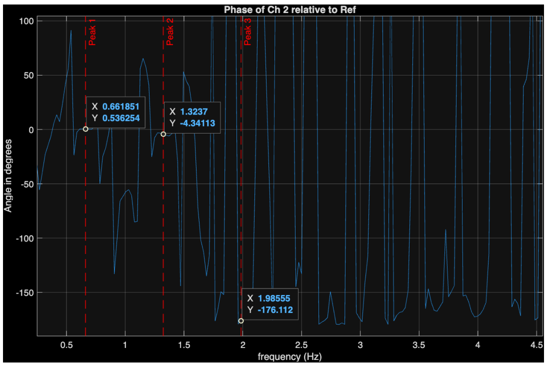
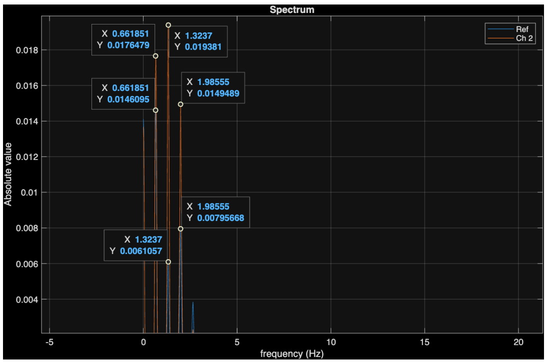
Lab 2 Analysis for 40 rpm

Undamped natural frequency ω_d of 1.6801 Hz and damping ratio ζ of 0.0083. Then I find 3 phases in phase frequency figure at 3 peaks frequency corresponding to amplitude frequency figure



3 points are (0.661851, 0.536254), (1.3237, -4.34113), (1.98555, -176.112)
amplitude frequency figure:



For base excitation, the frequency response function (FRF) is :

$$\frac{X}{X_b} = \frac{i2\zeta\omega_n\omega + \omega_n^2}{\omega_n^2 - \omega^2 + i2\zeta\omega_n\omega} = \frac{1 + i2\zeta\omega/\omega_n}{1 - (\omega/\omega_n)^2 + i2\zeta\omega/\omega_n}$$

phase angle of the frequency response is :

$$\phi(\omega) = tan^{-1}[\frac{-Im\, G(i\omega)}{Re\, G(i\omega)}] = tan^{-1}\frac{2\zeta\omega/\omega_n}{1 - (\omega/\omega_n)^2}$$

In [56]:
```
import numpy
from numpy import sqrt, zeros, abs, arctan2, rad2deg, linspace
# set parameter
f_d = 1.6801
zeta = 0.0083
f_n = f_d / sqrt(1-zeta**2)
# r = w/w_n = f/f_n
f_i = {0: 0.661851,
       1: 1.3237,
```

```
        2: 1.98555}

r = [f_i[i] / f_n for i in range(3)]
H_f = [(1 + 1j*2*zeta*ri) / (1 - ri**2 + 1j*2*zeta*ri) for ri in r]
H_mag = abs(H_f)
phi = [(arctan2(2*zeta*ri, (1-ri**2))) for ri in r ]
r_pts = r
display(H_mag)
display(rad2deg(phi))
# experimenral transibility
T1 = 0.0176479/0.0146095
T2 = 0.019381/0.0061057
T3 = 0.0149489/0.00795668
T_exp = [T1, T2, T3]
```

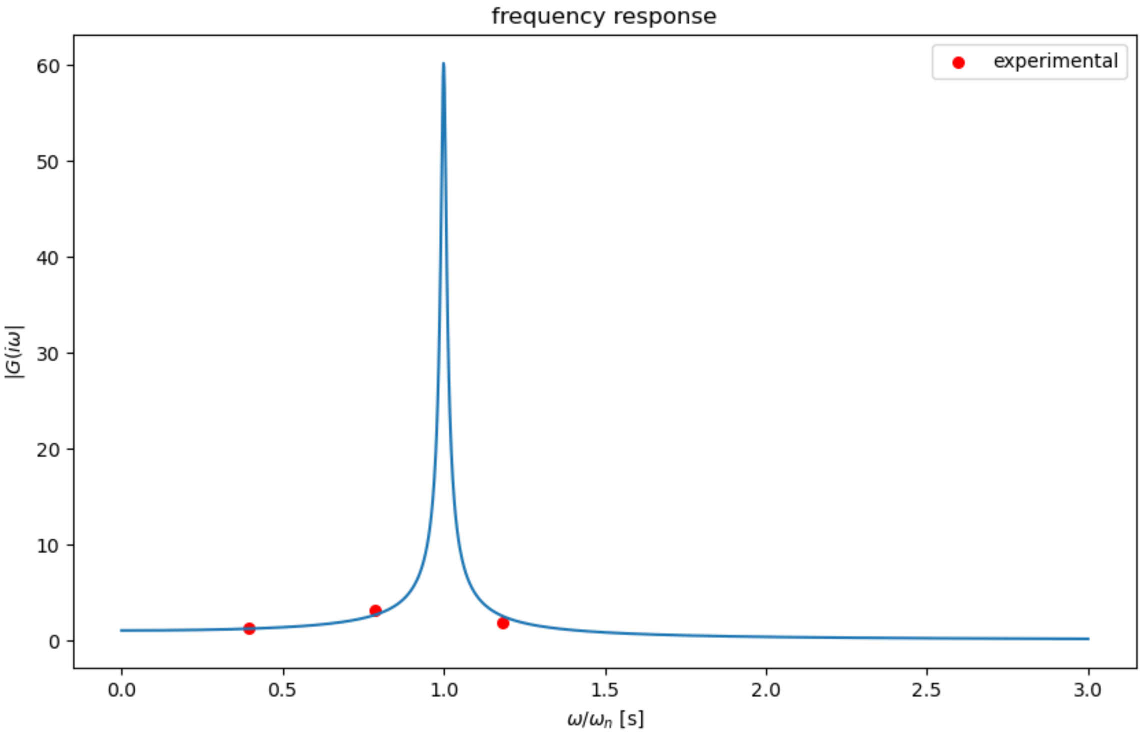
```
array([1.1836662 , 2.63506741, 2.51905309])
array([  0.44347093,   1.97474388, 177.16800764])
```

