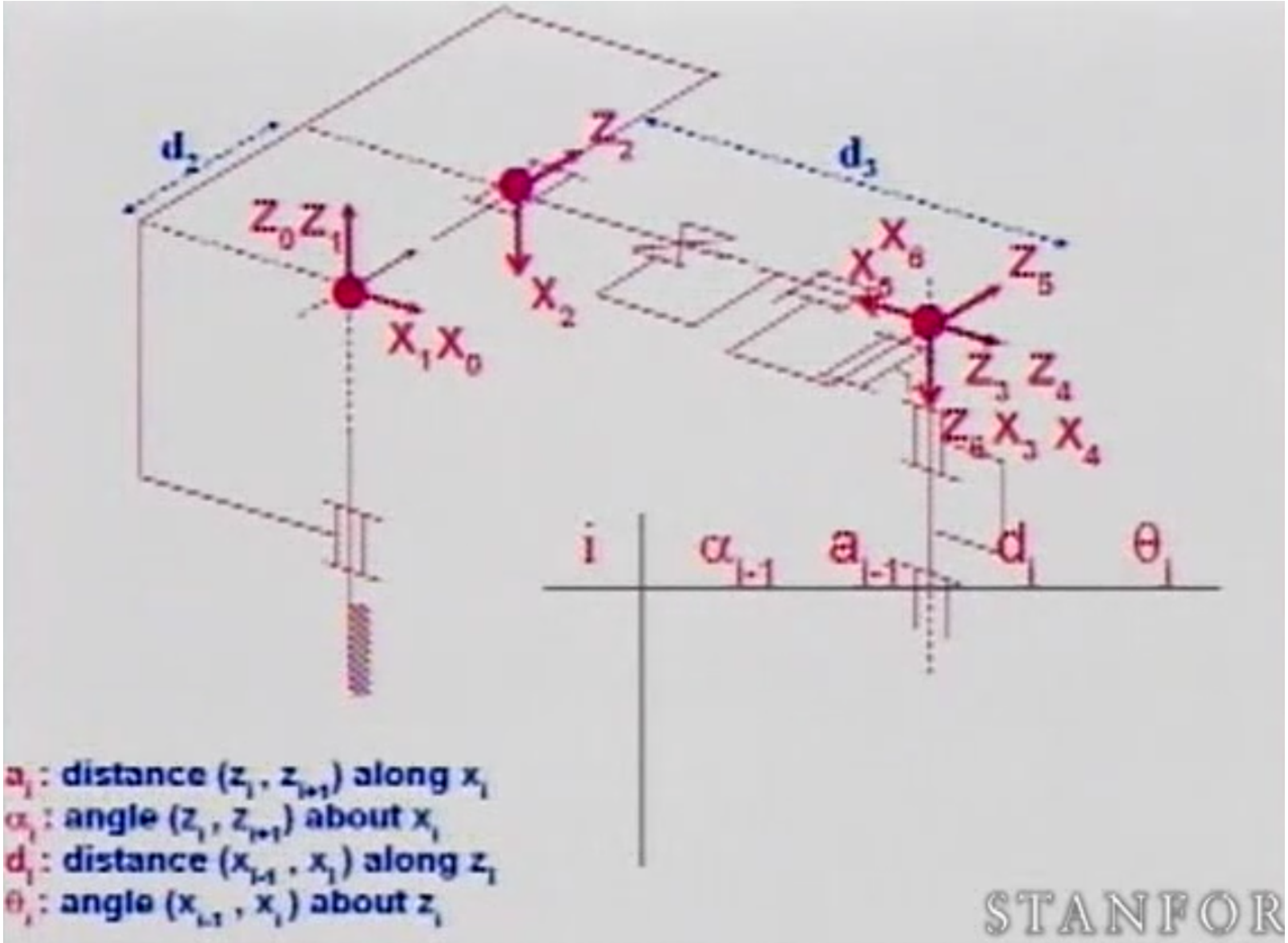
$$
{}^{2}_{3}T = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\therefore\ {}^{0}_{3}T = {}^{0}_{1}T\ {}^{1}_{2}T\ {}^{2}_{3}T
$$

