PCPKit Documentation

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1 Function Listing

Problem	Function	Section
Matrix Completion - Noise Free		
$\min_{\mathbf{A}} \ \tau\|\mathbf{A}\|_* + \frac{1}{2}\|\mathbf{A}\|_F^2$ s.t. $\mathcal{P}_\Omega(\mathbf{M}) = \mathcal{P}_\Omega(\mathbf{A})$	mc_svt	2.1.1
$\min_{\mathbf{A}} \ \tau\|\mathbf{A}\|_*$ s.t. $\mathcal{P}_\Omega(\mathbf{M}) = \mathcal{P}_\Omega(\mathbf{A})$	mc_ialm solve_lin	2.1.2 2.1.3
Matrix Completion - Noisy		
$\tau\|\mathbf{A}\|_* + \frac{\lambda}{2}\|\mathcal{P}_\Omega(\mathbf{A}) - \mathcal{P}_\Omega(\mathbf{M})\|_F^2$	mc_relax_lin mc_relax_lin_ext mc_relax_lin_acc	2.2
$\min_{\mathbf{A},\mathbf{E}} \ \tau\|\mathbf{A}\|_* + \frac{\lambda}{2}\|\mathbf{E}\|_F^2$ s.t. $\mathcal{P}_\Omega(\mathbf{M}) = \mathcal{P}_\Omega(\mathbf{A}) + \mathcal{P}_\Omega(\mathbf{E})$	mc_exact_fro	2.3
Principal Component Pursuit		
$\min_{\mathbf{A},\mathbf{E}} \ \|\mathbf{A}\|_* + \frac{\lambda}{2}\|\mathbf{E}\|_F^2$ s.t. $\mathbf{M} = \mathbf{A} + \mathbf{E}$	pcp_fro	3.1
$\min_{\mathbf{A},\mathbf{E}} \ \|\mathbf{A}\|_* + \lambda\|\mathbf{E}\|_1$ s.t. $\mathbf{M} = \mathbf{A} + \mathbf{E}$	pcp_l1	3.2
$\min_{\mathbf{A},\mathbf{E}} \ \|\mathbf{A}\|_* + \lambda\|\mathbf{E}\|_{1,2}$ s.t. $\mathbf{M} = \mathbf{A} + \mathbf{E}$	pcp_l1l2	3.3
Selective PCP		
$\min_{\mathbf{A},\mathbf{E}} \ \tau\|\mathbf{A}\|_* + \frac{\lambda}{2}\|\mathbf{E}\|_F^2$ s.t. $\mathcal{P}_\Omega(\mathbf{M}) = \mathcal{P}_\Omega(\mathbf{A})$ $\mathcal{P}_\Psi(\mathbf{M}) = \mathcal{P}_\Psi(\mathbf{A}) + \mathcal{P}_\Psi(\mathbf{E})$	sel_pcp	4

2 Matrix Completion

2.1 Noise Free Data

2.1.1 SVT

The function

$$[\mathbf{A}, \mathbf{f\_values}, \mathbf{stop\_vals}] = \text{mc\_svt}(\mathbf{M}, \Omega, \tau, \mu, iterations, tol)$$

solves the following

$$\begin{aligned} \min_{\mathbf{A}} \ & \tau\|\mathbf{A}\|_* + \frac{1}{2}\|\mathbf{A}\|_F^2 \\ \text{s.t. } & \mathcal{P}_\Omega(\mathbf{M}) = \mathcal{P}_\Omega(\mathbf{A}) \end{aligned} \tag{1}$$

as proposed by the authors of [1].

- $\mathbf{M}$ - matrix with observed entries
- Ω - vector of constrained matrix indices
- τ - regularisation (optional)
- μ - step size (optional)
- *iterations* - maximum number of iterations (optional)
- *tol* - stopping criteria tolerance (optional)

2.1.2 Inexact ALM

The function

$$[\mathbf{A}, \mathbf{f\_vals}, \mathbf{stop\_vals}] = \text{mc\_ialm}(\mathbf{M}, \Omega, \tau, \mu, iterations, tol)$$

solves the following

$$\begin{aligned} \min_{\mathbf{A}} \ & \tau\|\mathbf{A}\|_* \\ \text{s.t. } & \mathcal{P}_\Omega(\mathbf{M}) = \mathcal{P}_\Omega(\mathbf{A}) \end{aligned} \tag{2}$$

as proposed by the authors of [3].

- $\mathbf{M}$ - matrix with observed entries
- Ω - vector of constrained matrix indices
- τ - regularisation (optional)
- μ - step size (optional)
- *iterations* - maximum number of iterations (optional)
- *tol* - stopping criteria tolerance (optional)

2.1.3 Linearised ALM

The function

$$[\mathbf{A}, \mathbf{f\_vals}, \mathbf{stop\_vals}] = \text{mc\_lin}(\mathbf{M}, \Omega, \tau, \mu, \rho, iterations, tol)$$

solves the following

$$\begin{aligned} &\min_{\mathbf{A}} \ \tau\|\mathbf{A}\|_* \\ \text{s.t. } &\mathcal{P}_\Omega(\mathbf{M}) = \mathcal{P}_\Omega(\mathbf{A}) \end{aligned} \tag{3}$$

- $\mathbf{M}$ - matrix with observed entries
- Ω - vector of constrained matrix indices
- τ - regularisation (optional)
- μ - step size (optional)
- ρ - linearisation step size (optional)
- *iterations* - maximum number of iterations (optional)
- *tol* - stopping criteria tolerance (optional)