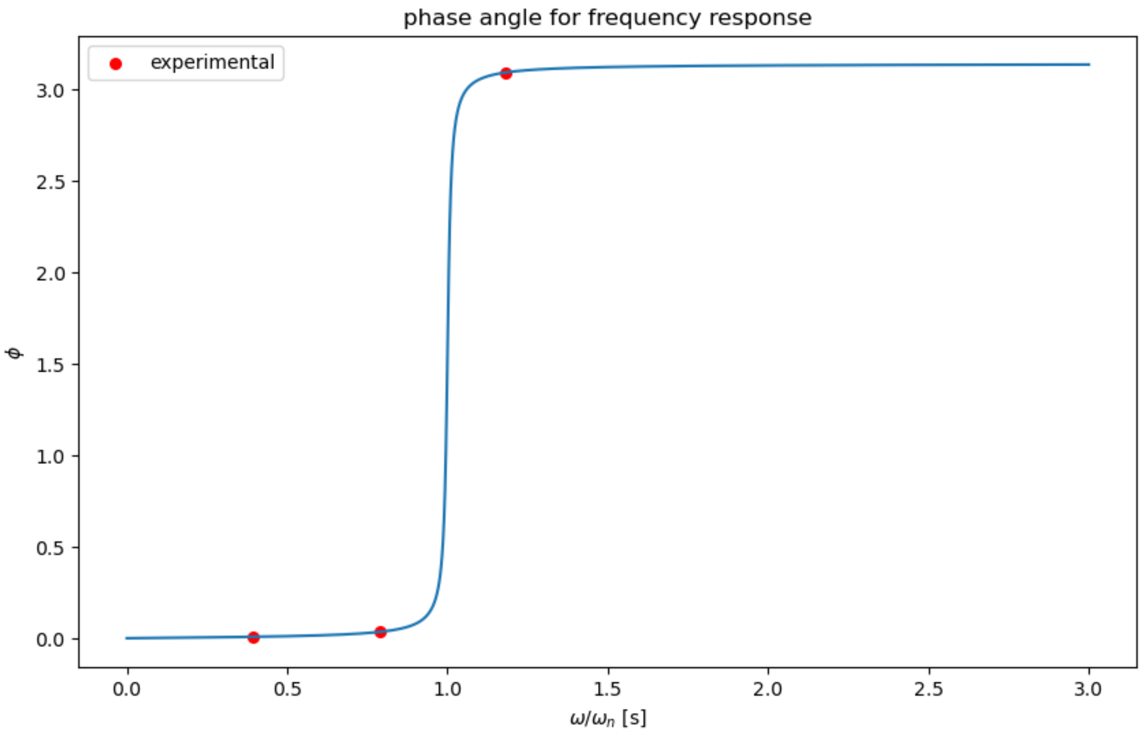
In [65]:

```
import matplotlib.pyplot as plt
r = linspace(0, 3, 2000);
h_f = abs((1 + 1j*2*zeta*r) / (1 - r**2 + 1j*2*zeta*r))
phi_sys = arctan2(2*zeta*r, (1- r**2))

#========================
# frequency response
plt.figure(figsize=(10, 6))
plt.plot(r, h_f)
plt.xlabel("$\omega/\omega_n$ [s]")
plt.ylabel("$|G(i\omega|$")
plt.scatter(r_pts, T_exp, color='red',s=30,label = 'experimental')
plt.legend()
plt.title("frequency response")

#========================
# phase angle for frequency response\
plt.figure(figsize=(10, 6))
plt.plot(r, phi_sys)
plt.xlabel("$\omega/\omega_n$ [s]")
plt.ylabel("$\phi$")
plt.scatter(r_pts, phi, color='red',s=30, label = 'experimental')
plt.legend()
plt.title("phase angle for frequency response")
```

Out[65]:

```
Text(0.5, 1.0, 'phase angle for frequency response')
```


frequency response
experimental
|G(iω)|
ω/ωn [s]
0
10
20
30
40
50
60
0.0
0.5
1.0
1.5
2.0
2.5
3.0


phase angle for frequency response
experimental
ϕ
ω/ωn [s]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0