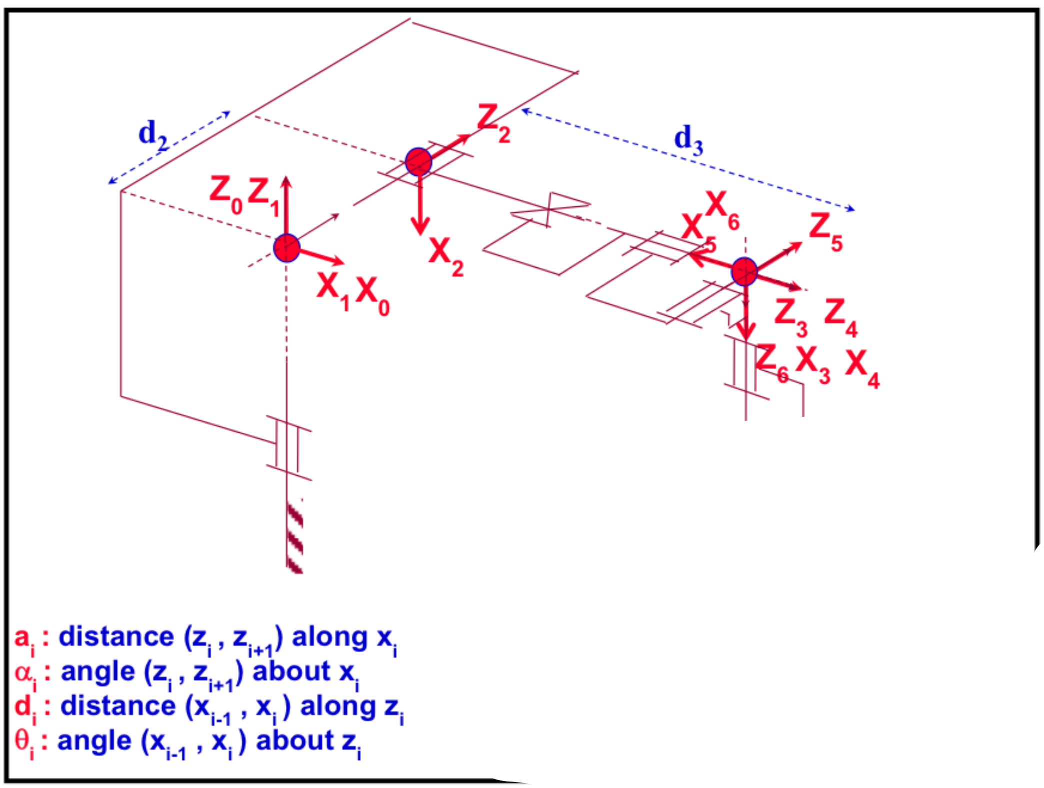
$$
\begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} \cos\theta_1\cos\theta_2 & -\cos\theta_1\sin\theta_2 & \sin\theta_1 & 0 \\ \sin\theta_1\cos\theta_2 & -\sin\theta_1\sin\theta_2 & -\cos\theta_1 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} \cos\theta_1\cos\theta_2 & \sin\theta_1 & \cos\theta_1\sin\theta_2 & a_2\cos\theta_1\cos\theta_2 + d_3\cos\theta_1\sin\theta_2 \\ \sin\theta_1\cos\theta_2 & -\cos\theta_1 & \sin\theta_1\sin\theta_2 & a_2\sin\theta_1\cos\theta_2 + d_3\sin\theta_1\sin\theta_2 \\ \sin\theta_2 & 0 & -\cos\theta_2 & a_2\sin\theta_2 - d_3\cos\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



d_2
d_3
Z_2
$Z_0 Z_1$
$X_1 X_0$
X_2
X_6
X_5
Z_5
Z_3 Z_4
Z_6 X_3 X_4
i
α_{i-1}
a_{i-1}
d_i
θ_i
a_i : distance (z_i , z_{i+1}) along x_i
α_i : angle (z_i , z_{i+1}) about x_i
d_i : distance (x_{i-1} , x_i) along z_i
θ_i : angle (x_{i-1} , x_i) about z_i
STANFOR

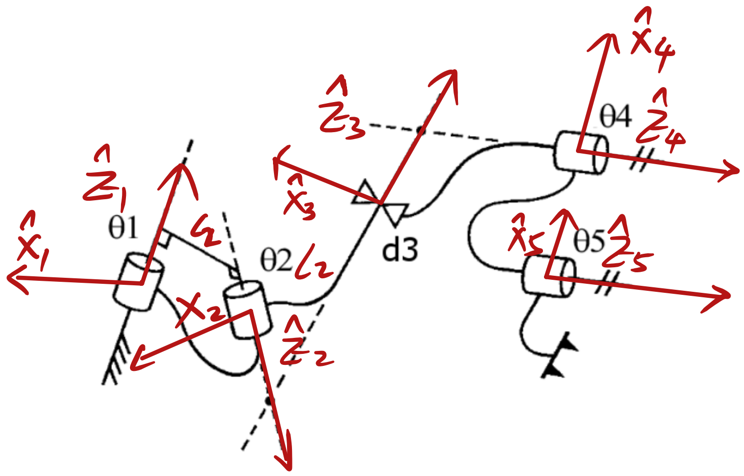


d_2
Z_2
d_3
$Z_0 Z_1$
X_2
X_6
X_5
Z_5
$X_1 X_0$
Z_3 Z_4
Z_6 X_3 X_4
a_i : distance (z_i , z_{i+1}) along x_i
α_i : angle (z_i , z_{i+1}) about x_i
d_i : distance (x_{i-1} , x_i) along z_i
θ_i : angle (x_{i-1} , x_i) about z_i

斜

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90°	0	d_2	θ_2
3	90°	0	d_3	0
4	0	0	0	0
5	-90°	0	0	θ_5
6	90°	0	0	θ_6

3.



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	α_1	a_1	d_2	θ_2
3	α_2	0	d_3	θ_3
4	α_3	0	d_4	θ_4
5	0	a_4	0	θ_5

相交