Linear Algebra

Report 1

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◇ Upper triangular matrix

A matrix is called upper triangular iff every entry below the main diagonal is zero. So if a matrix $K = [k_{ij}]$ is upper triangular, then every entry k_{ij} is zero when $i > j$.

Example

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix},$$

Non-example

$$\begin{bmatrix} 1 & 2 & 3 \\ 7 & 4 & 5 \\ 8 & 9 & 6 \end{bmatrix}.$$

◇ Scalar multiplication of matrices

let $A = [a_{ij}]$ be a matrix of size $m \times n$, where $m, n \in \mathbb{Z}$ and let c be a real number. The Scalar multiple of A by c is the $m \times n$ matrix cA obtained by multiplying every entry of the matrix A by c.

Example

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, 5A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}.$$

* Non-example cannot be found.

◇ Skew-symmetric matrix

A square matrix $A = [a_{ij}]$ is called *skew-symmetric matrix* iff $A = -A^T$. In other words $a_{ij} = -a_{ji}$ for every $1 \leq i, j \leq n$, which forces the main diagonal entries to be zeros.

Example

$$\begin{bmatrix} 0 & 5 & 4 \\ -5 & 0 & 3 \\ -4 & -3 & 0 \end{bmatrix},$$

Non-example

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}.$$

◇ A solution of a linear equation in 3 variables

A solution of a linear equation in 3 variables is a sequence of three real numbers (s_1, s_2, s_3) which satisfy the equation when the number s_i is substituted for the variable x_i, for every $1 \leq i \leq 3$.

Example

The sequence $(1, 1, 1)$ is a solution for the system of equations : $x + y + z = 3$.
The sequence $(0, 0, 0)$ is a Non-example as it does not satisfy the equation.

◇ Row-equivalent matrices

A matrix A is called *row equivalent* to a matrix B if and only if B is obtained from A by applying finitely many elementary row operations on A.

Example

$$A = \begin{bmatrix} 7 & 12 & 5 \\ 8 & 14 & 7 \\ 1 & 9 & 3 \end{bmatrix} \xrightarrow{(R_1 - R_3) \to R_1} \begin{bmatrix} 6 & 3 & 2 \\ 8 & 14 & 7 \\ 1 & 9 & 3 \end{bmatrix} = B.$$

A and B are row equivalent.

Non-example

$$A = \begin{bmatrix} 7 & 12 & 5 \\ 8 & 14 & 7 \\ 1 & 9 & 3 \end{bmatrix} . B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

A and B are not row equivalent.

◇ A matrix in row-echelon form

A matrix A is in row-echelon form iff it satisfies the three following conditions :

(i) All rows consisting entirely of zeros occur at the bottom of the matrix.

(ii) The first nonzero entry of each row is 1. (called the leading 1 or the pivot, and we call the column that has a leading 1, a pivot column.)

(iii) Any leading 1 is farther to the right than the leading 1 in the row above.

Example

$$A = \begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

A is in row-echelon form.

Non-example

$$B = \begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

B is not in row-echelon form.