Linear Algebra

Assignment 9

Yousef A. Abood

ID: 900248250

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4.6 Rank of a matrix

Problem 11

a)

$$\begin{bmatrix} -2 & -4 & 4 & 5 \\ 3 & 6 & -6 & -4 \\ -2 & -4 & 4 & 9 \end{bmatrix} \xrightarrow{\frac{-1}{2}R_1 \to R_1} \begin{bmatrix} 1 & 2 & -2 & \frac{-5}{2} \\ 3 & 6 & -6 & -4 \\ -2 & -4 & 4 & 9 \end{bmatrix}$$

$$\xrightarrow{(R_2 - 3R_1) \to R_2} \begin{bmatrix} 1 & 2 & -2 & \frac{-5}{2} \\ 0 & 0 & 0 & \frac{7}{2} \\ -2 & -4 & 4 & 9 \end{bmatrix}$$

$$\xrightarrow{(R_3 + 2R_1) \to R_3} \begin{bmatrix} 1 & 2 & -2 & \frac{-5}{2} \\ 0 & 0 & 0 & \frac{7}{2} \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$\xrightarrow{\frac{7}{2}R_2 \to R_2} \begin{bmatrix} 1 & 2 & -2 & \frac{-5}{2} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$\xrightarrow{(R_3 - 4R_2) \to R_3} \begin{bmatrix} 1 & 2 & -2 & \frac{-5}{2} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Thus, the basis for the row space is the set $\{(1, 2, -2, \frac{-5}{2}), (0, 0, 0, 1)\}$.

b) The rank of the matrix is the cardinality of basis for the row space. So, the rank is 2.

Problem 14

We obtain a matrix A from the vectors in S.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 3 & 4 \\ 2 & 3 & 1 \end{bmatrix}$$

We keep applying the ERO till we reach the REF.

$$\begin{bmatrix} 1 & 2 & 4 \\ -1 & 3 & 4 \\ 2 & 3 & 1 \end{bmatrix} \xrightarrow{(R_2+R_1)\to R_2} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 5 & 8 \\ 2 & 3 & 1 \end{bmatrix} \xrightarrow{(R_3-2R_1)\to R_3} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 5 & 8 \\ 0 & -1 & -7 \end{bmatrix}$$
$$\xrightarrow{\frac{1}{5}R_2\to R_2} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & \frac{8}{5} \\ 0 & -1 & -7 \end{bmatrix} \xrightarrow{(R_3+R_2)\to R_3} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & \frac{8}{5} \\ 0 & 0 & \frac{-27}{5} \end{bmatrix}$$
$$\xrightarrow{(\frac{-5}{27})R_3\to R_3} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & \frac{8}{5} \\ 0 & 0 & 1 \end{bmatrix}$$

Since the rows of the matrix are exactly the vectors in S, we get that the supspace of $R^3 = Span(S) = R(A)$. Therefore, the set $\{(1, 2, 4), (0, 1, \frac{8}{5}), (0, 0, 1)\}$ forms a basis for the subspace.

Problem 25

Problem 35

Problem 42