Linear Algebra

Assignment 6

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4.3 Subspaces of Vector Spaces

Problem 4

Proof. Suppose V is a vector space. Pick two matrices:

$$A = \begin{bmatrix} a & b \\ a-2b & 0 \\ 0 & c \end{bmatrix}, B = \begin{bmatrix} x & y \\ x-2y & 0 \\ 0 & z \end{bmatrix}$$

. We see:

$$A + B = \begin{bmatrix} a+x & b+y \\ a-2b+x-2y & 0 \\ 0 & c+z \end{bmatrix} = \begin{bmatrix} (a+x) & (b+y) \\ (a+x)-2(b+y) & 0 \\ 0 & (c+z) \end{bmatrix}$$

, so the set is closed under addition. Pick a scalar $k \in \mathbb{R}$. Observe:

$$cA = \begin{bmatrix} ka & kb \\ k(a-2b) & k(0) \\ k(0) & k(c) \end{bmatrix} = \begin{bmatrix} ka & kb \\ ka-2(kb) & 0 \\ 0 & kc \end{bmatrix},$$

clearly closed under scalar multiplication. The set is nonempty as it contains the zero matrix $\mathbf{0}_{3\times 2}$. Therefore, the set is a subspace of M_{32} ∎

Problem 8

We see $(2, 1, 1) + (2, 0, 0) = (4, 1, 1)$, so it is not closed under addition.

Problem 24

Proof. We know that $C(-\infty, \infty)$ is a vector space. Pick any two odd functions $f(x), g(x) \in C(-\infty, \infty)$. By definition, $(f + g)(x) = f(x) + g(x)$. Observe that by:

$$(f + g)(-x) = f(-x) + g(-x) = -f(x) - g(x) = -(f(x) + g(x)) = -(f + g)(x)$$

We deduce the set is closed under addition. Choose any scalar $c \in \mathbb{R}$. We see $cf(-x) = c(f(-x)) = c(-f(x)) = -cf(x)$, which clearly implies the set is closed under scalar multiplication. The set is non empty as it contains $f(x) = x^3$. Therefore, we proved that the set of all odd functions is subspace of the space of the continous functions. ■

Problem 28

Proof. Pick any two functions $f(x), g(x) \in C(-\infty, \infty)$ such that $f(0) = 1, g(0) = 1$. By definition, we know that $(f + g)(x) = f(x) + g(x)$, so $(f + g)(0) = f(0) + g(0) = 1 + 1 = 2$, clearly not closed under addition. Hence, the set of all functions such that $f(0) = 1$ is not a subspace. ■

Problem 41

Proof. Pick two elements $(a, b, ab), (x, y, xy) \in W$. Observe their addition is $(a + x, b + y, ab + xy)$. We know that $(a + x)(b + y) = ab + ay + xb + xy$. Let $a = 1, b = 2, x = 3, y = 4$, so $ab + xy = 2 + 12 = 14$ and $ab + ay + xb + xy = 2 + 4 + 6 + 12 = 24$, which clearly shows W is not closed under addition. Therefore, we proved that W is not a subspace of R^3. ■

Problem 54

Pick $x, y \in W$ and a matrix $A = [a_{ij}]$ of size $m \times n$. We know that $A(x + y) = Ax + Ay$ by the left distributivity of matrix multiplication over matrix addition. And we know $Ax = \mathbf{0}, Ay = \mathbf{0}$, so $Ax + Ay = \mathbf{0}$, so it is closed under vector addition. Choose any scalar $c \in \mathbb{R}$. It is clear that $A(cx) = c(Ax) = c(\mathbf{0}) = \mathbf{0}$. The set is not empty as it contains the zero matrix of size $n \times 1$. Therefore we proved that W is a subspace of R^n.