Linear Algebra

Report 2

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◇ Invertible matrix

A square matrix A of size $n \times n$ is called invertible iff we can find another matrix B such that $AB = I_n$ and $BA = I_n$. We call B the inverse of A and we denote it by A^{-1}.
*When a matrix is row equivalent to a matrix with row or column of zeros then it is noninvertible matrix.

Example

$$\begin{bmatrix} 1 & 2 & 5 \\ 5 & 6 & 10 \\ 11 & 12 & 22 \end{bmatrix} \begin{bmatrix} 0 & 2 & -2 \\ -1 & \frac{7}{2} & \frac{-3}{2} \\ \frac{1}{3} & \frac{-5}{3} & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$AB = I_n$.

Non-example

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 10 & 12 & 14 \end{bmatrix} \xrightarrow{(R_3 - 2R_2) \to R_3} \begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 0 & 0 & 0 \end{bmatrix}.$$

A is noninvertible matrix.

◇ Elementry matrix

A square matrix A of size $n \times n$ is called elementary iff it can be obtained from the identity matirx by applying exactly one elementary row operation.

Example

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{(R_3 - 5R_2) \to R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{bmatrix} = A, \text{ an elementary matrix.}$$

Example

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{(R_3 - 5R_2) \to R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -5 & 1 \end{bmatrix} = B, \text{ not an elementary matrix.}$$

◇ Determinant of a matrix

The determinant is a function that assaigns to every square matrix A a real number denoted by $det(A)$ or $|A|$. For a 1×1 matrix, we define $det([a]) = a$. The determinant of an $n \times n$ matrix, where $n \geq 2$, is the sum of the products of the entries of the first row with their corresponding cofactors.

$$det(A) = \sum_{j=1}^{n} (-1)^{1+j} a_{1j} det(A_{1j}) = \sum_{j=1}^{n} a_{1j} C_{1j} = a_{11}C_{11} + a_{12}C_{12} + ... + a_{1n}C_{1n}.$$

Example

$$A = \begin{bmatrix} 9 & 11 & 13 \\ 17 & 19 & 23 \\ 29 & 31 & 37 \end{bmatrix}.$$

$$det(A) = \begin{vmatrix} 9 & 11 & 13 \\ 17 & 19 & 23 \\ 29 & 31 & 37 \end{vmatrix} = 9 \begin{vmatrix} 19 & 23 \\ 31 & 37 \end{vmatrix} - 11 \begin{vmatrix} 17 & 23 \\ 29 & 37 \end{vmatrix} + 13 \begin{vmatrix} 17 & 19 \\ 29 & 31 \end{vmatrix} = -90 + 418 - 312 = 16.$$

* A nonexample cannot be found.