2.2 Noisey Data Relaxation

The functions

$$\begin{aligned} [\mathbf{A}, \mathbf{f\_vals}, \mathbf{stop\_vals}] &= \text{mc\_relax\_lin}(\mathbf{M}, \Omega, \tau, \lambda, \rho, iterations, tol) \\ [\mathbf{A}, \mathbf{f\_vals}, \mathbf{stop\_vals}] &= \text{mc\_relax\_lin\_ext}(\mathbf{M}, \Omega, \tau, \lambda, \rho, iterations, tol) \\ [\mathbf{A}, \mathbf{f\_vals}, \mathbf{stop\_vals}] &= \text{mc\_relax\_lin\_acc}(\mathbf{M}, \Omega, \tau, \lambda, \rho, iterations, tol) \end{aligned}$$

solve the following

$$\min_{\mathbf{A}} \ \tau\|\mathbf{A}\|_* + \frac{\lambda}{2}\|\mathcal{P}_\Omega(\mathbf{A}) - \mathcal{P}_\Omega(\mathbf{M})\|_F^2 \tag{4}$$

with increasing convergence speed based on [2].

- $\mathbf{M}$ - matrix with observed entries
- Ω - vector of constrained matrix indices
- τ - regularisation (optional)
- λ - regularisation (optional)
- ρ - linearisation step size (optional)
- *iterations* - maximum number of iterations (optional)
- *tol* - stopping criteria tolerance (optional)

2.3 Noisey Data Exact

The function

$$[\mathbf{A}, \mathbf{f\_vals}, \mathbf{stop\_vals}] = \text{solve\_exact\_fro}(\mathbf{M}, \Omega, \tau, \lambda, \rho, iterations, tol)$$

solve the following

$$\begin{aligned} \min_{\mathbf{A},\mathbf{E}} \; & \tau\|\mathbf{A}\|_* + \frac{\lambda}{2}\|\mathbf{E}\|_F^2 \\ \text{s.t. } & \mathcal{P}_\Omega(\mathbf{M}) = \mathcal{P}_\Omega(\mathbf{A}) + \mathcal{P}_\Omega(\mathbf{E}) \end{aligned} \tag{5}$$

- **M** - matrix with observed entries
- Ω - vector of constrained matrix indices
- τ - regularisation (optional)
- λ - regularisation (optional)
- ρ - linearisation step size (optional)
- *iterations* - maximum number of iterations (optional)
- *tol* - stopping criteria tolerance (optional)

3 Principal Component Pursuit

3.1 PCP Gaussian

The function

$$[\mathbf{A}] = \text{pcp\_fro}(\mathbf{M}, \lambda)$$

solve the following

$$\begin{aligned} \min_{\mathbf{A},\mathbf{E}} \ & \|\mathbf{A}\|_* + \frac{\lambda}{2}\|\mathbf{E}\|_F^2 \\ \text{s.t. } & \mathbf{M} = \mathbf{A} + \mathbf{E} \end{aligned} \tag{6}$$

- $\mathbf{M}$ - matrix with observed entries
- λ - regularisation

3.2 PCP Sparse

The function

$$[\mathbf{A}] = \text{pcp\_l1}(\mathbf{M}, \lambda)$$

solve the following

$$\begin{aligned} \min_{\mathbf{A},\mathbf{E}} \ & \|\mathbf{A}\|_* + \lambda\|\mathbf{E}\|_1 \\ \text{s.t. } & \mathbf{M} = \mathbf{A} + \mathbf{E} \end{aligned} \tag{7}$$

- $\mathbf{M}$ - matrix with observed entries
- λ - regularisation

3.3 PCP Column-wise Gaussian

The function

$$[\mathbf{A}] = \text{pcp\_l1l2}(\mathbf{M}, \lambda)$$

solves the following

$$\begin{aligned} \min_{\mathbf{A},\mathbf{E}} \ & \|\mathbf{A}\|_* + \lambda\|\mathbf{E}\|_{1,2} \\ \text{s.t. } & \mathbf{M} = \mathbf{A} + \mathbf{E} \end{aligned} \tag{8}$$

- $\mathbf{M}$ - matrix with observed entries
- λ - regularisation

4 Selective PCP

The function

$$[\mathbf{A}] = \text{sel\_pcp}(\mathbf{M}, \tau, \lambda, \Omega, \Psi)$$

solves the following

$$\begin{aligned} \min_{\mathbf{A},\mathbf{E}} \ \tau\|\mathbf{A}\|_* + \frac{\lambda}{2}\|\mathbf{E}\|_F^2 \\ \text{s.t. } \mathcal{P}_\Omega(\mathbf{M}) = \mathcal{P}_\Omega(\mathbf{A}) \\ \mathcal{P}_\Psi(\mathbf{M}) = \mathcal{P}_\Psi(\mathbf{A}) + \mathcal{P}_\Psi(\mathbf{E}) \end{aligned}$$

- $\mathbf{M}$ - matrix with observed entries
- τ - regularisation
- λ - regularisation
- Ω - uncorrupted indices
- Ψ - noisy indices

References

[1] Jian-Feng Cai, Emmanuel J Candès, and Zuowei Shen. A singular value thresholding algorithm for matrix completion. *SIAM Journal on Optimization*, 20(4):1956–1982, 2010.

[2] Shuiwang Ji and Jieping Ye. An accelerated gradient method for trace norm minimization. In *Proceedings of the 26th Annual International Conference on Machine Learning*, pages 457–464. ACM, 2009.

[3] Zhouchen Lin, Minming Chen, and Yi Ma. The augmented lagrange multiplier method for exact recovery of corrupted low-rank matrices. *arXiv preprint arXiv:1009.5055*, 2010.