Modern Algebra

Assignment 1

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1 Dihedral Groups

Problem 2

	e	R_{60°	R_{120°	s_1	s_2	s_3
e	e	R_{60°	R_{120°	s_1	s_2	s_3
R_{60°	R_{60°	R_{120°	e	s_3	s_1	s_2
R_{120°	R_{120°	e	R_{60°	s_2	s_3	s_1
s_1	s_1	s_2	s_3	e	R_{60°	R_{120°
s_2	s_2	s_3	s_1	R_{120°	e	R_{60°
s_3	s_3	s_1	s_2	R_{60°	R_{120°	e

D_3 is not Abelien, a counter example for commutativity is $R_{120^\circ} \cdot s_1 = s_3$ but $s_1 \cdot R_{120^\circ} = s_2$

Problem 3

a) $\{V\}$

b) $\{R_{270^\circ}\}$

c) $\{R_{0^\circ}\}$

d) $\{R_{0^\circ}, R_{180^\circ}, H, V, D, L\}$

e) $\emptyset$

Problem 5

For an n-gon, we have $2n$ of operations that preserve symmetry, these operations are rotations and reflections.

For rotations, there are n possible rotations for every n-gon including the identity element. The rotations other than the identity are the multiples of $\frac{360}{n}$ degrees. We can represent the rotations by r^c, where $1 \leq c \leq n-1$ is the number we can multiply $\frac{360}{n}$ with. Thus, the available rotations for an n-gon are: $e, r^1, r^2, r^3, ..., r^{n-1}$.

For reflections, the available reflections differ whether n is even or odd.

> ***n* is odd:** The available reflections are the reflections about the axes of symmetry, which are axes from one vertix to the midpoint of the opposite side. Then, there are n reflections as there are n verticies.
>
> ***n* is even:** The available reflections are the reflections about the axes of symmetry, $\frac{n}{2}$ of them are axes from one vertix to the opposite vertix,and the other $\frac{n}{2}$ are those from one midpoint of one side to the midpoint of the opposite side. Then there are n reflections.

Problem 11

When we do a reflection twice, we get the original orientation, so it is just like if we rotate by zero degree. Hence, any even number of similar reflections leads to a rotation. And if we rotate the shape any number of rotations we get a resultant rotation with the sum of angles of the rotations. If we applied a reflection followed by a rotation or vice versa, we get a reflection. We represent rotations in this problem by r and reflections by s.

Thus, for the given formula $a^2b^4ac^5a^3c$, we can expan it to $a^2b^4ac^4ca^2ac = rrarcrac = (rr)arcrac = rarcrac$. If both a, c are rotations **then the previous expression is rotation**.

However, let both a, c be reflections. Then, $rarcrac = rsrsrss = (rs)rsrss = srsrss = (sr)srss = ssrss = (ss)rss = rrss = (rr)ss = rss = (rs)s = ss = r$. **The result is rotation**.

Now, we let $a = r$ and $c = s$, we observe that $rarcrac = (rr)rsrrs = (rr)srrs = (rs)rrs = (sr)rs = (sr)s = ss = r$. **The result is reflection**.

Finally, let $c = r, a = s$, then $rarcrac = (rs)rrrsr = (sr)rrsr = (sr)rsr = (sr)sr = (ss)r = rr = r$. **The result is also a rotation**.

Therefore, we showed that the expression $a^2b^4ac^5a^3c$ is a rotation for every a, b, c in a dihedral group.

Problem 13

We observe the cayley table of D_4 in Modern Algebra Lecture Notes by Dr. Daoud Siniora. We find that $VR_{90^\circ} = R_{90^\circ}H$, but $V \neq H$.

Problem 21

If we applied any reflection twice, we get the identity element, so X cannot be a reflection. Since $Y \neq R_{90}$, then it can be e, R_{180}, R_{270}.It cannot be reflection as reflection after rotation leads to a reflection. By observing the same table in the Lecture Notes that we mentioned in Problem 13, we see that $R_{270}R_{180} = R_{180}R_{270} = R_{90}e = R_{90}$. But R_{270}, R_{90} cannot be represented in X^2 as $R_{270} = R_{90}R_{90}R_{90}$. and R_{90} is the smallest rotation we can do. Thus, $X = R_{90}$ and $X^2Y = R_{180}R_{270} = R_{90}R_{90}R_{270} = R_{90}$,and $Y = R_{270} \neq R_{90}$.