Linear Algebra

Report 4

Yousef A. Abood

ID: 900248250

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Definition 1: Basis of a vector space

Let V be any vector space, and S is a subset of that vector space. We call S a basis of a vector space if it is a spanning set of V and is a linearly independent set.

Example

An example is the set $S = \{7, 3x, 5x^2, x^3\}$, which is a basis of P_3.

Non-example

The set $\{(3, -9, 6), (1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ is not linear independent as we can get the first vector by a linear combination of the other vectors. Hence, it is clearly not a basis.

Definition 2: Dimension of a vector space

The dimension of a vector space V, denoted by $dim(V)$, is the cardinality of the basis. We also set $dim(\{\vec{\mathbf{0}}\}) = 0$.

Example

From the previous example, the dimension of $P_3 = dim(P_3) = |S| = 4$.

Definition 3: Rank of a matrix

The rank of a matrix A, denoted by $rank(A)$, is the dimension of the row space of A or the column space of A.

Example

Let A be the matrix

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & 7 & 6 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

and we see it is in REF. Thus, $R(A) = \{(1,2,4,5),(0,1,7,6),(0,0,1,3),(0,0,0,1)\}$ and $dim(R(A)) = 4$. Hence, the rank of the matrix is $rank(A) = 4$.